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A multi-objective robust optimization model to design a network for Emergency Medical Services under uncertainty conditions: A case study

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Abstract

The present research introduces a multi-objective robust optimization model to design emergency medical services network for uncertain costs and demands. The proposed model determines the location and the optimum capacity of relief medical service centers. In addition, the model determines the number and the type of ambulances that should be placed in each of the centers and allocated to demand zones. The multi-objective model attempts to maximize the coverage of demand zones, the availability of ambulances and minimizing the total costs simultaneously. A robust model is applied to our real word case study in an urban district.

Keywords: Emergency Medical Services (EMS), positioning, robust optimization, maximum coverage.

1-Introduction and literature review

Emergency medical services (EMS) provide effective and quick treatments to prevent sudden threats against human life. These centers improve health and general safety, attempt to aid affected people as fast as possible and reduce the risk of mortality and physical-financial damage. Occurrence of events like accidents, heart attack, stroke, suicide, different intoxications, etc. highlights the role of emergency more than before in quick and punctual transfer of injured and patients to the therapeutic centers in addition to therapeutic proceedings until reaching to the equipped centers like hospitals. In most of these cases, if an injured is not received the therapeutic proceedings at the specific time, it will cause severer problems, disability, maim or even death. One of the factors effective on punctual responding to the calls received by emergency centers is appropriate positioning for emergency stations in places where emergency vehicles can present in the accident place within standard time. Therefore, emergency centers should be located in places where their intervals to the accident zones are less than standard responding time or in the other words coverage radius. The point that should be considered is that in the real world we are faced with uncertainty. Parameters like the responding time or the number of demands should be considered as uncertain parameters. Moreover, on the issue of providing emergency services we face various and different goals such as minimizing responding time and costs, maximizing demands coverage and availability of emergency equipment, minimizing maximum travel time/ distance to each demand point and many other goals.

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Therefore, it is logical to consider multiple goals for optimizing the location of emergency station. Furthermore, the severity of injury and the type of injured need to medical facilities are different, for example, those who have stroke need to be recovered, while those who have an accident or fracture do not need heart resuscitation. Therefore, it is logical to provide emergency devices with different levels of facilities, send proper emergency device for each situation and prevent from allocating additional costs for providing emergency devices with high level equipment to respond the demands requiring low level equipment. Moreover, since it is possible to deploy more than one facility in each potential deployment location, therefore, we can determine the size of emergency stations based on the number of allocated facilities which could reduce costs. The main point in designing network for these centers is to make decision about the number and location of EMS centers in addition to decide about quality of allocating them to the demand zones. During past years, many researches had been conducted about location and allocation of demand for EMS. During this period, concentration of positioning studies for emergency facilities is on maximal covering problem. The maximal covering location problem was first introduced by Church and ReVelle (1974) in which the goal was defined as location of facilities to maximize demand covering. After that, many applied and theoretical developments were introduced based on this basic model (Berman and Krass, 2002; Shavandi and Mahlooji, 2006; Araz et al, 2007; Zhang and Jiang, 2014; Asiedu and Rempel, 2011; Berman et al, 2009; Alexandris and Giannikos, 2010; O'Hanley and Church, 2011; Sorensen and Church, 2010; Kanoun et al, 2010; Noyan, 2010; Ibri et al, 2012; Curtin et al, 2010). Berman and Krass (2002) considered the partial covering, which does not need covering of all demand zones in contrary to the generalized covering, for the maximal covering problem. Shavandi and Mahlooji (2006) introduced a fuzzy location model for congested systems and called it a fuzzy maximal covering location in queue theory framework. The multi-objective version of the maximal covering problem was proposed by Araz et al. (2007). The authors considered a multi-objective fuzzy ideal programming for location of emergency vehicles. Their objective functions (objectives) were to maximize the initial and backup covering and minimizing the distance of total trip from facilities location toward uncovering zones. In another research conducted by Alexandris and Giannikos (2010), the maximal covering model was developed using GIS and partial covering idea for better illustration of the demands covering. Ibri et al. (2012) introduced a multi-agent model for simultaneous decision-making about covering problem and vehicle dispatching. This model was formulated to minimize total trip time, costs of demand un-fulfillment and penalties of uncovering.

In addition to the above mentioned researches, there are several case studies in the literature of maximal covering (Ratick et al, 2009; Moore and ReVelle, 1982; Curtin et al, 2010; Murawski, and Church , 2009; Indriasari et al, 2010; Yin and Mu , 2012). Ratick et al. (2009) implemented the Moore and ReVelle (1982) hierarchical maximal covering model for medical facilities in Kohat region of Pakistan. To solve the covering problem, Murawski and Church (2009) proposed a model by assuming the allocated facilities are fixed in which accessibility to demand zones are improved. Their model which is called the maximal covering network improvement problem was formulated in form of an integer number programing problem and implemented in Ghana. Indriasari et al. (2010) introduced a model using GIS that its objective maximized the area serviced by specific emergency facilities. In contrary to other researches, Yin and Mu (2012) did not consider a fixed capacity for each facility but developed a model that let modular capacity levels selected among facilities.

In recent years, the maximal covering problem under uncertainty conditions has attracted some researches (de Assis Corrêa et al, 2009; Batanović et al, 2009; Berman and Wang, 2011; Geroliminis et al, 2009) [23-26]. Corrêa et al. (2009) studied the probabilistic maximal covering location-allocation problem using decomposition approach. Batanović et al. (2009) recommended location models of emergency facilities in the networks under uncertainty conditions. They studied a problem in which demand zones and their weights are important equally. In the case study conducted by Berman and Wang (2011), demands weight of network zones were assumed as random variables with unknown possibility distribution. The present article purpose was to find a location that minimizes the lost covering in the worst conditions. Geroliminis et al. (2009) proposed a queuing model for the emergency vehicle dispatching and location problem with the goal to maximize covering and minimize responding time. Navazi et al. (2018) introduced a multi-period location-allocation-inventory problem for ambulance and helicopter ambulance stations using robust possibilistic approach. The aims of this study are to locate the stations, allocate the accident-prone points to them, determine the required hospital bed capacities provided for patients with critical conditions, and decide about the inventory levels at opened stations including blood banks, medicines, etc. Finally, an augmented Ɛ-constraint method is used to cope with the bi-objective problem that minimizes the cost and the arrival time. Also the model is applied to a real case. A three-stage and multi-objective stochastic programming model to improve the sustainable rescue ability by considering secondary disasters in emergency logistics was proposed by Zhang et al. (2019). Model's results based on Wenchuan earthquake show that the solution in this paper outperforms some normal ways.

With respect to studying the above-mentioned research, we understand that although numerous location models for emergency facilities exist in the literature, just a few of them used multi-objective approach to consider covering/costs functions and accessibility of emergency vehicles. Similarly, most of the existing models assume that facilities capacity is constant and determined beforehand which cause reduction in their application in reality. In other words, previous research generally ignore technologies and different types of facilities and vehicles but assume that number of ambulances in medical centers is constant and determined previously. Moreover, most of the models in the literature were developed under certainty conditions or in conditions that probability distribution function of uncertain parameters are determined which is happened rarely in reality. Considering such research gaps and due to our case study, the present article attempts to introduce a robust model to design a network for EMS under uncertainty conditions. The model is capable of determining location and capacity of EMS centers in addition to making decision about type and number of ambulances in each center.

This paper moves toward innovation from different viewpoints. First, the model introduced in this article is capable of considering the increase in covering locations, increase in accessibility of ambulances and reduction of costs simultaneously through multi-objective optimization approach. Also, the ambulances are accessible and not busy while they are needed. Moreover, in addition to determination of type and number of ambulances, the proposed approach can consider different levels of capacity and technology for EMS and ambulances. In other words, this model provides the possibility of different type and number of ambulances in different stations. In contrary to the most of articles in the literature, the developed model is based on robust optimization approach that enables the mode to consider uncertainty in parameters without needing to know their distribution function. On the other hand, applying the introduced method in real problem of an urban district is considered as other considerable points in the present research.

This paper is organized as follows. In the second part, basics of robust and multi-objective optimization will be discussed. In the third part, the problem will be defined and modeled. In the fourth part, the case study will be explained and the calculated results will be presented. The final part will be about recommendations for further researches in addition to conclusion.

- Basics of robust and multi-objective optimization
- Basics of interval robust optimization

The model introduced in this article has been developed based on interval robust optimization. One of the first models of robust optimization was introduced by Soyster (1973) which is known as pessimistic approach. In this method, it is assumed that all data has the worst interval value. Therefore, decisions quality will be reduced remarkably. To solve such defection which is act conventionally, Ben-Tal and Nemirovski (2000) introduced another approach. Although their model solved the conventionality problem of Soyster method (Soyster, 1973), it made the problem more complicated and inapplicable to the optimization problems with discrete variables. To solve such problem, Bertsimas and Sim (2004) introduced the interval robust optimization approach by which it is possible to control the level of conventionality in addition to the advantage of applicability to the discrete optimization models. In the following, basics of the interval robust optimization method introduced by Bertsimas and Sim (2004), which is the base of present article, will be discussed briefly. For deeper study of robust optimization and

applying it to the location problems please refer to Refs. (Bertsimas and Sim, 2004; Nikulin, 2006; Snyder, 2006; Blanquero et al, 2011; Bardossy and Raghavan, 2013; Gülpınar et al, 2013; Zokaee et al, 2013).
Consider the following mixed integer programming (MIP) problem:

$$\begin{aligned} &\text{Minimize} \quad c'x \\ &\text{Subject to} \\ &Ax \le b \\ &l \le x \le u \\ &x_i \epsilon Z \\ &i=1,2,\ldots,m. \end{aligned} \tag{1}$$

Without losing generality of the problem, we assume that some parts of matrix A and vector C contain uncertain parameters. The L_i set for each line of matrix A is defined in a way it includes uncertain coefficients in *i*th constraint. Such uncertain parameters are denoted by $\tilde{a}_{ij}$. Moreover, $|L_i|$ is considered as the size of the L_i set. Similarly, coefficients of uncertain cost in of are denoted by $\tilde{c}_j$ and define the corresponding $\acute{L}$ set. Each uncertain parameter $\tilde{a}_{ij}$ can exceed its nominal value (denoted by a_{ij}) up to the specified radius (denoted by $\hat{a}_{ij}$) and takes value from symmetric range of $[a_{ij}-\hat{a}_{ij}, a_{ij}+\hat{a}_{ij}]$ independently. It is assumed that the cost coefficient $\tilde{c}_j$ takes value from the range $[c_j, c_j+\hat{c}_j]$ similarly. In addition, Γ_i and $\acute{\Gamma}$ parameters are defined as the level of conventionality in of and *i*th constraint (these parameters will be called degree of protection from now on). Where:

$$\acute{L} = \{j|\hat{c}j > 0\}, 0 \le \acute{\Gamma} \le |\acute{L}|, Li = \{j|\hat{a}ij > 0\}, 0 \le \Gamma i \le |Li| \tag{2}$$

In other words, Γ_i and $\acute{\Gamma}$ parameters show the number of parameters exceed their nominal value in of and i constraint. When $\Gamma_i = 0$, the model will be changed into certain mode but when $\Gamma_i = |L_i|$, all variables will be changed up to their radius and the model will be the same as Soyster (1973) pessimistic approach. Bertsimas and Sim (2004) formulated the robust model equal to linear programing (1).

$$\begin{aligned} &\text{Min} \quad \acute{c}x + \lambda' \Gamma' + \textstyle\sum_{j\epsilon \acute{L}} \acute{\mu} j \\ &\textstyle\sum_j a_{ij} x_j + \lambda_i \Gamma_i + \sum_{j\epsilon Li} \mu_{ij} \le b_i && \forall i \\ &\acute{\lambda} + \acute{\mu}_j \ge \hat{c}_j q_j && \forall j \epsilon L' \\ &\lambda_i + \mu_{ij} \ge \hat{a}_{ij} q_j && \forall i, j \epsilon L_i \\ &\mu_{ij} \ge 0 && \forall i, j \epsilon L_i \\ &\acute{\mu}_j \ge 0 && \forall j \epsilon L' \\ &\lambda_i \ge 0 && \forall i \\ &\acute{\lambda} \ge 0 \\ &g_j \ge 0 && \forall j \\ &-q_j \le x_j \le q_j && \forall j \\ &L_j \le x_j \le u_j && \forall j \end{aligned} \tag{3}$$

$X_i \epsilon Z$ $\forall i$

Note: if the numbers of uncertain coefficients of $\tilde{a}_{ij}$ are changed up to Γ_i, the robust answer will be still remained possible but if more than Γ_i coefficients are changed in *i*th constraint, the robust model will be remained possible with the following probability:

$$P(\sum_j \tilde{a}_{ij} x_j^* > b_i) \leq 1-\phi(\frac{1-\Gamma i}{\sqrt{n}}) \quad (4)$$

Where, x_j^* denotes optimized value of the robust model and $\phi(\Gamma)$ denotes the standard normal cumulative distribution function. Refer to the Bertsimas and Sim (2004) for more details.

1-1- Basics of ε – constraint method

The multi-objective approach of ε – constraint method was used in the present article in order to deal with multi objectives of a problem. Thus, this method will be introduced briefly in this part.
Assume that there is a multi-objective programing problem with p number of objective as follows:

$$\begin{aligned} &Max\ (f1(x), f2(x), \ldots, fp(x)) \\ &St: \\ &\quad X \epsilon S \end{aligned} \quad (5)$$

Where, X denotes decision variables vector and S denotes the possible space constraint. Moreover, $f_1(x), \ldots, f_p(x)$ denote p number of objective.

Dealing with multi- objectives in the ε – constraint method is through considering one of the objectives as the main objective and the other objectives entered the model as constraint:

$$\begin{aligned} &Max \quad f_1(x) \\ &St \\ &\quad f_2(x) \geq e_2 \\ &\quad f_3(x) \geq e_3 \\ &\quad \ldots \\ &\quad f_p(x) \geq e_p \\ &\quad X \epsilon S \end{aligned} \quad (6)$$

The problem answers will be obtained through making changes in right side parameters of the equations in the objectives exist in constraints (Mavrotas, 2009).

2-Defining and modeling the problem

A mathematical model will be introduced in this part to design a network for EMS. The network in this problem contains EMS centers and patients (demand zones) with different type of demands. Some ambulances with limited capacity are placed in each one of EMS centers. The ambulances are in different types and each one can only respond a specific type of patients' demand. The process is that a patient calls EMS centers and the appropriate ambulance will be dispatched to his location according to the received demand.

There are potential locations for establishment of EMS centers in the problem. In such potential locations, the EMS centers can be established by different technologies and capacities. Each one of different EMS centers is capable of placing specific number of ambulances. Naturally, the higher capacity (technology) of EMS centers, the higher cost is needed for their establishment. Similarly, the more equipped ambulances with more capacity placed in EMS centers, the costs will be increased more.

Several objectives are followed simultaneously in the present problem. First, time interval of EMS centers and demand zones will be as less as possible for ambulances to responds patients' needs as soon as possible and cover them. Thus, it is tried to prevent time interval between patients and EMS centers does not exceed the standard covering radius. Moreover, it is assumed that if the above-mentioned time interval exceeds the standard covering radius, the patients of that center are not covered by that center anymore. The second objective of the problem is about the possibility of increase in accessibility of ambulances for servicing different demand zones. In other words, it is tried to have at least one ambulance in an EMS center to respond a patient's demand. In fact, when an ambulance is dispatched for responding a demand in a zone, it will not available for other zones. Therefore, it is better to allocate more ambulances to a location for increasing accessibility of ambulances in a location and there are other ambulances to respond a demand if any one of ambulances is busy for a mission. The third objective of the problem is to minimize total costs including costs of establishing EMS centers and costs of allocating ambulances.

Obviously, the above-mentioned objectives can be contradictory. For instance, if more ambulances allocated to an EMS center, accessibility level will be increased in one hand but total costs will be increased on the other hand. In such situations and considering all of the three objectives, the present article introduces a mathematical model that determines the following decisions simultaneously:

A- The EMS centers should be established in which of the potential locations?

B- Each one of the EMS centers should be at which level of capacity (technology)?

C- How many ambulances of which type (technology) should be allocated to each one of EMS centers?

D- Each one of demand zones should be covered by which one of EMS centers?

2-1 The deterministic model

The following decision variables and parameters are defined to model the problem:

Sets

I demand locations with i index
J potential locations for establishment of EMS with j index
K ambulances with k index
H types (technology levels) of ambulances with h index
M types of demand
R types of (technology levels) of EMS centers with r index

Parameters

l_i standard covering radius (optimum distance or time interval between EMS centers and demand zone)
u_i the maximum acceptable covering radius (maximum acceptable distance or time interval between EMS centers and demand zone)

d_{ij} time distance between two points i and j
$\tilde{a}_i^m$ demand type m and i location
e^m_{ij} parameters of gradual covering which is obtainable from the following equation:

$$e^m_{ij}=\begin{cases} 1 & if \quad d_{ij} \le l_i \\ 1-\frac{dij}{max(dij)} & if \quad l_i \le d_{ij} \le u_i \\ 0 & if \quad d_{ij} \ge u_i \end{cases} \qquad (7)$$

This parameter takes value from 0 and 1 range. In other words, when a demand point placed inside the standard covering radius in proportion to an EMS center, its value becomes maximum that is 1. In addition, when a demand point placed out of the maximum acceptable standard covering radius, its value becomes minimum that is zero. If the distance from a demand point and EMS center placed between standard covering radius and maximum acceptable covering radius, it will take a value between 0 and 1 in proportion to the distance from EMS center.

$\tilde{F}^r_j$ cost of establishing EMS type r in potential location j
$\tilde{v}^h$ cost of placing ambulance type h in EMS
c^{hm} servicing capacity of ambulance type h to cover demand type m
g_{hr} maximum number of ambulances type h that an EMS type r can placed
b percentage of any busy ambulance (possibility of no ambulance available when needed)
q_k coefficient of confidence for a situation in which k ambulances allocated to service a demand point . In other words, this parameter is equal to the possibility of at least one ambulance available when a demand point needs. The more ambulances allocated to a demand point, the higher coefficient of confidence. In fact, the relationship between b and q_k will be as follows:

$$q_k= 1-(b)^k \qquad (8)$$

To justify the above equation, possibility of at least one ambulance available from k ambulances allocated to a demand point obtained from subtraction of the possibility of all k available ambulances that are not exist from one.

Decision variables

y^m_{ij} percentage of the demand point i type m allocated to the jth EMS center
z^h_j the number of ambulance type h placed in the jth EMS center
x^r_{jk} it is equal to 1 if the EMS center type r with k number of ambulances are placed in point j otherwise it is equal to zero
w^m_{ik} it is equal to 1 if the demand point i type m is covered by k number of ambulances otherwise it is equal to zero

The deterministic model is written as follows using the above symbols:

$$Max\ Z_1=\sum_{i\in I}\sum_{j\in J}\sum_{m\in M}\tilde{a}_i^m e^m_{ij} y^m_{ij} \qquad (9)$$
$$Max\ Z_2=\sum_{i\in I}\sum_{k\in K}\sum_{m\in M}\tilde{a}_i^m q_k w^m_{ik} \qquad (10)$$
$$Min\ Z_3=\sum_{i\in I}\sum_{k\in K}\sum_{r\in R}\tilde{f}_j^r x^r_{jk} + \sum_{j\in J}\sum_{h\in H}\tilde{v}^h z^h_j \qquad (11)$$

Subject to:

$$\sum_{i\in I}\tilde{a}^m_i y^m_{ij} \le \sum_{h\in H} c^{hm} z^h_j \qquad \forall j,m \qquad (12)$$
$$\sum_{j\in J} y^m_{ij}=1 \qquad \forall j,m \qquad (13)$$
$$Y^m_{ij} \le \sum_{k\in K}\sum_{r\in R} X^r_{jk} \qquad \forall i,j,m \qquad (14)$$
$$\sum_{k\in K}\sum_{r\in R} X^r_{jk} \le 1 \qquad \forall j \qquad (15)$$
$$\sum_{k\in K}\sum_{r\in R} kX^r_{jk}=\sum_{h\in H} Z^h_j \qquad \forall j \qquad (16)$$
$$\sum_{k\in K} kw^m_{ik} \le \sum_{h\in H}\sum_{i\in I} Z^h_j \qquad \forall i,m \qquad (17)$$

$Z^h_j \leq \sum_{k\in K} \sum_{r\in R} g_{hr} X^r_{jk}$ $\forall j,h$ (18)

$\sum_{k\in K} w^m_{ik} \leq 1$ $\forall i,m$ (19)

$0 \leq y^m_{ij} \leq 1$ $\forall i,m$ (20)

$Z^h_j \geq 0$ $\forall j,h$ (21)

$X^r_{jk} \in \{0,1\}$ $\forall j,k,r$ (22)

$W^m_{ik} \in \{0,1\}$ $\forall i,k,m$ (23)

The objective (9) tries to maximize demand covering through considering gradual covering parameter in equation (7). Obviously, the closer demand points allocated to the EMS centers, the more value of objective (9) will be increased through e^m_{ij}. The objective (10) maximizes the availability of ambulances through considering confidence coefficient parameter (refer to equation 7). The objective (11) tries to minimize the costs of establishing EMS centers and allocation of ambulances. Constraint (12) guarantees that total covered demands in each EMS center do not exceed the center capacity. Constraint (13) needs all (100%) demands of any demand point allocated to EMS centers. Constraint (14) prevents from allocation of demand points to the centers which have not been established.

Constraint (15) guarantees at any potential location, a maximum of one emergency medical services center to be established. Equation (16) requires that the number of ambulances established in each emergency medical center be equal to the number of ambulances of different types. Constraint (17) guarantees the number of ambulances allocated to each emergency medical services center does not exceed the sum of available ambulances. Constraint (18) prevents from the establishment of ambulances in an emergency medical services center more than the capacity of it. Constraint (19) guarantees a certain number of ambulances to be established. Constraint (20) requires the demand ration of each point to be between zero and one. Constraints (21) to (23) require positivity of variables.

2-2- The model in uncertain conditions

This section presents a robust model to deal with a state which is uncertain in the cost and demand parameters problem. It is assumed that the demand parameter ($\tilde{a}^m_i$) is in a distance with nominal value a^m_i and fluctuates by maximum variation $\hat{a}^m_i$. In other words, it is assumed that the uncertain parameter $\tilde{a}^m_i$ takes value from the range $[a^m_i - \hat{a}^m_i, a^m_i + \hat{a}^m_i]$. Similarly, it is assumed that the uncertain parameters $\tilde{f}^r_j$ and $\tilde{v}^h$ are fluctuated in $[f^r_j - \hat{f}^r_j, f^r_j + \hat{f}^r_j]$ and $[v^h - \hat{v}^h, v^h + \hat{v}^h]$ ranges respectively. In this case, using interval robust optimization approach explained in section 2-1, the robust form of the certain model introduced as follows:

$$Max\ \tilde{Z}_1 = \sum_{i\in I} \sum_{j\in J} \sum_{m\in M} a^m_i e^m_{ij} y^m_{ij} - \lambda^1 \Gamma^1 - \sum_{i\in I} \sum_{m\in M} \mu^1_{im} \quad (24)$$

$$Max\ \tilde{Z}_2 = \sum_{i\in I} \sum_{k\in K} \sum_{m\in M} a^m_i q_k w^m_{ik} - \lambda^2 \Gamma^2 - \sum_{i\in I} \sum_{m\in M} \mu^2_{im} \quad (25)$$

$$Min\ \tilde{Z}_3 = \sum_{i\in I} \sum_{k\in K} \sum_{r\in R} f^r_r x^r_{jk} + \sum_{j\in J} \sum_{h\in H} v^h z^h_j - \lambda^4 \Gamma^4 - \sum_{j\in J} \sum_{r\in R} \mu^4_{jr} - \lambda^5 \Gamma^5 - \sum_{h\in H} \mu^5_h \quad (26)$$

Subject to:

$\mu^1_{im} + \lambda^1 \geq \sum_{j\in J} \hat{a}^m_i e^m_{ij} y^m_{ij}$ $\forall i,m$ (27)

$\mu^2_{im} + \lambda^2 \geq \sum_{k\in K} \hat{a}^m_i q_k w^m_{ik}$ $\forall i,m$ (28)

$\mu^4_{jr} + \lambda^4 \geq \sum_{k\in K} \hat{f}^r_j x^r_{jk}$ $\forall j,r$ (29)

$\mu^5_h + \lambda^5 \geq \sum_{j\in J} \hat{v}^h z^h_j$ $\forall h$ (30)

$\sum_{i\in I} a^m_i y^m_{ij} + \lambda^3_{jm} \Gamma^3_{jm} + \sum_{i\in I} \mu^3_{ijm} \leq \sum_{h\in H} c^{hm} z^h_j$ $\forall j,m$ (31)

$\mu^3_{ijm} + \lambda^3_{jm} \geq \hat{a}^m_i y^m_{ij}$ $\forall i,j,m$ (32)

$$\lambda^1, \lambda^2, \lambda^3_{jm}, \lambda^4, \lambda^5, \mu^1_{im}, \mu^2_{im}, \mu^3_{ijm}, \mu^4_{jr}, \mu^5_h \geq 0 \qquad \forall i,j,h,m,r \qquad (33)$$

Constraints (14) to (23)
Note: objectives and constraints were written based on equation (3).

3-Model implementation and numerical results

Application of the recommended model in reality using real data from EMS center in Kerman city was studied. The model encoded in GAMS 24.1.2 software and then solved in all experiments by a computer of core i5 CPU and 6 gigabytes RAM within less than a minute.

3-1- Description of the case study

Kerman is one of the biggest cities in Iran country that experience a vulnerable condition at the present time. In this case study, the Kerman City was divided into 65 demand points and demand of each point approximated based on the calls to the EMS operator in Kerman City. Division of demand type was done according to the patients' deterioration intensity. The cases such as fractures and mild injuries categorized in normal group and those such as heart diseases, poisoning and sever injuries which need advanced equipment were categorized in special group. Two types of ambulances ordinary and special were allocated to respond patients' demands. Ordinary ambulances are capable of responding only ordinary demands and special ambulances for both demands.

In the problem, total of 30 potential locations to establish EMS centers were identified and their distance from demand points were calculated based on Euclidean distance. The standard covering radius according to the traffic load in Kerman City was considered as 1500 meters. The EMS centers were divided into 3 types of small, medium and large. According to the experts' opinion, averagely one hour was needed from the call until the end of mission to deal with a demand point. Since using time of an ambulance is 12 hours during a day averagely, dividing it by average time of dealing with a demand resulted in 12 as an ambulance capacity. The parameter of busyness coefficient was obtained through dividing available hours (12) by hours of a day which was 0.5. According to the experts' attitude, the cost of buying an ambulance type ordinary and special was considered as 1500 and 2200 million Rials respectively. The cost of establishing EMS centers type small, medium and large was assessed 1400, 2400 and 3000 million Rials respectively. Figure (1) shows dividing method of demand points and potential points for establishing EMS centers in Kerman city.

Fig. 1. Kerman city map and dividing demand points

3-2- Results of the model implementation

The problem solved through considering the covering objective as the main function and the other two functions (availability of ambulances and total costs) were considered as constraint in the ε – constraint method. The results obtained from application of robust model in case study of Kerman city show that using the introduced approach can result in 21.76% improvement in demand covering in proportion to the current conditions in Kerman city. Figure 2 shows optimized decisions obtained from using robust model including decisions of EMS locations and types of ambulances. Moreover, table 1 compares the number of EMS centers established before and after the model implementation. The applied point in table 1 is that although ordinary ambulances have not been used in current conditions of Kerman city (before the model implementation), using ordinary ambulances can be effective on improvement of the covering. In addition, it is noticeable that the model selected all centers from small type in the optimized answer.

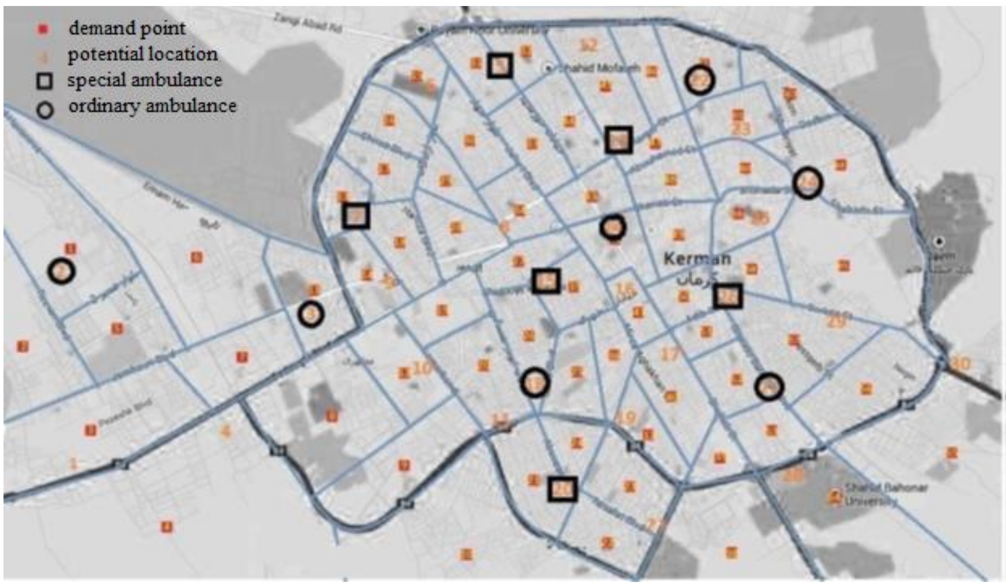

Fig. 2. Optimized decisions of EMS centers location and types of ambulances

Table 1 . comparing the number and type of ambulances allocated before and after the model implementation

	Before the model implementation	After the model implementation
No. of special ambulances	12	6
No. of ordinary ambulances	0	7
Total	12	13

3-3- Balance between cost, covering and availability objectives

The relationship between cost, covering and availability objectives is studied in this section. To do so, the above relationship and quality of balance between them are studied by ε – constraint method introduced in section 1-1. Figure 3 shows the relationship between covering and cost rates in a constant level of availability.

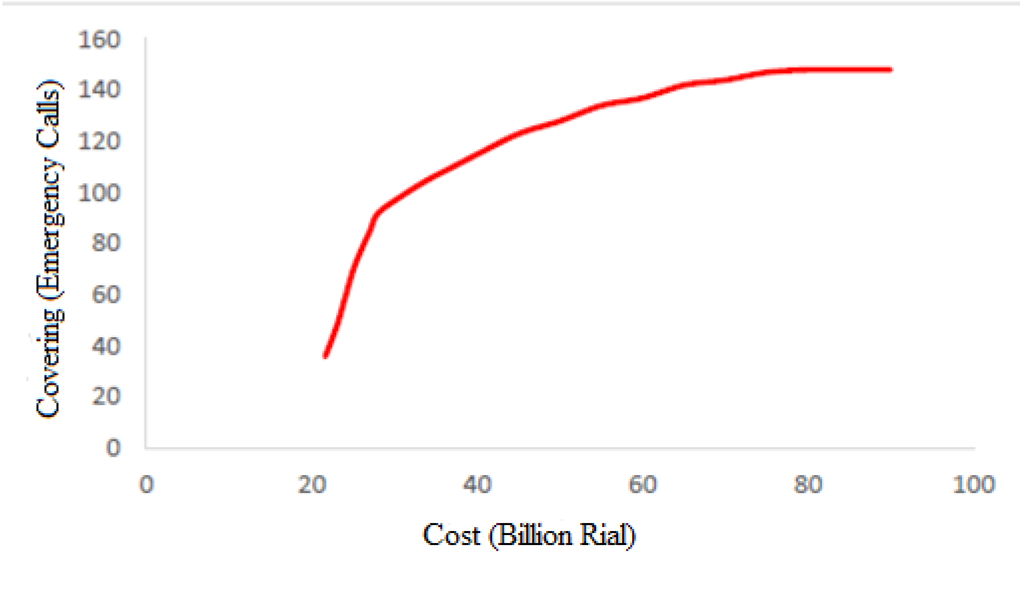

Fig. 3 . The relationship between covering and cost rates in constant level of availability

According to figure 3 there is a direct relationship between the demand covering and the cost.

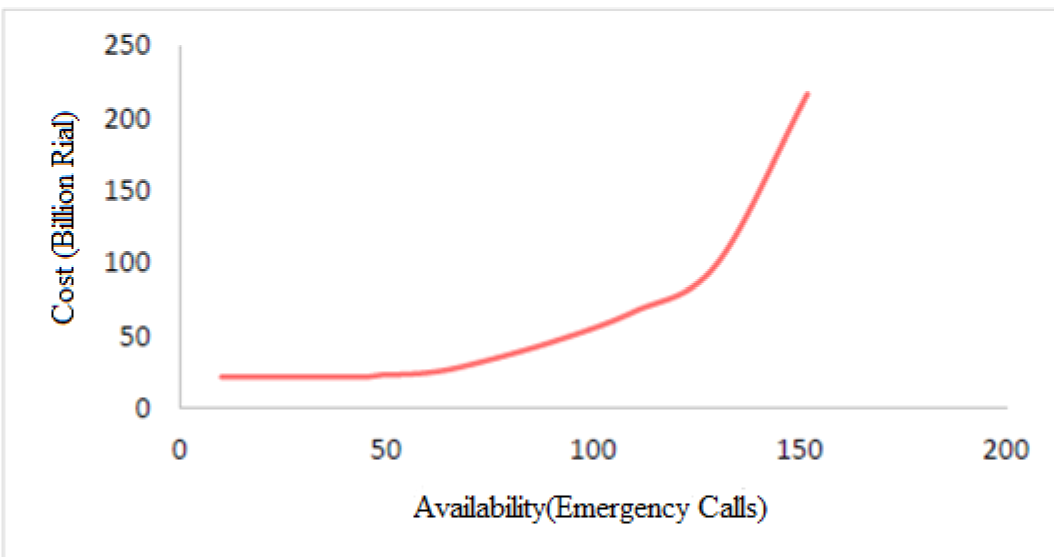

Fig. 4 . The relationship between cost and availability in constant covering level

Figure 4 shows that more increase in availability causes more increase in the curve slope; achieving higher availability needs more increase in costs rate. In figures 3 and 4, the objectives are studied in pairwise form and their relationship is identified. Now, all three objectives are studied simultaneously in figure 5 in which the covering objective is considered as the main objective and the other two objectives (cost and availability) are considered as constraints. Then, values of covering objective are obtained according to the cost objective for constant values of 50, 70, 110 and 130 of availability function.

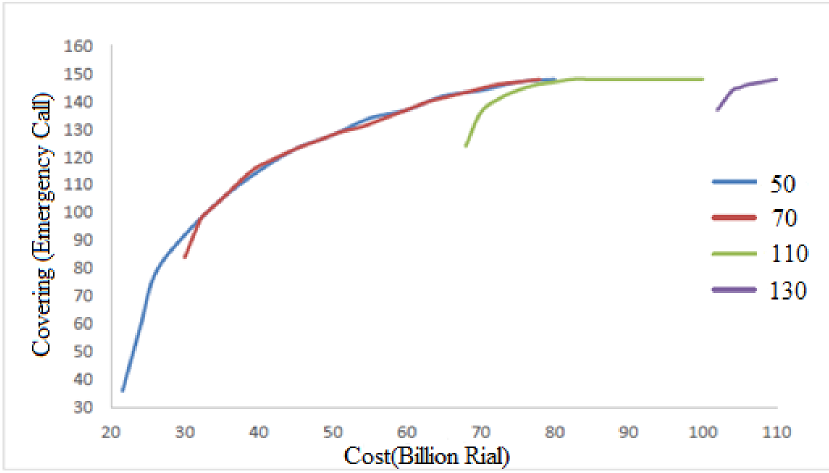

Fig. 5 . The relationship between covering and cost rates in different levels of availability

As expected, figure 5 delineated that the covering and cost objectives keep their direct relationship (the value of availability is assumed to be constant). However, slope and range of the curves are influenced by availability rate. On this basis, it can be said that if the budget becomes less than 30 billion Rials, the availability level can get 50 maximally.

Obviously, the curve slope has a higher value at first and reduced gradually. For instance, increase in budget from 30 to 40 billion Rials results in 16% increase in covering rate while increase in budget from 70 to 80 billion Rials just results in 0.013% increase in covering rate. Therefore, increase in budget at first is very economical. If it is just decided to increase covering, the budget more than 80 billion Rials does not seem economical because more than 0.007% improvement in covering is not achievable for it. However, it should be said that when covering reaches to maximum, achieving availability more than level 50 units is still possible.

3-4- Analysis of covering sensitivity versus variations of protection degree and demand fluctuations

Table 2 shows that change in protection degree causes change in covering objective.

Table 2 . values of objective for different values of protection degree

	Γ1	Γ2	Γ3	Covering objective
A	16	16	1	98
B	32	32	1	82
C	65	65	1	66
D	1	1	2	113
E	48	48	2	66
F	16	16	3	98
G	16	16	6	98

Since Γ is the protection degree parameter related to the objective and affects the objective rate directly, the most objective variations from optimized value can be because of it. Another factor effective on objective variations related to the limitation of possibility space of the problem trough tightening of existing constraints and addition of uncertainty constraints. A glance at table 1 delineates that the role of Γ in reduction of objective is less but Γ can play an important role in changing possibility space of the model on the other hand.

Studying the possibility space of the problem and analysis of covering objective sensitivity to demand fluctuations rate is conducted through calculations for 5%, 10%, 15% and 20% fluctuations of demand from nominal value and the results are shown in figure 6. The perpendicular axis in left side of figure 6 shows the rate covering objective influenced by different demand fluctuations of nominal value. In other words, if ZN and ZR denote optimized covering value in certain and uncertain model respectively, this axis shows the reduction percentage of demand covering objective using the equation (ZN-ZR)/ZN. Moreover, the perpendicular axis on right side of figure 6 shows the possibility of constraints violation in terms of different values of protection degree. It is noticeable that value of this possibility is calculated by equation (10).

Figure 6 shows that the worst objective value is occurred when the protection degree takes its maximum value. Also by comparing demand variations, the 20% fluctuation of demand had the most influence (19.65%) on objective value. In other words, more variations in nominal value of demand data cause covering rate to become worst.

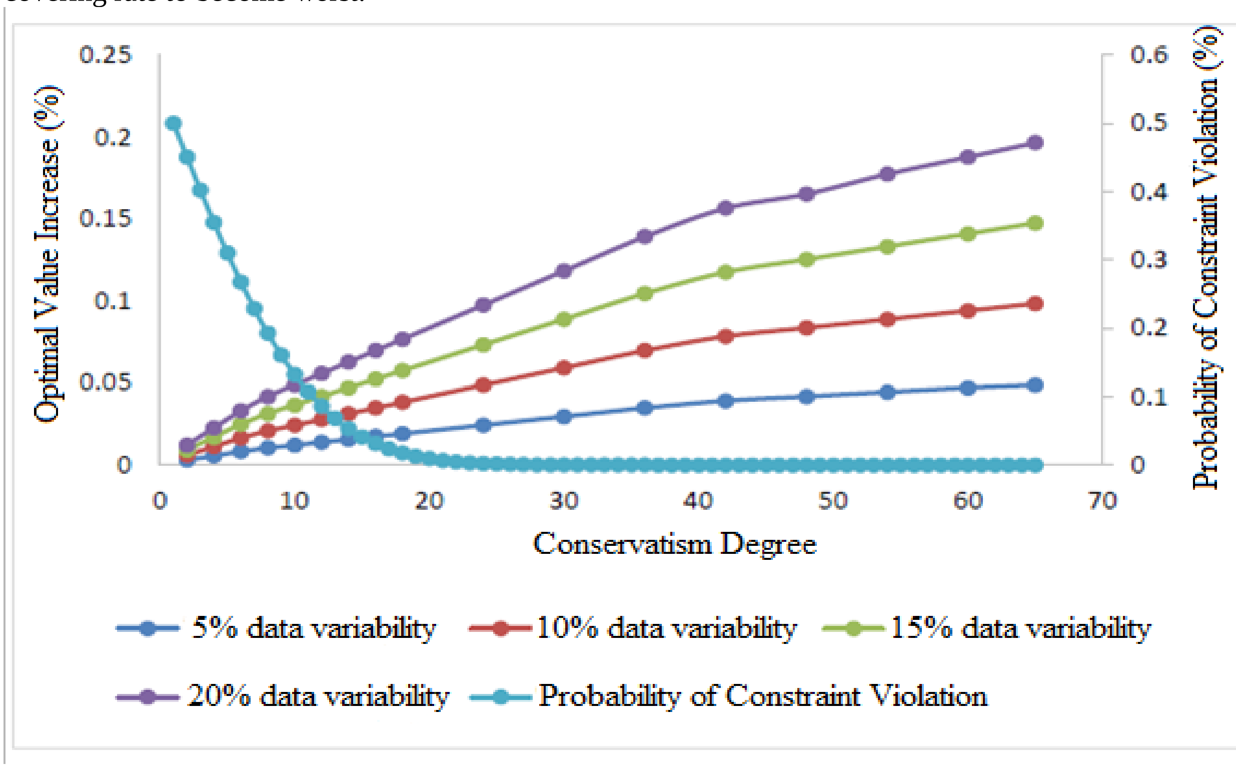

Fig. 6 . analysis of the model sensitivity to fluctuations in demand values

Table 3 helps to understand that what minimum value of protection degree is needed to achieve a specific possibility for not violating the constraints.

Possibility of violating constraint	Protection degree	Protection degree
		Max protection degree
α = 1%	19	29
α = 5%	14	21.5
α = 10%	11	16.9
α = 30%	5	7.7
α = 40%	3	4.6
α = 50%	1	1.5

4-Conclusion

In this article, a robust multi-objective model to design a network for EMS under uncertainty conditions is introduced. The multi-objective model attempts to optimize contradictory objectives simultaneously: maximizing covering of demand points, minimizing costs and maximizing the availability of ambulances. An interval robust optimization approach, which doesn't need possibility distribution functions of uncertain parameters, is used to deal with the uncertainty of cost and demand. The other contribution of the presented model is its capability to determine simultaneously the location and the capacity for EMS centers, in addition to determine the type and the number of ambulances. Real data of an urban district is used for the robust model. The results of sensitivity analysis and managerial insights are presented in the paper. Results show that the introduced model can be very effective in real word and leads to improving the conditions.

The introduced approach is developable from different aspects. (1) The problems related to the disaster and risk management in location problems of EMS systems can be added to the introduced problem. (2) Considering aerial and land EMS in the model simultaneously. (3) Assuming different conditions during days and nights and using dynamic programming for modeling the problem.

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