Journal of Industrial and Systems Engineering
Vol. 12, No. 2, pp. 151-173
Spring (April) 2019

Measuring performance of a three-stage structure using data envelopment analysis and Stackelberg game

Ehsan Vaezi[1], Seyyed Esmaeil Najafi[1*], Seyyed Mohammad Hajimolana[1], Farhad Hosseinzadeh Lotfi[2], Mahnaz Ahadzadeh Namin[3]

[1]*Department of Industrial Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran*
[2]*Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran*
[3]*Department of Mathematics, Shahr.e Qods Branch, Islamic Azad University, Tehran, Iran*

ehsan.vaezi@srbiau.ac.ir, e.najafi@srbiau.ac.ir, Molana@srbiau.ac.ir, farhad@hosseinzadeh.ir, mahnazahadzadehnamin@gmail.com

Abstract

In this paper, we consider a three-stage network comprised of a leader and two followers in respect to the additional desirable and undesirable inputs and outputs. We utilize the non-cooperative approach multiplicative model to measure the efficiency of the overall system and the performances of decision-making units (DMUs) from both, the optimistic and pessimistic views. Moreover, we utilize the concept of a goal programming and define a kind of cooperation between the leader and followers, so that the objectives of the managers are capable of being inserted in the models. In actual fact, a kind of collaboration is considered in a non-cooperative game. The non-cooperative models from this view cannot be converted into linear models. Therefore, a heuristic method is proposed to convert the nonlinear models into linear models. After obtaining the efficiencies based on the double-frontier view, the DMUs are ranked and classified into three clusters by the k-means algorithm. Finally, this paper considers a genuine world example, in relevance to production planning and inventory control, for model application and analyzes it from the double-frontier view. The proposed models are simulations of a factory in a real world, with a production area as leader and a warehouse and a delivery point as two followers. This factory has been regarded as a dynamic network with a time period of 24 intervals.

Keywords: Network DEA, game theory, Stackelberg game, goal programming, double-frontier, undesirable output.

1- Introduction

Evaluation and the measurement of performance lead to smart or intelligent systems with incentives for individuals for the desired behavior. Performance measurement is one of the fundamental managerial processes, for analyzing their own performance and likewise, surveying the conformity between the performance and the set of goals. The outcome of the evaluation can provide the grounds for taking the correct measures in decision-making for the future. Performance appraisal is a key part in the formulation and implementation of organizational policies.

---

*Corresponding author
ISSN: 1735-8272,

Today, all organizations have somehow, depicted the importance, of having a measuring system, for performance. As a principle, every organization should and till wherever feasible, measure its performance capacities.

The absence of an effective assessment or evaluation system is directly related to the disintegration of an organization and this shortcoming is considered an organizational disease; for without measuring, there shall be no basis for judgments, opinions and evaluations. As whatever cannot be evaluated, cannot be even fittingly managed. So as to ensure a correct management, every organization must use scientific models for the evaluation of performance, so that its efforts and the results achieved from its performance can be appraised. Several factors have an impact on the growth and development of countries. Researches executed in this arena indicate that efficiency impacts enhance the speed of economic development. These surveys have revealed that in the past years, there is a difference in the economic growth and development of countries due to modification in the level of efficiency and productivity of factors relative to production. Thence, an increment in performance and efficiency of organizations is an inevitable necessity, for survival, in global markets today. This issue is not confined to a particular sector or industry and in a limited period of time shall encompass all the sectors of economy. A performance assessment is a process which appraises measures, evaluates and judges the performance of an organization during a given period. This measuring of performance is carried out by comparing the present circumstances with that of the desirable or ideal conditions, which are based on pre-determined indexes. In general, the objectives of assessing the performance are a response to the results in specifying quality improvement measures and to reduce costs, as well as comprehend, as to what is being evaluated. The performance evaluation topics can be examined from different views. There are two traditional and modern views in this regard. The traditional view focuses solely on the work of the past period and is shaped by the requirements of the past. In this view, the time and space conditions of the system are ignored and may cause deviations as a result of work. A new view has targeted education, growth and development of evaluated capacities and performance improvements. This approach identifies the weaknesses and strengths of the systems. In the new view, the problem is studied in the context of time and a systemic attitude is dominant. Organizational units are only a part of the whole system. As a result, a new view leads to growth and development, improvement of performance, and the realization of the goals of the organization. In recent years, several models and approaches have been proposed for measuring efficiency, based on two general parametric and non-parametric methods. In this research, the Data Envelopment Analysis (DEA) is used as a nonparametric approach. This method selects the efficient units and provides the efficiency frontier. This frontier is a criterion for the evaluation of other units. In this paper, we will measure the performance by using the data envelopment analysis method for the following five reasons. First, it evaluates the performance of the organization on the basis of a logical model with a flexible structure. Second, it detects inefficient units. Third, the degree of inefficiency of the units is determined. Fourth, there is no prior standard level and the comparison criterion is another unit that operates under the same conditions. Fifth, DEA determines the patterns and references for the inefficient units among of the efficient units.

The DEA is a theoretical framework which discusses the analyzing of efficiency and its application in the arena of production planning and inventory control is observed very poorly. In the past two decades, the manufacturing or production sector has grown significantly and being attentive towards production is one of the key goals of Iran’s programs. An increment in the importance of the production sector, during the recent years and anxiety as to efficiency growth in this sphere, has a direct correlation with the economic system. A rise in costs, has led to pressurizing the production units to increase their organizational efficiency. A rise in costs, has led to haul, the production units towards incrementing their organizational performance. The best manner to ensure an efficiency increase would be to carry out a correct and logical use of the resources available. This could only be accomplished by ensuring a correct managerial performance, including a coherent evaluation of the returns attained. In continuation, the paper unfolds as follows: Section (2) reviews the literature on the data envelopment analysis approach. Section (3), describes the methodology and model formulation. In section (4), the heuristic approach has been described, so as to resolve the non-cooperative view of the network analysis. Section (5) of the paper describes a factory, evaluating it dynamically and section (6) concludes the paper.

2- Literature review

Data Envelopment Analysis (DEA) is a non-parametric method for measuring the relative efficiency of a set of analogous decision-making units (DMUs), with multiple inputs and outputs (Hwang et al., 2013). This method is considered a frontier or boundary function surrounding and involving the input and output factors. It not only determines the most efficient units, but it also analyses the inefficient ones (Kritikos, 2017). Charnes et al. (1978) developed the initial DEA task of Farrel (1957), the said model was known as (Charnes-Cooper Rhodes) or the "CCR Model". Banker et al. (1984) expanded the DEA models and presented the (Banker-Charnes-Cooper) or the "BCC Model". The classical data envelopment analysis models, such as, the CCR and BCC, assume that the systems are considered as black boxes; and due to the shortcomings in considering the intermediary variables and the internal interactions of the system, valuable information is eliminated (Lee et al., 2016). Fare and Grosskopf (2000) indicated to the disadvantages and weak points of the classical DEA models and referred to the Network Data Envelopment Analysis Model (NDEA). These models defined the interactions and intermediate variables and similarly, by utilizing the series and parallel sub-divisions, dealt with evaluating the efficiency of complex systems. Since the NDEA Models take into account the internal interactions of systems, hence, a more realistic performance of the systems can be demonstrated. In network models the performance of the entire system is calculated in relevance to the constraints or restrictions of the internal processes and the interactions between the general efficiency and that of the processes is established. Though, in the classic DEA Models, if the DMU has internal processes, the efficiency of these internal processes and the general process is computed independently and the correlations between the general efficiency and that of the processes is not conventional (Chen & Yan, 2011). Kao (2009) categorized the network models into three sets, namely, series, parallel and hybrid. Kao stated that, when activities in a system are protracted in respect to each other, the system is of a series structure; and whenever activities are in a parallel form alongside each other, the system has a parallel structure. Similarly, when there is a hybrid condition between the series and parallel aspects, a hybrid mode is engaged. In order to calculate the efficiency of the entire network, both, in the series or parallel mode, usually, the efficiency coefficient attained in the stages relative to each other and the weighted average efficiency or the stages are normally and respectively utilized. In a series or parallel structure, a DMU is efficient when all its sub-processes are efficient (Kou et al., 2016). Several studies have been carried out in relevance to NDEA and in respect to which, the task of Cook et al. (2010) can be indicted. They developed a multi-stage model, in which each stage is able to consider the additional inputs and outputs. In fact, in this model, the outputs of each stage can be regarded as the final product and exit the system and or enter the next stage as an input. Thereby, each stage can take the additional inputs into consideration, as not being the outputs of the prior stage. Zhou et al. (2018), review the literature on network data envelopment analysis (NDEA) applications in sustainability using citation-based approaches from 1996 to 2016. In the past few years, in relevance to network analysis, new discussions have been contributed in view of the game theory, such that this theory has become one of the vital methods in the analysis of NDEA or have been converted into multi-stage models (Liang et al., 2008). Li et al. (2012) rendered a model for a two-stage structure, a phase of which holds a more important standpoint for managers. They have named this phase as "leader" and the other phase as "follower". In order to calculate the efficiency, initially, the efficiency of the leader phase was maximized to the optimum and then the efficiency of the follower phase was brought to hand, by maintaining a constant efficiency for the leader phase. This model was known as a decentralized controlled or a Stackelberg game, which has been widely used by researchers in the recent years. An et al. (2017), took a network, comprising of two stages with a collaborative condition between them into consideration and computed the efficiency of this network in the cooperative and non-cooperative conditions on a (leader-follower) basis. The results demonstrated that, the overall efficiency in cooperative conditions was higher than that of the non-cooperative one. Wu et al. (2016) contemplated on and computed the efficiency of a two-stage network, in another similar research, with undesirable outputs in cooperative and non-cooperative conditions. The results of this research, which considers the total efficiency as the sum of the efficiency component, denotes that, the efficiency of the sub-DMUs is in the condition of a leader in the maximal and as a follower in the minimal. In yet another research by Zhou et al. (2018), a network consisting of a leader and some followers were evaluated in a black box and non-cooperative modes and the results were compared. In this study which aimed at minimizing costs, the CCR data envelopment analysis model was utilized. In other researches that were performed in the grounds of leader-follower, the research by Du et al. (2015) can be designated. They analyzed a parallel network in the cooperative and non-cooperative mode. Rezaee et al. (2016) combine DEA and Nash bargaining game as a cooperative game theory approach to evaluate the performance of two stage network. Shafiee (2017) considers a two-stage network and use non-cooperative Stackelberg game with rough set theory to evaluate the performance of DMUs under uncertainty. Amirkhan et al. (2018) rendered a model for a three-stage structure that all of the stages cooperate together to improve the overall efficiency of main DMU. In this study, a new three-stage DEA model is developed using the concept of three-player Nash bargaining game for PSTS processes.

In the recent years, special attention has been paid to undesirable factors in DEA Models. Such that, Liu et al. (2016) utilized the clustering methods and described this sphere as one of the four critical spheres or domains of DEA, from the researchers' viewpoint. Fare and Grosskopf (1989), for the initial time, mentioned the aspect of undesirable factors, in evaluating efficiency performance. Seiford and Zhu (2002) considered a network structure and proposed a model for efficiency evaluation that increased the desirable output and decreased the undesirable output. A non-radial network DEA model is suggested by Jahanshahloo et al. (2005), for considering the undesirable outputs. Badiezadeh and Farzipoor (2014) reflected on a production line, as a system with undesirable outputs and measured the overall efficiency of the system under consideration and the internal interactions of DMUs. Lu and Lo (2007) classified the undesirable outputs within a framework of three modes: The first method was to overlook all the undesirable outputs. The second method was to restrict the expansion of the undesirable outputs, or by considering these undesirable outputs as a nonlinear DEA model. The third method taken under contemplation for the undesirable outputs, was as an input, or signified with a negative sign, as an output and or by imposing a single downward conversion. In the past few years, the role of the undesirable factors in DEA models has made considerable progress and the tasks of Wang et al. (2013) and Wu et al. (2015) can be indicated to.

The DEA with a double-frontier studies two efficiencies for each DMU. One is called the optimistic efficiency or best relative efficiency and other efficiency is known as the pessimistic efficiency or the poorest efficiency (Amirteimoori, 2007). In the optimistic efficiency each DMU is compared with a set of efficient DMUs that are located on the efficiency frontier; whereas, in the pessimistic efficiency the comparison of each DMU is made with a set of inefficient DMUs that are located on the inefficiency frontier (Parkan and Wang, 2000). The value of the optimistic approach is less than or equates to (1); and from the pessimistic viewpoint is more than (1) or equal to (1). The efficiency value of the optimistic approach is less than (1), when the DMU under evaluation is not on the efficiency frontier; whereas, it equates to (1) when the DMU under assessment or evaluation is on the efficiency frontier. The pessimistic value approach is more than (1) when the DMU under evaluation is not on the inefficiency frontier; but is equivalent to (1) when the DMU under evaluation is on the efficiency frontier (Azizi and Wang, 2013; Jahanshahloo and Afzalinejad, 2006). In actual fact, the double-frontier, views each DMU from two perspectives and any conclusion which implies to only one of these two viewpoints shall result in a one-sided and an incomplete perspective (Azizi and Ajirlu, 2011). The measurement of efficiency, based on the optimistic and pessimistic views in a mutual fashion, shall lead to an increment in accuracy for the purpose of ranking the DMUs (Badiezadeh et al., 2018). Doyle et al. (1955) for the first time obtained the efficiency of DMUs from the two optimistic and pessimistic viewpoints. Entani et al. (2002) attained the double-frontier in order to measure the efficiency for each lower bound and upper bound DMU as optimistic and pessimistic efficiencies respectively. So as to combine the results of the optimistic and pessimistic approaches, which would usher a general or overall efficiency, several other researchers suggested mathematical combinations (i.e. averaging between the optimistic and pessimistic values) (Azizi, 2014). Wang and Chen (2009) used a geometric mean to combine the results of an optimistic and pessimistic viewpoint for ranking the DMUs. In the recent years, numerous other researchers have utilized the double-frontier to measure efficiency and in this relative Jiang et al. 2012; Wang and Lan 2013; Yang and Morita 2013; Azizi et al. 2015; Jahed et al. 2015 and Badiezadeh et al. 2018 can be indicated.

The researches carried out utilized and were based on DEA, which were mainly in static environments. For the initial time, Sengupta (1995), dealt with efficiency evaluations in dynamic environments. Dynamic models are models where, data is continuously changing over several incessant periods or cycles; and each time period is considered as a DMU. Similarly, the correlation between the periods in these models, utilizes additional inputs and outputs amid these periods (Jafarian Moghaddam and Ghoseiri 2011). Since (the epoch of) Sengupta's task, several articles have been published in the sphere of dynamic networks, which differ in relevance to case studies and the manner in which the efficiency of the DMUs are calculated. In other words, models in relative to Kawaguchi et al. (2014) and Wang et al. (2014) can be mentioned respectively, for performance or efficiency evaluation in hospital environments and banks in a dynamic genre.

A multiple criteria decision-making can be divided into two groups, consisting of multi-criterion and multi-objective decision-making. A goal programming is one of the multi-objective decision-making techniques, which assists in encompassing several aims synchronously; and by minimizing the deviation between these objectives, the optimal solution can be determined. In this method, the objective function of the key problem is somehow formulated by the auxiliary variables that are namely deviations from the goal condition, so that the total set of undesirable deviations of the ideals are minimized (Ransikarbum and Mason, 2016). This technique specifies as to the goals achieved and the ones which have not been so. In addition to which, by utilizing a goal programming, the amount of deviation of each of these goals from their ideal level comes to hand (Shabanpour et al., 2017). A goal programming was performed by Charnes and Cooper in 1961 (Dhahri and Chabchoub, 2007). In the past few years, numerous researchers have used the goal programming method and rendered new models and for such models, one can refer to Chen et al. (2017); Trivedi and Singh (2017) and He et al. (2016). Methods in relevance to goal programming modes are extremely diverse and even make provisions to optimize contradictory goals. Jolai et al. (2011) set up and utilized goal programming for three kinds of analysis: 1-Specifying the essential resources to fulfill a set of goals under consideration, 2-Determining the intensity of attaining goals, 3-Determining the optimal and substantial response with due attention to the amount of resources available and the priority of objectives or goals. Yousefi et al. (2017) suggest a hybrid goal programming-data envelopment analysis model in a network structure to present improvement solutions and rank units (all efficient and inefficient) based on experts' requirements.

In accordance with the points mentioned, most of the researches performed in the network deliberate on two stages, but the current research takes a three-stage process into consideration, which, in addition to the intermediary variables, has additional and undesirable inputs and outputs as well. We utilize an optimistic and pessimistic viewpoint, to secure efficiency and increase accuracy. Moreover, we utilize the concept of a goal programming and define a kind of cooperation between the leader and followers, so that the objectives of the managers are capable of being inserted in the models. The chief goal of this paper is to impose the opinions of the managers in the models and analyze them, as well as compare results. Hence, a kind of collaboration is considered in a non-cooperative game. The non-cooperative models cannot be turned into linear models, from the optimistic and pessimistic views, because of the additional inputs and outputs. Therefore, we use a heuristic technique to convert the nonlinear models into linear models. Finally, this paper proposed a clustering method based on the double-frontier view by using the k-means algorithm.

3- Methodology

Each research is a systematic activity, in which either knowledge develops, or a situation is described and explained, or ultimately a particular problem is solved. Given that each research begins with a specific problem and purpose at hand, therefore, researches are of different types. Accordingly, this research is an applied research. The statistical population of this research includes the production, maintenance and distribution network of a factory (Nasiri Dairy factory), which is defined as an annual planning horizon in 24 periods. In this study, the methodology is designed in four steps. In the first step, the variables and data are collected based on the observation, interview and library studies. In the second step, a network data envelopment analysis (NDEA) approach is designed to measure the performance of DMUs based on the optimistic, pessimistic and double-frontier views. In the third step, a heuristic approach is designed, so as to resolve the optimistic and pessimistic models. Finally, in the fourth step, the decision-making units are ranked and classified by the k-means algorithm. In Fig. 1, the methodology is shown in four steps.

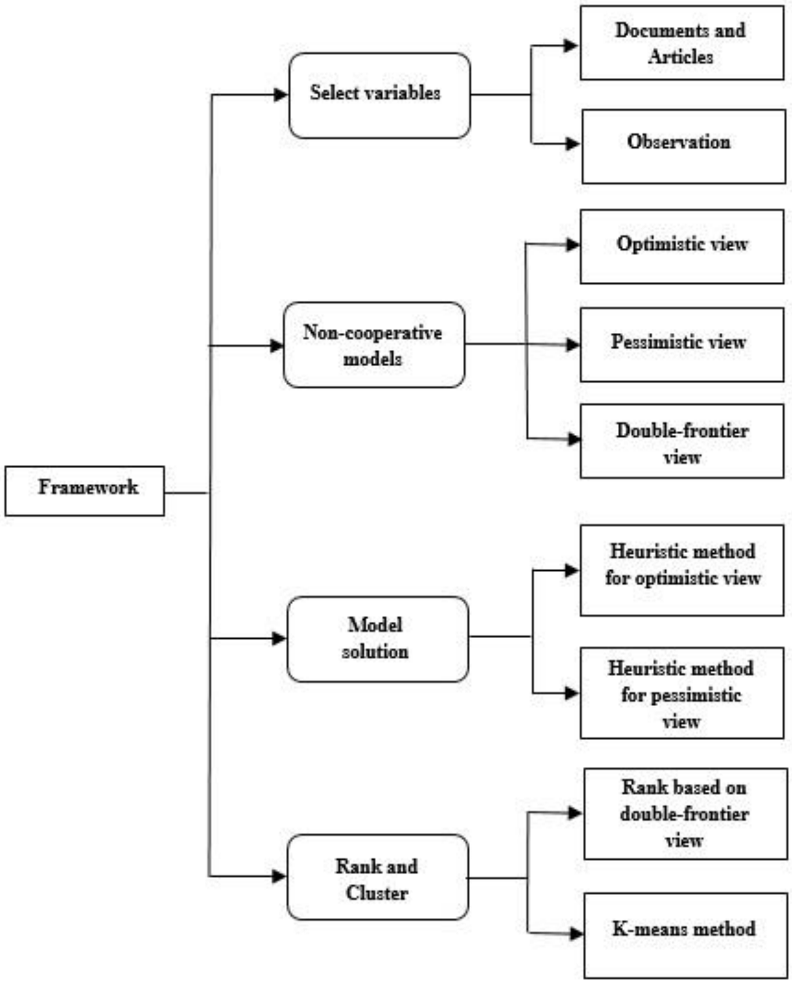

Fig 1. Steps of methodology

3-1- Model description

We consider a set of n homogeneous decision making units (DMUs) that are denote by DMU_j (j=1,..., n), and each DMU_j (j=1,…,n) has three-stage, as shown in Fig. 2, where all the stages are connected together in series. We denote, the inputs of the first stage by $x^1_{i_1j}$ ($i_1=1,\ldots,I_1$) and the undesirable outputs of the first stage by $y^1_{r_1j}$ ($r_1=1,\ldots,R_1$). We denote, the intermediate measures between first stage and second stage by $z^1_{d_1j}$ ($d_1=1,\ldots,D_1$) and between second stage and third stage by $z^2_{d_2j}$ ($d_2=1,\ldots,D_2$). The additional inputs and outputs of the second stage are denoted by $x^2_{i_2j}$ ($i_2=1,\ldots,I_2$) and $y^2_{r_2j}$ ($r_2=1,\ldots,R_2$), respectively. Finally, we denote, the additional inputs of the third stage by $x^3_{i_3j}$ ($i_3=1,\ldots,I_3$) and the outputs of the third stage by $y^3_{r_3j}$ ($r_1=1,\ldots,R_3$). We adopt $v^1_{i_1}$, $v^2_{i_2}$ and $v^3_{i_3}$ as the weights of the inputs to the first, second and third stages, respectively. Kao and Hwang (2008) used the same weights for the intermediate measures. In accordance with this, we value the intermediate measures in this research, irrespective of its dual role (as an input in one stage or as an output in the next stage). We assume that the weights relative to the intermediate measures between stages 1 and 2 and similarly, weights in relevance with the intermediate measures between stages 2 and 3 are uniform. Therefore, we adopt $w^1_{d_1}$ and $w^2_{d_2}$ as the weights of the intermediate measures between stage 1, stage 2 and stage 3, respectively. The weights of the outputs for the first, second and third stages second stage are denoted by $u^1_{r_1}$, $u^2_{r_2}$ and $u^3_{r_3}$, respectively.

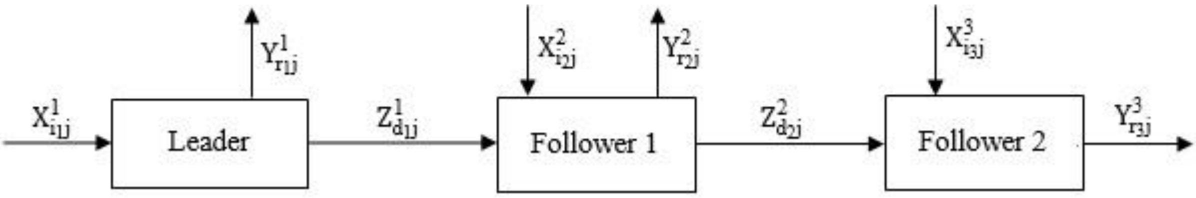

Fig 2. Structure of three-stage leader-follower system with additional inputs and undesirable outputs

Researchers are more inclined to utilize input-oriented models for efficiency analysis, mainly for three reasons. The first is that, demand reveals a growing trend, the estimation of which is an intricate matter. The second is that, managers have a better control on the inputs, rather than the outputs. The third is that, the model reflects the initial objectives of policy-makers, on the basis of being responsible in responding to the demands of the people. Furthermore, the units must reduce costs and or restrict the use of resources. Thereby, in this research, an input- oriented model is utilized. According to the opinions of managers, we shall describe and consider the first stage in the role of a "leader", the second stage as the "first follower" and the third stage as the "second follower". Thence, we demonstrate the optimistic and pessimistic efficiencies of the leader's stage with θ_o^L and φ_o^L respectively; the optimistic and pessimistic efficiencies of the second and third stages as θ_o^{1F}, θ_o^{2F} and φ_o^{1F}, φ_o^{2F} respectively; and the optimistic and pessimistic efficiencies of the second and third stages together are shown as θ_o^{12F} and φ_o^{12F} respectively. In this section, which comprises of the proposed approach of this paper, efforts have been made to insert the goals of the managers into the models. We designate the first stage as the "leader" and assume that the second and third stages together, are in the form of a "follower". Under these conditions, the leader optimizes its efficiency so that the efficiency of the followers does not reduce from a certain level, or in actual fact, the leader maximizes its efficiency to forestall the eradication of the followers. Actually, the leader-follower characteristic is a non-cooperative game, which we hybrid with a cooperative approach in this section. In accordance with this, we describe the maximal efficiency of the leader stage from the optimistic viewpoint as hereunder:

$$\theta_o^{L*}=\max\left\{\theta_o^L \middle| \ \theta_o^{1F}\geq c_1,\ \theta_o^{2F}\geq c_2,\ \theta_j^L\leq 1,\ \theta_j^{1F}\leq 1,\ \theta_j^{2F}\leq 1,\ j=1,\ldots,n\ \right\} \tag{1}$$

All the variables in the model (1) are non-negative. Model (1) secures the maximal efficiency of the leader stage, on condition that, the efficiency of none of the stages is more than (1); and for DMU_0 the follower stages (second and third stages) are not lower than the values of c_1 and c_2 respectively. The values of c_1 and c_2 are actually the minimal efficiency of the second and third stages which are numerals at intervals of (0 and 1) in accordance with the goals of managers. It should be noted that if the values of $c_1= c_2\ \varepsilon$ =are considered such, so that they are closer to (0), then the two constraints $\theta_o^{1F}\geq c_1$ and $\theta_o^{2F}\geq c_2$are simply redundant. So the model (1) is feasible. But there could be a possibility that in reality, the goals of managers is not capable of being attained and the model turns into a superfluous one. Hence, we utilized the concept of 'goal programming' and the two assigned values $\alpha_1$and α_2 are reduced ($\theta_o^{1F}\geq c_1$-α_1 $\theta_o^{2F}\geq c_2$-α_2) from the opinion of managers under contemplation, so that by using model (2), conditions for securing the goal of managers is surveyed. Model (1) is a fractional model and by utilizing the Charnes-Cooper conversion (1962), as well as contemplating on the goal programming concept, as illustrated hereunder, it is converted into a linear model.

$$\theta_o^{L*}=\max\ \sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1o}^1-\sum_{r_1=1}^{R_1} u_{r_1}^1\, y_{r_1o}^1-M(\alpha_1+\alpha_2)$$

$$\text{s.t.}\quad \sum_{i_1=1}^{I_1} v_{i_1}^1\, x_{i_1o}^1=1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1j}^1-\sum_{r_1=1}^{R_1} u_{r_1}^1\, y_{r_1j}^1-\sum_{i_1=1}^{I_1} v_{i_1}^1\, x_{i_1j}^1\leq 0,\qquad j=1,\ldots,n \tag{2}$$

$$j=1,\ldots,n\sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2j}^2+\sum_{r_2=1}^{R_2} u_{r_2}^2\, y_{r_2j}^2-\sum_{i_2=1}^{I_2} v_{i_2}^2\, x_{i_2j}^2-\sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1j}^1\leq 0,$$

$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 \le 0, \quad j=1,\dots,n$

$(c_1 - \alpha_1) \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 + (c_1 - \alpha_1) \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 - \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2 \le 0$

$(c_2 - \alpha_2) \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3 + (c_2 - \alpha_2) \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 - \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3 \le 0$

$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \ge \varepsilon;\ r_1=1,\dots,R_1;\ r_2=1,\dots,R_2;\ r_3=1,\dots,R_3;$

$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \ge \varepsilon;\ i_1=1,\dots,I_1;\ \ i_2=1,\dots,I_2;\ \ i_3=1,\dots,I_3;\ j=1,\dots,n.$

$w_{d_1}^1, w_{d_2}^2 \ge \varepsilon;\ d_1=1,\dots,D_1;\ \ d_2=1,\dots,D_2.$

In the model (2) the optimum efficiency has been demonstrated with the symbol (*) and "M" denotes a large numeral, which factually is a penalty that causes the manager's goal to be achievable. It should be mentioned that in the case where, $\alpha_1=0$, $\alpha_2=0$, the model (2) is feasible from the point of the manager's goal and if this is not the issue, we request the manager to reduce his goals (c_i) to the measurement of α_i to make the model possible. On the basis of the task of Wang et al. (2005), we modify model (2), as hereunder to obtain the efficiency of the leader stage from the pessimistic view. Similar to our optimistic approach, we obtain the pessimistic efficiency of the leader stage, under conditions where the follower stages are at a distance from the inefficient frontier, *i.e.* $\varphi_o^{2F} \ge c_4 - \alpha_4$, $\varphi_o^{1F} \ge c_3 - \alpha_3$ in which case, $c_3, c_4 \ge 1$.

$$\varphi_o^{L*} = \min \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1 + M(\alpha_3+\alpha_4)$$

$$\text{s.t.} \quad \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1 = 1$$

$$\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1 \ge 0, \quad j=1,\dots,n \tag{3}$$

$$j=1,\dots,n \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 \ge 0,$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 \ge 0, \quad j=1,\dots,n$$

$$\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2 - (c_3 - \alpha_3) \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 - (c_3 - \alpha_3) \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2 \ge 0$$

$$\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3 - (c_4 - \alpha_4) \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3 - (c_4 - \alpha_4) \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 \ge 0$$

$$u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \ge \varepsilon;\ r_1=1,\dots,R_1;\ r_2=1,\dots,R_2;\ r_3=1,\dots,R_3;$$

$$v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \ge \varepsilon;\ i_1=1,\dots,I_1;\ \ i_2=1,\dots,I_2;\ \ i_3=1,\dots,I_3;\ j=1,\dots,n.$$

$$w_{d_1}^1, w_{d_2}^2 \ge \varepsilon;\ d_1=1,\dots,D_1;\ \ d_2=1,\dots,D_2.$$

Analogous to the optimistic approach of "M" that is a large numerical, which in this circumstance and with due attention to the type of objective function has been supplemented to the model in order to fulfill the manager's goal. It should be observed that in the case where $\alpha_1=0$, $\alpha_2=0$, the model (3) is feasible in respect to the opinion of the manager or else we shall request the manager to reduce his goals of (c_i) to the measurement of α_i to make the model possible. Therefore, the maximal optimistic efficiency of the leader stage θ_o^{L*} and the minimal pessimistic efficiency of the leader stage $\varphi_o^{L*}$is brought to hand respectively, from models (2 and 3). To compute the efficiency of the followers we shall assume the second and third stages as one stage and obtain the efficiency of the follower stage. We hybrid the efficiencies of the second and third stages, being attentive to the fact that they are in series and define them as figures $\theta_o^{12F} = \theta_o^{1F} . \theta_o^{2F}$ in accordance with the tasks of Kao and Hwang (2008). Hence, the maximal efficiency together for the follower stages from the optimistic viewpoint is brought to hand as rendered hereunder:

$$\theta_o^{12F*}=\max \frac{\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1} \cdot \frac{\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2}$$

$$\begin{aligned}
\text{s.t.}\quad & \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1}\leq 1, \quad j=1,\ldots,n \\
& \frac{\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1}\leq 1, \quad j=1,\ldots,n \\
& \frac{\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2}\leq 1, \quad j=1,\ldots,n \\
& \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1}=\theta_o^{L*} \\
& u_{r_1}^1, u_{r_2}^2, u_{r_3}^3\geq \varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3; \\
& v_{i_1}^1, v_{i_2}^2, v_{i_3}^3\geq \varepsilon;\ i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;\ j=1,\ldots,n. \\
& w_{d_1}^1, w_{d_2}^2\geq \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.
\end{aligned} \tag{4}$$

The maximal and overall efficiency of the second and third stages is gained by model (4), on condition that, the efficiency of none of the stages equates to more than (1); and to the approach of Li et al. (2012), the efficiency of the leader's stage should remain constant. On the founding's of the tasks of Wang et al. (2005), we describe model (4) as given below, in order to attain the minimal efficiency of the overall follower stages from the pessimistic view.

$$\varphi_o^{12F*}=\min \frac{\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1} \cdot \frac{\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2}$$

$$\begin{aligned}
\text{s.t.}\quad & \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1}\geq 1, \quad j=1,\ldots,n \\
& \frac{\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2}{\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1}\geq 1, \quad j=1,\ldots,n \\
& \frac{\sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3}{\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2}\geq 1, \quad j=1,\ldots,n \\
& \frac{\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1}{\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1}=\varphi_o^{L*} \\
& u_{r_1}^1, u_{r_2}^2, u_{r_3}^3\geq \varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3; \\
& v_{i_1}^1, v_{i_2}^2, v_{i_3}^3\geq \varepsilon;\ i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;\ j=1,\ldots,n. \\
& w_{d_1}^1, w_{d_2}^2\geq \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.
\end{aligned} \tag{5}$$

Models (4 and 5) are nonlinear and in the fourth section of this paper, an innovative approach in resolving it is utilized. In assuming that, the models are solved and given that the stages are in series (figure 2), we define the total and maximal optimistic efficiency and the minimal and total pessimistic efficiency are respectively specified as below:

$$\theta_o^{overall*} = \theta_o^{L*} . \theta_o^{12F*}, \varphi_o^{overall*} = \varphi_o^{L*} . \varphi_o^{12F*} \quad (6)$$

Wang and Chin (2009) used an approach for ranking DMUs from both, the optimistic and pessimistic views. We then define the overall efficiency according to the double-frontier in formula (7) as below:

$$\emptyset_o^* = \sqrt{\theta_o^{overall*} . \varphi_o^{overall*}} \quad (7)$$

3-2- Clustering

With the result of Formula (7), we can rank the DMUs. This ranking is based on the optimistic and pessimistic views. In the following, we use the k-means algorithm to cluster the DMUs into several groups. K-means clustering is a simple unsupervised learning algorithm that is used to solve clustering problems. It follows a simple procedure of classifying a given data set into a number of clusters, defined by the letter "k," which is fixed beforehand. The clusters are then positioned as points and all observations or data points are associated with the nearest cluster, computed, adjusted and then the process starts over using the new adjustments until a desired result is reached. The groups are determined in such a way that the similarity between the members of a group is high and the similarity between members of different groups is low. Given a set of observations (x_1, x_2, …, x_n), where each observation is a d-dimensional real vector, k-means clustering aims to partition the n observations into k ($\leq$ n) sets S = {s_1, s_2, …, s_k} so as to minimize the within-cluster sum of squares (WCSS) (i.e. variance). Formally, the objective is to find: $\arg\min_{s} \sum_{i=1}^{k} \sum_{x \in s_i} \|x - \mu_i\|^2 = \arg\min_{s} \sum_{i=1}^{k} |s_i| \mathrm{Var}\, s_i$ where μ_i is the mean of points in s_i. In the K-Means algorithm, the k-member is randomly selected from among the n members as cluster centers. Then the n-k remaining members are assigned to the nearest cluster. After assigning all members, the cluster centers are recalculated and the members are assigned to the clusters according to the new centers, and this continues until the centers of each cluster remain constant. In this paper, we use the k-means technique to cluster the results of the described models (cluster based on the result of the formula (6)) and these results are shown in the case study section. In order to select the best cluster, based on expert opinions and previous studies, a suggested range for the number of clusters was initially identified. In accordance with the opinions of managers, we suggest to cluster the DMUs into three groups (k=3) with similar characteristics.

4- Heuristic approach to solve nonlinear models

In this section we will use a solution to gain the efficiency of the followers. Due to the presence of additional inputs and outputs in the first, second and third stages, models (4 and 5) are nonlinear. To solve these models we use a heuristic approach as hereunder:

4-1- A heuristic method from optimistic view

We are aware that the objective function of model (4) is the multiplicative efficiency of the two-stages, *i.e.* $\theta_o^{12F*} = \max \theta_o^{1F} . \theta_o^{2F}$. We take θ_o^{1F} as a variable in the objective function which modifies between the $[0, \theta_o^{1F\text{-max}}]$ interval. We describe θ_o^{1F} as given below, so that we are able to move it between intervals.

$$\theta_o^{1F} = \theta_o^{1F\text{-max}} - k_1 \Delta\varepsilon, \qquad k_1 = 0,1,\ldots,\left\lceil \frac{\theta_o^{1F\text{-max}}}{\Delta\varepsilon} \right\rceil + 1 \quad (8)$$

We take $\Delta\varepsilon$ as a step size and consider it an extremely small amount and describe $\theta_o^{1F\text{-max}}$ as the maximum efficiency of the first follower stage and its value is capable of being computed by the model below.

$$\theta_o^{1F\text{-max}} = \max \left\{ \theta_o^{1F} \,\middle|\, \theta_j^{L} \leq 1, \theta_j^{1F} \leq 1, \theta_j^{2F} \leq 1, \; j=1,\ldots,n \right\} \quad (9)$$

Model (9) secures the maximum efficiency of the first follower stage, under conditions where the efficiency of all the stages is less than (1). In actual fact, this model irrespective, of the leader-follower correlations, attributes the highest efficiency to the second stage. This model is a fractional model and by utilizing the Charnes-Cooper conversion (1962), as illustrated hereunder, it is converted into a linear model.

$$
\begin{aligned}
\theta_o^{1F\text{-max}} = \max\ & \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2 \\
\text{s.t.}\quad & \sum_{i_1=1}^{I_1} v_{i_2}^2 x_{i_2o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 = 1 \\
& \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1 \le 0, \quad j=1,\dots,n \qquad (10) \\
& j=1,\dots,n \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 \le 0, \\
& \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 \le 0, \quad j=1,\dots,n \\
& u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \ge \varepsilon;\ r_1=1,\dots,R_1;\ r_2=1,\dots,R_2;\ r_3=1,\dots,R_3; \\
& v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \ge \varepsilon;\ i_1=1,\dots,I_1;\ i_2=1,\dots,I_2;\ i_3=1,\dots,I_3;\ j=1,\dots,n. \\
& w_{d_1}^1, w_{d_2}^2 \ge \varepsilon;\ d_1=1,\dots,D_1;\ d_2=1,\dots,D_2.
\end{aligned}
$$

In determining the value of $\theta_o^{1F\text{-max}}$ by model (10), we convert model (4) into the following model.

$$
\theta_o^{12F*} = \max \left\{ \theta_o^{1F}.\theta_o^{2F} \,\middle|\, \theta_j^{L} \le 1,\ \theta_j^{1F} \le 1,\ \theta_j^{2F} \le 1,\ \theta_o^{L} = \theta_o^{L*}, \theta_o^{1F} = \frac{O_o^2}{I_o^2},\ \theta_o^{1F} \in \left[0,\ \theta_o^{1F\text{-max}}\right],\ j=1,\dots,n \right\} \qquad (11)
$$

In the model (11) we considered θ_o^{1F} in the objective function as a variable and the constraint which specified this variable, together with its interval of modification was added to the model. In model (11), we have demonstrated the efficiency of the second stage or θ_o^{1F} briefly, in a form of output to an input. The model (11) is a fractional one and by utilizing the Charnes-Cooper conversion (1962), as illustrated hereunder, it is converted into a linear model.

$$
\begin{aligned}
\theta_o^{12F*} = \max\ & \theta_o^{1F} . \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3 \\
\text{s.t.}\quad & \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3 + \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 = 1 \\
& \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1 - \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1 \le 0, \quad j=1,\dots,n \qquad (12) \\
& j=1,\dots,n \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2 - \sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2 - \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 \le 0, \\
& \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3 - \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3 - \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 \le 0, \quad j=1,\dots,n \\
& \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 - \sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1 - \theta_o^{L*} \sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1 = 0 \\
& \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2 + \sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2 - \theta_o^{1F*} \left(\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2 + \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1 \right) = 0 \\
& \theta_o^{1F} \in \left[0,\ \theta_o^{1F\text{-max}}\right] \\
& u_{r_1}^1, u_{r_2}^2, u_{r_3}^3 \ge \varepsilon;\ r_1=1,\dots,R_1;\ r_2=1,\dots,R_2;\ r_3=1,\dots,R_3; \\
& v_{i_1}^1, v_{i_2}^2, v_{i_3}^3 \ge \varepsilon;\ i_1=1,\dots,I_1;\ i_2=1,\dots,I_2;\ i_3=1,\dots,I_3;\ j=1,\dots,n. \\
& w_{d_1}^1, w_{d_2}^2 \ge \varepsilon;\ d_1=1,\dots,D_1;\ d_2=1,\dots,D_2.
\end{aligned}
$$

In model (12) and by utilizing formula (8), we increase the value of k_1 from (0) to its higher level, in order to solve the new model each time with θ_o^{1F} . We solve the returns of the entire conditions of the k_1 model and the responses of the model is assigned as $\theta_o^{12F}(k_1)$. By comparing all the values of $\theta_o^{12F}(k_1)$, we define θ_o^{12F*} as the maximal efficiency of the total sum of the follower stages from the optimistic view. It should be noted that, we have tested our proposed approach under two conditions and each time have considered a stage as a variable. Given that the efficiency of a stage is somewhat unique, thereby, the results of these two methods have come to hand with an extremely good approximation and in order to explain our approach, we have denoted one of these two conditions above.

3-2- A heuristic method from pessimistic view

We know that the objective function of model (5) is the multiplicative efficiency of two stages, *i.e.* $\varphi_o^{12F*}=\min \varphi_o^{1F}.\ \varphi_o^{2F}$. Similar to our optimistic view, we take φ_o^{1F} as a variable in the objective function that modifies between the $[\varphi_o^{1F\text{-min}}, M]$ interval. We describe φ_o^{1F} as rendered below so that we can move it within the interval.

$$\varphi_o^{1F} = \varphi_o^{1F\text{-min}} + k_1 \Delta\varepsilon, \qquad k_1=0,1,\ldots,\left[\frac{M-\varphi_o^{1F\text{-min}}}{\Delta\varepsilon}\right]+1 \tag{13}$$

We consider "M" to be a large amount and alike the optimistic approach, $\Delta\varepsilon$ as a step size and an extremely small amount. $\varphi_o^{1F\text{-min}}$ is described as the minimum efficiency of the first follower stage and its sum can be computed by the following formula.

$$\varphi_o^{1F\text{-min}}= \min \left\{\varphi_o^{1F} \,\middle|\, \varphi_j^{L} \geq 1,\ \varphi_j^{1F} \geq 1,\ \varphi_j^{2F} \geq 1,\ \ j=1,\ldots,n \right\} \tag{14}$$

Model (14), secures the minimum efficiency of the first follower stage, on condition that the efficiency of all the stages is more than (1). In fact, this model, regardless to the leader-follower correlation, attributes the least amount of efficiency to the second stage. This model is a fractional model and by employing the Charnes-Cooper conversion (1962), it is converted into a linear model as given hereunder:

$$\begin{aligned}
\varphi_o^{1F\text{-min}}= \min\ & \textstyle\sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2o}^2+\sum_{r_2=1}^{R_2} u_{r_2}^2\, y_{r_2o}^2 \\
\text{s.t.}\quad & \textstyle\sum_{i_2=1}^{I_2} v_{i_2}^2\, x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1o}^1= 1 \\
& \textstyle\sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1j}^1-\sum_{r_1=1}^{R_1} u_{r_1}^1\, y_{r_1j}^1-\sum_{i_1=1}^{I_1} v_{i_1}^1\, x_{i_1j}^1 \geq 0, \quad j=1,\ldots,n \\
& \textstyle j=1,\ldots,n\sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2j}^2+\sum_{r_2=1}^{R_2} u_{r_2}^2\, y_{r_2j}^2-\sum_{i_2=1}^{I_2} v_{i_2}^2\, x_{i_2j}^2-\sum_{d_1=1}^{D_1} w_{d_1}^1\, z_{d_1j}^1 \geq 0, \\
& \textstyle\sum_{r_3=1}^{R_3} u_{r_3}^3\, y_{r_3j}^3-\sum_{i_3=1}^{I_3} v_{i_3}^3\, x_{i_3j}^3-\sum_{d_2=1}^{D_2} w_{d_2}^2\, z_{d_2j}^2 \geq 0, \quad j=1,\ldots,n \\
& u_{r_1}^1,u_{r_2}^2,u_{r_3}^3\geq \varepsilon;\ r_1=1,\ldots,R_1;\ r_2=1,\ldots,R_2;\ r_3=1,\ldots,R_3; \\
& v_{i_1}^1,v_{i_2}^2,v_{i_3}^3\geq \varepsilon;\ i_1=1,\ldots,I_1;\ \ i_2=1,\ldots,I_2;\ \ i_3=1,\ldots,I_3;\ j=1,\ldots,n. \\
& w_{d_1}^1,w_{d_2}^2\geq \varepsilon;\ d_1=1,\ldots,D_1;\ \ d_2=1,\ldots,D_2.
\end{aligned} \tag{15}$$

In specifying the value of $\varphi_o^{1F\text{-min}}$ by model (15), model (5) is modified and converted to the model below:

$$\varphi_o^{12F*}=\min \left\{\varphi_o^{1F}.\varphi_o^{2F} \,\middle|\, \varphi_j^{L} \geq 1,\ \varphi_j^{1F} \geq 1,\ \varphi_j^{2F} \geq 1,\ \varphi_o^{L}=\varphi_o^{L*},\ \varphi_o^{1F}= \frac{O_o^2}{I_o^2},\ \varphi_o^{1F} \in[\varphi_o^{1F\text{-min}}, M],\ \ j=1,\ldots,n \right\} \tag{16}$$

It should be brought to attention that, in the model (16), we take φ_o^{1F} in the objective function as a variable and alike the optimistic approach, constraints which specify this variable, along with its interval of modification is supplemented to the model. The model (16) is a fractional model and by using the Charnes-Cooper conversion (1962), it is converted into a linear model as given hereunder:

$$\varphi_o^{12F*}= \min \varphi_o^{1F} . \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3o}^3$$

$$
\begin{aligned}
\text{s.t.}\quad & \sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3o}^3+\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2= 1\\
& \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1j}^1-\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1j}^1 \geq 0, \quad j=1,\dots,n \\
& j=1,\dots,n\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2j}^2-\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2j}^2-\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1j}^1 \geq 0,\\
& \sum_{r_3=1}^{R_3} u_{r_3}^3 y_{r_3j}^3-\sum_{i_3=1}^{I_3} v_{i_3}^3 x_{i_3j}^3-\sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2j}^2 \geq 0, \quad j=1,\dots,n\\
& \sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1-\sum_{r_1=1}^{R_1} u_{r_1}^1 y_{r_1o}^1-\varphi_o^{L*}\sum_{i_1=1}^{I_1} v_{i_1}^1 x_{i_1o}^1= 0\\
& \sum_{d_2=1}^{D_2} w_{d_2}^2 z_{d_2o}^2+\sum_{r_2=1}^{R_2} u_{r_2}^2 y_{r_2o}^2-\varphi_o^{1F*}\left(\sum_{i_2=1}^{I_2} v_{i_2}^2 x_{i_2o}^2+\sum_{d_1=1}^{D_1} w_{d_1}^1 z_{d_1o}^1\right)=0\\
& \varphi_o^{1F} \in\left[\varphi_o^{1F\text{-min}} ,M\right]\\
& u_{r_1}^1,u_{r_2}^2,u_{r_3}^3\geq \varepsilon;\ r_1=1,\dots,R_1;\ r_2=1,\dots,R_2;\ r_3=1,\dots,R_3;\\
& v_{i_1}^1,v_{i_2}^2,v_{i_3}^3\geq \varepsilon;\ i_1=1,\dots,I_1;\ i_2=1,\dots,I_2;\ i_3=1,\dots,I_3;\ j=1,\dots,n.\\
& w_{d_1}^1,w_{d_2}^2\geq \varepsilon;\ d_1=1,\dots,D_1;\ d_2=1,\dots,D_2.
\end{aligned}
\tag{17}
$$

In model (17) and by employing formula (13), we increment the value of k_1 to its utmost level, in order to solve the model each time with the new φ_o^{1F}. we resolve the entire the returns of the conditions of the k_1 model and the responses of the model is denoted by $\varphi_o^{12F}(k_1)$. By comparing all the values of $\varphi_o^{12F}(k_1)$, we define φ_o^{12F*}as the minimal efficiency of the total sum of the follower stages from the pessimistic view. It should be noted that, similar to the optimistic approach, we have tested our proposed approach under two conditions and each time have considered a stage as a variable; with due attention to the fact that, the efficiency of a stage is somewhat unique, thereby, the results of these two methods have come to hand with an extremely good approximation and in order to explicate our approach, we have represented one of these two conditions above.

5- Case Study description

In the authentic world, a factory produces three products. This factory has a production area, a warehouse area and a delivery point. We consider each one of these as a stage. The production area plays the role of the "leader" and the other two, the role of "followers". We have contemplated on this factory for within a length of 24 time periods and as a dynamic network. In this network, a number of outputs during a time period of t in the second stage are converted to a number of inputs in the second stage during a time period of t+1. We assume each time period to be a DMU. Hence, the inputs and outputs of each DMU are according to the following. We assign the production costs of the three products produced as an input of the first stage and denote it as (x_1^1, x_2^1, x_3^1). The transport costs for produce from the first to the second stage is described as an undesirable output of the first stage, which we show as y_1^1. The intermediary produce between the first and second stages, is the quantity of produce of each commodity, which is demonstrated as $(z_1^1,\ z_1^2,\ z_1^3)$. The additional outputs for the second stage are respectively, the cost of reserving storage location x_1^2 cost of holding goods x_2^2 and the goods remaining in the warehouse from the previous period, which is illustrated as (x_3^2, x_4^2, x_5^2). We define the output of the second stage as the quantity of the remaining goods in the warehouse for the subsequent period of time, and represent it with (y_1^2, y_2^2, y_3^2). The intermediary products between the second and third stages are the quantities of delivery of each commodity, which is demonstrated by (z_1^4, z_1^5, z_1^6). We describe the additional inputs of the third stage as the transport costs of goods to the third stage and this is illustrated as x_1^3. Finally, the output of the third stage is the profit from the sale of goods, which is indicated by y_1^3. In continuation, we illustrate the input values for the 24 time periods in table 1 and the mean values and outputs in table 2.

Table 1. The inputs of the factory for 24 period in 2016

DMU	Production cost			Cost of reserving storage location	Cost of holding goods	Goods remaining from last period			Cost of Transport goods to delivery points
	x_1^1	x_2^1	x_3^1	x_1^2	x_2^2	x_3^2	x_4^2	x_5^2	x_1^3
1	29120000	36160000	51520000	1700000	1430000	0	0	0	3680000
2	50960000	63280000	77280000	1700000	1430000	0	0	0	6235000
3	80080000	99440000	128800000	1700000	1430000	0	0	0	9915000
4	101920000	126560000	180320000	1700000	1430000	0	0	0	12880000
5	43680000	54240000	77280000	1700000	1430000	0	0	0	5520000
6	50960000	63280000	103040000	1700000	1430000	0	0	0	6645000
7	94640000	126560000	154560000	1700000	1670000	0	0	0	11755000
8	145600000	180800000	257600000	1700000	3620000	0	2	0	15435000
9	145600000	180800000	257600000	1700000	3170000	6	8	4	19115000
10	145600000	180800000	257600000	1700000	1730000	4	6	4	20555000
11	145600000	180800000	257600000	1700000	1430000	0	0	2	19220000
12	145600000	180800000	257600000	1700000	1430000	0	0	0	16815000
13	87360000	99440000	128800000	1700000	1430000	0	0	0	10290000
14	50960000	63280000	77280000	1700000	1430000	0	0	0	6235000
15	50960000	63280000	103040000	1700000	1430000	0	0	0	6645000
16	43680000	54240000	77280000	1700000	1430000	0	0	0	5520000
17	80080000	99440000	128800000	1700000	1430000	0	0	0	9915000
18	94640000	117520000	154560000	1700000	1430000	0	0	0	11755000
19	72800000	90400000	128800000	1700000	1430000	0	0	0	9200000
20	87360000	108480000	154560000	1700000	1430000	0	0	0	11040000
21	87360000	108480000	128800000	1700000	1430000	0	0	0	10630000
22	109200000	135600000	180320000	1700000	3830000	0	0	0	9915000
23	145600000	180800000	257600000	1700000	1430000	8	8	4	22080000
24	145600000	180800000	257600000	1700000	1430000	0	0	0	18400000

In the table 1, the values of (0), for each period indicate that, the goods have not remained in the warehouse since the previous period (columns 7 to 9). The following table 2, also shows values with (0), which illustrate that the goods for the subsequent period have not remained in the warehouse (columns 9 to 11).

Table 2. The outputs and the intermediate measures of the factory for 24 periods in 2016

DMU	Quantity of goods produced			Quantity of goods delivered			Cost of Transport goods to warehouse	Goods remaining for next period			Profit
	z_1^1	z_2^1	z_3^1	z_1^2	z_2^2	z_3^2	y_1^1	y_1^2	y_2^2	y_3^2	y_1^3
1	8	8	4	8	8	4	1960000	0	0	0	31800000
2	14	14	6	14	14	6	3310000	0	0	0	51110000
3	22	22	10	22	22	10	5270000	0	0	0	82910000
4	28	28	14	28	28	14	6860000	0	0	0	111300000
5	12	12	6	12	12	6	2940000	0	0	0	47700000
6	14	14	8	14	14	8	3550000	0	0	0	60190000
7	26	28	12	26	26	12	6460000	0	2	0	98810000
8	40	40	20	34	34	16	9800000	6	8	4	130610000
9	40	40	20	42	42	20	9800000	4	6	4	162410000
10	40	40	20	44	46	22	9800000	0	0	2	177380000
11	40	40	20	40	40	22	9800000	0	0	0	166880000
12	40	40	20	34	40	20	9800000	0	0	0	153510000
13	24	22	10	24	22	10	5430000	0	0	0	83640000
14	14	14	6	14	14	6	3310000	0	0	0	51110000
15	14	14	8	14	14	8	3550000	0	0	0	60190000
16	12	12	6	12	12	6	2940000	0	0	0	47700000
17	22	22	10	22	22	10	5270000	0	0	0	82910000
18	26	26	12	26	26	12	6250000	0	0	0	98810000
19	20	20	10	20	20	10	4900000	0	0	0	79500000
20	24	24	12	24	24	12	5880000	0	0	0	95400000
21	24	24	10	24	24	10	5640000	0	0	0	86320000
22	30	30	14	22	22	10	7230000	8	8	4	82910000
23	40	40	20	48	48	24	9800000	0	0	0	190800000
24	40	40	20	40	40	20	9800000	0	0	0	159000000

In continuation, we secure the efficiency of the factory from a leader-follower scenario. For this purpose, $c_1 = c_2 = 0.6$ and $c_3 = c_4 = 1.05$ are considered as goals of managers. The values of ($\alpha_i = 0$, $i=1,2,3,4$) have come to hand from models (2 and 3) and show that the goals of the managers has been attained. In the leader-follower scenario, on the basis of the opinions of managers, $\Delta\varepsilon = 0.01$ and M=3 have been considered. Similarly, the value for ε in all the models has been considered as 0.05 by the managers. We have executed the heuristic method expressed in the section (4). The values achieved for k_1 together with the maximal optimistic efficiency and the minimal pessimistic efficiency of the first stage has been illustrated in the table 3.

Table 3. Results of the maximum and minimum efficiencies of the first stage and k values

DMU	leader-follower scenario			
	Optimistic View		Pessimistic View	
	k_1	$\theta_o^{1F\text{-max}}$	k_1	$\varphi_o^{1F\text{-min}}$
1	26	0.94507	0	1.05
2	17	0.9691	0	1.05
3	19	0.98283	1	1.06867
4	17	0.9794	5	1.17167
5	20	0.95193	0	1.05
6	18	1	0	1.05
7	17	1	2	1.05053
8	0	1	0	1.08231
9	0	1	0	1.05
10	9	1	0	1.05
11	39	1	7	1.08356
12	26	0.99375	20	1.09597
13	17	1	0	1.05
14	17	0.9691	0	1.05
15	18	1	0	1.05
16	20	0.95193	0	1.05
17	19	0.98283	1	1.06867
18	17	0.9897	2	1.103
19	20	0.96567	2	1.103
20	15	0.97253	3	1.13734
21	19	1	0	1.05
22	0	1	0	1.05
23	40	1	0	1.05
24	33	1	13	1.27467

In studying the values of k, we were aware that, in this case study, that the pessimistic efficiency of the first follower, in most of the cases, is optimized, when the values of k are low (column 4). This signifies that, the optimal efficiency value of the second stage or the first follower are proximate to their minimum values (columns 5), whereas, in the case of the optimistic efficiency value of the second stage, are far from their maximum value, in most circumstances (columns 3). Table 4 gives the overall efficiency and the efficiencies of stages based on the optimistic and pessimistic views.

Table 4. Results based on the optimistic and pessimistic views

DMU	Optimistic View				Pessimistic View			
	$\theta_o^{overall*}$	θ_o^{L*}	θ_o^{1F*}	θ_o^{2F*}	$\varphi_o^{overall*}$	φ_o^{L*}	φ_o^{1F*}	φ_o^{2F*}
1	0.49711	1	0.68507	0.72563	1.1025	1	1.05	1.05
2	0.49315	1	0.7991	0.61713	1.1025	1	1.05	1.05
3	0.50159	1	0.79283	0.63265	1.1326	1	1.07867	1.05
4	0.48928	1	0.8094	0.60449	1.28275	1	1.22167	1.05
5	0.50488	1	0.75193	0.67144	1.1025	1	1.05	1.05
6	0.55827	1	0.82	0.68081	1.1025	1	1.05	1.05
7	0.50264	1	0.83	0.60559	1.12406	1	1.07053	1.05
8	0.72931	1	1	0.72931	1.13643	1	1.08231	1.05
9	0.68483	1	1	0.68483	1.1025	1	1.05	1.05
10	0.61525	1	0.91	0.67609	1.1025	1	1.05	1.05
11	0.36931	1	0.61	0.60542	1.21794	1	1.15356	1.0558
12	0.44027	1	0.73375	0.60002	1.36077	1	1.29597	1.05
13	0.49891	1	0.83	0.60109	1.1025	1	1.05	1.05
14	0.49315	1	0.7991	0.61713	1.1025	1	1.05	1.05
15	0.55827	1	0.82	0.68081	1.1025	1	1.05	1.05
16	0.50488	1	0.75193	0.67144	1.1025	1	1.05	1.05
17	0.50159	1	0.79283	0.63265	1.1326	1	1.07867	1.05
18	0.49219	1	0.8197	0.60045	1.17915	1	1.123	1.05
19	0.51182	1	0.76567	0.66846	1.17915	1	1.123	1.05
20	0.51117	1	0.82253	0.62146	1.22571	1	1.16734	1.05
21	0.48724	1	0.81	0.60153	1.1025	1	1.05	1.05
22	0.79076	1	1	0.79076	1.1025	1	1.05	1.05
23	0.36	1	0.6	0.6	1.1025	1	1.05	1.05
24	0.40516	1	0.67	0.60471	1.4749	1	1.40467	1.05

From the second column of table 4, we note that the efficiency scores of period 22 is the highest and the efficiency scores of period 23 is the lowest, from the optimistic view. The sixth column of Table 4, show that the efficiency scores of period 24 is the highest and the efficiency scores of periods 1,2,5,6,9,10,13,14,15,16,21,22 and 23 is the lowest, from the pessimistic view. By comparing the results, we observe that the difference in optimistic and pessimistic views in some cases, for example, by looking at the second column of table 4, we find that, the efficiency scores of period 10 is higher than period 11 (0.61525 < 0.36931) from the optimistic view. But, from the sixth column of Table 4, it can be noted that, period 11 is higher than period 10 (1.1025 < 1.21794) from pessimistic view. Therefore, for the final ranking of DMUs, we use the double-frontier or the optimistic and pessimistic views that we have explained in the section (3) by formula (7). Table 5 gives the overall efficiency and clustering results based on the double-frontier view.

Table 5. The efficiency evaluation and clustering results based on the double-frontier view

DMU	$\emptyset_o^{overall}$	cluster	DMU	$\emptyset_o^{overall}$	cluster
1	0.74031	2	13	0.74165	2
2	0.73735	2	14	0.73735	2
3	0.75372	2	15	0.78453	2
4	0.79222	2	16	0.74607	2
5	0.74607	2	17	0.75372	2
6	0.78453	2	18	0.76181	2
7	0.75166	2	19	0.77686	2
8	0.91038	1	20	0.79154	2
9	0.86892	1	21	0.73292	2
10	0.82359	2	22	0.9337	1
11	0.67066	3	23	0.63	3
12	0.77401	2	24	0.77302	2

Based on the second and fifth columns of table 5, the performance of 24 DMUs is rated as follows:

$$\mathrm{DMU}_{22} > \mathrm{DMU}_{8} > \mathrm{DMU}_{9} > \mathrm{DMU}_{10} > \mathrm{DMU}_{4} > \mathrm{DMU}_{20} > \mathrm{DMU}_{6} = \mathrm{DMU}_{15} > \mathrm{DMU}_{19} >$$
$$\mathrm{DMU}_{12} > \mathrm{DMU}_{24} > \mathrm{DMU}_{18} > \mathrm{DMU}_{3} = \mathrm{DMU}_{17} > \mathrm{DMU}_{7} > \mathrm{DMU}_{5} = \mathrm{DMU}_{16} > \mathrm{DMU}_{13} >$$
$$\mathrm{DMU}_{1} > \mathrm{DMU}_{2} = \mathrm{DMU}_{14} > \mathrm{DMU}_{21} > \mathrm{DMU}_{11} > \mathrm{DMU}_{23},$$

Where, symbol " $>$ " means that the performance is better than and symbol "$=$ " means that the performance is equal. It should be noted that, in some cases, for example, in DMU_3 and DMU_{17}, the rank of the DMUs are equal. This is due to the fact that the demand, the amount of production of each good, the amount of delivery and maintenance of each good and other item during periods 3 and 17 were absolutely equivalent and this factory has the same performance. The third and sixth columns of table 5 report that units 8,9,22 are in the first cluster. Units 1...7,10,12,13...21, 24 are located in the second cluster. Finally units 11, 23 are located in the third cluster. In the figure 3 we identified three groups of DMUs based on the double-frontier view. Since there are no important differences in the inputs across the three groups, it implies that groups 2 and 3 have all abilities in place, but are poor in executing these capabilities and changing them into high level of performance. Thus, these groups must benchmark themselves against group 1 and identify ways to execute their abilities better. We have put this research at the disposal of the managers, so that the best decisions can be adopted for the abovementioned factory.

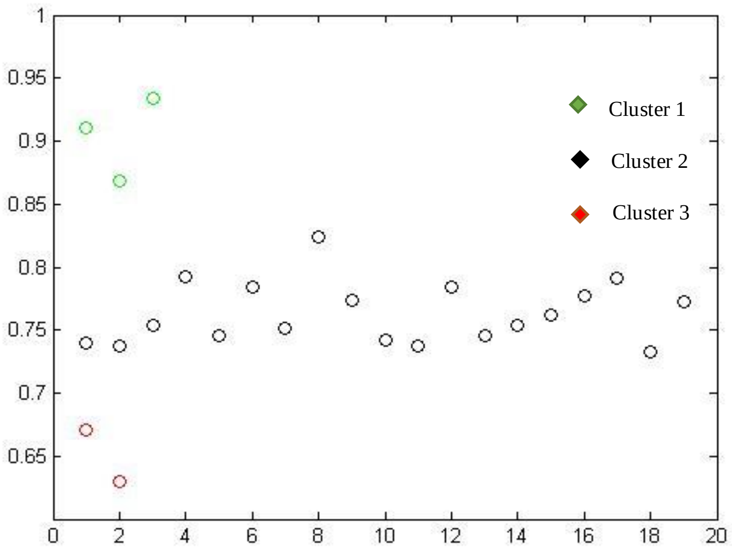

Fig 3. The three groups' classification of the DMUs

6- Conclusions

The black box approach neglects the internal activities of systems and evaluates performance based on the final inputs and outputs. According to the belief of many researchers, this task causes a lack of confidence in the evaluation results. The network DEA models can simulate systems with complex internal structures by using stages and sub-DMUs and then evaluate the overall efficiencies of systems, stages and sub-DMUs, respectively. In this paper, we considered a three-stage network, in respect to the additional desirable and undesirable inputs and outputs. Then, we obtained the efficiency of this network from the non-cooperative approach. In the leader-follower scenario, we considered one stage as the leader and the other two stages together, as a follower. We have made efforts to assist managers in network analysis by utilizing diverse non-cooperative approaches; and also insert the goals of managers in the models. It was for this purpose that we pursued the goal programming concept in the leader-follower scenario; and defined a leader-follower collaboration based on the goals of managers. Due to the fact that, a conclusion implying only one of these two, optimistic or pessimistic views is one-sided and incomplete, so, in this paper we used the double-frontier to analyze the network. Moreover, a heuristic technique was used to convert non-linear models into linear models.

DEA application in relevance to production planning and inventory control has been observed to an extremely slight degree. In this paper, we have contemplated on an example, in the authentic world in the grounds of production planning and inventory control. In this paper, a factory producing dairy products, with a production area, warehouse premises and a delivery point, including the total costs, pertaining to production, storage, warehouse reservation, transport costs from the production area to the warehouse and from the warehouse to the delivery point, as well as the profits from sale of goods have been considered and simulated. This factory has been regarded as a dynamic network with a time period of 24 intervals. The results of the ranking based on double-frontier view showed that, the time periods, (22) and (23) were the best and poorest respectively, in context to the efficiency within 24 phases of time. Similarly, we detected that between the time period of (1) and (24), a fluctuating condition occurred and there was an absence, of a specific system, to alleviate efficiency. We used the k-means clustering algorithm for clustering units into three clusters based on the double-frontier. The clustering results are shown as 3, 19 and 2 units, which are located in the first, second and third clusters, respectively.

The proposed heuristic approach in this paper was performed for two stages and due to the presence of additional inputs and outputs, the stages increase, thereby, making the model more complicated. As a result the solving period is extremely elevated. So as to decrease this period, the ("$\Delta\varepsilon$)" step size can be increased. Hence, the value of the step size ("$\Delta\varepsilon$)" , which determines the accuracy and the time for resolving the problem can be reflected on by managers. We put the results of this research at the disposal of the managers, so that they procure the best decision for the abovementioned factory. Modeling with inaccurate and random data is suggested for research in the future.

References

Amirkhan, M., Didehkhani, H., Khalili-Damghani, K., & Hafezalkotob, A. (2018). Measuring Performance of a Three-Stage Network Structure Using Data Envelopment Analysis and Nash Bargaining Game: A Supply Chain Application. *International Journal of Information Technology & Decision Making*, *17*(05), 1429-1467.

Amirteimoori, A. (2007). DEA efficiency analysis: Efficient and anti-efficient frontier. Applied Mathematics and Computation, 186(1), 10-16.

An, Q., Yang, M., Chu, J., Wu, J., & Zhu, Q. (2017). Efficiency evaluation of an interactive system by data envelopment analysis approach. Computers & Industrial Engineering, 103, 17-25.

Azizi, H. (2014). DEA efficiency analysis: A DEA approach with double frontiers. International Journal of Systems Science, 45(11), 2289-2300.

Azizi, H., & Ajirlu, H. G. (2011). Measurement of the worst practice of decision-making units in the presence of non-discretionary factors and imprecise data. Applied Mathematical Modelling, 35(9), 4149-4156.

Azizi, H., & Wang, Y. M. (2013). Improved DEA models for measuring interval efficiencies of decision-making units. Measurement, 46(3), 1325-1332.

Azizi, H., Kordrostami, S., & Amirteimoori, A. (2015). Slacks-based measures of efficiency in imprecise data envelopment analysis: An approach based on data envelopment analysis with double frontiers. Computers & Industrial Engineering, 79, 42-51.

Badiezadeh, T., & Farzipoor, R. (2014). "Efficiency evaluation of production lines using maximal balance index", International Journal Management and Decision Making, Vol. 13, No. 3, pp. 302-317.

Badiezadeh, T., Saen, R. F., & Samavati, T. (2018). Assessing sustainability of supply chains by double frontier network DEA: A big data approach. Computers & Operations Research, 98, 284-290.

Banker, R. D., Charnes, A., & Cooper, W. W. (1984). Some models for estimating technical and scale inefficiencies in data envelopment analysis. Management science, 30(9), 1078-1092.

Charnes, A., Cooper, W. W., & Rhodes, E. (1978). Measuring the efficiency of decision making units. European journal of operational research, 2(6), 429-444.

Chen, C., & Yan, H. (2011). Network DEA model for supply chain performance evaluation. European Journal of Operational Research, 213 (1): 147–155.

Chen, L. H., Ko, W. C., & Yeh, F. T. (2017). Approach based on fuzzy goal programing and quality function deployment for new product planning. European Journal of Operational Research, 259(2), 654-663.

Cook, W.D., Zhu, J., Bi, G. & Yang, F. (2010). Network DEA: additive efficiency decomposition. European Journal of Operational Research, 207 (2): 1122– 1129.

Dhahri, I., & Chabchoub, H. (2007). Nonlinear goal programming models quantifying the bullwhip effect in supply chain based on ARIMA parameters. European Journal of Operational Research, 177(3), 1800-1810.

Doyle, J. R., Green, R. H., & Cook, W. D. (1995). Upper and lower bound evaluation of multiattribute objects: Comparison models using linear programming. Organizational Behavior and Human Decision Processes, 64(3), 261-273.

Du, J., Zhu, J., Cook, W. D., & Huo, J. (2015). DEA models for parallel systems: Game-theoretic approaches. Asia-Pacific Journal of Operational Research, 32(02), 1550008.

Entani, T., Maeda, Y., & Tanaka, H. (2002). Dual models of interval DEA and its extension to interval data. European Journal of Operational Research, 136(1), 32-45.

Fare, R., & Grosskopf, S. (2000). "Network DEA", Socio Economics Planning Science, Vol. 4, No. 1, pp. 35–49.

Fare, R., Grosskopf, S., Lovell, K., & Pasurka, C., (1989). Multilateral productivity comparisons when some outputs are undesirable: A nonparametric approach. The Review of Economics and Statistics,71, 90–98.

Farrell, M.J. (1957). The Measurement of Productive Efficiency Journal of the Royal Statistical Society Series A (General) 120:253-290.

He, Y., Gao, S., Liao, N., & Liu, H. (2016). A nonlinear goal-programming-based DE and ANN approach to grade optimization in iron mining. Neural Computing and Applications, 27(7), 2065-2081.

Hwang, S. N., Chen, C., Chen, Y., Lee, H. S., & Shen, P. D. (2013). Sustainable design performance evaluation with applications in the automobile industry: Focusing on inefficiency by undesirable factors. Omega, 41(3), 553-558.

Jafarian Moghaddam, A.R., & Ghoseiri, K., (2011). Fuzzy dynamic multi-objective Data Envelopment Analysis model. Expert Systems with Applications, 38, 850–855.

Jahanshahloo, G.R., HosseinzadehLotfi, F., Shoja, N., Tohidi, G., & Razavyan, S. (2005). "Undesirable inputs and outputs in DEA models", Applied Mathematics and Computation, Vol. 169, No. 2, pp. 917–925.

Jahanshahloo, G. R., & Afzalinejad, M. (2006). A ranking method based on a full-inefficient frontier. Applied Mathematical Modelling, 30(3), 248-260.

Jahed, R., Amirteimoori, A., & Azizi, H. (2015). Performance measurement of decision-making units under uncertainty conditions: An approach based on double frontier analysis. Measurement, 69, 264-279.

Jiang, J. L., Chew, E. P., Lee, L. H., & Sun, Z. (2012). DEA based on strongly efficient and inefficient frontiers and its application on port efficiency measurement. OR spectrum, 34(4), 943-969.

Jolai, F., Yazdian, S. A., Shahanaghi, K., & Khojasteh, M. A. (2011). Integrating fuzzy TOPSIS and multi-period goal programming for purchasing multiple products from multiple suppliers. Journal of purchasing and Supply Management, 17(1), 42-53.

Kao, C., & Hwang, S. N. (2008). Efficiency decomposition in two-stage data envelopment analysis: An application to non-life insurance companies in Taiwan. European journal of operational research, 185(1), 418-429.

Kawaguchi, H., Tone, K., & Tsutsui, M. (2014). Estimation of the efficiency of Japanese hospitals using a dynamic and network data envelopment analysis model. Health care management science, 17(2), 101-112.

Kritikos, M. N. (2017). A full ranking methodology in data envelopment analysis based on a set of dummy decision making units. Expert Systems with Applications, 77, 211-225.

Kou, M., Chen, K., Wang, S., & Shao, Y. (2016). Measuring efficiencies of multi-period and multi-division systems associated with DEA: An application to OECD countries' national innovation systems. Expert Systems with Applications, 46, 494-510.

Lee, T., Zhang, Y., & Jeong, B. H. (2016). A multi-period output DEA model with consistent time lag effects. Computers & Industrial Engineering, 93, 267-274.

Li, Y., Chen, Y., Liang, L., & Xie, J. (2012). DEA models for extended two-stage network structures. Omega, 40(5), 611-618.

Liang, L., Cook, W. D., & Zhu, J. (2008). DEA models for two-stage processes: Game approach and efficiency decomposition. Naval Research Logistics (NRL), 55(7), 643-653.

Liu, J., Lu, L., & Wen-Min L.u. (2016). Research fronts in data envelopment analysis, Omega, Volume 58 ,2016, Pages 33-45, ISSN 0305-0483.

Lu, W. M., & Lo, S. F. (2007). A closer look at the economic-environmental disparities for regional development in China. European Journal of Operational Research, 183(2), 882-894.

Parkan, C., & Wang, Y. M. (2000). Worst efficiency analysis based on inefficient production Frontier. Department of Management Sciences, City University of Hong Kong Hong Kong Working Paper.

Ransikarbum, K., & Mason, S. J. (2016). Goal programming-based post-disaster decision making for integrated relief distribution and early-stage network restoration. International Journal of Production Economics, 182, 324-341.

Rezaee, M. J., Izadbakhsh, H., & Yousefi, S. (2016). An improvement approach based on DEA-game theory for comparison of operational and spatial efficiencies in urban transportation systems. *KSCE Journal of Civil Engineering*, *20*(4), 1526-1531.

Seiford, L.M., & Zhu, J. (2002). "Modeling undesirable factors in efficiency evaluation," European Journal of Operational Research, Vol. 142, No. 1, pp. 16–20.

Sengupta, J.K., (1995). Dynamic of Data Envelopment Analysis: Theory of Systems Efficiency. Springer Science & Business Media, Netherlands.

Shabanpour, H., Yousefi, S., & Saen, R. F. (2017). Future planning for benchmarking and ranking sustainable suppliers using goal programming and robust double frontiers DEA. Transportation Research Part D: Transport and Environment, 50, 129-143.

Shafiee, M. (2017). Supply Chain Performance Evaluation With Rough Two-Stage Data Envelopment Analysis Model: Noncooperative Stackelberg Game Approach. *Journal of Computing and Information Science in Engineering*, *17*(4), 041002.

Trivedi, A., & Singh, A. (2017). A hybrid multi-objective decision model for emergency shelter location-relocation projects using fuzzy analytic hierarchy process and goal programming approach. International Journal of Project Management, 35(5), 827-840.

Wang, K., Yu, S., & Zhang, W. (2013). China's regional energy and environmental efficiency: A DEA window analysis based dynamic evaluation. Mathematical and Computer Modelling, 58(5-6), 1117-1127.

Wang, W. K., Lu, W. M., & Liu, P. Y. (2014). A fuzzy multi-objective two-stage DEA model for evaluating the performance of US bank holding companies. Expert Systems with Applications, 41(9), 4290-4297.

Wang, Y. M., & Chin, K. S. (2009). A new approach for the selection of advanced manufacturing technologies: DEA with double frontiers. International Journal of Production Research, 47(23), 6663-6679.

Wang, Y. M., & Lan, Y. X. (2013). Estimating most productive scale size with double frontiers data envelopment analysis. Economic Modelling, 33, 182-186.

Wang, Y. M., Greatbanks, R. and Yang, J.B. Interval efficiency assessment using data envelopment analysis, Fuzzy Sets and Systems, 153(3), (2005), 347–370.

Wu, J., Lv, L., Sun, J., & Ji, X. (2015). A comprehensive analysis of China's regional energy saving and emission reduction efficiency: from production and treatment perspectives. Energy Policy, 84, 166-176.

Wu, J., Zhu, Q., Ji, X., Chu, J., & Liang, L. (2016). Two-stage network processes with shared resources and resources recovered from undesirable outputs. European Journal of Operational Research, 251(1), 182-197.

Yang, X., & Morita, H. (2013). Efficiency improvement from multiple perspectives: An application to Japanese banking industry. Omega, 41(3), 501-509.

Yousefi, S., Soltani, R., Saen, R. F., & Pishvaee, M. S. (2017). A robust fuzzy possibilistic programming for a new network GP-DEA model to evaluate sustainable supply chains. *Journal of Cleaner Production*, *166*, 537-549.

Zhou, H., Yang, Y., Chen, Y., & Zhu, J. (2018). Data envelopment analysis application in sustainability: The origins, development and future directions. *European Journal of Operational Research*, *264*(1), 1-16.

Zhou, X., Luo, R., Tu, Y., Lev, B., & Pedrycz, W. (2018). Data envelopment analysis for bi-level systems with multiple followers. Omega, 77, 180-188.