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Multi-period network data envelopment analysis to measure efficiency of a real business

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Abstract

Measuring the efficiency of real businesses is not a simple task, because a real business may involve several processes and sub-processes, forming a very complicated dynamic network of interactions. In this paper a customized dynamic network data envelopment analysis (NDEA) model is proposed to measure the efficiency of the sub-processes in a real business. The proposed dynamic NDEA model is fully designed and customized for IMI which is a leading institute in providing consulting management, publication, and educational services. First, we have identified the network of the Industrial Management Institute (IMI) which includes educational, consulting, and publication sub-processes. Then, the most important sub-processes and the associated dynamic interactions are determined. Afterwards, a dynamic NDEA model is proposed to measure the efficiency of sub-processes. The main theoretical properties of proposed dynamic NDEA model are also discussed through theorems. Assessing the performance of IMI's sub-processes is not a trivial task due to the complexity of sub-processes in IMI. The proposed dynamic NDEA model is applied using real operational data of the IMI gathered through a sixty-month planning horizon. An attempt has been accomplished to form a relationship between the total efficiency of process and the efficiency of each sub-process by a regression analysis. The managers of IMI can monitor the efficiency score of main process and sub-processes during planning horizon which can help to improve inefficient sub-process.

Keywords: Linear programming, network data envelopment analysis, performance measurement, multi-period performance analysis.

1-Introduction

Business performance measurement is an important task in order to illustrate the efficacy of process and sub-process during multiple periods of time. Performance measurement in real business especially service organizations are more sensitive as the processes and sub-processes usually make high-interactive, dynamic and complicated network structures. Identification of processes and sub-processes as well as the relationship between them is not a trivial task in service organizations. Measuring the performance of processes and sub-processes in service organizations help managers to maintain the strength and improve the weak.

The Industrial Management Institute (IMI) is active in the fields of management and comprehensively in the education, consulting and publishing processes in Iran.

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IMI helps the public and private organizations and businesses improving the management systems and methods, promoting the efficiency through implementing the training programs and providing the managers and policy makers to detect the executive shortages and executive restraints of the plans through providing consultancy services and researches. IMI is a ground-breaking organization that supports the nation's management potential and draws upon world-class partners as a premiere firm in the MENA region. IMI offers management consulting, education, and research services to national and regional firms, aiming at assisting the sustained growth and development of the economy and economic subsystems. We support our clients in all public, private, and non-profit sectors so that they can discover and realize their business potentials and identify their main challenges. Our objective is to help develop management capacity in firms, nationwide and beyond. IMI has representation in 27 cities throughout Iran. IMI's branches are privately owned firms that offer their services under the license of IMI's headquarters. Offices are in Shiraz, Isfahan, Tabriz, Mashhad, Yazd, among others. IMI has also undertaken a variety of management consulting and education projects in conjunction with international partners. The managers of IMI must be aware of the fact that how to achieve the relative success and best performance in terms of utility in relation to their competitors. In other words, they must be aware of their own success in comparison with other similar institutes and previous years. The process of performance assessment provides the opportunity for IMI to identify the problems and take the proper action before the problems increase.

The complicated relations between the different units of IMI and different types of inputs and outputs (desirable or undesirable) create a high-interactive, dynamic and complicated network structure. Therefore, the assessment of the overall efficiency of the main process by taking into consideration different types of inputs and outputs and also determining the efficient and inefficient processes and sub-processes can be useful for improving the effectiveness of the education, consulting and publishing processes.

Regarding the review of the literature of the past researches, we can state that identification of the main processes and sub-processes in such large scale service business and assessing the efficiency of them in the form of a dynamic network has not been reported. Although there some researches have been accomplished in the field of education and research performance, there are no independent research on the education, consulting and publishing industry in the literature of past work.

The next section of this article will be organized as follows. In section 2, a review of the relevant literature is presented. In section 3, the main problem of the study is defined. In section 4, the mathematical modeling of performance assessment is presented. In section 5 the real case study of Industrial Management Institute and the results are presented. Conclusion and future research directions are presented in section 6.

2- Literature review

In this section, relevant literature is briefly reviewed.

2-1- DEA literature

The Data Envelopment Analysis (DEA) was first proposed by charnes et al (1978). DEA is a non-parametric linear programming method to determine the relative efficiency of the decision making units (DMU) (Ramanathan, 2003).

The classic DEA models examines the single-stage process where the internal structures of DMUs are ignored. On the other hand, Network Data Envelopment Analysis (NDEA) is used to examine the multistage processes considering the internal structure (Chen and Zhu, 2004; Kao and Hwang, 2008; Liang et al., 2008; Chen et al., 2009; Chen et al., 2009; Cook et al., 2010; Liang et al., 2011). The simplest form of the internal structure of the systems is series relation. This structure indicates some of the processes that are connected in row (Kao, 2014). In the parallel structures, all stages are worked independently. Network structures contain some processes that are not series or parallel. Jie & Liang (2012) identified the critical input-output combinations for each DMU international tourist hotel. So DEA can be used to evaluate performance and the critical input-output measure. Khalili-Damghani and Molayee (2018) proposed a hybrid approach based on DEMATEL, ANP, and DEA-based Malmquist Productivity Index to measure the performance of detection and governmental punitive agency. Khalili-Damghani et al. (2019) measured the productivity of the bank branches using data envelopment analysis and Malmquist index.

2-2- Network DEA

Network DEA basically addresses the internal processes of a DMU through taking into consideration Sub-DMUs. As opposed to the common DEA models, the network models do not have a constant formulation, and they are generally modeled based on network structure and internal processes of the units under study. Kao, (2014) has classified network DEA models into nine groups. Kao and Hwang (2008) have proposed a two-stage model with a series structure where all outputs of the first stage are used as the input of the second stage. Liang et al., (2008) has extended the two-stage model considering extra input in the second stage. The real network structure is composed of several series and parallel structures. Kao, (2009) proposed a network DEA model to assess the performance of the overall network system with a linear combination in the sub-processes. Kao, (2014) developed network DEA models for multistage systems by taking into consideration the separate intermediate products.

Yong et al., (2017) have used a network slack-based model and an undesirable distance model to measure overall and sub-process supply chain efficiency in china’s E-retailing industry.

Amirkhan et al., (2018a) developed a network DEA model to measure the efficiency of a pure serial three-stage process. All of the stages cooperate together to improve the overall efficiency of main DMU. Amirkhan et al., (2018a) applied the proposed model to measure the efficiency of a supply chain network. Amirkhan et al., (2018b) proposed an expert system in form of an uncertain DEA approach. So, proposed fuzzy-robust DEA models, which simultaneously served the advantage of each of the fuzzy and robust approaches, calculated the upper and lower bounds of the efficiency scores of DMUs under CRS and VRS conditions. Sharahi and Khalili (2019) have proposed a decision support system for allocation of resources and setting the targets across a set of entities in an equitable manner in presence of uncertainty. They used De-Novo programming to optimally determine the resources and targets of DMUs in network DEA rather than optimizing existing DMUs. Tavana et al., (2019) proposed a multi-objective multi-period network DEA model to evaluate the efficiency of oil refineries. Sharahi et al., (2019) proposed a new network data envelopment analysis models to measure the efficiency of natural gas supply chain. The proposed model was used to measure the efficiency of a gas supply chain and the associated efficiency of all elements in the chain during a 5-year planning horizon. Tavana et al., (2019) proposed a Malmquist productivity index for network production systems in the energy sector. They used the proposed model to measure the productivity of several Iranian oil refineries.

2-3- Education industry applications of DEA

The DEA has widely been used in education and teaching industry. For instances the researches works by Meek, (2000); Avkiran, (2001); Kyvik, (2004); Carlos et al., (2006); Casu & Thanassoulis, (2006); Glass et al., (2006); Leitner et al., (2007); Celik & Ecer, (2009); Agasisti & PerezEsparrells, (2010); Abramo et al., (2012); Monfared & Safi, (2013) are important to be mentioned here.

Antreas and Estelle, (1997) measured the efficiency of the educational institutes in UK using DEA. Antreas and Estelle, (1997) measured the cost efficiency in the operations of the postgraduate educational institutes. In the first step the costs and incomes of the universities were considered to establish the headquarters. In the second step, the resources and abilities of the student were considered to achieve adequate result. Avkiran (2001) examined the technical and scale efficiencies of the Australian universities using BCC[1]-DEA method. The findings of the research showed that the Australian universities suitably performed in terms of technical and scale efficiency. Lopez and lanzer, (2002) assessed the performance of 58 departments in a Brazilian university. They classified the outputs into four groups including the quantitative, qualitative, research and service. They assessed the groups using the fuzzy DEA model, and obtained the correlation among the four educational, research, qualitative and service aspects using correlation coefficient. Martin (2003) examined the performance of Zaragoza university departments using DEA. Martin (2003) divided the inputs into 3 groups as the financial, human and physical resources. Martin (2003) also classified the outputs into two educational and research levels.

[1]Banker, Charnes, Cooper: BCC

The students' views are examined and considered as the necessary factor in monitoring the quality at the universities (Hill and Lomas, 2003). The students are the main educational clients who have attracted the most attention (Sirvanci, 2004). Toumady and Ris., (2005) examined the efficiency of 209 educational institutes in 8 European countries. They used 3 models to carry out the assessment. The first model, which was called the competency model, focused on the characteristics, knowledge levels of the students to provide proper educational services. The comparative model was also proposed to assess the quality of the services offered to the students by the universities during the education period and to attract them in the labor market. Finally, the last model, which was called the comprehensive model, considered the inputs and outputs of two previous models simultaneously and examined the overall performance of the universities.

The higher education institutes that are strong in research and instruction have focused on the research power through merging with the strong research institutes in order to boost the progress. These institutes focused on instruction power through merging with the similar educational institutes in order to obtain economic scale (Glass et al., 2006).

(Kao and Hung, 2008) assessed the relative efficiency of 6 scientific departments including 41 educational departments affiliated with Taiwan Cheng Chung National University using output oriented BCC-DEA model.

There are two views on the importance of the student as a client: the first perspective states that the students cooperate in the learning process as inputs and outputs. The second view states that the potential employers consider the students as the primary clients and they believe that the economic reality of the market must be taken into consideration. In both views, the student is considered an important client of the institute education department (Yeo, 2008). There are several papers focused on teaching quality to assess the performance of the educational institutes. Among them Colbert et al., (2000); Agasisti & Bianco, (2009); Meng et al., (2008); Ahn et al., (1998); Sinuany-Stern et al., (1994); Beasley, (1995); Gander, (1995); Abbott & Doucouliagos, (2003); Kao & Hung (2008); Tyagi et al., (2009); Agasisti & Perez- Esparrells, (2010) have worth to be mentioned here.

Daneshvar and Serpil Erol, (2009) used the DEA-ANP method to assess the performance of the departments of Amir Kabir University of Technology. The combination of DEA-ANP was also used to assess the performance of other institutes in several researches (Zhang and Cui, 1999; Bowen, 1990; Yang and Kuo, 2003; Takamura and Tone, 2003; Saen et al., 2005).

3- Problem definition

In this paper, a customized network structure is proposed to assess the cause process in the Iran Industrial Management Institute (IMI). The cause process consists of three sub-processes including education, consulting and publication. The complicated process among the units and high amount of inputs and outputs of cause process form a complicated network for the IMI. Using dynamic network DEA made it possible to determine the IMI's regress/progress over the time. The common DEA do not consider the internal structure of each decision making units (DMUs). Therefore, the score of the efficiency of each DMU cannot be decomposed into the efficiency of its sub-processes. Regarding the dynamic network, the cause process is considered as a DMU in period t. The inputs and outputs of DMUs will be change over the assessment period.

All these structures have been modeled based on a real process in IMI. The main goal of this modeling is to measure the relative efficiency of the cause process of IMI over a five-year planning horizon using the real monthly operational data. Figure 1 briefly shows the main modules of the research.

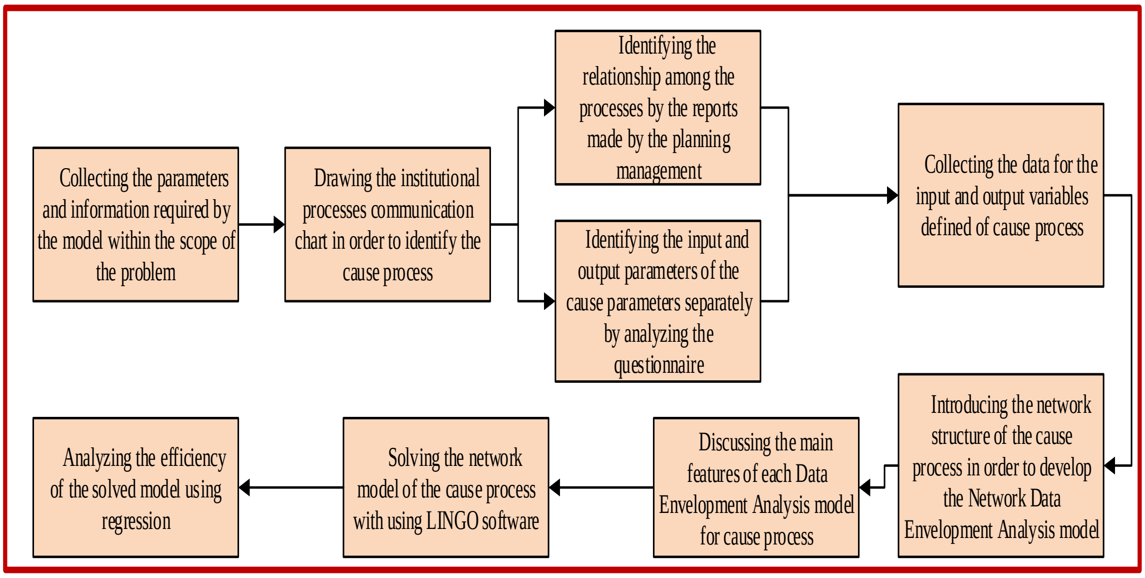

Fig1. Main steps of the research

3-1- Cause process in IMI

The cause process at IMI is determined as follows. A brief description is prepared for the tasks of each department. The organizational structure of the department is prepared. The interaction including the inputs and outputs between departments is determined. The importance of each activity performed by each department is determined using the cost/benefit analysis of cause process and activities considering the added value of each task. On the other hand, the importance of each activity in a given process is determined on the basis of the cost of the activity divided by total cost of all activities in the same process. Then, using Pareto rule, the most important and less important activities are selected as the cause activities.

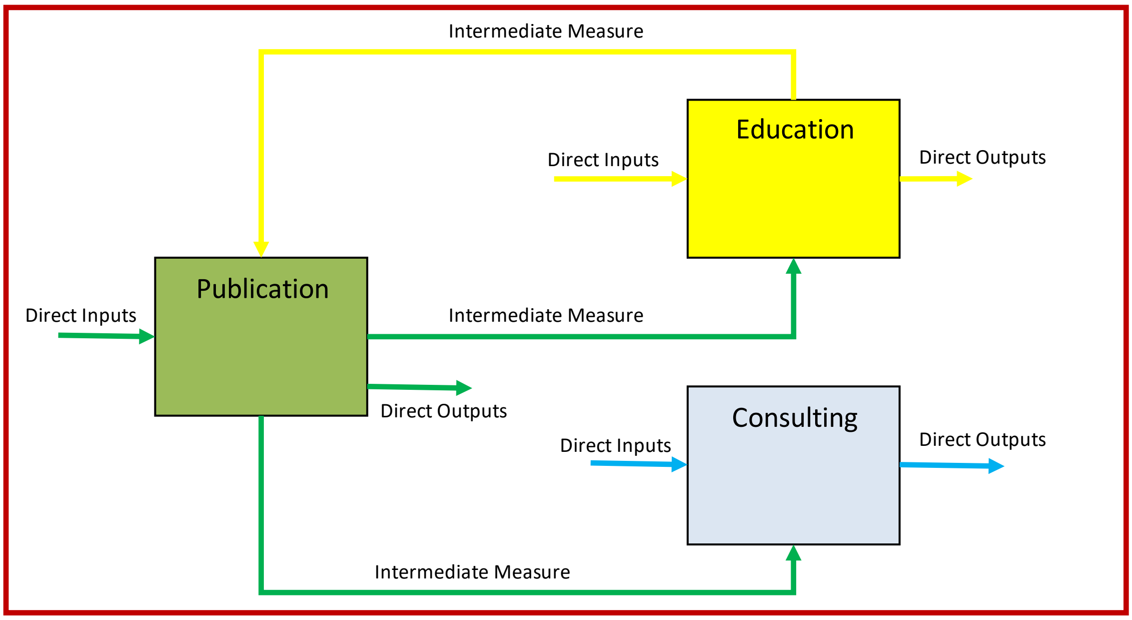

Fig 2. Network of the cause activities for single period

At the next step, on the basis of organizational chart of the IMI, a network including all inputs and outputs and interactions between activities is plotted for cause activities. The networks have been validated in a brain storming session incorporating the opinion of the managing directors and the chief of the systems and methods department. The schematic view of the cause activities for a single period is presented in figure 2. The multi period cause process is presented in figure 3.

At the next step, some criteria including inputs, outputs and intermediate measures are also selected on the basis of experiences of the chief executive of IMI. The indices were added to the indices obtained from the relationship among the processes and those selected form literature review. At the final step, the conceptual model for cause process is formed. In this research, the qualitative indices are quantified using a 5-point Likert Scale. Using the network of process in cause activities in IMI, the total efficiency of the IMI as well as the efficiency of sub-processes can be calculated during a multiple-planning horizon. In the next section the modeling procedure for this aim is developed.

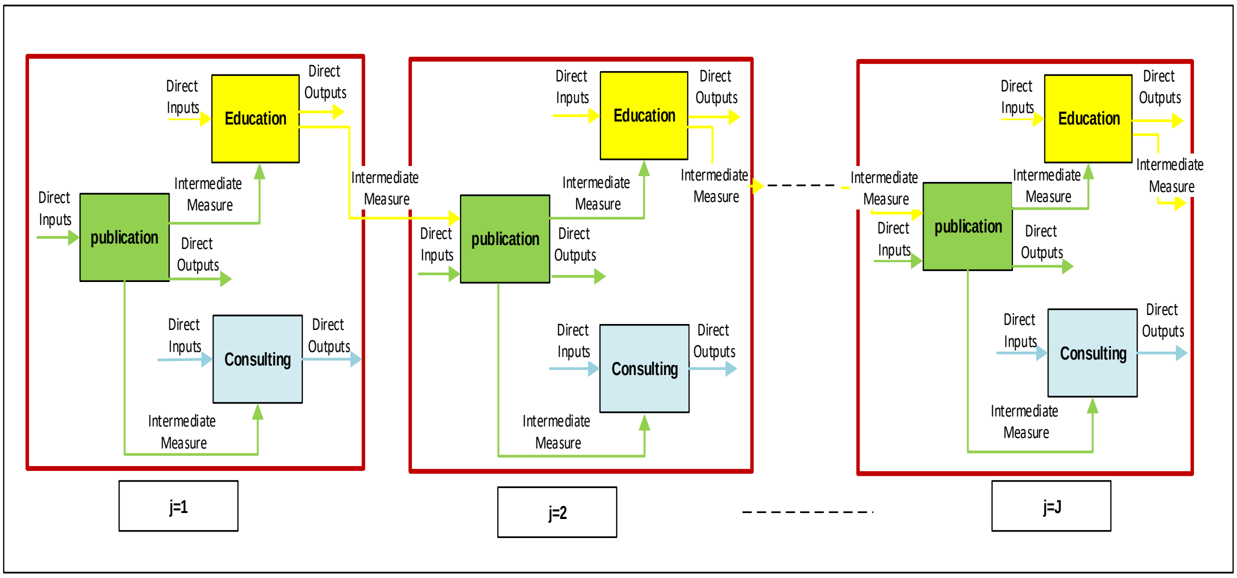

Fig 3. Multi-period network of the cause activities

4- Proposed network DEA model

In this section a customized NDEA model is developed to measure the efficiency score of cause network depicted in figure 2. The schematic view of associated DMUs for cause process is shown in Figure 4. The notations which are used in the mathematical model, is also presented in figures 3. For sake of generality, the sub-processes presented in figure 2 and figure 4 are named using sub-process1, sub-process2 and sub-process 3. In this way more general forms of DMUs are presented in figures 4 and 5. It is notable that each DMU is assessed during multiple planning periods.

4-1- Network DEA model for Cause Process

As mentioned, figure 4 presents the schematic view of DMU associated with cause process.

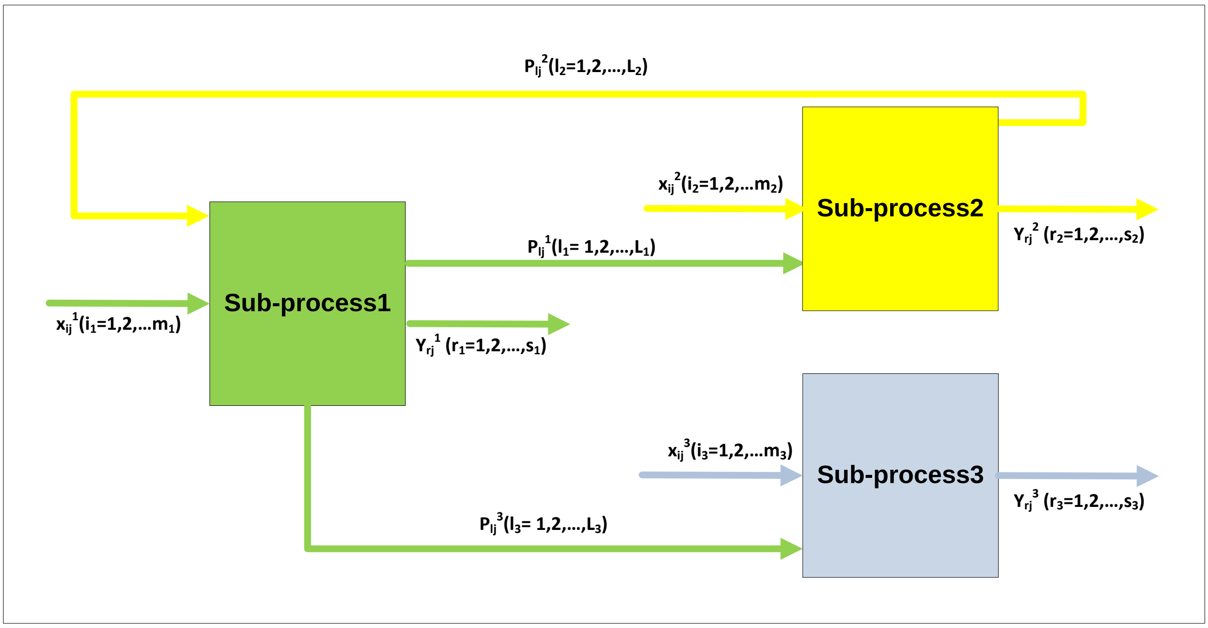

Fig 4. Schematic view of a DMU associated with cause process for single period

In figure 4, each DMU$_j$ (j=1,2,…,J) consists of sub-process1, sub-process2 and sub-process3. Sub-process1 consumes, m_1 inputs $x_{ij}^{1}(i_1=1,2,\ldots,m_1)$ and L_2 intermediate measures $P_{lj}^{2}(l_2=1,2,\ldots,L_2)$ to produce s_1 outputs $Y_{rj}^{1}(r_1=1,2,\ldots,s_1)$ and L_1 intermediate measures $P_{lj}^{1}(l_1= 1,2,\ldots,L_1)$ and L_3 intermediate measures $P_{lj}^{3}(l_3= 1,2,\ldots,L_3)$. Sub-process2 consumes L_1 intermediate measures $P_{lj}^{1}(l_1=1,2,\ldots,L_1)$ and m_2 inputs $x_{ij}^{2}(i_2=1,2,\ldots m_2)$ to produce s_2 outputs Y_{rj}^{2} $(r_2=1,2,\ldots,s_2)$. Sub-process3 consumes L_3 intermediate measures $P_{lj}^{3}(l_3= 1,2,\ldots,L_3)$ and m_3 inputs $x_{ij}^{3}(i_3=1,2,\ldots m_3)$ to produce s_3 outputs Y_{rj}^{3} $(r_3=1,2,\ldots,s_3)$.

The efficiency of total process, sub-process1, sub-process2 and sub-process3 in period *t* are parameterized using e_j, e_j^{1}, e_j^{2}, e_j^{3}, respectively.

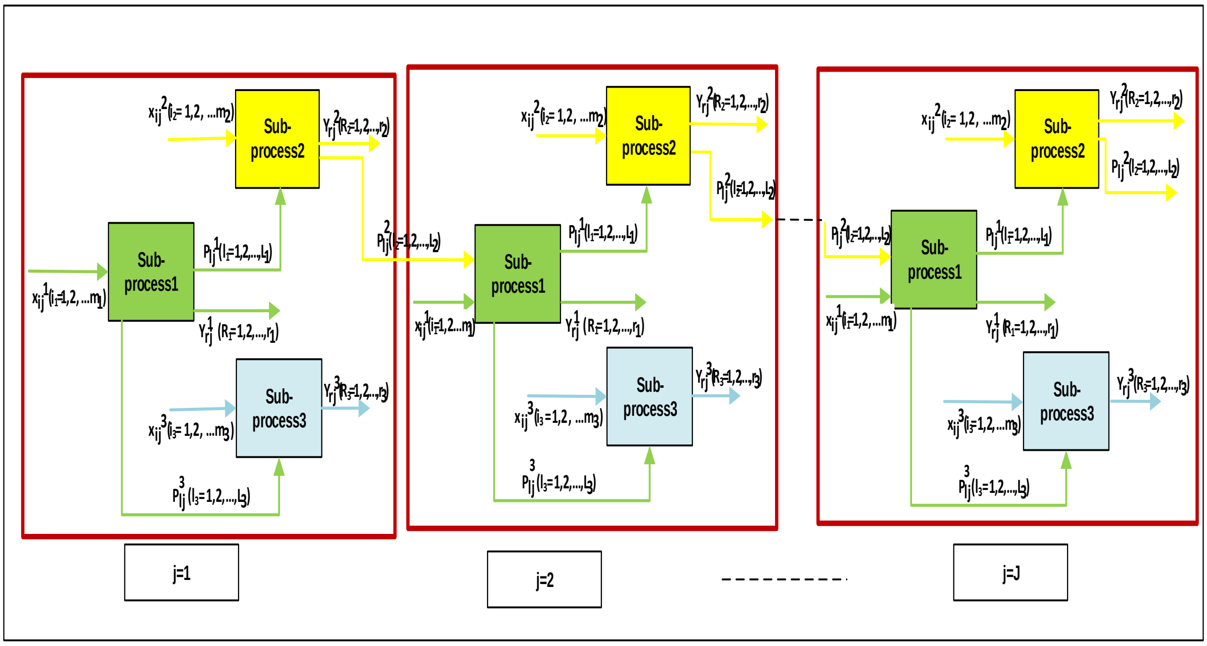

Fig 5. Schematic view of a network DMU associated with cause process for multi period

In figure 5, a network of cause process with inputs, outputs and intermediate products is presented for multi periods. The index t is the same as the DMUs being evaluated in the context of time. All indices, parameters and decision variables used in the proposed network DEA model is presented in table1.

Table 1. Indices, parameters and decision variables used in network DEA model

Indices		
J	The number of DMUs	J=1,…,j
o	DMU which is under assessment	
m_1	The number of inputs of sub- Process1	$i_1=1,\dots,m_1$
m_2	The number of inputs of sub- Process2	$i_2=1,\dots,m_2$
m_3	The number of inputs of sub- Process3	$i_3=1,\dots,m_3$
L_1	The number of outputs of sub- Process1; the number of inputs of sub- Process2;	$l_1=1,\dots,L_1$
L_2	The number of outputs of sub- Process2; the number of inputs of sub- Process1;	$l_2=1,\dots,L_2$
L_3	The number of outputs of sub- Process1; the number of inputs of sub- Process3;	$l_3=1,\dots,L_3$
L_4	The number of outputs of sub- Process2; the number of inputs of sub- Process3;	$l_4=1,\dots,L_4$
L_5	The number of outputs of sub- Process3; the number of inputs of sub- Process1;	$l_5=1,\dots,L_5$
L_6	The number of outputs of sub- Process3; the number of inputs of sub- Process2;	$l_6=1,\dots,L_6$
r_1	The number of outputs of sub- Process1	$r_1=1,\dots,s_1$
r_2	The number of outputs of sub- Process2	$r_2=1,\dots,s_2$
r_3	The number of outputs of sub- Process3	$r_3=1,\dots,s_3$
Parameters		
X^1_{ij}	The ith input of sub-process 1 of DMU_j	
X^2_{ij}	The ith input of sub-process 2 of DMU_j	
X^3_{ij}	The ith input of sub-process 3 of DMU_j	
P^1_{lj}	The lth output of sub-process 1 of DMU_j; the lth input of sub-process 2 of DMU_j	
P^2_{lj}	The lth output of sub-process 2 of DMU_j ; the lth input of sub-process 1 of DMU_j	
P^3_{lj}	The lth output of sub-process 1 of DMU_j ; the lth input of sub-process 3 of DMU_j	
P^4_{lj}	The lth output of sub-process 2 of DMU_j; the lth input of sub-process 3 of DMU_j	
P^5_{lj}	The lth output of sub-process 3 of DMU_j; the lth input of sub-process 1 of DMU_j	
P^6_{lj}	The lth output of sub-process 3 of DMU_j; the lth input of sub-process 2 of DMU_j	
Y^1_{rj}	The rth output of sub-process 1 of DMU_j	
Y^2_{rj}	The rth output of sub-process 2 of DMU_j	
Y^3_{rj}	The rth output of sub-process 3 of DMU_j	
Decision variables		
v_i^1	The multiplier of the ith input of sub-process1	$i_1=1,\dots,m_1$
v_i^2	The multiplier of the ith input of sub-process2	$i_2=1,\dots,m_2$
v_i^3	The multiplier of the ith input of sub-process3	$i_3=1,\dots,m_3$
w_l^1	The multiplier of the lth output of sub-process1; the multiplier of the lth input of sub-process2	$l_1=1,\dots,L_1$
w_l^2	The multiplier of the lth output of sub-process2; the multiplier of the lth input of sub-process1	$l_2=1,\dots,L_2$
w_l^3	The multiplier of the lth output of sub-process1; the multiplier of the lth input of sub-process3	$l_3=1,\dots,L_3$
w_l^4	The multiplier of the lth output of sub-process2; the multiplier of the lth input of sub-process3	$l_4=1,\dots,L_4$
w_l^5	The multiplier of the lth output of sub-process3; the multiplier of the lth input of sub-process1	$l_5=1,\dots,L_5$
w_l^6	The multiplier of the lth output of sub-process3; the multiplier of the lth input of sub-process2	$l_6=1,\dots,L_6$
u_r^1	The multiplier of the rth output of sub-process1	$r_1=1,\dots,s_1$
u_r^2	The multiplier of the rth output of sub-process2	$r_2=1,\dots,s_2$
u_r^3	The multiplier of the rth output of sub-process3	$r_3=1,\dots,s_3$
e_j	The overall efficiency score of DMU_j	j=1,…,J
e_j^k	The efficiency score of sub-process k of DMU_j	k=1,2,3; j=1,…,J

Model (1) is proposed to measure the efficiency score of first DMU presented in figure 5.

$$
\begin{aligned}
&Max \quad e_o = \left(\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 o}^1 + \sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 o}^2 + \sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 o}^3 + \sum_{l_2=1}^{l_2} w_{l_2}^2 p_{l_2 o}^2 \right) \Big/ \left(\sum_{i_1=1}^{m_1} v_{i_1}^1 x_{i_1 o}^1 + \sum_{i_2=1}^{m_2} v_{i_2}^2 x_{i_2 o}^2 + \sum_{i_3=1}^{m_3} v_{i_3}^3 x_{i_3 o}^3 \right) \\
&s.t. \\
&\left(\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j}^1 + \sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 j}^2 + \sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 j}^3 + \sum_{l_2=1}^{l_2} w_{l_2}^2 p_{l_2 j}^2 \right) \Big/ \left(\sum_{i_1=1}^{m_1} v_{i_1}^1 x_{i_1 j}^1 + \sum_{i_2=1}^{m_2} v_{i_2}^2 x_{i_2 j}^2 + \sum_{i_3=1}^{m_3} v_{i_3}^3 x_{i_3 j}^3 \right) \le 1, \quad j=1 \\
&\left(\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j}^1 + \sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 j}^2 + \sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 j}^3 + \sum_{l_2=1}^{l_2} w_{l_2}^2 p_{l_2 j}^2 \right) \Big/ \left(\sum_{i_1=1}^{m_1} v_{i_1}^1 x_{i_1 j}^1 + \sum_{i_2=1}^{m_2} v_{i_2}^2 x_{i_2 j}^2 + \sum_{i_3=1}^{m_3} v_{i_3}^3 x_{i_3 j}^3 + \sum_{l_2=1}^{l_2} w_{l_2}^2 p_{l_2 (j-1)}^2 \right) \le 1, \quad j=2,\dots,J \\
&\left(\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j}^1 + \sum_{l_1=1}^{l_1} w_{l_1}^1 p_{l_1 j}^1 + \sum_{l_2=1}^{l_2} w_{l_3}^3 p_{l_3 j}^3 \right) \Big/ \left(\sum_{i_1=1}^{m_1} v_{i_1}^1 x_{i_1 j}^1 \right) \le 1, \quad j=1 \\
&\left(\sum_{r_1=1}^{s_1} u_{r_1}^1 y_{r_1 j}^1 + \sum_{l_1=1}^{l_1} w_{l_1}^1 p_{l_1 j}^1 + \sum_{l_2=1}^{l_2} w_{l_3}^3 p_{l_3 j}^3 \right) \Big/ \left(\sum_{i_1=1}^{m_1} v_{i_1}^1 x_{i_1 j}^1 + \sum_{l_2=1}^{l_2} w_{l_2}^2 p_{l_2 (j-1)}^2 \right) \le 1, \quad j=2,\dots,J \\
&\left(\sum_{r_2=1}^{s_2} u_{r_2}^2 y_{r_2 j}^2 + \sum_{l_2=1}^{l_2} w_{l_2}^2 p_{l_2 j}^2 \right) \Big/ \left(\sum_{i_2=1}^{m_2} v_{i_2}^2 x_{i_2 j}^2 + \sum_{l_1=1}^{l_1} w_{l_1}^1 p_{l_1 j}^1 \right) \le 1, \quad j=1,2,\dots,J \\
&\left(\sum_{r_3=1}^{s_3} u_{r_3}^3 y_{r_3 j}^3 \right) \Big/ \left(\sum_{i_3=1}^{m_3} v_{i_3}^3 x_{i_3 j}^3 + \sum_{l_3=1}^{l_3} w_{l_3}^3 p_{l_3 j}^3 \right) \le 1, \quad j=1,2,\dots,J \\
&v_{i_k}^k \ge \varepsilon, \quad i_k = 1,\dots,m_k, k=1,2,3. \\
&u_{r_k}^k \ge \varepsilon, \quad r_k = 1,\dots,s_k, k=1,2,3. \\
&w_{l_k}^k \ge \varepsilon, \quad l_k = 1,\dots,L_k, k=1,2,3.
\end{aligned}
\tag{1}
$$

Model (1) is a fractional programming network DEA considering constant return to scale (CRS) assumptions period one. As Model (1) is a non-linear mathematical programming, so it is difficult to find its global optimum solution. The following procedure is proposed to convert it into a linear programming. The variable change (2) is accomplished in order to resolve the non-linearity.

$$
b = 1 \Big/ \left(\sum_{i_1=1}^{m_1} v_{i_1}^1 x_{i_1 o}^1 + \sum_{i_2=1}^{m_2} v_{i_2}^2 x_{io}^2 + \sum_{i_3=1}^{m_3} v_{i_3}^3 x_{i_3 o}^3 \right) \tag{2}
$$

Replacing the variable exchange (2) in model (1) will result in model (3).

$$Max \quad e_o = \sum_{r_1=1}^{s_1} bu_{r_1}^1 y_{r_1 o}^1 + \sum_{r_2=1}^{s_2} bu_{r_2}^2 y_{r_2 o}^2 + \sum_{r_3=1}^{s_3} bu_{r_3}^3 y_{r_3 o}^3 + \sum_{l_2=1}^{l_2} bw_{l_3}^2 p_{l_3 o}^2$$

$s.t.$

$$\sum_{r_1=1}^{s_1} bu_{r_1}^1 y_{r_1 j}^1 + \sum_{r_2=1}^{s_2} bu_{r_2}^2 y_{r_2 j}^2 + \sum_{r_3=1}^{s_3} bu_{r_3}^3 y_{r_3 j}^3 + \sum_{l_2=1}^{l_2} bw_{l_2}^2 p_{l_2 j}^2 - \sum_{i_1=1}^{m_1} bv_{i_1}^1 x_{i_1 j}^1$$
$$- \sum_{i_2=1}^{m_2} bv_{i_2}^2 x_{i_2 j}^2 - \sum_{i_3=1}^{m_3} bv_{i_3}^3 x_{i_3 j}^3 \le 0, \quad j = 1$$

$$\sum_{r_1=1}^{s_1} bu_{r_1}^1 y_{r_1 j}^1 + \sum_{r_2=1}^{s_2} bu_{r_2}^2 y_{r_2 j}^2 + \sum_{r_3=1}^{s_3} bu_{r_3}^3 y_{r_3 j}^3 + \sum_{l_2=1}^{l_2} bw_{l_2}^2 p_{l_2 j}^2 - \sum_{i_1=1}^{m_1} bv_{i_1}^1 x_{i_1 j}^1 - \sum_{i_2=1}^{m_2} bv_{i_2}^2 x_{i_2 j}^2$$
$$- \sum_{i_3=1}^{m_3} bv_{i_3}^3 x_{i_3 j}^3 - \sum_{l_2=1}^{l_2} bw_{l_2}^2 p_{l_2 (j-1)}^2 \le 0, \quad j = 2, \dots, J$$

$$\sum_{r_1=1}^{s_1} bu_{r_1}^1 y_{r_1 j}^1 + \sum_{l_1=1}^{l_1} bw_{l_1}^1 p_{l_1 j}^1 + \sum_{l_3=1}^{l_3} bw_{l_3}^3 p_{l_3 j}^3 - \sum_{i_1=1}^{m_1} bv_{i_1}^1 x_{i_1 j}^1 \le 0, \quad j = 1$$

$$\sum_{r_1=1}^{s_1} bu_{r_1}^1 y_{r_1 j}^1 + \sum_{l_1=1}^{l_1} bw_{l_1}^1 p_{l_1 j}^1 + \sum_{l_3=1}^{l_3} bw_{l_3}^3 p_{l_3 j}^3 - \sum_{i_1=1}^{m_1} bv_{i_1}^1 x_{i_1 j}^1 - \sum_{l_2=1}^{l_2} bw_{l_2}^2 p_{l_2 (j-1)}^2 \le 0, \quad j = 2, \dots, J$$

$$\sum_{r_2=1}^{s_2} bu_{r_2}^2 y_{r_2 j}^2 + \sum_{l_2=1}^{l_2} bw_{l_2}^2 p_{l_2 j}^2 - \sum_{i_2=1}^{m_2} bv_{i_2}^2 x_{i_2 j}^2 - \sum_{l_1=1}^{l_1} bw_{l_1}^1 p_{l_1 j}^1 \le 0, \quad j = 1, 2, \dots, J$$

$$\sum_{r_3=1}^{s_3} bu_{r_3}^3 y_{r_3 j}^3 - \sum_{i_3=1}^{m_3} bv_{i_3}^3 x_{i_3 j}^3 - \sum_{l_3=1}^{l_3} bw_{l_3}^3 p_{l_3 j}^3 \le 0, \quad j = 1, 2, \dots, J$$

$$\sum_{i_1=1}^{m_1} bv_{i_1}^1 x_{i_1 o}^1 + \sum_{i_2=1}^{m_2} bv_{i_2}^2 x_{i_2 o}^2 + \sum_{i_3=1}^{m_3} bv_{i_3}^3 x_{i_3 o}^3 = 1$$

$$v_{i_k}^k \ge \varepsilon, \quad i_k = 1, \dots, m_k, k = 1, 2, 3.$$
$$u_{r_k}^k \ge \varepsilon, \quad r_k = 1, \dots, s_k, k = 1, 2, 3.$$
$$w_{l_k}^k \ge \varepsilon, \quad l_k = 1, \dots, L_k, k = 1, 2, 3.$$
$$b \ge 0 \tag{3}$$

Model (3) is also a non-linear mathematical programming due to product of two decision variables. Variable change (4) is proposed to resolve the non-linearity.

$$v'^k_{i_k} = bv^k_{i_k}, \quad i_k = 1, \dots, m_k, k = 1, 2, 3.$$
$$u'^k_{r_k} = bu^k_{r_k}, \quad r_k = 1, \dots, s_k, k = 1, 2, 3.$$
$$w'^k_{l_k} = bw^k_{l_k}, \quad l_k = 1, \dots, L_k, k = 1, 2, 3. \tag{4}$$

Variable change (4) will result in linear programming model (5).

$$
\begin{aligned}
&Max \quad e_o = \sum_{k=1}^{3}\sum_{r_k=1}^{s_k} u'^{k}_{r_k} y^{k}_{r_k o} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 o} \\
&s.t. \\
&\left(\sum_{k=1}^{3}\sum_{r_k=1}^{s_k} u'^{k}_{r_k} y^{k}_{r_k o} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 j}\right) - \left(\sum_{k=1}^{3}\sum_{i_k=1}^{m_k} v'^{k}_{i_k} x^{k}_{i_k j}\right) \le 0, \quad j=1 \\
&\left(\sum_{k=1}^{3}\sum_{r_k=1}^{s_k} u'^{k}_{r_k} y^{k}_{r_k o} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 j}\right) - \left(\sum_{k=1}^{3}\sum_{i_k=1}^{m_k} v'^{k}_{i_k} x^{k}_{i_k j} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 (j-1)}\right) \le 0, \quad j=2,\dots,J \\
&\sum_{r_1=1}^{s_1} u'^{1}_{r_1} y^{1}_{r_1 j} + \sum_{l_1=1}^{l_1} w'^{1}_{l_1} p^{1}_{l_1 j} + \sum_{l_3=1}^{l_3} w'^{3}_{l_3} p^{3}_{l_3 j} - \sum_{i_1=1}^{m_1} v'^{1}_{i_1} x^{1}_{i_1 j} \le 0, \quad j=1 \\
&\sum_{r_1=1}^{s_1} u'^{1}_{r_1} y^{1}_{r_1 j} + \sum_{l_1=1}^{l_1} w'^{1}_{l_1} p^{1}_{l_1 j} + \sum_{l_3=1}^{l_3} w'^{3}_{l_3} p^{3}_{l_3 j} - \sum_{i_1=1}^{m_1} v'^{1}_{i_1} x^{1}_{i_1 j} - \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 (j-1)} \le 0, \quad j=2,\dots,J \\
&\sum_{r_2=1}^{s_2} u'^{2}_{r_2} y^{2}_{r_2 j} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 j} - \sum_{i_2=1}^{m_2} v'^{2}_{i_2} x^{2}_{i_2 j} - \sum_{l_1=1}^{l_1} w'^{1}_{l_1} p^{1}_{l_1 j} \le 0, \quad j=1,2,\dots,J \\
&\sum_{r_3=1}^{s_3} u'^{3}_{r_3} y^{3}_{r_3 j} - \sum_{i=1}^{m_3} v'^{3}_{i_3} x^{3}_{i_3 j} - \sum_{l_3=1}^{l_3} w'^{3}_{l_3} p^{3}_{l_3 j} \le 0, \quad j=1,2,\dots,J \\
&\sum_{k=1}^{3}\sum_{i_k=1}^{m_k} v'^{k}_{i_k} x^{k}_{i_k o} = 1 \\
&v'^{k}_{i_k} \ge \varepsilon, \quad i_k = 1,\dots,m_k, k=1,2,3. \\
&u'^{k}_{r_k} \ge \varepsilon, \quad r_k = 1,\dots,s_k, k=1,2,3. \\
&w'^{k}_{l_k} \ge \varepsilon, \quad l_k = 1,\dots,L_k, k=1,2,3.
\end{aligned}
\tag{5}
$$

Model (5) is solved in order to achieve the relative efficiency score of period one in figure 5. Using the global optimum value of the decision variables obtained from model (5), the efficiency of each sub-process can be calculated using equations (6).

$$
\begin{aligned}
e^{(1)}_j &= \left(\sum_{r_1=1}^{s_1} u^{1*}_{r_1} y^{1}_{r_1 j} + \sum_{l_1=1}^{l_1} w^{1*}_{l_1} p^{1}_{l_1 j} + \sum_{l_3=1}^{l_3} w^{3*}_{l_3} p^{3}_{l_3 j}\right) \Big/ \left(\sum_{i_1=1}^{m_1} v^{1*}_{i_1} x^{1}_{i_1 j}\right), \quad j=1 \\
e^{(2)}_j &= \left(\sum_{r_2=1}^{s_2} u^{2*}_{r_2} y^{2}_{r_2 j} + \sum_{l_2=1}^{l_2} w^{2*}_{l_2} p^{2}_{l_2 j}\right) \Big/ \left(\sum_{i_2=1}^{m_2} v^{2*}_{i_2} x^{2}_{i_2 j} + \sum_{l_1=1}^{l_1} w^{1*}_{l_1} p^{1}_{l_1 j}\right), \quad j=1 \\
e^{(3)}_j &= \left(\sum_{r_3=1}^{s_3} u^{3*}_{r_3} y^{3}_{r_3 j}\right) \Big/ \left(\sum_{i_3=1}^{m_3} v^{3*}_{i_3} x^{3}_{i_3 j} + \sum_{l_3=1}^{l_3} w^{3*}_{l_3} p^{3}_{l_3 j}\right), \quad j=1
\end{aligned}
\tag{6}
$$

The similar procedure is conducted in order to achieve the relative efficiency of the DMU_j, j=2,…,J. To this aim model (7) is proposed.

$$
\begin{aligned}
&Max \quad e_o = \sum_{k=1}^{3}\sum_{r_k=1}^{s_k} u'^{k}_{r_k} y^{k}_{r_k o} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 o} \\
&s.t. \\
&\left(\sum_{k=1}^{3}\sum_{r_k=1}^{s_k} u'^{k}_{r_k} y^{k}_{r_k o} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 j}\right) - \left(\sum_{k=1}^{3}\sum_{i_k=1}^{m_k} v'^{k}_{i_k} x^{k}_{i_k j}\right) \le 0, \quad j=1 \\
&\left(\sum_{k=1}^{3}\sum_{r_k=1}^{s_k} u'^{k}_{r_k} y^{k}_{r_k o} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 j}\right) - \left(\sum_{k=1}^{3}\sum_{i_k=1}^{m_k} v'^{k}_{i_k} x^{k}_{i_k j} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 (j-1)}\right) \le 0, \quad j=2,\dots,J \\
&\sum_{r_1=1}^{s_1} u'^{1}_{r_1} y^{1}_{r_1 j} + \sum_{l_1=1}^{l_1} w'^{1}_{l_1} p^{1}_{l_1 j} + \sum_{l_3=1}^{l_3} w'^{3}_{l_3} p^{3}_{l_3 j} - \sum_{i_1=1}^{m_1} v'^{1}_{i_1} x^{1}_{i_1 j} \le 0, \quad j=1 \\
&\sum_{r_1=1}^{s_1} u'^{1}_{r_1} y^{1}_{r_1 j} + \sum_{l_1=1}^{l_1} w'^{1}_{l_1} p^{1}_{l_1 j} + \sum_{l_3=1}^{l_3} w'^{3}_{l_3} p^{3}_{l_3 j} - \sum_{i_1=1}^{m_1} v'^{1}_{i_1} x^{1}_{i_1 j} - \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 (j-1)} \le 0, \quad j=2,\dots,J \\
&\sum_{r_2=1}^{s_2} u'^{2}_{r_2} y^{2}_{r_2 j} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 j} - \sum_{i_2=1}^{m_2} v'^{2}_{i_2} x^{2}_{i_2 j} - \sum_{l_1=1}^{l_1} w'^{1}_{l_1} p^{1}_{l_1 j} \le 0, \quad j=1,2,\dots,J \\
&\sum_{r_3=1}^{s_3} u'^{3}_{r_3} y^{3}_{r_3 j} - \sum_{i=1}^{m_3} v'^{3}_{i_3} x^{3}_{i_3 j} - \sum_{l_3=1}^{l_3} w'^{3}_{l_3} p^{3}_{l_3 j} \le 0, \quad j=1,2,\dots,J \\
&\sum_{k=1}^{3}\sum_{i_k=1}^{m_k} v'^{k}_{i_k} x^{k}_{i_k o} + \sum_{l_2=1}^{l_2} w'^{2}_{l_2} p^{2}_{l_2 (o-1)} = 1 \\
&v'^{k}_{i_k} \ge \varepsilon, \quad i_k = 1,\dots,m_k, k=1,2,3. \\
&u'^{k}_{r_k} \ge \varepsilon, \quad r_k = 1,\dots,s_k, k=1,2,3. \\
&w'^{k}_{l_k} \ge \varepsilon, \quad l_k = 1,\dots,L_k, k=1,2,3.
\end{aligned} \tag{7}
$$

The efficiency of each sub-process can also be calculated using equations (8).

$$
\begin{aligned}
e^{(1)}_j &= \left(\sum_{r_1=1}^{s_1} u^{1*}_{r_1} y^{1}_{r_1 j} + \sum_{l_1=1}^{l_1} w^{1*}_{l_1} p^{1}_{l_1 j} + \sum_{l_3=1}^{l_3} w^{3*}_{l_3} p^{3}_{l_3 j}\right) \Big/ \left(\sum_{i_1=1}^{m_1} v^{1*}_{i_1} x^{1}_{i_1 j} + \sum_{l_2=1}^{l_2} w^{2*}_{l_2} p^{2}_{l_2 (j-1)}\right), \quad j=2,\dots,J \\
e^{(2)}_j &= \left(\sum_{r_2=1}^{s_2} u^{2*}_{r_2} y^{2}_{r_2 j} + \sum_{l_2=1}^{l_2} w^{2*}_{l_2} p^{2}_{l_2 j}\right) \Big/ \left(\sum_{i_2=1}^{m_2} v^{2*}_{i_2} x^{2}_{i_2 j} + \sum_{l_1=1}^{l_1} w^{1*}_{l_1} p^{1}_{l_1 j}\right), \quad j=2,\dots,J \\
e^{(3)}_j &= \left(\sum_{r_3=1}^{s_3} u^{3*}_{r_3} y^{3}_{r_3 j}\right) \Big/ \left(\sum_{i_3=1}^{m_3} v^{3*}_{i_3} x^{3}_{i_3 j} + \sum_{l_3=1}^{l_3} w^{3*}_{l_3} p^{3}_{l_3 j}\right), \quad j=2,\dots,J
\end{aligned} \tag{8}
$$

4-1-1-The theoretical characteristics of network model for the cause process

The theoretical properties of the proposed models are discussed in this sub-section.

Theorem 1: Model (5) is always feasible and the value of its objective function is bounded.

Proof 1: Let's consider $\lambda^{(1)}_j, \lambda^{(2)}_j, \lambda^{(3)}_j$ and θ the associated dual variables of linear model (5). Therefore, the dual form of model (5) can be written as model (9).

$$
\begin{aligned}
& Min \quad \theta \\
& s.t. \\
& \sum_{j=1}^{J} \lambda_j^{(1)} x_{i_1 j}^1 \le \theta x_{i_1 o}^1, \quad i_1 = 1,\dots,m_1 \\
& \sum_{j=1}^{J} \lambda_j^{(1)} p_{l_1 j}^1 - \sum_{j=1}^{J} \lambda_j^{(2)} p_{l_1 j}^1 \ge 0, \quad l_1 = 1,\dots,L_1 \\
& \sum_{j=1}^{J} \lambda_j^{(1)} p_{l_2 j}^2 - \sum_{j=1}^{J} \lambda_j^{(2)} p_{l_2 j}^2 \ge 0, \quad l_2 = 1,\dots,L_2 \\
& \sum_{j=1}^{J} \lambda_j^{(1)} y_{r_1 j}^1 \ge y_{r_1 o}^1, \quad r_1 = 1,\dots,s_1 \\
& \sum_{j=1}^{J} \lambda_j^{(2)} x_{i_2 j}^2 \le \theta x_{i_2 o}^2, \quad i_2 = 1,\dots,m_2 \\
& \sum_{j=1}^{J} \lambda_j^{(2)} w_{l_3 j}^3 - \sum_{j=1}^{J} \lambda_j^{(1)} w_{l_3 j}^3 \ge 0, \quad l_3 = 1,\dots,L_3 \\
& \sum_{j=1}^{J} \lambda_j^{(2)} y_{r_2 j}^2 \ge y_{r_2 o}^2, \quad r_2 = 1,\dots,s_2 \\
& \sum_{j=1}^{J} \lambda_j^{(3)} x_{i_3 j}^3 \le \theta x_{i_3 o}^3, \quad i_3 = 1,\dots,m_3 \\
& \sum_{j=1}^{J} \lambda_j^{(3)} y_{r_3 j}^3 \ge y_{r_3 o}^3, \quad r_3 = 1,\dots,s_3 \\
& \theta = free \\
& \lambda_j^{(1)}, \lambda_j^{(2)}, \lambda_j^{(3)} \ge 0, \quad j = 1,\dots,J
\end{aligned} \tag{9}
$$

Consider arbitrary solution (10) for model (9).

$$
\begin{cases}
\lambda_j^{(1)} = \lambda_j^{(2)} = \lambda_j^{(3)} = 0, \quad j = 1,\dots,J, \quad j \ne o \\
\theta = 1 \\
\lambda_o^{(1)} = \lambda_o^{(2)} = \lambda_o^{(3)} = 1
\end{cases} \tag{10}
$$

It can be concluded that independent of the inputs, intermediate and outputs, solution (10) is always a feasible solution for model (9). Hence, model (5) is also feasible. Since the objective function of model (9) is minimizing, it can be concluded that the optimum value of the objective function of model (9) is always less than or equal to the objective function of any feasible solution, so $\theta^* \le \theta \le 1$. Moreover, on the basis of the dual theorem in linear programming the optimal objective value of primal and dual models are equal (i.e., $e^* = \theta^*$). This completes the proofs.

Theorem 2: Model (7) is always feasible and the value of the objective function is bounded.

Proof 2: According to theorem 1, the proof is straightforward.

5- Case study and results

Industrial Management Institute (IMI), founded in 1962, has been worked on publishing, consulting, and education areas. The mission of the IMI is to leaders, operational managers, and specialists. The IMI's mission in the education area is to create professional leaders, operational managers, and specialists in Iran's industrial and service sectors. IMI's purpose is to develop management capacity and facilitate change and improvement in public and private sectors.

Comprehensiveness, integration, and customization are the factors that, to this day, differentiate IMI's services. IMI offers its integrated services in the management consulting, education, research and publication areas. Since its inception, IMI has taken on a variety of challenges in an ever-widening range of fields and contexts. Armed with a world-class professional cadre of over 60 resident consultants and a vast network of management professionals, IMI's experience spans over 3,000 consulting & research projects and the training of over 200,000 Iranian top executives, general managers, and functional experts during the past five decades. The education, consulting and publication processes are of great importance in IMI and measuring the efficiency and service quality provided by these processes are so vital. Therefore, the assessment of the performance and offering systematic and scientific solutions to analyze the processes of IMI is in a high priority.

On the other hand, it is also very important and necessary for the managers to measure the efficiency of IMI in order to identify and remove the causes of inefficiency. The measurement of the efficiency results in competition among different units and this competition results in the attempts to improve the efficiency. In this section, using the proposed model, the total network efficiency of the cause process of IMI is measured during a five-year period using the monthly data beginning from 2011 to 2015.

5-1- Data gathering

We focus on 60 homogeneous DMUs (i.e., the monthly data for a five-year planning period). The values of input/output variables and the intermediate products have been collected from the monthly reports made by IMI from 2011 to 2015. For sake of brevity, the details of the information are not provided, although a descriptive statistics of the information along with a list of the variables identified in cause has been presented in Appendices A, B and C.

5-2- The results of the cause process

In the first structure a network model of the cause process including three sub-processes as education, publishing, and consulting as depicted in figures 2-3 are considered. Model (5) and Model (7) which have been proposed for cause process is coded and run using LINGO software. The efficiency scores for the main DMUs and 3 sub-processes were obtained. The results have been presented in the in figure 6.

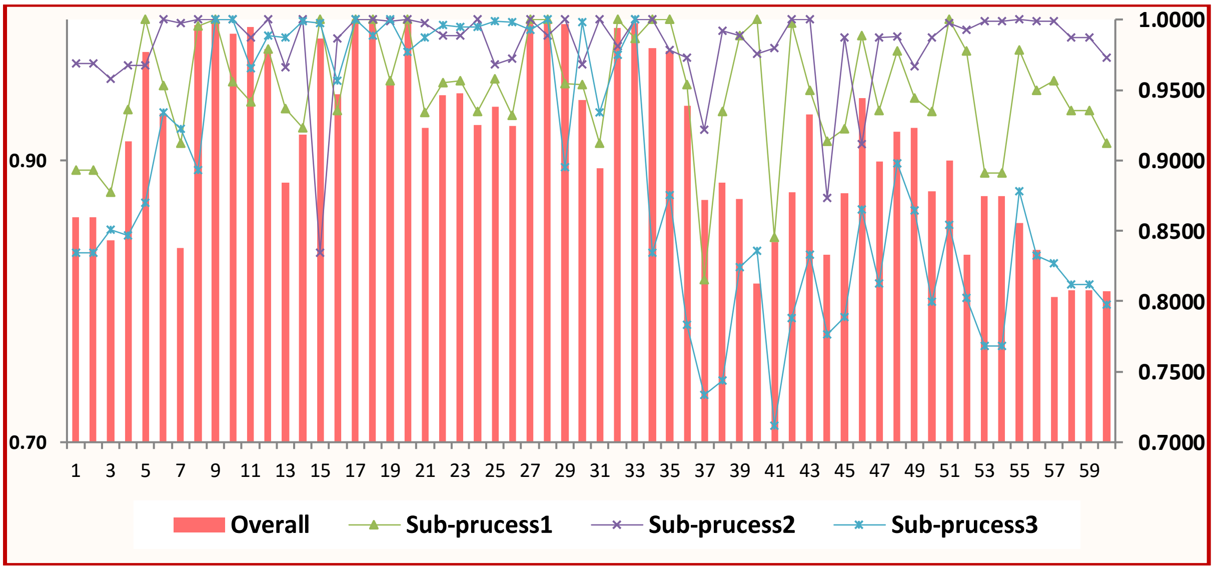

Fig 6. Efficiency scores of the cause processes in 60 periods

According to the obtained results, the findings have been stated as follows:

- DMU_9 and DMU_{17} are efficient in all stages. On the other hand, the educational, consulting and publication sub-processes are also efficient in these DMUs.

- DMU_{20}, DMU_{27} and DMU_{34} are efficient in the stages 1 and 2. However, it was because of their performance in the third stage (97.6%, 99.23%, and 83.45%) that they failed to achieve complete efficiency.
- Similarly, DMU_{10}, and DMU_{33} are efficient in the second and third stages, and DMU_{28} is also efficient in the first and third stages.
- Average scores of efficiency of the sub-processes of education, consulting and publication in the cause processes of IMI are (95.15%, 98.13%, and 89.22%), where the third stage (consulting sub-process) has the least efficiency score in comparison with the efficiency scores of the first and second stages.
- As it can be seen in the figure 6, there is a strong relationship between the total efficiency scores and the efficiency of the first sub-process (publication). Figure 7 shows the average efficiency scores of the three sub-processes in 60 periods.

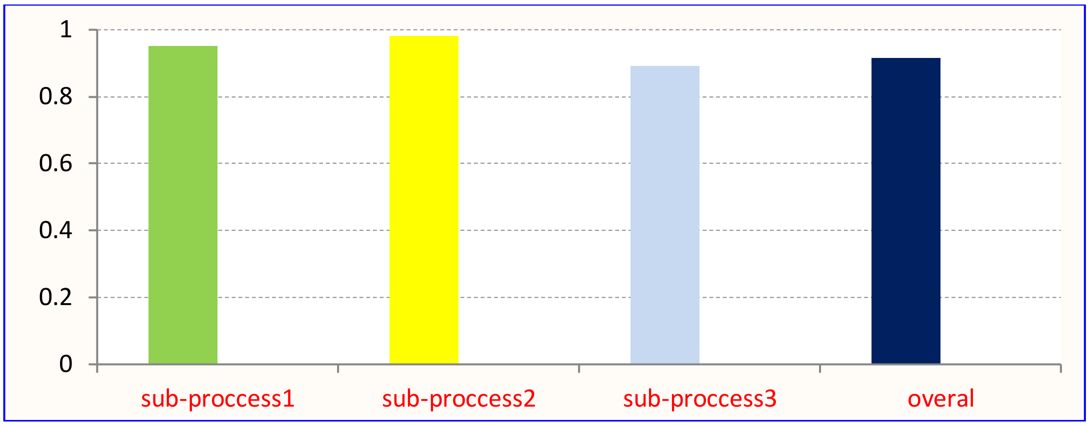

Fig 7. the average efficiency scores of the cause process in 60 period

5-3- Decomposition of Total Efficiency into sub-Efficiencies

One of the main issues in network DEA models is a method where the total efficiency score of main DMU is decomposed into the efficiency score of the sub-processes. Some classical analysis approaches have been proposed when there are no intermediate measures. These are weighted average of the efficiency of the stages, the geometrical average of the efficiency of the stages, harmonic average of the efficiency of the stages. These hypotheses are noteworthy when there are no intermediate products in the structure, but it is possible that we face a complicated network structure with undesirable outputs, intermediate products, and extra inputs. According to DMUs (i.e., cause process) which has included several intermediate products, the decomposition process should be accomplished using a customized approach. In this research statistical analysis of the efficiency scores, has been provided to decompose the efficiency scores. On the other hand, a curve fitting method incorporating a regression analysis in which the efficiency score of the sub-processes is assumed as an independent variable, while the efficiency score of the whole network of the cause process has been considered as an independent variable is used to decompose the efficiency scores.

On the other hand, a curve fitting method incorporating a regression analysis in which the efficiency scores of the sub-processes are assumed as independent variables, while the efficiency score of the whole network has been considered as dependent variable is used to decompose the efficiency scores. Several curves have been tested in order to determine the best fit. The results of the five regression models including linear regression, logarithmic regression, quadratic regression, cubic regression and growth regression are presented in table 2.

Table2. Model summery and parameter estimated

Equation	R square	F	sig
Linear	0.309	25.920	0.000
Logarithmic	0.299	24.720	0.000
Quadratic	0.388	18.041	0.000
Cubic	0.386	17.917	0.000
Growth	0.303	25.216	0.000

It can be concluded form table 2 that the significance level of the F test all of the regression models is smaller than 0.05, so all models are meaningful. The fittest model has the highest F value. As shown in table 2, linear regression with value F=25.92 is the best regression model. Table 3 presents the estimation linear regression function.

Table3. The results of regression analysis in cause process

Regression mode	Beta	R2	Sig
Total Efficiency=0.316 E + 0.251 C + 0.541 P + 0.183	0.736	0.541	0.000

The notations E, C and P have been selected for the efficiency of education, consulting and publishing, respectively. Coefficient of determination (R^2) indicates the extent to which the fitted function corresponds to the real observations. Also, this coefficient shows the extent, to which the changes in the dependent variable, i.e., total efficiency, is affected by the relevant independent variables, i.e. E, C, and P. R^2 value is acceptable. The significance level 0.05 (P-value) confirms the significance of the regression analysis and the estimated parameters. As it can be observed in table 3, Beta is standardized coefficients that make the scale of the variables and also makes it possible to compare the variables. Therefore, the standardized coefficients are used to compare the effects of several independent variables on the dependent variable.

6- Conclusions and future researches

The conceptual model including the main cause process of Industrial Management institute was analyzed using network DEA. The cause process was selected in the basis of the cost analysis. The classic DEA models cannot consider the internal processes of the real systems with complicated structures. So, the total efficiency score of a DMU cannot be decomposed into the efficiency of sub-processes. The efficiency of cause process and the associated sub-processes were calculated using the proposed network DEA model in the five-year planning period from 2011 to 2015 based on monthly data. The network of the cause process includes three sub-processes as publication, education and consulting.

The mathematical model was developed for these a network. The proposed model was linear programming easily was solved by operations research software. The main properties of the proposed model including feasibility and the limitation of the objective function were discussed through several theorems. A statistical analysis was also carried out for the estimated regression function where the efficiency score of the sub-processes were considered as independent variable and the total efficiency score of the cause process network was considered the dependent variable.

The proposed model was applied on a case study in the Industrial Management institute business processes over a sixty-month planning period. The results obtained from the case study showed the efficiency and applicability of the proposed model. The results obtained from the proposed model showed that the publication sub-process had the greatest effect on the efficiency of the whole cause process network in a five-year period of this study.

The total efficiency score of DMU and total efficiency score of sub-DMUs were used to recognize the significant relationship with the help of statistical analysis and based on regression method. Fitting method was used to discover a function of the efficiency scores of each sub-process and also as the main process in a five-year period. The efficiency scores of the stages (peripheral processes) were considered as independent variable while the total efficiency score of the cause process was considered dependent variables. The results of regression analysis showed the extent to which the changes in the dependent variable were affected by the relevant independent variable.

A developed version of proposed model of this study on the basis of variable return to scale assumptions can be assumed in future research. The inputs and outputs of this study can be changed and measured using the fuzzy sets in future research. Developing a method to identify the inefficient DMUs and sub-DMUs and projecting them toward efficient frontier can be considered in future researches. The proposed model of this study can be customized and used in other applications such as universities, teaching centers and service organizations.

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Appendices

Appendix A. Input and outputs of publication sub process

Sub-Process Name	Indices	Input	Output	Intermediate Product	Measurement Unit	Average	Standard Deviation	Max	Min	parameters
Publication	Goods sent from the warehouse and procurement section	*			number	42	35.11	212	11	x_{11j}^{1}
	Ability and potential of the institute to meet the clients' needs	*			amount	3.60	0.49	4	3	x_{21j}^{1}
	The expectations and requests of the clients	*			amount	4.60	0.49	5	4	x_{31j}^{1}
	The provided legal services	*			number	2	1.86	7	0	x_{41j}^{1}
	Performance of annual plans	*			amount	3.80	0.99	5	3	x_{51j}^{1}
	The requests sent to receive financial resources		*		number	3	2.29	11	0	y_{11j}^{1}
	The requests sent to formulate the instructions and processes		*		number	2	1.82	7	0	y_{21j}^{1}
	The sent requests to formulate instructions and processes		*		number	1	0.80	3	0	y_{31j}^{1}
	Conveying knowledge and information		*		amount	3.40	0.49	4	3	y_{41j}^{1}
	The requests sent to attract man force, change the personnel status and providing training for the publication unit		*		number	2	1.42	5	0	y_{51j}^{1}
	Books and journals sent to education department			*	number	280	303.58	925	30	p_{11j}^{1}
	New edition announced to the education unit			*	number	3	1.19	4	0	p_{21j}^{1}
	New edition announced to the consultant			*	number	3	1.22	4	0	p_{12j}^{2}
	Requests for books and journals from education to publication			*	number	275	304.48	924	25	p_{12j}^{3}

Appendix B. Input and outputs of education sub process

Sub-Process Name	Indices	Input	Output	Intermediate Product	Measurement Unit	Average	Standard Deviation	Max	Min	parameters
education	Ability and potential of the institute to meet the needs of the clients	*			amount	3.20	1.18	5	2	x^2_{12j}
	The expectations and requests of the clients	*			amount	4.80	0.40	5	4	x^2_{22j}
	The legal services provided	*			number	2	1.86	7	0	x^2_{32j}
	Performance of annual plans	*			amount	3.20	0.85	4	2	x^2_{42j}
	The goods sent from the warehouse and procurement section	*			number	206	251.46	850	18	x^2_{52j}
	The trained learners	*			number	296	300.12	910	42	x^2_{62j}
	Visiting professors	*			number	234	14.27	264	218	x^2_{72j}
	Educational experts	*			number	38	9.36	49	25	x^2_{82j}
	Specific educational contracts	*			number	3	1.77	8	0	x^2_{92j}
	New courses	*			number	2	2.04	7	0	x^2_{102j}
	Abolished courses	*			number	1	0.74	3	0	x^2_{112j}
	The educational level of current students	*			amount	4.60	0.49	5	4	x^2_{122j}
	Faculty members	*			number	29	1.04	31	28	x^2_{132j}
	New sub-disciplines	*			number	1	0.72	4	0	x^2_{142j}
	The attracted man force that have been trained and developed for IT section	*			number	5	3.27	11	0	x^2_{152j}
	The analyzed opinions and requests of the education clients	*			number	10	6.21	22	1	x^2_{162j}
	The advertisements	*			number	5	3.09	10	0	x^2_{172j}
	The sent ideas and suggestions	*			number	3	1.93	6	0	x^2_{182j}
	The formulated processes and instructions	*			number	4	2.21	7	0	x^2_{192j}
	The requested books and journals	*			number	198	132.57	412	20	x^2_{202j}
	Receiving the required financial resources	*			number	10	3.19	15	5	x^2_{212j}
	The formulated processes and instructions	*			number	4	2.21	7	0	x^2_{222j}
	The announced new edition	*			number	5	3.50	11	0	x^2_{232j}
	The request sent to provide legal services		*		number	3	1.83	7	0	y^2_{12j}
	Conveying knowledge and information		*		amount	3.40	0.49	4	3	y^2_{22j}
	The M.A. theses sent to the library		*		number	6	3.68	16	0	y^2_{32j}
	The sent requests to attract man force, change the personnel status and training the publication unit		*		number	4	2.46	9	0	y^2_{42j}
	The sent requests to receive financial resources		*		number	9	5.17	29	2	y^2_{52j}
	The sent requests to formulate the instructions and processes		*		number	2	1.47	6	0	y^2_{62j}
	The graduates of the courses		*		number	236	235.58	917	13	y^2_{72j}
	Professors' assessment		*		score	73	4.48	82	65	y^2_{82j}
	Educational certificates		*		number	210	159.13	823	33	y^2_{92j}
	Assessment of the course organizers		*		score	70	8.70	79	56	y^2_{102j}
	Assessment of courses		*		score	65	10.81	79	48	y^2_{112j}
	International certificates		*		number	5	6.26	22	0	y^2_{122j}
	Research plans		*		number	1	0.85	3	0	y^2_{132j}
	Published books written by M.A. students		*		number	1	0.49	2	0	y^2_{142j}
	Students' theses manuscript in preparation		*		number	7	3.69	18	1	y^2_{152j}
	Books and journals sent from publication to education department	*			number	3	3.28	15	0	y^2_{162j}
	Request for books and journals for the publication unit	*	*		number	283	318.22	924	2	y^2_{172j}
	Clients' opinions sent from the education department			*	number	26	14.30	56	1	p^3_{12j}
	Requesting the information systems and hardware and software services			*	number	4	3.48	10	0	p^4_{12j}

Appendix C. Input and outputs of consulting sub process

Sub-Process Name	Indices	Input	Output	Intermediate Product	Measurement Unit	Average	Standard Deviation	Max	Min	parameters
consulting	Ability and potential of the institute to meet the clients' needs	*			amount	3.40	1.37	5	2	x_{13j}^{3}
	The expectations and requests of the clients	*			amount	4.60	0.49	5	4	x_{23j}^{3}
	The provided legal services	*			number	2	1.76	7	0	x_{33j}^{3}
	The announced new edition			*	number	5	3.5	11	0	p_{12j}^{2}
	Performance of annual plans	*			amount	3	1.28	5	2	x_{43j}^{3}
	The goods sent from the warehouse and procurement section	*			number	31	47.43	312	5	x_{53j}^{3}
	Consulting unit experts	*			number	24	2.17	26	17	x_{63j}^{3}
	Consulting unit consultants	*			number	37	1.75	39	32	x_{73j}^{3}
	Employers	*			number	3	1.64	7	0	x_{83j}^{3}
	Consulting agreements	*			number	2	1.70	7	0	x_{93j}^{3}
	Affiliated experts	*			number	18	2.77	22	13	x_{103j}^{3}
	Internal colleagues	*			number	11	1.51	13	8	x_{113j}^{3}
	External colleagues	*			number	5	0.67	6	4	x_{123j}^{3}
	The provided services	*			number	5	2.66	12	1	x_{133j}^{3}
	The sent consulting requests	*			number	4	1.79	8	0	x_{143j}^{3}
	Conveying knowledge and information		*		amount	3.40	0.49	4	3	y_{13j}^{3}
	The requests sent to provide legal services		*		number	2	1.72	7	0	y_{23j}^{3}
	The requests sent to formulate instructions and processes		*		number	1	1.29	5	0	y_{33j}^{3}
	The sent requests to receive financial resources		*		number	5	2.55	13	1	y_{43j}^{3}
	The files of the completed project that were sent to the library of the institute		*		number	2	1.32	5	0	y_{53j}^{3}
	The requests sent to attract man force, change the personnel status and training		*		number	2	1.78	6	0	y_{63j}^{3}