

A mathematical model for the electric vehicle routing with time windows considering queuing system at charging stations and alternative paths

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Abstract

Due to many damages that human activities have imposed on the environment, authorities, manufacturers, and industry owners have taken into account the development of the supply chain more than ever. One of the most influential and essential human activities in the supply chain is transportation which green vehicles such as electric vehicles (EVs) are expected to generate higher economic and environmental impact. To this end, designing an efficient routing scheme for the fleet of EVs is significant. A remarkable issue about EVs is their need for stations to charge their battery. Due to the existence of time limitations, more attention should be paid to time spent at charging station, so considering the queuing system at charging stations makes more precise time calculations. Furthermore, multigraphs are more consistent with the characteristics of the transportation network. Hence, we consider alternative paths including two criterion cost and energy consumption in the network. First, we develop a mixed integer linear programming for the electric vehicle routing problem on a multigraph with the queue in charging stations to minimize traveling and charging costs. Since the proposed problem is NP-hard in a strong sense, we provide a simulated annealing algorithm to search the solution space efficiently and explore a large neighborhood within short computational time. The efficiency of the model is investigated with numerical and illustrative examples. Then the sensitivity analysis is performed on the proposed model to indicate the importance of the queuing system and the impact of battery capacity on the penetration of EVs.

Keywords: Electric vehicle routing, charging station, queuing system, multigraph, alternative paths, simulated annealing

1-Introduction

One of the most fundamental human-induced damages to the environment is the increase in Green House Gas (GHG) emissions, resulting in global warming, pollution, environmental damages, and animal health risks. Nowadays, attention to the environment and applying environmental laws are important to reduce the impact of human activities on the environment. Further, researchers' interest in environmental issues is growing increasingly. Transportation through the use of fossil fuels has a great impact on increasing GHG emissions and environmental pollution. For instance, transportation accounted for the largest portion (28%) of total U.S. GHG emissions in 2016 (www.epa.gov/greenvehicles/fast-facts-transportation-greenhouse-gas-emissions).

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So that proper routing can be effective in reducing emissions and harming the environment. In the current research, in addition to the main goal of routing, namely finding the most optimal routes, we pay attention to reducing environmental damages and pollution. To this end, we focus on the route planning for the fleet of Battery Electric Vehicles (BEVs) with limited battery capacity. BEVs produce lower air and noise pollution in comparison with Internal Combustion Engine Vehicles (ICEVs) because they do not use any kind of fossil fuel or fuel with limited natural resources. Instead, they move by inserting the battery and using the driving force generated by the electricity. A BEV is propelled by an electric motor and only uses the power provided by its battery pack, which can be charged from the electricity grid. Other advantages of BEVs compared to ICEVs are as follows: Their motors can produce great torque at low speeds and are much more efficient than ICEVs. BEVs also require less maintenance than ICEVs and do not need oil, so maintenance costs are further reduced (Pelletier et al. 2016).

The remainder of the paper is organized as follows: Section 2 discusses the related literature of electric vehicle routing problem (E-VRP) and works on vehicle routing in multigraph. Section 3 describes the problem and formulates the mathematical model. An illustrative example is also clarified in section 3. The proposed SA heuristic is presented in Section 4. Section 5 provides the sensitivity analysis and computational study and discusses the results. Finally, the conclusion and future works direction are given in section 6.

2- Literature review

We divide the literature section into three sub-sections. In the first sub-section, the electric vehicle routing problem is described. In the second part of this section, related works on vehicle routing in multigraph is discussed in more details. Finally, the contributions of the current study are presented in section 2-3.

2-1- Related works on electric vehicle routing

Schneider et al. (2014) focused on BEVs and formulated the electric vehicle routing problem with time windows (E-VRPTW) which seek to minimize the total distance and number of EVs. They also considered full charge strategy and developed a hybrid composed of Variable Neighborhood Search (VNS) and Tabu Search (TS) heuristic for E-VRPTW that makes use of the strong diversification effect of the VNS and involves a TS heuristic to search the solution space efficiently. One of the major barriers to the use of EVs is their limited driving range. The limited driving range means that a fully charged electric vehicle can only move a short distance, which makes the vehicle constantly visits the charging station, and spends a long time to charge its battery. Some factors that temporarily reduce vehicle range include extreme temperatures, high speeds, quick acceleration, carrying heavy loads, and upward slopes (Pelletier et al. 2016). In another study, Bruglieria et al. (2015) designed a Variable Neighborhood Search Branching (VNSB) to solve the E-VRPTW.

In the electric vehicle routing problem, the importance of battery charging stations is remarkable. First, the depot must be equipped to charge the battery, so that the battery is fully charged at night. In the following, the routing process starts with a fully charged battery. This action reduces charging costs. Secondly, in addition to the depot, public charging stations should be established in different locations of the geographic network. One of the main points that differentiate the E-VRP through other vehicle routing models is the calculation of the battery charge level at each node. As a result, if the battery charge level is less than the required amount, the battery charging station should be visited. In particular, it should be planned to charge up to the next station or completion of the entire route before it is discharged. At some stations, in order to reduce the spent time at the station, instead of recharging the battery, the battery pack is swapped. As an example in Yang & Sun (2015), an electric vehicle battery swap station location routing problem (BSS-EV-LRP) has been addressed, which aims to determine the location plan of battery swap stations (BSSs) and the routing plan of a fleet of EVs. They proposed two heuristic method to solve the problem. One heuristic SIGALSN is a four-phase algorithm, including modified sweep heuristic for initialization, iterative greedy method for location sub-problem, adaptive large neighborhood search (ALNS) for routing sub-problem and another one is TS-MCWS which combines TS and modified Clarke–Wright saving method. Hof et al. (2016) extended solution methods for vehicle routing problems with intermediate stops by extending the

adaptive variable neighborhood search algorithm to solve the recently introduced battery swap station location-routing. It should be noted that a plentitude of this type of stations is less than recharging stations.

Shao et al. (2017) proposed another effective way of reducing the time spent on the charging station by the partial charging strategy. Keskin and Catay (2016) considered partial charge strategy for E-VRPTW (E-VRPTW-PR) and aimed to minimize the total travel, waiting and recharging time plus the number of the employed EVs. They proposed the ALNS algorithm with several removal and insertion mechanisms by selecting them dynamically and adaptively based on their past performances, including new mechanisms specifically designed for E-VRPTW and E-VRPTW-PR. Felipe et al. (2014) also considered the possibility of performing a partial recharge at a station, multiple technologies for recharging an EV that implying different recharging time and cost and the cost due to battery amortization in the objective function. In fact, they expanded Green vehicle routing problem, introduced by Erdogan and Miller-Hooks (2012), for EVs and presented constructive and deterministic local search algorithms as well as a metaheuristic extension based on a Simulated Annealing (SA) framework. Bruglieria et al. (2017) implemented a three-phase metaheuristic in which the first two phases are based on mixed integer linear programs to generate feasible solutions and the third one is based on a Variable Neighborhood Search local Branching (VNSB) for the time-effective E-VRP-PR.

Wang et al. (2017) investigated the route choice problem in the traveling and charging of multiple BEVs and three objective functions are proposed to minimize total traveling cost components, including travel times, energy consumption and charging costs. The fuzzy programming approach and fuzzy preference relations are introduced to transform the three objective functions into a single objective function. Sweda et al. (2016) developed efficient algorithms for finding an optimal prior routing and recharging policy and then present solution approaches to an adaptive problem that builds on a priori policy. Wang et al. (2017) develop a modeling framework to optimize electric bus recharging schedules, which determines both the planning and operational decisions while minimizing total annual costs and presented a real-world data as a case study. Agrawal et al. (2016) analyzed the differences between the route choice behaviors of BEVs and ICEVs in a mixed traffic context with the potential for BEV range anxiety. Shao et al. (2017) presented EVRP with charging time and variable travel time where the traffic condition is not constant. They proposed a genetic algorithm to solve the problem.

Goeke and Schneider (2015) addressed an ALNS approach to solving the routing of a mixed fleet of EVs and ICEVs with an expansion of the energy consumption function.

Wang Li-ying and Song Yuan-bin (2015) developed a hybrid heuristic that incorporates an adaptive variable neighborhood search (AVNS) with the tabu search algorithm for the multiple charging station location-routing problem with time windows of EV to optimize the routing plan of capacitated EVs and the strategy of charging stations. The difference between this article and the previous research is to consider the possibility of using various types of infrastructure to charge and optimize the choice of Infrastructure type in accordance with the decision of the location of the station and the routing.

Penna et al. (2016) presented the integration of a multi-start hybrid algorithm based on the Iterated Local Search (ILS) metaheuristic and a set partitioning formulation to solve the electric fleet size and mix vehicle routing problem with time windows and recharging stations (E-FSMFTW). Hiermann et al. (2016) proposed a hybrid heuristic by means of branch-and-price, which combines an ALNS with an embedded local search and labeling procedure for intensification for the E-FSMFTW.

Shengyin Li et al. (2016) dealt a multi-period optimization model for the deployment of public electric vehicle charging stations on the network which is formulated as a mixed integer linear program and solved by a heuristic based algorithm using a genetic algorithm.

In order to highlight the contributions of this research, the contents of mentioned E-VRP studies and other similar studies are summarized in table 1.

Table 1. E-VRP studies.

Authors	Objective function		Fleet of EVs		Type of CSs		Charging strategy		Battery cons. & charging rate		Customer demand		Time window structure		Multi-graph structure	Considering queuing sys.
	Single obj.	Multi obj.	Homo.	Heter.	Chrg. sta.	Swap. sta.	Full	Partial	Constant	Variable	Deter.	Dynamic	Hard	Soft		
Schneider et al.(2014)	*		*		*		*		*		*		*			
Yang & Sun (2014)	*			*	*		*				*					
Bruglieria et al.(2015)	*		*		*			*		*			*			
Felipe et al.(2014)	*		*		*			*		*						
Keskin et al.(2016)	*		*		*			*		*			*			
Li-ying et al.(2015)	*		*		*			*		*			*			
H. Yang et al.(2015)	*		*		*			*		*			*			
Penna et al. (2016)	*			*	*			*		*			*			
Hiermann et al.(2016)	*			*	*			*		*						
Hof et al. (2017)	*			*		*		*				*				
Bruglieri et al.(2017)	*			*	*				*	*			*			
S. Li et al. (2016)	*		*		*			*		*			*			
Roberti et al.(2016)	*		*		*			*	*	*			*			
This paper	*		*		*			*		*			*		*	*

Single obj., Single objective. Multi obj., Multi objective. Homo., Homogenous. Heter., Heterogeneous. Chrg. sta., Battery Cons. & Chrging rate, Battery consumption and charging rate. Deter., Deterministic. Considering queuing sys., Considering queuing system

2-1- Related works on vehicle routing in multigraph

The classic vehicle routing problems are usually designed with one edge (or arc) between two nodes (Setak et al. 2015). However, according to the complexities of urbanism in many transportation networks there exist more than one edge between the nodes. Setak et al. (2015) addressed a new extension of the routing problem which considers more than one edge between the nodes. In their model, the parallel arcs are differentiated with different traffic pattern during the day. They proposed a TS algorithm to solve the problem. Huang et al. (2017) formulated and solved the time-dependent vehicle routing problem with path flexibility (TDVRP-PF) under deterministic and stochastic traffic conditions. In this paper, each arc in the customer graph corresponds to multiple paths in the geographical graph. Having both path flexibility and time-dependent travel time seems to be a good

representation of a wide range of stochasticity and dynamics in the travel time. Setak et al. (2017) extended the time-dependent pollution routing problem in the multigraphs (TDPRPM) problem, which considered more than one edge between nodes, and minimized emitted pollution cost. This problem is also solved by a TS heuristic. Garaix et al. (2010) considered vehicle routing problem for freight or passenger transportation activities with alternative paths where optimization was not done based on a single criterion, but according to the various criterion such as travel time, travel cost, distance and so on. They extended a dynamic programming solution method for this problem and introduced the Fixed Sequence Arc Selection Problem (FSASP) method that efficiently determines the arc selection for the fixed sequence of vertices. Lai et al. (2016) studied the routing problem of heterogeneous vehicles on multigraphs. They considered different criteria such as time, cost, distance, depreciation, fuel consumption, etc. lead to alternative paths. This paper focuses on the development of a metaheuristic solution method based on a TS heuristic and FSASP method. Tikani and Setak (2018) formulated a reliable time-dependent routing problem with time windows in a multigraph based network. They presented efficient methods to solve their proposed model.

2-3- Contributions of the study

Concerning reviewing the related literature, the contributions of this study can be elaborated as follows:

1. We extend the E-VRPTW (proposed by Schneider et al. 2014) according to a special representation of the transportation network introduced in the literature called a multigraph (see Setak et al. 2015). The impact of the multigraph on the solution quality of E-VRPTW is evaluated using different computational experiments.
2. We consider the queuing system in charging stations, so we add the waiting time to time constraints to make more precise time calculations. It may affect the route planning for EVs.
3. The basic of E-VRPTW belongs to the class of NP-hard problems. However, the existence of alternative paths in the network brings more computational challenges to the problem. Herein, we provide an efficient SA algorithm for solving the proposed model in an efficient way.

3- Problem description and model formulation

We formulate the electric vehicle routing problem with time windows considering the queuing system at charging stations and alternative paths as a mixed integer linear programming (MILP). We describe the multigraph representation of the network in subsections 3-1. Queuing system description at the charging station is described in subsections 3-2. Finally, problem modeling is stated in Subsections 3-3.

3-1- Multigraph representation of the network

We consider a multigraph with alternative paths including two criterion distance and cost. Since battery charge consumption function is considered as a linear function in terms of distance, therefore different distance edges have different energy consumption. In other words, by increasing or decreasing distance, the energy consumption of the edge increases (or decreases) and the travel time increases (decreases) in terms of the average speed. Figure 2 shows the representation of a simple graph and a multigraph. As depicted in figure the e_{th} parallel arc between two nodes i and j is represented by (i, j, e_{ij}) .

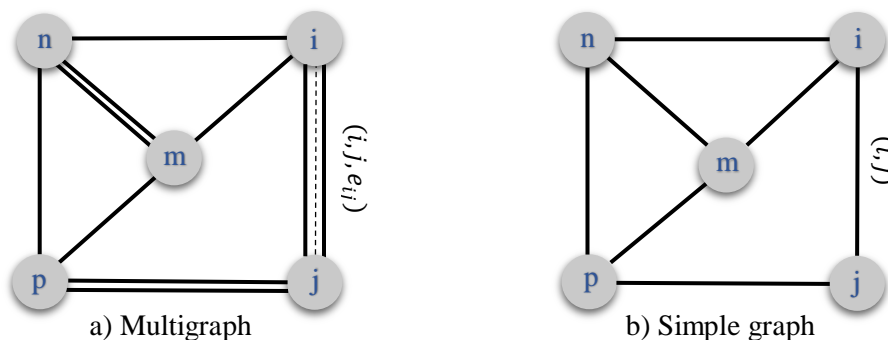


Fig 1. Representation of simple graph vs. multigraph

3-2- Queuing system description at the charging station

The time windows make the problem modeling more difficult, because of that special attention should be given to the time calculation in this problem. As stated, the number of chargers is limited per station and the battery charging time is too long. If the number of EVs is out of the charging station capacity, they should wait in the queue to get service (Qui et al. 2013). Thus considering the queuing system at charging stations causes the problem to be near to the real world data. In this case, due to applying the queuing model indexes to the E-VRPTW mathematical model, the waiting time of customers can be saved (Qui et al. 2013) and total travel time and travel costs can be reduced. In this study the $M / M / s$ queuing system at charging stations is considered. Figure 2 shows a schematic view of the queuing system at the charging station (Said et al. 2013).

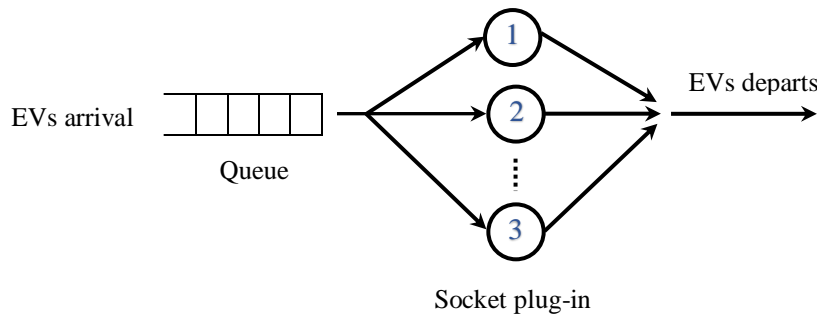


Fig 2. Schematic view of queuing system at the charging station

Yang et al. (2013) investigated the EV charging problem and proposed CS selection algorithms. The travel time from the point of origin to a charging station, queuing time for charging and charging time were considered as the objectives of their model.

3-3- Problem modeling

The vertex set which included depot, customers and charging stations is as follows:

V : Set of customers, $V = \{1, 2, \dots, n\}$

F : Set of charging stations

\hat{F} : Set of dummy vertices generated to permit several visits to each station

\hat{V} : Set of customers and visits to charging stations (Set of customer vertices including visits to recharging stations), $\hat{V} = V \cup \hat{F}$

Vertices 0 and $n+1$ denote the same depot, and every route starts at 0 and ends at $n+1$.

V_0 : Set of customers and start depot, $V_0 = V \cup \{0\}$

V_{n+1} : Set of customers and end depot, $V_{n+1} = V \cup \{n+1\}$

\hat{F}_0 : Set of visits to charging stations and start depot, $\hat{F}_0 = \hat{F} \cup \{0\}$

\hat{F}_{n+1} : Set of visits to charging stations and end depot, $\hat{F}_{n+1} = \hat{F} \cup \{n+1\}$

\hat{V}_0 : Set of customers, visits to charging stations and start depot, $\hat{V}_0 = V \cup \hat{F} \cup \{0\}$

\hat{V}_{n+1} : Set of customers, visits to charging stations and end depot, $\hat{V}_{n+1} = V \cup \hat{F} \cup \{n+1\}$

The problem can be defined on a complete graph $G = (\hat{V}_{0,n+1}, A)$.

A : Set of arcs, $A = \{(i, j, e_{ij}) \mid i, j \in \hat{V}_{0,n+1}, i \neq j\}$

e_{ij} : The e th parallel edge between nodes i and j

E_{ij} : Number of parallel edges between nodes i and j

Each arc has a distance d_{ij}^e , a travel time t_{ij}^e , an average speed v , and a travel cost c_{ij}^e . The total travel cost includes the cost of travel between two nodes and the variable cost. The driver's income, the cost of vehicle and battery depreciation, etc., can be considered as the traveling cost which is calculated per unit of distance. The variable cost is paid for the alternative paths, therefore the path with less distance and travel time should pay the variable cost as a toll charge. In other words, the shorter edge between two nodes in comparison with the original route between them has a more variable cost. Also, c_{cs} is considered as the battery charge cost when referring to the station node.

Each traveled arc consumes the amount $h \cdot d_{ij}^e$ of the remaining battery charge, where h denotes the constant charge consumption rate, so the battery charge consumption function is assumed to be linear. A travel time of each arc is calculated according to equation (1).

$$t_{ij}^e = \frac{d_{ij}^e}{v} \quad (1)$$

A set of homogeneous vehicles with a maximal load capacity of Q and a maximal battery capacity of B is positioned at the depot. In each problem, up to K vehicles are required for routing. Each vertex $i \in \hat{V}_{0,n+1}$ is assigned a positive demand $q_i = \begin{cases} 0 & \text{if } i \notin V \\ q_i & \text{if } i \in V \end{cases}$ and a hard time window $[e_i, l_i]$ in which service should start in this interval, but may end after this interval. All vertices $i \in V_{0,n+1}$ have a service time S_i ($S_0, S_{n+1}=0$). At a charging station, the difference between the current charge level and the battery capacity B is recharged with a charging rate of g . So that the charging time depends on the charge level of the vehicle when arriving at the respective station (Schneider et al. 2014). For simplification reasons, we also assume a linear charging function.

The definition, parameters and required equations for the queuing model of M/M/s at charging stations and the calculation of waiting time in the station are as follows:

The arrival time of an electric vehicle to the station is independent of others. Therefore, electric vehicles visit the charging station according to the Poisson process, and their arrival time is assumed to be negative exponential distribution. Since the battery charge level of each vehicle is different and independent from others, the time required for charging its battery is also different and memory less. Therefore, the battery charge time, in other words, the duration of service at the station also follows a negative exponential distribution. Each charger at the station at any moment can serve only one vehicle. So, if all the servers are busy, other vehicles will wait in line until the service providers are available.

First come First Serve (FCFS) system

System status is the number of EVs in the charging station.

λ : Average arrival rate of EVs

μ : Average service rate of chargers

$ns(i)$: Number of servers (chargers) in i th charging station, $i \in \hat{F}$

$\rho(i)$: Utilization of the i th server; also the probability that the i th server is busy or the probability that someone is being served.

$L_q(i)$: The long-run average number in queue in the i th station

$P_0(i)$: Probability that the system (i th charging station) is empty in the long-run

$W_q(i)$: The long-run average delay in queue per customer in the i th station

$$\rho(i) = \frac{\lambda}{ns(i)\mu} \quad (2)$$

$$P_0(i) = \left[\frac{\left(\frac{\lambda}{\mu}\right)^{ns(i)}}{ns(i)! (1-\rho(i))} + \sum_{v=0}^{ns(i)-1} \frac{\left(\frac{\lambda}{\mu}\right)^v}{v!} + 1 \right]^{-1} \quad (3)$$

$$L_q(i) = \frac{\left(\frac{\lambda}{\mu}\right)^{ns(i)} \rho(i)}{ns(i)! (1-\rho(i))^2} P_0(i) \quad (4)$$

$$W_q(i) = \frac{L_q(i)}{\lambda} \quad (5)$$

The decision variables of the problem define as a follows:

X_{ij}^e : Binary decision variable, $i \in \hat{V}_0, j \in \hat{V}_{n+1}, i \neq j$:

$$X_{ij}^e = \begin{cases} 1 & \text{if one of the vehicles travels through the } e\text{th edge from node } i \text{ to node } j \\ 0 & \text{otherwise.} \end{cases}$$

Y_i : The remaining battery capacity on arrival at vertex $i, i \in \hat{V}_{0,n+1}$

τ_i : The time of arrival at vertex $i, i \in \hat{V}_{0,n+1}$

u_i : The remaining cargo on arrival at vertex $i, i \in \hat{V}_{0,n+1}$

The mathematical model of the problem is as follows:

$$\min \sum_{i \in \hat{V}_0, j \in \hat{V}_{n+1}, i \neq j} \sum_{e=1}^{E_{ij}} c_{ij}^e X_{ij}^e + c_{cs} \sum_{i \in \hat{V}_0, j \in \hat{F}, i \neq j} \sum_{e=1}^{E_{ij}} X_{ij}^e \quad (6)$$

$$\sum_{e=1}^{E_{ij}} X_{ii}^e = 0 \quad \forall i \in \hat{V}_{0,n+1}, \quad (7)$$

$$\sum_{j \in \hat{V}_{n+1}, i \neq j} \sum_{e=1}^{E_{ij}} X_{ij}^e = 1 \quad \forall i \in \hat{V}, \quad (8)$$

$$\sum_{j \in \hat{V}_{n+1}, i \neq j} \sum_{e=1}^{E_{ij}} X_{0j}^e \leq K, \quad (9)$$

$$\sum_{j \in \hat{V}_{n+1}, i \neq j} \sum_{e=1}^{E_{ij}} X_{ij}^e \leq 1 \quad \forall i \in \hat{F}, \quad (10)$$

$$\sum_{i \in \hat{V}_{n+1}, i \neq j} \sum_{e=1}^{E_{ij}} X_{ji}^e - \sum_{i \in \hat{V}_0, i \neq j} \sum_{e=1}^{E_{ij}} X_{ij}^e = 0 \quad \forall j \in \hat{V}, \quad (11)$$

$$\tau_j \geq \tau_i + \sum_{e=1}^{E_{ij}} (t_{ij}^e + s_i) X_{ij}^e - l_0 \left(1 - \sum_{e=1}^{E_{ij}} X_{ij}^e\right) \quad \forall i \in \hat{V}_0, j \in \hat{V}_{n+1} \text{ and } i \neq j, \quad (12)$$

$$\tau_j \geq \tau_i + \sum_{e=1}^{E_{ij}} t_{ij}^e X_{ij}^e + g(B - y_i) + W_q(i) - (l_0 + gB) \left(1 - \sum_{e=1}^{E_{ij}} X_{ij}^e\right) \quad (13) \forall i \in \hat{F}, j \in \hat{V}_{n+1} \text{ and } i \neq j,$$

$$e_j \leq \tau_j \leq l_j \quad \forall j \in \hat{V}_{0,n+1}, \quad (14)$$

$$0 \leq u_j \leq u_i - q_i \sum_{e=1}^{E_{ij}} X_{ij}^e + Q \left(1 - \sum_{e=1}^{E_{ij}} X_{ij}^e\right) \quad \forall i \in \hat{V}_0, j \in \hat{V}_{n+1}, i \neq j, \quad (15)$$

$$0 \leq u_0 \leq Q \quad (16)$$

$$0 \leq y_j \leq y_i - \sum_{e=1}^{E_{ij}} (h \cdot d_{ij}^e) X_{ij}^e + B \left(1 - \sum_{e=1}^{E_{ij}} X_{ij}^e\right) \quad \forall j \in \hat{V}_{n+1}, i \in \hat{V}, i \neq j, \quad (17)$$

$$0 \leq y_j \leq B - \sum_{e=1}^{E_{ij}} (h \cdot d_{ij}^e) X_{ij}^e \quad \forall j \in \hat{V}_{n+1}, i \in \hat{F}_0, i \neq j, \quad (18)$$

$$X_{ij}^e \in \{0,1\}, \quad \forall i \in \hat{V}_0, j \in \hat{V}_{n+1}, i \neq j, \quad (19)$$

The objective function (6) minimizes total traveling and charging costs. Constraint (7) ensures that the origin and destination nodes of each vehicle are not similar. Constraint (8) ensures that each customer vertex has exactly one successor. Constraint (9) determines the maximum number of routes that can be started from the depot. It is equal to the maximum number of EVs. Constraint (10) ensures that each station has at most one successor vertex. Constraint (11) or flow constraint notes that the number of arrivals at a vertex must be equal to the number of departures. Constraint (12) calculates arrival time to each vertex from depot or customers. Constraint (13) calculates arrival time to each vertex from charging station which includes the waiting time in the queue. Constraints (14) enforces that every vertex is visited within its time window. In addition, Constraints (12)–(14) prevent the formation of sub tours. Constraint (15) calculates the remaining cargo on arrival at each vertex. Constraint (16) ensures that total cargo does not exceed the capacity of the vehicle. Constraint (17) calculates the battery charge level on arrival at each vertex. Constraint (18) calculates the battery charge level at the node after the start depot or the station because the battery is fully charged at the start depot or station. Constraints (17) and (18) ensure that the battery never be overcharged. Constraint (19) defines the decision variable.

3-4- Illustrative example

We explain the problem with an illustrative example. We consider three types of problems. The first and second examples present E-VRPTW on a simple graph and a multigraph considering queuing system at stations (See figure 3 and figure 4) and the third one presents the importance of queuing system at charging station (See figure 5). These examples include 10 customer nodes, 3 charging station nodes, and a depot. The distance between nodes is calculated according to their geographic location in the network. Average speed and battery consumption rate is considered to be 1. The maximum number of available EVs is also assumed to be 2. In table 2, the information obtained from solving the examples and the best Solution values are expressed.

In figure 3, E-VRPTW on a simple graph with a specified value of λ and chargers, are implemented. In this example, a low λ value is considered, so that the queuing system does not have any effect on the routing scheme. In figure 4, the preceding example is implemented on a multigraph. In figure 4, between nodes 2 and 5 as well as nodes 7 and 3 due to traffic congestion, two parallel routes are considered. By solving the example, it is found that edge 2 between nodes (2,5) and edge 1 between nodes 3 and 7 is more economical. On the edges 2 and 5, due to the long distance and as a result more energy consumption, moving on a shorter edge but more costly is economical, but in the edges (3,7), the high cost of the edge 2 prevents the selection of edge 2 and the electric vehicle continues its route from edge 1. In example 3, charging stations are visited 3 times and in example 4, twice. Each time visiting the charging station increases the charging cost and travel time because a relatively long time is spent waiting in the queue and charging the battery. Then, by examining examples 3 and 4, it can be concluded that the existence of parallel edges causes different route planning and reduces the objective function (total cost).

In example 5, EVRPTW, as in example 3, is considered on a simple graph. The only difference between them is the value of λ . In example 5, the average arrival rate of EVs is considered to be more so that the necessity to increase the number of chargers at each station and the number of charging stations in the network should be emphasized. Due to the increase in the arrival rate, the waiting time in the queue at the station also increases. Compared to example 3 routing changes due to the existence of time windows, and more charging stations would be visited, so the total battery charge time and cost of the charge would be increased and as a result, the objective function, which is equal to the total cost, would be increased. As can be seen in figure 5, each station can be visited more than once.

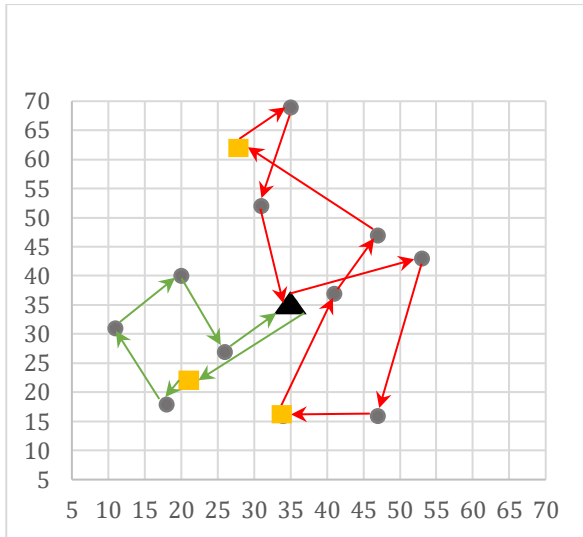


Fig 3. Example solution of E-VRPTW on a simple graph

▲ Depot ■ Charging station ● Customer

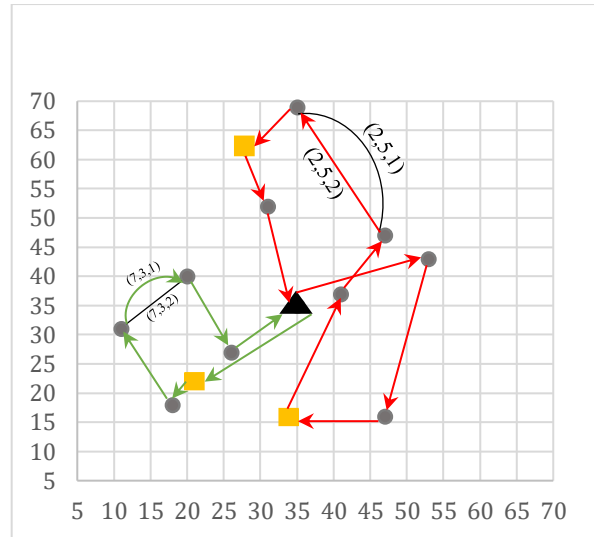


Fig 4. Example solution of E-VRPTW on a simple graph

▲ Depot ■ Charging station ● Customer

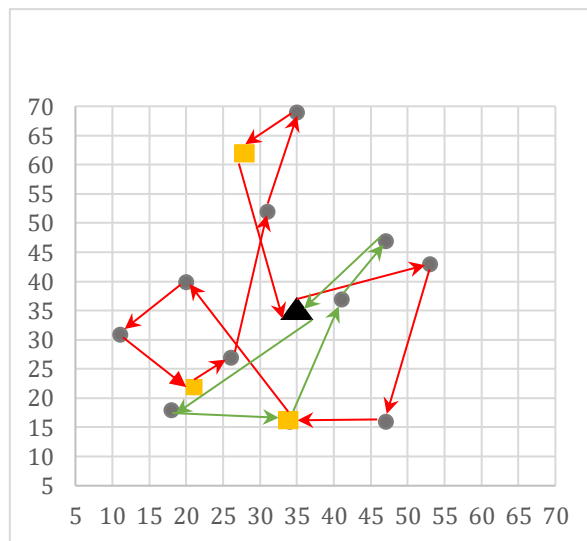


Fig 5. Example solution of E-VRPTW on a simple graph with emphasis on queuing system.

▲ Depot ■ Charging station ● Customer

As stated, each example has a different routing scheme. That means the queue and alternative paths affect the routing and total cost.

Table 2. The resulting information of example 4, 5 and 6

Example 4	1. Route	Vertex	D	C.1	C.8	CS.3	C.4	C.2	CS.1	C.5	C.6	D		
	2. Route	Vertex	D	CS.2	C.10	C.7	C.3	C.9	D					
	Total cost		256.147											
Example 5	1. Route	Vertex	D	C.1	C.8	CS.3	C.4	C.2	C.5	CS.1	C.6	D		
	2. Route	Vertex	D	CS.2	C.10	C.7	C.3	C.9	D					
	Total cost		250.477											
Example 6	1. Route	Vertex	D	C.1	C.8	CS.3	C.3	C.7	CS.2	C.9	C.6	C.5	CS.1	D
	2. Route	Vertex	D	C.10	CS.3	C.4	C.2	D						
	Total cost		313.080											

4- Solution procedure based on SA heuristic

The proposed MILP model can be used to solve small-scale instances and a simulated annealing algorithm is provided to solve medium-scale and large-scale problem instances. The aim of a metaheuristic algorithm is to explore the solution space efficiently, without undercounting all the solutions. In this section, we propose a SA algorithm to solve the proposed E-VRPTW considering the queuing system and alternative paths.

Simulating annealing approach has been proposed by Kirkpatrick et al. (1983), and independently by Cerny (1985) for optimization problems. It is one of the commonly used metaheuristics and has been successfully applied to solve several types of VRP (Breedam (1995); Chiang and Russell (1996); Osman (1993); Bent and Hentenryck (2004); Tavakkoli-Moghaddam et al. (2006); Tavakkoli-Moghaddam et al. (2011); Afifi et al. (2013); Vincent et al. (2017)).

SA has also been applied to the green vehicle routing problem in different studies. For example, Xiao et al. (2012), Kassem and Chen (2013), Yasin and Vincent (2015).

In fact, SA is based on the analogy with the behavior of physical annealing processes in solids (Johnson et al. 1989; Yang et al. 2005). This method uses a stochastic approach to search for solutions and move to neighborhood solutions. If a neighborhood solution is better, the current solution S will be replaced by neighborhood solution S_{new} . However, in SA, moving to a worse neighborhood solution would be accepted with a certain probability according to a random number, shown in equation 21, in order to avoid being trapped in local optimum space.

$$\Delta Z = Z_{S_{new}} - Z_S \quad (20)$$

$$p = e^{\left(-\frac{\Delta Z}{T}\right)} \quad (21)$$

The accepted probability is based on two parameters T and ΔZ called the temperature, and gradually reduces, or cools, in the search process. At the beginning of the search, the temperature is higher, thus the accepted probability of the move is higher. However, when nearing the end of the search process, the temperature is reduced, and the accepted probability of the move is smaller. Moreover, if the neighborhood solution is much worse than the current solution, the accepted probability of the move will be smaller. Therefore, the initial temperature, the cooling function, and the final temperature will affect the results of SA (Yiyo Kuo, 2010).

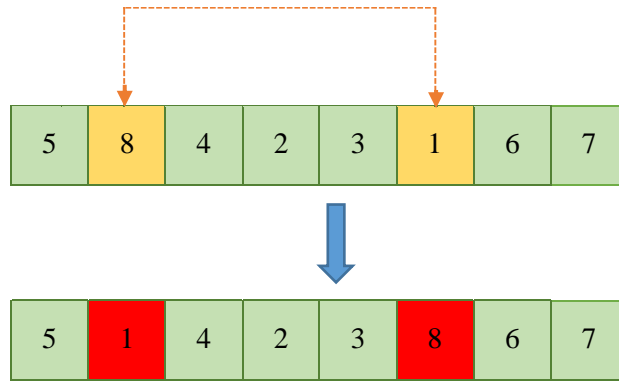
Note that in each iteration, the temperature is gradually decreased in terms of the positive ratio, as well as the search process continues until the temperature is smaller or equal to the final ratio. The steps of the proposed SA algorithm are described in algorithm 1.

Algorithm 1. Proposed SA algorithm for E-VRPTW

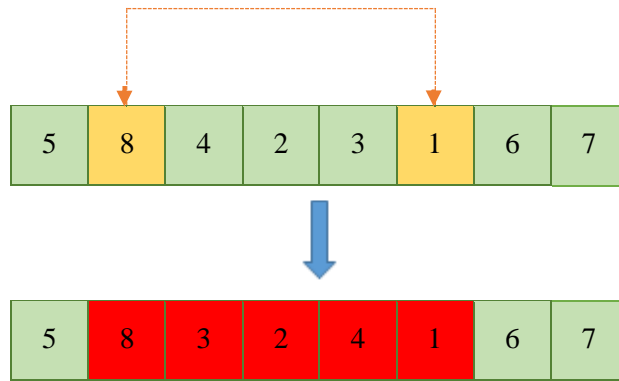
- Step 1:** Randomly choose a sequence of customers as an initial solution S . Each solution can be divided into K routes by splitters which equals one unit less than the number of EVs.
 - Step 2:** Set $\text{Iteration} = \text{Iteration} + 1$ and calculate the cost function of the initial solution.
 - Step 3:** Generate neighborhood solutions S_{new} randomly and calculate the cost function of the neighborhood solution.
 - Step 4:** If $\Delta Z \leq 0$
 $S = S_{new}$
else
set $S = S_{new}$, with probability p
end
 - Step 5:** Choose the best solution with the best cost function and set $x^* = x$ as the best solution and $f(x^*)$ as the best cost.
 - Step 6:** Update T .
 - Step 7:** If a stopping condition is met (T is smaller or equal to the final ratio) then stop. Else go to Step 1.
-

4-1- Neighborhood structures

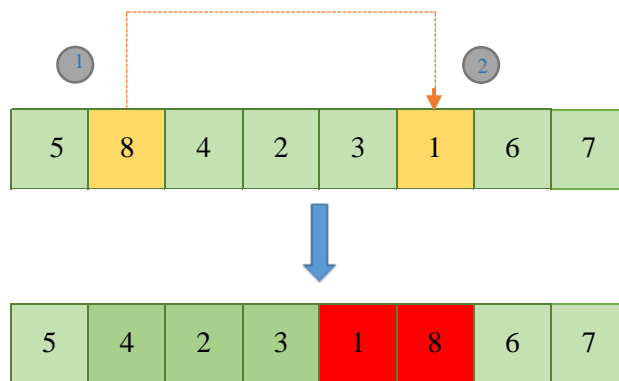
In this study, three kinds of neighborhood search are exerted in the initial solution: (i) the swap operator select a pair of customer nodes in the current solution and exchange their position in the sequence. (ii) The reversion operator chooses a pair of customer nodes and reverses the sequence between these two nodes. (iii) The insertion operator by preserving the order of the two selected nodes removes the first node from its position and then inserts it in position after the second node. Figure 6 shows an illustration of the operators. In any inner iteration, the neighborhood structure is randomly selected. For this purpose, numbers 1 - 3 are assigned to operators, and for each structure, one of these numbers is randomly generated and the related operator is applied to the current solution.



(i) Illustrative of the swap operator



(ii) Illustrative of the reversion operator



(iii) Illustrative of the insertion operator

Fig 6. Representation of neighborhood searches

4-2- Penalized objective function

We evaluate a solution by the following penalized cost function:

$$f(S) = Z(1 + \alpha MeanCV(S) + \beta MeanTWV(S)) \quad (22)$$

Where Z denotes the total cost, $MeanCV(S)$ the mean of capacity violation, $MeanTWV(S)$ the mean of the time windows violation and α, β are factors for weighting the violations.

4-3- Unfeasibly condition check in the proposed SA

A solution is infeasible if one of the following conditions holds:

K : Number of routes in the solution

R_k : Subset of all vertices in the k th route

$$k \in K \wedge \sum_{i \in R_k} q(i) > Q \quad (23)$$

$$k \in K, i \in R_k \setminus \{\hat{F}\}, j \in R_k \wedge \tau(i) + s(i) + t^e(i, j) < e(j) \quad (24)$$

$$k \in K, i \in R_k \setminus \{\hat{F}\}, j \in R_k \wedge \tau(i) + s(i) + t^e(i, j) > l(j) \quad (25)$$

$$k \in K, i \in R_k \setminus \{V_{0,n+1}\}, j \in R_k \wedge \tau(i) + s(i) + t^e(i, j) + Wq(i) < e(j) \quad (26)$$

$$k \in K, i \in R_k \setminus \{V_{0,n+1}\}, j \in R_k \wedge \tau(i) + s(i) + t^e(i, j) + Wq(i) > l(j) \quad (27)$$

$$k \in K \wedge \sum_{i \in R_k, j \in R_k} h. d_{ij}^e > B \quad (28)$$

Equation (23) refers to violations of the EVs capacity. If the total demand of the route is higher than EVs capacity, this route can be labeled infeasible. Equations (24)-(27) refer to violations of the time windows. In equations (26) and (27) the service time refers to battery charging duration. Equation (28) is based on EVs battery capacity violation. If total charge consumption of the route is higher than EVs battery capacity and charging station cannot be inserted in the route, this route can be labeled as infeasible.

The following equations calculate the constraint violations.

$$\forall k \in K, TC(k) = \sum_{i \in R_k} q(i) \quad (29)$$

$$CV(k) = \max\left(\frac{TC(k)}{Q} - 1, 0\right) \quad (30)$$

$$MeanCV = \text{mean}(CV) \quad (31)$$

The capacity violation of the route $k \in K$ is calculated by equation (30) and the total capacity penalty of the solution S is calculated by mean of the individual violations of all routes as equation (31) (Tikani and Setak, 2019).

$$\forall i \in R_k, TWV(i) = \max\left([0, 1 - \frac{\tau(i)}{e(i)}, \frac{\tau(i)}{l(i)} - 1\right] \quad (32)$$

$$MeanTWV = \text{mean}(TWV) \quad (33)$$

The time windows violation of the route $k \in K$ and the total time windows penalty of the solution S is calculated by equations (32) and (33) respectively.

4-4-The process of insertion or removing a charge station into a solution

In the following, we describe how to calculate the total cost of a solution and how to insert a charging station if needed or remove it. First, solution S is divided into K routes. Then, $TC(k)$ and $CV(k)$ for $k \in K$, $\tau(i)$ for $i \in R_k$ and the total violation of capacity and time windows $MeanCV$ and $MeanTWV$ are calculated. Here is the process of calculating the total cost of VRP. If the value of $MeanCV$ and $MeanTWV$ are zero, the solution is feasible and the process continues otherwise, the total cost is considered as the total cost of the infeasible solution. Further, VRP should be converted to E-VRP. To this end, after confirming the feasibility of route $k(k \in K)$, the total charge consumption (TCC) of the route is calculated, if it is less than the capacity of the battery, the path will be released in the same way and at the same cost, else the required charging station number (NCS) is calculated.

We insert these stations respectively. The first station is inserted between the pairs of nodes considering the following hints:

- 1) The charge consumption from origin to the first station is less than battery capacity.
- 2) The constraint of time windows is met.
- 3) The cost of the detouring is at least possible.

Other charging stations are inserted considering the following hints:

- 1) The charge consumption from the previous station to this one is less than battery capacity.
- 2) The constraint of time windows is met.
- 3) The cost of the detouring is at least possible.

If one of the above hints is not met, the charging station is removed from the route.

After inserting NCS number of the charging station, the total charge consumption between the last station and the end depot is calculated. If this value of a solution is higher than battery capacity, a new charging station is inserted according to the above.

Figure 7 shows a schematic of insertion and removal process of E-VRP (Schneider et al. 2014).

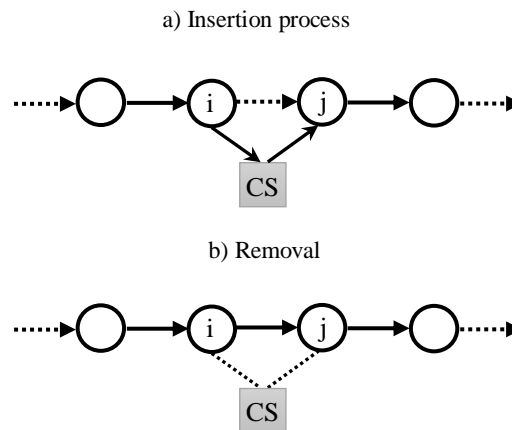


Fig 7. Insertion and Removal of a charging station

Finally, the total cost of the route is calculated by adding the cost of detouring and the charging cost to the $C(k)$.

4-5- Handling the parallel links in the proposed SA

The above description is used for solving an E-VRPTW on a simple graph. For converting the graph to a multigraph, we randomly generate the sequence of positive integer numbers which is smaller or equal to the number of parallel edges. Each member of the sequence represents the parallel edge of the pairs of nodes. Therefore the value of distance, time and charge consumption between pairs of

nodes is calculated based on the related sequence. Figure 8 shows a schematic of the selection process of parallel edges in the proposed approach.

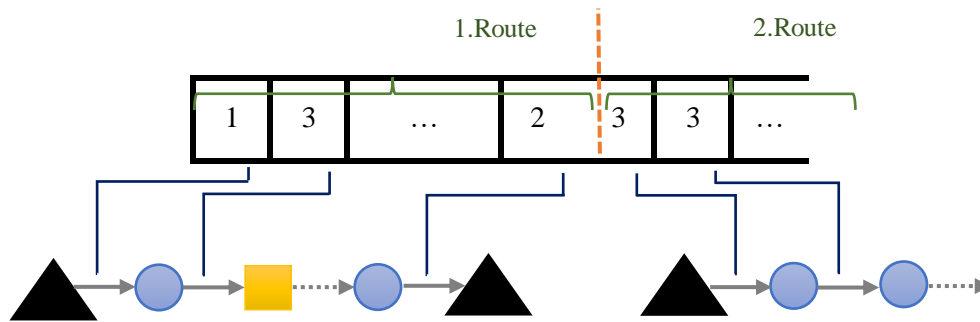


Fig 8. Selection process of parallel edges

5- Computational study

In this section, we express the computational results obtained from solving instances with different scales and sensitivity analysis on some parameters of the mathematical model.

5-1- Numerical results

We perform the presented model on a computer equipped with an Intel Core i5 processor with 2.5 GHz speed and 4 GB RAM and operating on Windows 8.1. We also analyze the performance of a SA algorithm on the small-scale instances solved by using GAMS software version 23.5.1 with CPLEX solver. The value of parameters related to the E-VRPTW is addressed in Schneider et al. (2014). The parameters of the queuing system and alternative paths are presented in table 3 and table 4 respectively.

Yang. J. et al. (2017) calculated the average battery charge time at public charging stations for the various power of charging devices and average daily service rates. According to the data of this paper, the average service rate is considered to be 2. Due to the queuing system stability, the value of $\rho(i)$ should be less than 1. Otherwise, the system is unstable and the queue length is infinite. Moreover, the average arrival rate of EVs is randomly generated, so that it should be satisfied the system stability and also the waiting time at charging station does not exceed the time windows of station and depot nodes.

Table 3. Queuing system parameter values for the numerical experiments.

Parameter	Description	Values
$ns(i)$	Number of chargers	Uniform[20,35], Integer
λ	Average arrival rate	Uniform $[\frac{\mu. \min ns(i)}{2}, \mu. \min ns(i)]$, Integer

According to table 4, we solve three types of instances which are different from the perspective of alternative paths. The main data of instances are produced in Schneider et al. (2014). It should be noted that instances of type 1, 2 and 3 have one, two and three parallel edges between pairs of nodes, respectively. Also, each type of instances has specific parallel edges distance and variable cost. Moreover, the pairs of nodes are randomly chosen.

Table 4. Alternative paths parameter values for the numerical experiments

Ins. type	# of parallel edges	d^1	d^2	c^1	c^2	c_{cs}
1	1	d^1	$0.5d^1$	d^1	$d^2 + 0.65d^1$	5
2	2	d^1	$0.6d^1$	d^1	$d^2 + 0.42d^1$	5
3	3	d^1	$0.4d^1$	d^1	$d^2 + 0.70d^1$	5

We assume the average speed of EVs on each arc as a constant value which is addressed in Schneider et al. (2014). The charging cost is also assumed to be 5. The results of solving different scaled instances in the simple graph and multigraph are presented in Table 5.

We found the best solution for all instances based on the number of customers (C), the number of Charging stations (CSs) and the number of EVs required (K). Furthermore, the best solution value is calculated by selecting the minimum number of vehicles. The obtained answers for simple graphs are more or equal to multigraphs. Because in simple graphs, due to the long distance of edges, the battery consumption is much higher. $\% \Delta f$ shows a percentage of cost reduction. It implies that using a multigraph network may decrease the total cost of EVs routing. Moreover, visiting the station increases the charging cost as well as the objective function. Multigraphs may reduce the number of visits to the charging station.

Table 5. Numerical results

Inst.	data	Inst. type	#of C	#of CSs	#of K	λ	Best Solution				Time(s)*		%Δf
							GAMS		SA		GAMS	SA	
							S.	M.	S.	M.	M.		
1	c103c5	1	5	1	2	30	177.97	177.97	177.97	177.97	0.56	2.2	0
2	c101c5	1	5	2	3	31	257.15	245.23	257.15	245.23	0.27	3.9	4.64
3	rc204c5	3	5	3	2	38	190.16	183.34	190.16	183.34	0.43	5	3.59
4	r202c5	2	5	2	2	28	147.65	147.65	147.65	147.65	0.48	1.9	0
5	rc105c5	3	5	4	2	37	256.3	247.04	256.3	247.04	1	2.6	3.61
6	r103c10	2	8	2	2	37	157.91	150.89	157.91	150.89	0.62	4.3	4.45
7	r203c10	2	8	4	1	45	217.06	217.06	217.06	217.06	20.2	4.42	0
8	rc204c5	3	8	2	2	29	349.84	232.16	349.84	232.16	5.99	4.6	33.63
9	r201c10	2	10	3	2	25	256.15	250.48	256.15	250.48	5.49	8.8	2.14
10	c202c10	1	10	4	2	34	305.69	303.47	305.69	303.47	6.8	6.17	0.73
11	r103c10	2	10	2	3	41	211.87	200.63	211.87	200.63	20.5	12.2	5.31
12	rc205c10	3	10	3	2	36	349.79	344.75	349.79	344.75	1.43	9.16	1.44
13	c104c10	1	10	4	2	27	288.93	284.29	288.93	284.29	10.8	9.5	1.61
14	c208c15	1	12	3	2	23	310.28	310.28	310.28	310.28	4.73	8.31	0
15	r103c15	2	12	4	3	20	315.57	315.57	315.57	315.57	5.1	9.41	0
16	rc103c15	2	12	4	4	29	404.55	402	404.55	402	42	11.6	0.63
17	c208c15	1	15	4	2	41	315.55	315.55	315.55	315.55	12.1	19.8	0
18	rc103c15	3	15	5	4	22	417.67	412.05	417.67	412.05	574	22.3	1.35
19	c202c15	1	15	5	2	35	408.62	408.62	408.62	408.62	954	38.11	0
20	r102c15	2	15	7	5	32	459.34	449.50	459.34	449.50	721	31.7	2.14
21	c101-21	2	18	5	3	47	-	-	196.1	191.13	-	16.07	2.53
22	c101-21	3	20	5	3	41	-	-	206.99	201.8	-	24.27	2.5
23	c202-21	1	20	4	2	43	-	-	217.88	198.06	-	26.06	9.1
24	c101-21	3	22	5	3	41	-	-	222.16	222.16	-	37.18	0
25	c101-21	3	24	6	4	36	-	-	253.8	251.03	-	49.9	1.09
26	c101-21	2	26	5	4	48	-	-	255.18	255.18	-	117.5	0
27	c202-21	1	26	3	3	40	-	-	264.49	264.49	-	24.32	0
28	c101-21	3	28	6	4	39	-	-	259.18	256.41	-	91.8	1.07
29	c202-21	3	30	4	3	40	-	-	277.67	277.67	-	40.79	0
30	rc108-21	3	30	3	3	29	-	-	423.08	419.12	-	134.6	0.94
Avg.									279.15	271.19	-	-	2.86

* t(min)

5-2- Sensitivity analysis

In order to illustrate the impact of some parameters, sensitivity analysis is performed in this section. The sensitivity analysis of the waiting time in the queue (see equation (5)) in terms of service rates and different arrival rates are displayed in figure (9) and figure (10). We consider 20, 30 and 40 number of the server at each station as well as 2 and 1 per hour for service rates.

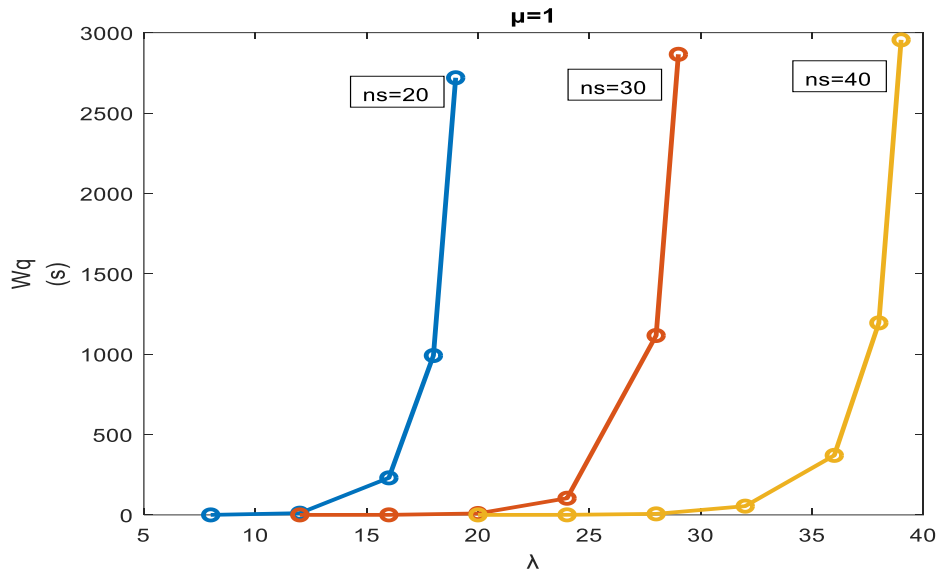


Fig 9. The waiting time in the queue at the charging station for the service rate of 1

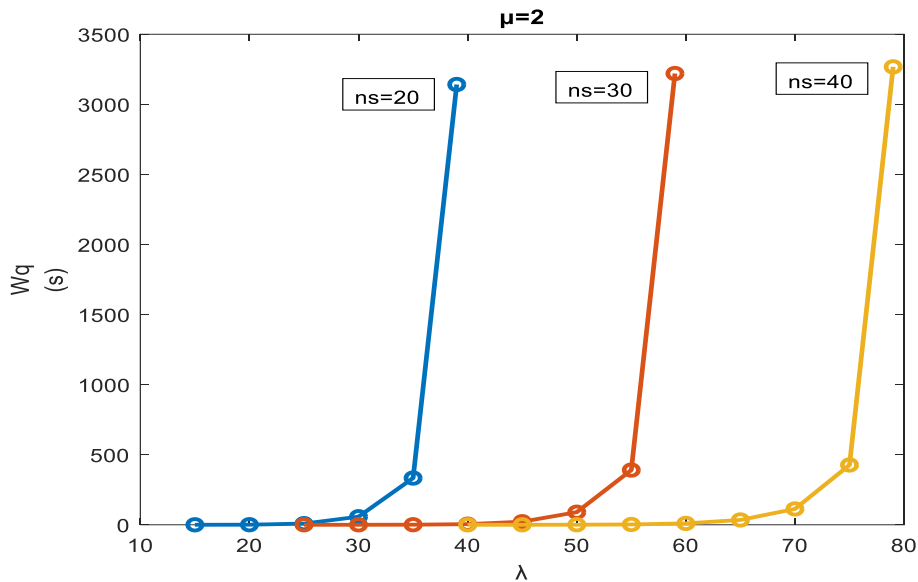


Fig 10. The waiting time in the queue at the charging station for the service rate of 2

According to the above figures, in order to a specific number of server, the higher service rate can service the higher value of λ , so that the stability of the queuing system is retained. Also, for the specified λ , the waiting time in the queue in figure (10) is less than figure (9), since its service rate is higher. As the popularity of these vehicles increases, the arrival rate is expected to rise in the near future. So the number of stations and charging devices in each station and service rates should be increased to allow the system to meet demand. As shown in figures (9) and figure (10), by increasing the average arrival rate of EVs, at a fixed service rate, the number of charging devices (chargers) should be increased. Moreover, by increasing the power of the charging devices, the charging time of the battery and as a result, the service rate will be increased, so that the system will be able to service more vehicles.

In figure (11), the sensitivity analysis of the EV battery capacity and its effect on the objective function are shown. The higher battery capacity improves the driving range of EV, so the charging cost is reduced. Therefore, special attention should be paid to the battery industry in order to prepare high quality and high capacity batteries.

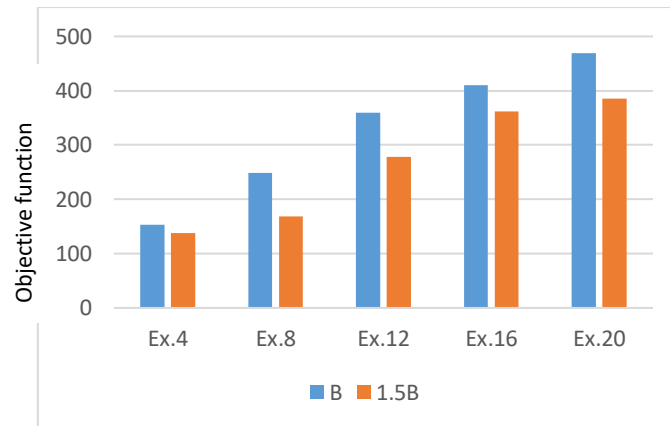


Fig 11. The effect of battery capacity on the objective function

6- Conclusion and future works

We propose the modeling of electric vehicle routing on multigraphs (distance-cost/energy consumption/cost) considering the queuing system in battery charging stations. Then the proposed model was solved by CPLEX solver and SA algorithm for the instances of different scales and the results were presented in a table. The demand for EVs is expected to rise in the near future. Therefore the importance of investigating issues related to them is also remarkable. One of the advantages of this research is the use of a queuing system in battery charging stations. Nowadays, one of the most important weaknesses in these vehicles is the shortage of their charging stations in the network. So considering the queuing system in modeling shows the importance of increasing the number of stations and the power of charging devices, as well as the effort to reduce the battery charge time. Another advantage of this paper is the use of multigraph which keeps available alternative paths with several criteria in the network. To this end, we develop E-VRPTW with alternative paths based on two criteria of energy consumption and cost because multigraphs are more consistent with transport networks. So these two cases bring the E-VRPTW model closer to reality. The proposed directions for future research are as follows:

1. Proposing an exact algorithm like benders algorithm or column generation method to solve the large instances in an acceptable computational time.
2. Incorporating the decisions about the location of charging stations in the proposed model.
3. Utilizing two types of battery charging stations and swapping stations in the model.
4. Developing useful research in the field of battery charging and consumption rates.

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