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An approach for solving two- person zero- sum matrix games in neutrosophic environment

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Abstract

Neutrosophic set is considered as a generalized of crisp set, fuzzy set, and intuitionistic fuzzy set for representing the uncertainty, inconsistency, and incomplete knowledge about a real world problem. This paper aims to develop two-person zero- sum matrix games in a single valued neutrosophic environment. A method for solving the game problem with indeterminate and inconsistent information is proposed. Finally, two examples are given to illustrate the practically and the efficiency of the method.

Keywords: Matrix games, payoff matrix, neutrosophic set, linear programming, neutrosophic optimal strategy

1- Introduction

Game theory is a mathematical modeling technique used for decision problems when there are two or more decision makers in conflict or cooperation with each other. Each decision maker plays the game to outsmart the others. Game theory provides many effective and efficient tools and techniques to mathematically formulate and solve many multi person *i*th strategies in tractions among multiple rational DMs (Krishnaveni and Ganesan, 2018). Game theory is widely applied in many fields, such as economic and management, social policy and international and national policies (Von Neumann and Morgenstern, 1944). Simple necessary and sufficient conditions for the comparison of information structures in zero- sum games have been introduced by Peski (2008). The traditional game theory assumes that all data of game are known exactly by players. However, there are some games in which players are not able to evaluate exactly some data in our realistic situations. In these, the imprecision is due to inaccuracy of information and vague comprehension of situations by players. For these, many researchers have made contribution and introduced some techniques for finding the equilibrium strategies of these games (Berg and Engel, 1998), and Takahashi, 2008).

In many scientific areas, such as system analysis and operations research, a model has to be set up using data which is only approximately known. Fuzzy sets theory, introduced by Zadeh (1965), makes this possible. Dubois and Prade (1980) extended the use of algebraic operations on real numbers to fuzzy numbers by the use a fuzzification principle. Bellman and Zadeh (1970) introduced the concept f a maximizing decision making problem. Selvakumari and Lavanya (2015) and Thirucheran et al. (2017) accelerated fuzzy game.

Based on the expected value operator and the trust measure of variables under roughness, Xu and Yao (2010) discussed a class of two- person zero- sum games payoffs represented as rough.

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Campos (1989) studied game problem under fuzziness in the goal and payoffs. Sakawa and Nishizaki (1994) used max- min principle of game theory to study single and multiobjective matrix games with fuzzy goals and payoffs. Kumar (1988) investigated the max- min solution for the multiobjective game problem. Bector et al. (2004a, 2004b) showed that a two- person zero- sum matrix game with having fuzzy goals and fuzzy payoffs is equivalent to a pair of LPPs, each of them are the dual to the other in fuzzy sense. Vijay et al. (2005, 2007) based on the fuzzy duality; fuzzy relation approach and ranking function for solving fuzzy matrix game. Pandey and Kumar (2010) proposed a modified approach based on new order function for solving multiobjective matrix game with vague payoffs. Nan et al. (2010) studied fuzzy matrix game and a Lexicographic methodology for finding the solution for it. Sahoo (2017) proposed a solution methodology for solving fuzzy matrix game based on signed distance method. Li and Hong (2012) proposed an approach for solving constrained matrix games with triangular fuzzy numbers payoffs. Sahoo (2015) introduced a new technique based on parametric representation of interval number to solve game problem. Bandyopadhyay et al. (2013a) studied matrix game with triangular intuitionistic fuzzy number payoff. Bandyopadhyay and Nayak (2013b) studied symmetric trapezoidal fuzzy number matrix game payoffs, where they transformed it into different lengths interval fuzzy numbers. Chen and Larboni (2006) defined matrix game with triangular membership function and proved that two person zero-sum game with fuzzy payoff matrices is equivalent to two linear programming problems. Seikh et al. (2013) proposed an alternative approach for solving matrix game.

In this paper, based on the concept of single valued trapezoidal neutrosophic numbers introduced by Thamaraiselvi and Santhi (2016), we discuss a simple game, namely, the game in which the number of players is two and single valued trapezoidal neutrosophic payoffs which one player receives are equal to single valued trapezoidal neutrosophic payoffs which the other player loses. Here, linear programming problems and their implementation is being used in two- person zero- sum matrix game theory.

The outlay of the paper is organized as follows: In section 2; basic concepts and results related to fuzzy numbers, trapezoidal fuzzy numbers, intuitionistic trapezoidal fuzzy numbers, and neutrosophic set are recalled. In section 3, two- person zero sum game in neutrosophic environment is formulated. In section 4, an approach for solving matrix game is proposed. In section 5, two numerical examples are given to illustrate the efficiency of the solution approach. Finally, some concluding remarks are reported in section 6.

2- Preliminaries

In order to discuss our problem conveniently, basic concepts and results related to fuzzy numbers, trapezoidal fuzzy numbers, intuitionistic trapezoidal fuzzy numbers, and neutrosophic set are recalled.

Definition1. (Zadeh , 1965). A fuzzy set $\tilde{A}$ defined on the set of real numbers R is said to be fuzzy number, if its membership function $\mu_{\tilde{A}} : R \to [0,1]$ has the following properties

(i) $\tilde{A}$ is an upper semi- continuous membership function;

(ii) $\tilde{A}$ is convex , i. e., $\mu_{\tilde{A}}(\lambda x + (1-\lambda)) \geq \min \{\mu_{\tilde{A}}(x), \mu_{\tilde{A}}(y)\}$, $\lambda \in [0,1]$, for all $x, y \in R$.

(iii) $\tilde{A}$ is normal, i. e., $\exists\, x_0 \in R$ for which $\mu_{\tilde{A}}(x_0) = 1$;

(iv) Supp$(\tilde{A}) = \{x \in R : \mu_{\tilde{A}}(x) > 0\}$ is the support of the $\tilde{A}$, and its closure cl (supp$(\tilde{A})$) is compact set.

Definition 2. (Kaufmann and Gupta, 1988). A fuzzy number $\tilde{A}(a_1, a_2, a_3, a_4)$ on R is a trapezoidal fuzzy number if its membership function $\mu_{\tilde{A}}(x) : R \to [0,1]$ has the following characteristics

$$\mu_{\tilde{A}}(x) = \begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_4-x}{a_4-a_3}, & a_3 \le x \le a_4 \\ 0, & Otherwise \end{cases}$$

Definition 3. (Atanassov , 1986). Let X be a nonempty set. An intuitionistic fuzzy set $\tilde{A}_I$ of X is defined as $\tilde{A}_I = \{\langle x, \mu_{\tilde{A}_I}(x), \eta_{\tilde{A}_I}(x) : x \in X \rangle\}$, where $\mu_{\tilde{A}_I}(x)$ and $\eta_{\tilde{A}_I}(x)$ are membership and non-membership function, respectively such that $\mu_{\tilde{A}_I}(x)$, $\eta_{\tilde{A}_I}(x) : X \to [0,1]$ and $0 \le \mu_{\tilde{A}_I}(x) + \eta_{\tilde{A}_I}(x) \le 1$, for all $x \in X$.

Definition 4. (Atanassov , 1986). An intuitionistic fuzzy subset $\tilde{A}_I = \{\langle x, \mu_{\tilde{A}_I}(x), \eta_{\tilde{A}_I}(x) : x \in X \rangle\}$ of R is called an intuitionistic fuzzy number if the following conditions hold:

(i) There exists $m \in R$ such that $\mu_{\tilde{A}_I}(m) = 1$, and $\eta_{\tilde{A}_I}(m) = 0$,

(ii) $\mu_{\tilde{A}_I}$ is continuous function from $R \to [0,1]$ such that $0 \le \mu_{\tilde{A}_I}(x) + \eta_{\tilde{A}_I}(x) \le 1$, for all $x \in X$, and

(iii) The membership and non-membership functions of $\tilde{A}_I$ are

$$\mu_{\tilde{A}_I}(x) = \begin{cases} 0, & -\infty < x \le a_1 \\ f(x), & a_1 \le x \le a_2 \\ g(x), & a_2 \le x \le a_3 \\ 0, & a_3 \le x < \infty; \end{cases}$$

$$\eta_{\tilde{A}_I}(x) = \begin{cases} 0, & -\infty < x \le a_1^{\circ} \\ f^{\circ}(x), & a_1^{\circ} \le x \le a_2 \\ g^{\circ}(x), & a_2 \le x \le a_3^{\circ} \\ 0, & a_3^{\circ} \le x < \infty, \end{cases}$$

Where, $f, f^{\circ}, g, g^{\circ}$ are functions from $R \to [0,1]$, f, and g° are strictly increasing functions, and g, and f° are strictly decreasing functions with $0 \le f(x) + f^{\circ}(x) \le 1$, and $0 \le g(x) + g^{\circ}(x) \le 1$.

Definition5. (Jianqiang and Zhong, 2009) A trapezoidal intuitionistic fuzzy number is denoted by $\tilde{A}_{IT} = (a_1, a_2, a_3, a_4), (a_1^{\circ}, a_2, a_3, a_4^{\circ})$, where $a_1^{\circ} \le a_1 \le a_2 \le a_3 \le a_4 \le a_4^{\circ}$, with membership and non-membership functions are

$$\mu_{\tilde{A}_{IT}}(x)=\begin{cases}\dfrac{x-a_1}{a_2-a_1}, & a_1\le x\le a_2\\ \dfrac{a_4-x}{a_4-a_3}, & a_3\le x\le a_4\\ 0, & Otherwise,\end{cases}$$

$$\eta_{\tilde{A}_{IT}}(x)=\begin{cases}\dfrac{a_2-x}{a_2-a_1^{\circ}}, & a_1^{\circ}\le x\le a_2\\ \dfrac{x-a_3}{a_4^{\circ}-a_3}, & a_3\le x\le a_4^{\circ}\\ 1, & Otherwise.\end{cases}$$

Definition6. (Smarandache, 1998). Let X be a nonempty set. A neutrosophic set is defined as $\overline{A}^N=\left\{\left\langle x,U_{\overline{A}^N}(x),V_{\overline{A}^N}(x),W_{\overline{A}^N}(x)\right\rangle : x\in X, U_{\overline{A}^N}(x),V_{\overline{A}^N}(x),W_{\overline{A}^N}(x)\in\left]^-0,1^+\right[\right\}$, where $U_{\overline{A}^N}(x)$, $V_{\overline{A}^N}(x)$, and $W_{\overline{A}^N}(x)$ are truth, indeterminacy, and falsity membership functions, respectively, and there is no restriction on the summation of them, so $^-0\le U_{\overline{A}^N}(x)+V_{\overline{A}^N}(x)+ W_{\overline{A}^N}(x)\le 3^+$ and $\left]^-0,1^+\right[$ is nonstandard unit interval.

Definition7. (Wang et al. 2010). Let X be a nonempty set. The single valued neutrosophic set $\overline{A}^{N_{SV}}$ of X is defined as

$\overline{A}^{N_{SV}}=\left\{\left\langle x,U_{\overline{A}^N}(x),V_{\overline{A}^N}(x),W_{\overline{A}^N}(x)\right\rangle : x\in X\right\}$, where $U_{\overline{A}^N}(x)$, $V_{\overline{A}^N}(x)$, and $U_{\overline{A}^N}(x)\in[0,1]$ for each $x\in X$ and $0\le U_{\overline{A}^N}(x)+V_{\overline{A}^N}(x)+W_{\overline{A}^N}(x)\le 3$.

Definition8. (Thamaraiselvi and Santhi , 2016). Let $u_{\tilde{a}},\zeta_{\tilde{a}},\xi_{\tilde{a}}\in[0,1]$, and $a_1,a_2,a_3,a_4\in R$ such that $a_1\le a_2\le a_3\le a_4$. The single valued trapezoidal neutrosophic number $\overline{a}=\left\langle (a_1,a_2,a_3,a_4); u_{\overline{a}},\zeta_{\overline{a}},\xi_{\overline{a}}\right\rangle$ is a special neutrosophic set on the real numbers R, whose truth, indeterminacy, and falsity membership functions are

$$\mu_{\overline{a}}(x)=\begin{cases}u_{\overline{a}}\left(\dfrac{x-a_1}{a_2-a_1}\right), & a_1\le x\le a_2\\ u_{\overline{a}}, & a_2\le x\le a_3\\ u_{\overline{a}}\left(\dfrac{a_4-x}{a_4-a_3}\right), & a_3\le x\le a_4\\ 0, & otherwise,\end{cases}$$

$$
v_{\bar{a}}(x)=\begin{cases}\dfrac{a_2-x+\zeta_{\bar{a}}(x-a_1)}{a_2-a_1}, & a_1\le x\le a_2\\ \zeta_{\bar{a}}, & a_2\le x\le a_3\\ \dfrac{x-a_3+\zeta_{\bar{a}}(a_4-x)}{a_4-a_3}, & a_3\le x\le a_4\\ 1, & otherwise,\end{cases}
$$

$$
\pi_{\bar{a}}(x)=\begin{cases}\dfrac{a_2-x+\xi_{\bar{a}}(x-a_1)}{a_2-a_1}, & a_1\le x\le a_2\\ \xi_{\bar{a}}, & a_2\le x\le a_3\\ \dfrac{x-a_3+\xi_{\bar{a}}(a_4-x)}{a_4-a_3}, & a_3\le x\le a_4\\ 1, & otherwise,\end{cases}
$$

Where, $u_{\bar{a}},\zeta_{\bar{a}}$, and $\xi_{\ddot{a}}$ are the maximum truth, minimum indeterminacy, and minimum falsity membership degrees, respectively. A single valued trapezoidal neutrosophic number $\bar{a}=\langle(a_1,a_2,a_3,a_4);u_{\bar{a}},\zeta_{\bar{a}},\xi_{\bar{a}}\rangle$ may be expressed an ill- defined quantity about a, which is approximately equal to $[a_2,a_3]$.

Definition 9. (Thamaraiselvi and Santhi, 2016) Let $\bar{a}=\langle(a_1,a_2,a_3,a_4);u_{\bar{a}},\zeta_{\bar{a}},\xi_{\bar{a}}\rangle$, and $\bar{b}=\langle(b_1,b_2,b_3,ba_4);u_{\bar{b}},\zeta_{\bar{b}},\xi_{\bar{b}}\rangle$ be two single valued trapezoidal neutrosophic (SVTRN) numbers and $c\neq 0$, then

(1) $\bar{a}(+)\bar{b}=\langle(a_1+b_1,a_2+b_2,a_3+b_3,a_4+b_4);u_{\bar{a}}\wedge u_{\bar{b}},\zeta_{\bar{a}}\vee\zeta_{\bar{b}},\xi_{\bar{a}}\vee\xi_{\bar{b}}\rangle$,

(2) $\bar{a}(-)\bar{b}=\langle(a_1-b_4,a_2-b_2,a_3-b_3,a_4-b_1);u_{\bar{a}}\wedge u_{\bar{b}},\zeta_{\bar{a}}\vee\zeta_{\bar{b}},\xi_{\bar{a}}\vee\xi_{\bar{b}}\rangle$,

(3) $\bar{a}\otimes\bar{b}=\begin{cases}\langle(a_1b_1,a_2b_2,a_3b_3,a_4b_4);u_{\bar{a}}\wedge u_{\bar{b}},\zeta_{\bar{a}}\vee\zeta_{\bar{b}},\xi_{\bar{a}}\vee\xi_{\bar{b}}\rangle, & a_4>0,b_4>0\\ \langle(a_1b_4,a_2b_3,a_3b_2,a_4b_1);u_{\bar{a}}\wedge u_{\bar{b}},\zeta_{\bar{a}}\vee\zeta_{\bar{b}},\xi_{\bar{a}}\vee\xi_{\bar{b}}\rangle, & a_4<0,b_4>0\\ \langle(a_4b_4,a_3b_3,a_2b_2,a_1b_1);u_{\bar{a}}\wedge u_{\bar{b}},\zeta_{\bar{a}}\vee\zeta_{\bar{b}},\xi_{\bar{a}}\vee\xi_{\bar{b}}\rangle, & a_4<0,b_4<0,\end{cases}$

(4) $\dfrac{\bar{a}}{\bar{b}}=\begin{cases}\langle(a_1/b_4,a_2/b_3,a_3/b_2,a_4/b_1);u_{\bar{a}}\wedge u_{\bar{b}},\zeta_{\bar{a}}\vee\zeta_{\bar{b}},\xi_{\bar{a}}\vee\xi_{\bar{b}}\rangle, & a_4>0,b_4>0\\ \langle(a_4/b_4,a_3/b_3,a_2/b_2,a_1/b_1);u_{\bar{a}}\wedge u_{\bar{b}},\zeta_{\bar{a}}\vee\zeta_{\bar{b}},\xi_{\bar{a}}\vee\xi_{\bar{b}}\rangle, & a_4<0,b_4>0\\ \langle(a_4/b_1,a_3/b_2,a_2/b_3,a_1/b_4);u_{\bar{a}}\wedge u_{\bar{b}},\zeta_{\bar{a}}\vee\zeta_{\bar{b}},\xi_{\bar{a}}\vee\xi_{\bar{b}}\rangle, & a_4<0,b_4<0,\end{cases}$

(5) $c\bar{a}=\begin{cases}\langle(ca_1,ca_2,ca_3,ca_4);u_{\bar{a}},\zeta_{\bar{a}},\xi_{\bar{a}}\rangle, & c>0\\ \langle(ca_4,ca_3,ca_2,ca_1);u_{\bar{a}},\zeta_{\bar{a}},\xi_{\bar{a}}\rangle, & c<0,\end{cases}$

(6) $\bar{a}^{-1} = \left\langle \left(\frac{1}{a_4}, \frac{1}{a_3}, \frac{1}{a_2}, \frac{1}{a_1} \right); u_{\bar{a}}, \zeta_{\bar{a}}, \xi_{\bar{a}} \right\rangle, \bar{a} \neq 0.$

Definition10. Let $\bar{a} = \langle (a_1, a_2, a_3, a_4); u_{\bar{a}}, \zeta_{\bar{a}}, \xi_{\bar{a}} \rangle$ be a SVTRN number, then

(i) Score function $S(\bar{a}) = (1/16) * (a_1 + a_2 + a_3 + a_4) * (\mu_{\bar{a}} + (1 - v_{\bar{a}}) + (1 - \pi_{\bar{a}}));$

(ii) Accuracy function $B(\bar{a}) = (1/16) * (a_1 + a_2 + a_3 + a_4) * (\mu_{\bar{a}} + (1 - v_{\bar{a}}) + (1 + \pi_{\bar{a}})).$

Definition11. Let $\bar{a}, \bar{b}$ be any two SVTRN numbers, then

(i) If $S(\bar{a}) < S(\bar{a})$ then $\bar{a} < \bar{b}$,

(ii) If $S(\bar{a}) = S(\bar{a})$, and if

(1) $B(\bar{a}) < B(\bar{b})$, then $\bar{a} < \bar{b}$,

(2) (2) $B(\bar{a}) > B(\bar{b})$ then $\bar{a} > \bar{b}$, and

(3) (3) $B(\bar{a}) = B(\bar{b})$ then $\bar{a} = \bar{b}$.

3- Problem definition and solution concepts

The two- person zero- sum game is the simplest case of game theory in which how much one player receives is equal to how much the other loses. Parthasarathy and Raghavan (2010) studied the case when both players gave pure, mixed strategies. Nevertheless, the noncooperation between players may be vague.

There are three types of two- person zero- sum single valued trapezoidal neutrosophic matrix games:

1. Two- person zero- sum matrix games with single valued trapezoidal neutrosophic goals,
2. Two- person zero- sum matrix games with single valued trapezoidal neutrosophic payoffs,
3. Two- person zero- sum matrix games with single valued trapezoidal neutrosophic goals and single valued trapezoidal neutrosophic payoffs.

Consider a two player zero sum game in which the entries in the payoff matrix $\overline{P_{NI}}$ are single valued trapezoidal neutrosophic numbers. The neutrosophic pay-off matrix is

$$\bar{P}^N = \text{Player } I \begin{array}{c} \text{Player } II \\ \begin{pmatrix} \bar{p}^N_{11} & \bar{p}^N_{12} \cdots & \bar{p}^N_{1j} \cdots & \bar{p}^N_{1n} \\ .. & .. & .. & .. \\ \bar{p}^N_{i1} & \bar{p}^N_{i2} \cdots & \bar{p}^N_{ij} \cdots & \bar{p}^N_{in} \\ .. & .. & .. & .. \\ \bar{p}^N_{m1} & \bar{p}^N_{m2} & \bar{p}^N_{mj} \cdots & \bar{p}^N_{mn} \end{pmatrix} \end{array} \tag{1}$$

Players I, and II have n and m strategies, respectively denoted by M , and M', respectively and are defined as

$$M = \left\{ x \in R^{m:} : x_i \geq 0, \sum_{i=1}^{m} x_i = 1 \right\}, \tag{2}$$

and,

$$M^{'} = \left\{ y \in R^{n} : y_j \ge 0, \sum_{j=1}^{n} y_j = 1 \right\}. \tag{3}$$

The mathematical expectation for player I is

$$\overline{Z}^{N} = \sum_{j=1}^{n} \sum_{i=1}^{m} \overline{p}_{ij}^{N} x_i \, y_j \,, \quad \text{and for player } II \text{ is}$$

$$\overline{Z}^{N} = \sum_{i=1}^{m} \sum_{j=1}^{n} \overline{p}_{ij}^{N} x_i \, y_j$$

Where, $\overline{p}_{ij}^{N} = \left\langle \left(\overline{p}_{ij,1}^{N}, \overline{p}_{ij,2}^{N}, \overline{p}_{ij,3}^{N}, \overline{p}_{ij,4}^{N} \right); u_{\overline{p}_{ij}^{N}}, \zeta_{\overline{p}_{ij}^{N}}, \xi_{\overline{p}_{ij}^{N}} \right\rangle$.

Definition12. (Saddle point): If the min- max value equals to the max- min value then the game is called a saddle point (or equilibrium) and the corresponding strategies are said optimum strategies. The amount of payoff at an equilibrium point is the game value.

Remark 1. It is clear that the two mathematical expectations are the same since the sums are finite. Because of vagueness of payoffs single valued trapezoidal neutrosophic numbers, it is very difficult for the players to choose the optimal strategy. So, we consider how to maximize player's or minimize the opponent's neutrosophic payoffs. Upon this idea, let us propose the maximum equilibrium strategy as in the following definition.

Definition13. In one two- person zero- sum game, player I 's mixed strategy $x^{\bullet}$ player II 's mixed Strategy $y^{\bullet}$ are said to be optimal neutrosophic strategies if $x^{T} \overline{P}^{N} y^{\bullet} \le x^{\bullet T} \tilde{P} y^{\bullet} \le x^{\bullet T} \overline{P}^{N} y$ for any mixed strategies x and y.

Remark 2. The optimal neutrosophic strategy of player I is the strategy which maximizes $\overline{Z}^{N}$ irrespective of II ' s strategy. Also, the optimal neutrosophic strategy of player II is the strategy which minimizes $\overline{Z}^{N}$ irrespective of I 's strategy.

According to definition 10, and based on definition of the score value of each neutrosophic payoff $\overline{p}_{ij}^{N}$, the neutrosophic payoff matrix defined in (1) is reduced to the classical payoff matrix game as

$$\begin{array}{cc} & \text{Player } II \\ P = \text{Player } I & \begin{pmatrix} p_{11} & p_{12} \cdots & p_{1j.} & \cdots & p_{1n} \\ .. & .. & .. & & .. \\ p_{i1} & p_{i2} \cdots & p_{ij} \cdots & & p_{in} \\ .. & .. & .. & & .. \\ p_{m1} & p_{m2} & p_{mj} & \cdots & p_{mn} \end{pmatrix} \end{array} \tag{4}$$

Let us consider the game with deterministic payoff matrix (4), and the mixed strategies of players I, and II defined in (2) and (3), respectively. If E is the optimum value of the game of a player II, then the linear programming model for player II becomes

$$\begin{aligned} & \min \ \mathrm{E} \\ & \text{Subject to} \\ & \sum_{j=1}^{n} p_{ij} y_j \le E; \ y_j \ge 0, \ j = 1,2,\ldots,n. \end{aligned} \tag{5}$$

Putting $y_j^{'} = y_j / \mathrm{E}$,. Then problem (5) becomes

$$\max_j \left(\sum_{j=1}^{mn} y_j^{'} \right)$$

Subject to (6)

$$\sum_{j=1}^{mn} p_{ij} y_j^{'} \le 1;$$

$$y_j^{'} \ge 0, \forall j$$

Similarly, the linear programming model for player I is as

$$\max \Phi$$

Subject to (7)

$$\sum_{i=1}^{nm} p_{ij} x_i \ge \Phi;$$

$$x_i \ge 0, i = 1,2,...,m.$$

Putting $x_i^{'} = x_i / \Phi, i = 1,2,...,m$. Then problem (7) becomes

$$\min_i \left(\sum_{i=1}^{m} x_i^{'} \right)$$

Subject to (8)

$$\sum_{i=1}^{nm} p_{ij} x_i^{'} \ge 1;$$

$$x_i^{'} \ge 0; \forall i.$$

4-Solution procedure

In this section, a solution procedure for solving the two person zero sum matrix game problem is introduced as in the following steps:

Step1: Calculate the score value of each neutrosophic payoff $\overline{p}_{ij}^{N}$ in (1) and obtain the classical matrix game (4)

Step2: Translate the payoff matrix (4) into the corresponding problem (5) into problem (7)

Step3: Convert all of problems (5) and (6) into the corresponding linear programming problems (6), and (8), respectively.

Step4: Apply the simple method or any software (Lingo) to solve problems (6),and (8), to obtain the optimal strategies and the corresponding neutrosophic matrix game value for players II, and I , respectively.

5- Numerical examples

Example 5-1- Solve the two- person zero- sum matrix game with trapezoidal neutrosophic numbers

$$\overline{P}^{N} = \left(\overline{p}_{ij}^{N}\right)_{3\times 3} = \begin{pmatrix} (0,1,3,6);.7,.5,.3 & (5,8,10,14);.3,.6,.6 & 0 \\ (9,11,14,16);.5,.4,.7 & (12,15,19,22);.6,.4,.5 & (3,5,6,8);.6,.5,.4 \\ (5,8,10,14);.3,.6,.6 & (3,5,6,8);.6,.5,.4 & (15,17,19,22);.4,.8,.4 \end{pmatrix}$$

The corresponding crisp payoff:

$$P=\left(p_{ij}\right)_{3\times 3}=\begin{array}{cc} & \textit{Player II} \\ \textit{Player I} & \begin{pmatrix} 1 & 3 & 0 \\ 4 & 7 & 2 \\ 3 & 5 & 6 \end{pmatrix} \end{array}$$

Referring to problem (6) , we get

$$\max\left(y_1^{'}+y_2^{'}+y_3^{'}\right)$$

Subject to:

$$y_1^{'}+3y_2^{'}+0y_3^{'}\le 1,$$
$$4y_1^{'}+7y_2^{'}+2y_3^{'}\le 1,$$
$$3y_1^{'}+5y_2^{'}+6y_3^{'}\le 1,$$
$$y_1^{'},y_2^{'},y_3^{'}\ge 0.$$

Table1. The optimal strategy of player II

Variables	Optimal strategy	Game value
$y_1^{'}$	0.8	3.52412
$y_2^{'}$	0	
$y_3^{'}$	0.2	

Referring to problem (8), we have

$$\min\left(x_1^{'}+x_2^{'}+x_3^{'}\right)$$

Subject to:

$$1x_1^{'}+4x_2^{'}+3x_3^{'}\ge 1,$$
$$3x_1^{'}+7x_2^{'}+5x_3^{'}\ge 1,$$
$$0x_1^{'}+2x_2^{'}+6x_3^{'}\ge 1,$$
$$x_1^{'},x_2^{'},x_3^{'}\ge 0.$$

Table2. The optimal strategy of player *I*

Variables	Optimal strategy	Game value
$x_1^{'}$	0.0271	3.52412
$x_2^{'}$	0.4233	
$x_3^{'}$	0.5496	

Thus

Table3. The optimal neutrosophic strategy of Example5.1

Player I	Player II	Game value
$x_1^{'} = 0.0271$ $x_2^{'} = 0.4233$ $x_3^{'} = 0.5496$	$y_1^{'} = 0.8$ $y_2^{'} = 0$ $y_3^{'} = 02$	$(7.14894, 9.5561, 11.79924, 14.79936); 0.3, 0.8, 0.7$

Example 5-2- Consider the following payoff matrix game in neutrosophic environment

$$\overline{P}^N = \left(\overline{p}_{ij}^N\right)_{3\times 4} = \begin{pmatrix} (9,11,14,16);.5,.4,.7 & (5,8,10,14);.3,.6,.6 & (4,8,11,15);.6,.3,.2 & (15,17,19,22);.4,.8,.4 \\ (15,18,22,30);.8,.2,.6 & (16,19,26,32);.9,.2,.7 & (11,13,20,25);.8,.2,.6 & (14,17,21,28);.8,.2,.6 \\ (0,1,3,6);.7,.5,.3 & (3,5,6,8);.6,.5,.4 & (25,30,32,45);.7,.3,.5 & (12,15,19,22);.6,.4,.5 \end{pmatrix}$$

Referring to the definition of score value the above payoff neutrosophic matrix game can be reduced to the corresponding deterministic payoff as:

$$P = \left(p_{ij}\right)_{3\times 4} = \begin{array}{c} \\ Player\ I \end{array} \begin{array}{c} Player\ II \\ \begin{pmatrix} 4 & 3 & 5 & 6 \\ 11 & 12 & 9 & 10 \\ 1 & 2 & 19 & 7 \end{pmatrix} \end{array}$$

According problem (6) , we have

$$\max \left(y_1^{'} + y_2^{'} + y_3^{'} + y_4^{'}\right)$$

Subject to:

$$4y_1^{'} + 3y_2^{'} + 5y_3^{'} + 6y_4^{'} \le 1,$$
$$11y_1^{'} + 12y_2^{'} + 9y_3^{'} + 10y_4^{'} \le 1,$$
$$y_1^{'} + 2y_2^{'} + 19y_3^{'} + 7y_4^{'} \le 1,$$
$$y_1^{'}, y_2^{'}, y_3^{'}, y_4^{'} \ge 0.$$

Using the simplex method (Phase2), we get

Table4. The optimal strategy of player II

Variables	Optimal strategy	Game value
$y_1^{'}$	0	9.769
$y_2^{'}$	0	
$y_3^{'}$	0.231	
$y_4^{'}$	0.769	

Referring to problem (8), we have

$\min\left(x_1^{'} + x_2^{'} + x_3^{'}\right)$

Subject to:

$$4x_1^{'} + 11x_2^{'} + x_3^{'} \geq 1,$$
$$3x_1^{'} + 12x_2^{'} + 2x_3^{'} \geq 1,$$
$$5x_1^{'} + 9x_2^{'} + 19x_3^{'} \geq 1,$$
$$6x_1^{'} + 10x_2^{'} + 7x_3^{'} \geq 1; x_1^{'}, x_2^{'}, x_3^{'} \geq 0.$$

Table5. The optimal strategy of player I

Optimal strategy	Game value
$x_1^{'} = 0$ $x_2^{'} = 0.923$ $x_3^{'} = .077$	9.769

It is clear from the duality theorem that the optimal strategy of player I is:

$$\left(x_1^{'}, x_2^{'}, x_3^{'}\right) = (0, 0.923, 0.077),$$

which are the coefficients slack variables in the final table of the simplex method .

Thus,

Table6. The optimal neutrosophic strategy of example5.2

Player I	Player II	Game value
$x_1^{'} = 0$ $x_2^{'} = 0.923$ $x_3^{'} = 0.077$	$y_1^{'} = 0$ $y_2^{'} = 0$ $y_3^{'} = 0.231$ $y_4^{'} = 0.769$	$(13.526527, 16.348888, 21.024101, 27.414184); 0.6, 0.4, 0.6$

6-Conclusions

Neutrosophic sets are being more generalization of intuitionistic fuzzy set, which provide an additionally to introduce the indeterminacy through the uncertainty. In this paper, we have considered a two- person zero- sum matrix games with single valued trapezoidal neutrosophic numbers. Firstly, we have given the definition of the game with trapezoidal neutrosophic payoffs and then proposed an equilibrium strategy. Secondly, we proposed a solution procedure. Lastly, two numerical examples illustrated our research methods. We have discussed only one kind of games with uncertain payoffs. But of course, there are games with uncertain payoffs which will be take in our consideration the future.

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