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An economic-statistical model for production and maintenance planning under adaptive non-central chi-square control chart

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Abstract

Most of the inventory control models assume that quality defect never happens, which means production process is perfect. However, in real manufacturing processes, the production process starts its operation in the in-control state; but after a period of time, shifts to the out-of-control state because of occurrence of some disturbances. In this paper, in order to approach the model to real manufacturing conditions, a process is considered in which quality defect and machine deterioration may occur. Since the adaptive control charts detect the occurrence of assignable cause quicker than the traditional control charts, an adaptive non-central chi-square control chart is designed, which monitors the process mean and variance, simultaneously. In addition, to reduce the failure rate of the machine, two types of maintenance policies consisting of reactive and preventive are planned. Then, the particle swarm optimization algorithm is employed to minimize the overall cost per cycle involving inventory cost, quality loss cost, inspection cost and maintenance cost subject to statistical quality constraints. Finally, to demonstrate the effectiveness of the suggested approach, two comparative studies are presented. The first one confirms that integration of production planning, maintenance policy and statistical process monitoring leads to a significant increase in the cost savings. The second one indicates superiority of the developed adaptive control chart in comparison with the control chart with the fixed parameters.

Keywords: Production planning, maintenance policy, economic-statistical design, non-central chi-square chart, adaptive control chart.

1-Introduction

The economic production quantity (EPQ) model has been widely investigated in recent years. Most of the existing models were made based on this assumption that all of the produced items are of perfect quality. However, in real conditions the operating state of production process may shift from the in-control state to the out-of-control state due to occurrence of an assignable cause that lead to quality defect.

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This issue results in increasing the costs of production system due to increment of producing non-conforming items. In order to reduce these costs, the quality control and the maintenance planning must be considered in the production system. In the other word; inventory control, quality control, and maintenance are three interrelated operational factors affecting the performance of manufacturing systems. Despite of the significant interaction effect among inventory control, quality control and maintenance, there are few studies in which these three interrelated fields are investigated, simultaneously. In this regard, Porteus (1986) added some assumptions to the classical EPQ model to make a more realistic model. Rosenblatt and Lee (1986) investigated the effects of producing defective items and machine deterioration on the optimal production cycle time.

Some researchers suggested the integration of inventory control and quality control. Rahim (1994) presented a model for finding the optimal economic production quantity and control chart parameters by considering the occurrence of a non-Markovian shock and an increasing failure rate. Rahim and Ben-Daya (1998) expanded the model proposed by Rahim (1994) for situations where the production process stops during the investigating false alarms. Pan et al (2011) introduced an integrated model for determining the economic production quantity and designing the control chart. Rivera-Gómez et al (2013) considered a process in which the machine deteriorations and the failure rate increases over time continuously. Alamri et al (2016) considered an imperfect production process and developed a EOQ model. In that model, a 100 percent screening process of the lot is conducted and the percentage of defective items per lot reduces according to a learning curve. It is should be noted that one good example of manufacturing systems, which is common in practice, is imperfect robotic systems. Accordingly, Mohamadi et al (2011), Foumani et al (2017, a), Foumani et al (2017, b) and Foumani et al (2015) concentrating on inspection cost and quality loss cost in the imperfect robotic systems.

Boukas and Liu (2001) studied the production and maintenance planning for a manufacturing system with a failure prone machine. They considered three operating states consisting of good; average and bad; as well as a failure state for the machine. Gharbi and Kenne (2003) addressed the issue of the production planning in a process with multiple machines subject to breakdowns and repairs. Jiang et al. (2015) suggested a production model for a deteriorating system integrated with maintenance strategy. Some researchers focused on relationship between quality control and maintenance such as Tagaras (1988), Ben-Daya and Rahim (2000), Xiang (2013), Ardakan et al (2015).

A few number of researchers proposed models in which three fields of inventory control, quality control and maintenance are considered, simultaneously. Ben-Daya and Makhdoum (1998) presented an integrated model for determining the economic production quantity and control chart design parameters. Then, they studied the effects of implementing various maintenance policies on that model. Lam and Rahim (2002) developed an integrated model for joint optimization of production run length, control chart parameters, and maintenance policies. Pan et al (2012) considered an imperfect production process and combined the concepts of EPQ, quality control and maintenance in a unified model. Bouslah et al (2016) considered an imperfect production system is in which the machine deteriorations over time and then optimized the sampling plan, make-to-stock production and preventive maintenance, simultaneously. Nourelfath et al (2016) proposed a model based on integration of production, maintenance, and quality for an imperfect process in a lot-sizing context ignoring the statistical properties of the process. Salmasnia et al (2017) integrated design of production run length, maintenance policy and control chart by assuming that occurrence of assignable cause only leads to mean shift while variance of quality characteristic remains unchanged.

When an assignable cause occurs in the process, the process mean, process variance or both of them may change. Joint monitoring of the process mean and variance was rarely considered in models that integrate three concepts of inventory control, quality control, and maintenance. Rahim and Ohta (2005) developed a joint optimization model of economic production quantity and control chart parameters in which shift in both the process mean and variance is considered. The characteristics of the existing methods in the literature are summarized in table 1.

The $\bar{X}$ control chart is usually used for monitoring the process mean and the R chart is applied for monitoring the process variability, especially when the sample size is small. In the previous studies, the joint $\bar{X}$ and R chart has been extensively applied for monitoring the process mean and variability simultaneously (see Jones and Case, 1981; Saniga, 1989; Rahim, 1989; Costa, 1993; Rahim and Costa, 2000; Ohta et al, 2002). Costa and Rahim (2004) developed a non-central chi-square (NCS) control chart and demonstrated this chart operates more efficient than the $\bar{X}$ and R charts in detecting assignable cause(s) which change the process mean and/or increase the process variability.

The control charts with fixed parameters are slow in detecting small and moderate shifts and also are not economically attractive. There are other types of control chart that are faster in detecting small and moderate shifts and have lower costs than the traditional control charts. In these charts that are called adaptive control charts, at least one of the design parameters consisting of sample size (n), sampling interval (h), and the coefficient of control limits (l) are allowed to change in time depending on the value of statistic in the previous sample. Reynolds et al (1988) first introduced the variable sampling interval (VSI) control chart. After that, the adaptive control chart with variable sample size (VSS) was proposed by Prabhu et al (1993) and Costa (1994) separately. Prabhu et al (1994) investigated the chart with variable sample size and sampling interval (VSSI). Finally, Costa (1998) and Costa (1999b) proposed control charts in which all of the design parameters (n, h, l) are variables. This chart was called variable parameters (VP) control chart. Costa and De Magalhaes (2007) studied the variable parameters NCS control chart and conclude that the use of this chart not only is simpler than the joint $\bar{X}$ and R chart with variable parameters, but also has a better performance in shift detection when the process mean and variance may shift from their initial values.

In this study, to fill the research gaps and overcome the above-mentioned drawbacks, an integrated model with the following properties is presented:

1. This study integrates three interrelated concepts of inventory, maintenance and statistical monitoring in a unified model.
2. In contrast to the most of existing approaches in the literature that assumes the process variance remains unchanged when an assignable cause occurs, the proposed method monitors both the process mean and variability using a non-central chi-square control chart.
3. In contrast to most of the existing models that employ the control charts with fixed parameters, this study uses a control chart with the variable parameters to monitor the process. This means that the sample size, sampling interval, warning and control limits varies according to the current statistic value.
4. The proposed method aims to optimize the expected total cost subject to statistical constraints.

The rest of this paper is structured as follows: In section 2, the assumptions and notations of the production process are presented and the VP non-central chi-square control chart is briefly reviewed. Section 3 contains the methodology for joint optimization of production run length, control chart design parameters, and decision related to maintenance. In section 4, a solution algorithm to optimize the suggested mathematical programming is described. Section 5 provides a numerical illustration for showing the application of the proposed model. Next, two comparison studies are presented to show the superiority of the proposed model compared to the state-of-the-art.

Table1. Literature review

Paper	Optimization model			Optimization characteristics		Control chart			Shift in	
	Inventory control	Quality control	Mainte nance	Cost	Statistical properties	Control chart has not been used	Non-adaptive control chart	Adaptive control chart	Mean	variance
Rosenblatt and Lee (1986)	✓			✓		✓				
Rahim (1994)	✓	✓		✓			✓		✓	
Rahim and Ben-Daya (1998)	✓	✓		✓			✓		✓	
Rahim and Ohta (2005)	✓	✓		✓	✓		✓		✓	✓
Pan et al (2011)	✓	✓		✓			✓		✓	
Rivera-Gomez et al (2013)	✓	✓		✓		✓				
Alamri et al (2016)	✓	✓		✓		✓				
Boukas and Liu (2001)	✓		✓	✓		✓				
Gharbi and Kenne (2003)	✓		✓	✓		✓				
Tagaras (1988)		✓	✓	✓			✓			
Ben-Daya and Rahim (2000)		✓	✓	✓			✓		✓	
Xiang (2013)		✓	✓	✓			✓		✓	
Ardakan et al (2015)		✓	✓	✓			✓		✓	
Ben-Daya and Makhdoum (1998)	✓	✓	✓	✓			✓		✓	
Lam and Rahim (2002)	✓	✓	✓	✓			✓		✓	
Pan et al (2012)	✓	✓	✓	✓	✓		✓		✓	
Bouslah et al (2016)	✓	✓	✓	✓		✓				
Nourelfath et al (2016)	✓	✓	✓	✓		✓				
This paper	✓	✓	✓	✓	✓			✓	✓	✓

2-Problem description

As mentioned earlier, traditional EPQ model aims to minimize holding and set up costs with this assumption that the production process is perfect, which means the non-conforming items never produce. In this paper, to make the model more adapted to real production conditions, an imperfect production process including two states of in-control and out-of-control is considered. The imperfect process begins its operation in the in-control state, which means the quality characteristic under consideration follows a normal distribution with mean μ_0 and standard deviation σ_0. As time progresses, an assignable cause (such as operator errors, undesirable materials, changes in the environment, etc) may occur that leads to a shift in either the process mean ($\mu_1 = \mu_0 + \delta\sigma_0$) or the process variability ($\sigma_1 = \gamma\sigma_0$), or in both together. In other words, the process situation changes from in-control state to the out-of-control state. In this situation, the quality loss cost that is imposed to the manufacturer dramatically growths due to producing more non-conforming items. To reduce the out-of-control period, a non-central chi square control chart with the variable parameters is designed that monitors the process mean and variability simultaneously.

In the imperfect production process, three conditions may occur for a production cycle. Condition1 occurs if the process remains in the in-control state from the starting of the cycle until the end. In such situation, a perfect preventive maintenance activity is conducted on the process at the end of the production cycle, which results in restoration of the process to as-good-as new condition. Condition 2 is a situation in which an assignable cause occurs and control chart issues a true alarm during production cycle. In this moment, the production process is stopped and a reactive maintenance (RM) is implemented to detect and remove the assignable cause. Condition 3 is similar to condition 2 with this difference that control chart does not issue an alarm until the end of the production cycle. When scenario 3 happens, at the end of the production cycle, the process shift is identified and PM activity is replaced with a CM activity.

2-1-Notations

The notations applied to constructing the proposed model are presented in table 2.

Table 2. Notations in alphabetical order

Notation	Description
a	Scale parameter of the Weibull distribution
A	Startup cost
ARL_0	Average run length in the in-control state
ARL_1	Average run length in the out-of-control state
ATS_0	Average time to signal in the in-control state
ATS_1	Average time to signal in the out-of-control state
ATS_l	Acceptable lower bound for ATS_0
ATS_u	Acceptable upper bound for ATS_1
b	Shape parameter of the Weibull distribution
B	Holding cost per time unit
C_F	Fixed cost of sampling
C_P	Preventive maintenance cost
C_R	Reactive maintenance cost
C_V	Variable cost of sampling
C_Y	Cost of false alarm investigation
d	Daily demand rate
D	Annual demand

Table 2. Continued

Notation	Description
e_l	The difference between l^{th} sample mean and the target value for the process mean
E	Time needed to record each sample
$E(M)$	Expected maintenance cost
$E(Q)$	Expected quality loss cost
$E(S)$	Expected inspection cost
$E(T_{in}\|C_1)$	Expected time that the process is in the in-control state in condition 1
$E(T_{in}\|C_2)$	Expected time that the process is in the in-control state in condition 2
$E(T_{in}\|C_3)$	Expected time that the process is in the in-control state in condition 3
$E(T_{out}\|C_2)$	Expected time that the process is in the out-of-control state in condition 2
$E(T_{out}\|C_3)$	Expected time that the process is in the out-of-control state in condition 3
ETC	Expected total cost
f_1	Proportion of taken samples during in-control period when using loose control limits
f_2	Proportion of taken samples during in-control period when using strict control limits
h_1	Longer inspection interval
h_2	Shorter inspection interval
k	Number of sampling points in a production cycle when one of the conditions 1 or 3 happen
l_1	Coefficient used in determining the loose control limit
l_2	Coefficient used in determining the strict control limit
n_1	Small sample size
n_2	Large sample size
p	Daily production rate
p_1	Proportion of time spent during the in-control period when using loose control limits
p_2	Proportion of time spent during the in-control period when using strict control limits
P	Annual production
$P(C_1)$	Occurrence probability of condition 1
$P(C_2)$	Occurrence probability of condition 2
$P(C_3)$	Occurrence probability of condition 3
$P(sig)$	Probability of issuing a signal by control chart
Q_1	Quality loss cost per unit in the in-control state
Q_2	Quality loss cost per unit in the out-of-control state
s_1	Expected number of samples in the in-control period when using lose control limits
s_2	Expected number of samples in the in-control period when using strict control limits
T	Production cycle time
T_1	Time needed to search an assignable cause
UCL_1	Upper control limit when using lose control
UCL_2	Upper control limit when using strict control
UWL_1	Upper warning limit when using lose control
UWL_2	Upper warning limit when using strict control
w_1	Coefficient used in determining the loose warning limit
w_2	Coefficient used in determining the strict warning limit
Y_l	The statistic of the chi-square control chart
α_1	Probability of Type I error when using lose control
α_2	Probability of Type I error when using strict control
β_1	Probability of Type II error when using lose control
β_2	Probability of Type II error when using strict control
λ_{01}	Non-centrality parameter in the in-control state when using lose control
λ_{02}	Non-centrality parameter in the in-control state when using strict control

Table 2. Continued

Notation	Description
λ_{11}	Non-centrality parameter in the out-of-control state when using lose control
λ_{12}	Non-centrality parameter in the out-of-control state when using strict control
μ_0	The mean of the process in the in-control state
μ_1	The mean of the process in the out-of-control state
σ_0	The Standard deviation of process in the in-control state
σ_1	The Standard deviation of process in the out-of-control state
τ_1	Expected time between last taken sample in the in-control period until the occurrence of an assignable cause when using lose control
τ_2	Expected time between last taken sample in the in-control period until the occurrence of an assignable cause when using strict control

2-2-Model assumptions

The used assumptions in order to model the introduced problem are as follows:

1. The time that the process remains in the in-control state follows a Weibull distribution with shape parameter b and scale parameter a.
2. The quality characteristic of interest follows a normal distribution with mean μ_0 and standard deviation σ_0 when the process is in the in-control state. However, the occurrence of an assignable cause leads to changing the process mean from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0$, where $\delta \neq 0$, and/or shifting the process variability from σ_0 to $\sigma_1 = \gamma\sigma_0$, where $\gamma \neq 1$.
3. Each production cycle starts its operation in the in-control state.
4. A production cycle terminates either with a true alarm or after $(k + 1)$ thsampling interval.
5. If an assignable cause occurs in the l^{th}sampling interval $(0 < l < k)$, a search process is performed to detect the assignable cause and then reactive maintenance is accomplished to remove the effect of the occurred assignable cause.
6. The samples are taken independently.
7. The annual production (daily production rate) is assumed to be greater than the annual demand (daily demand rate).

2-3-Non-central chi-square control chart with variable parameters

Assume that the quality characteristic of process,x, is normally distributed in the in-control state with mean μ_0 and standard deviation σ_0. As time progresses, the process goes to the out-of-control state along with occurring an assignable cause. In this situation, the process mean changes from μ_0 to $\mu_1 = \mu_0 + \delta\sigma_0$ with $\delta \neq 0$, and/or the process variability shifts from σ_0 to $\sigma_1 = \gamma\sigma_0$ with $\gamma \neq 1$.

Consider x_{lj}, $l = 1,2,3, \dots$, $j = 1,2, \dots, n_i$ is the j^{th} observed value of quality characteristic x in l^{th} sampling of size n_i, $i = 1,2$ and $\bar{X}_l = (x_{l1} + x_{l2} + \cdots + x_{li})/n_i$ is the mean of sample l. Hence, the difference between l^{th} sample mean and the target value for process mean is $e_l = (\bar{X}_l - \mu_0)$. For l^{th} sample, the non-central chi-square control chart statistic is obtained according to Equation (1).

$$Y_l = \sum_{j=1}^{n_i} \left(x_{lj} - \mu_0 + \xi_l\sigma_0\right)^2 \qquad l = 1,2, \dots \tag{1}$$

where the parameter ξ_l as a function of the value of error e_l is defined as $\xi_l = \begin{cases} d & if \quad e_l \geq 0 \\ -d & if \quad e_l < 0 \end{cases}$.

During the in-control period, Y_l / σ_0^2 has non-central chi-square distribution with n_i degrees of freedom and non-centrality parameter $\lambda_{0i} = n_i d^2$. Hence, the probability of type I error is calculated as:

$$\alpha_i = P\left[Y_l > l_i\sigma_0^2 \mid Y_l / \sigma_0^2 \sim \chi^2_{n_i}(n_i d^2)\right] \qquad i = 1,2 \tag{2}$$

where l_i is the coefficient used in determining the upper control limit.

During the out-of-control period, Y_l / σ_1^2 has non-central chi-square distribution with n_i degrees of freedom and non-centrality parameter $\lambda_{1i} = n_i(\delta + \xi_l)^2/\gamma^2$. Hence; the power of the control chart is as follows:

$$1 - \beta_i = P\left[Y_l > l_i\sigma_0^2 \mid Y_l / \sigma_1^2 \sim \chi^2_{n_i}(n_i(\delta + \xi_l)^2/\gamma^2)\right] \qquad i = 1,2 \tag{3}$$

As mentioned before, since the adaptive control charts detect small and moderate shifts quicker than the control charts with the fixed parameters, this study designs a non-central chi-square control chart with variable parameters in which all of the design parameters are allowed to change depending on the position of the current sample point on chart. To do this, the interval $[0, UCL]$ in the chart is partitioned in to two regions $[0, UWL]$ and $[UWL, UCL]$, which are called central region and warning region, respectively. The UCL and UWL are defined as follows:

$$UCL_i = l_i\sigma_0^2 \qquad i = 1,2 \tag{4}$$

$$UWL_i = w_i\sigma_0^2 \qquad i = 1,2 \tag{5}$$

where $w_i \leq l_i$ .Thus, this study considers two strategies for sampling as follows: (1) Loose control that uses (n_1, h_1, w_1, l_1) as control chart parameters and (2) Strict control that the parameters values in this strategy are (n_2, h_2, w_2, l_2), where $n_1 < n_2$, $h_1 > h_2, w_1 > w_2$ and $l_1 > l_2$. Based on the position of the current sample on the control chart, one of these two strategies is chosen in the next sampling. If the previous sample is plotted in the central region, likely the process remains in the in-control state until next sampling and we can use the loose control. In the loose control, sample with smaller size (n_1) after a longer time interval (h_1) is collected from the process and the related statistic is plotted on the control chart with loose warning and control limits (w_1, l_1). On the contrary, if a sample point is placed on the warning region, there are evidences that the process may shift to an out-of-control state. In this case, the strict control strategy is applied in the next sampling. In this strategy, sample of larger size (n_2) after a shorter time intervals (h_2) is taken from the process and the strict warning and control limits (w_2, l_2) are used.

The NCS chart with variable parameters (VP) can be constructed with two scales to simplify implementation of the chart. The left side scale is used for loose control strategy and the right scale is applied for the strict control strategy. By designing the control chart in this way, the worker can avoid the use of two separate control charts. The values of Y related to the loose control strategy are plotted on the left side and the values of Y related to the strict control strategy are plotted on the right side. In order to coincide the warning and the control limits for both loose and strict control strategies, the scales on the left side and on the right side are related piecewise linearly. The graphical representation of the introduced VP non-central chi-square chart is illustrated in figure1.

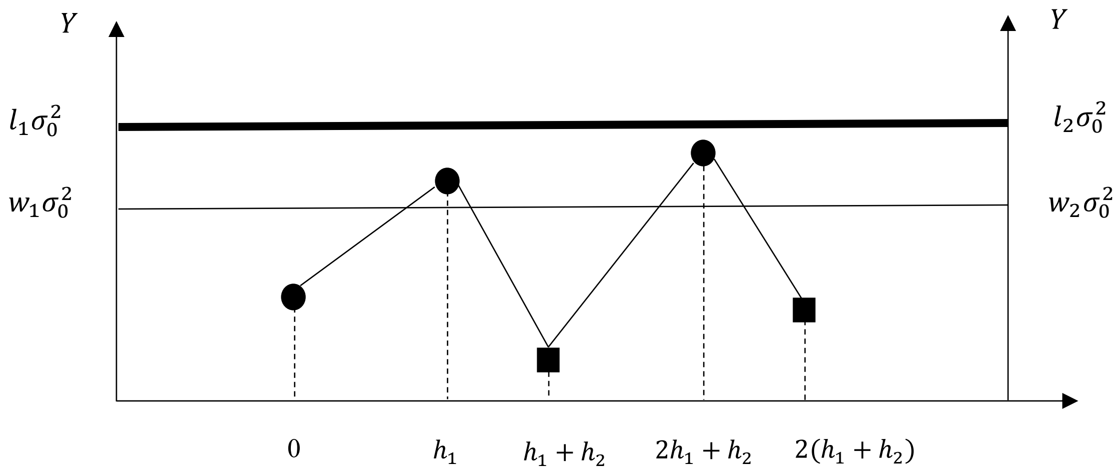


Fig 1. The adaptive NCS control chart

At the beginning of the process or after each false alarm, the sampling strategy is selected randomly. f_1 and f_2 are considered as proportion of samples during the in-control period using the loose and strict control strategies, respectively. Moreover, the proportions of time spent during the in-control period using the loose and strict control strategies can be calculated as equations (8) and (9).

$$f_1 = P\left[Y_l < w_i\sigma_0^2 \mid Y_l < l_i\sigma_0^2;\ Y_l/\sigma_0^2 \sim \chi_{n_i}^2(n_i d^2)\right] \qquad i = 1,2 \tag{6}$$

$$f_2 = 1 - f_1 \tag{7}$$

$$p_1 = \frac{f_1 h_1}{f_1 h_1 + f_2 h_2} \tag{8}$$

$$p_2 = \frac{f_2 h_2}{f_1 h_1 + f_2 h_2} \tag{9}$$

3-Economic-statistical model

In the production process described in the previous Section, three different conditions might happen based on the occurring time of the assignable cause. In this Section, these conditions are explained and the costs associated with the model in each condition are calculated. These costs include the startup cost, inventory holding cost, inspection cost, maintenance cost, and the cost of producing non-conforming items.

3-1- Condition 1 (C_1)

In this condition, the process remains in the in-control state during the production cycle and a planned preventive maintenance is performed at the end of $(k+1)^{th}$ sampling interval. Therefore, the expected in-control time is equal to summation of $(k+1)$ sampling interval and the expected out-of-control time is zero. Moreover, the happening probability of this condition can be calculated according to equation (11). A given production cycle in this condition is illustrated in Figure 2.

$$E(T_{in}|C_1) = (k+1)h_1 \times p_1 + (k+1)h_2 \times p_2 \quad (10)$$

$$P(C_1) = 1 - [F((k+1)h_1) \times f_1 + F((k+1)h_2) \times f_2] \quad (11)$$

where F(.) is the cumulative function of Weibull distribution.

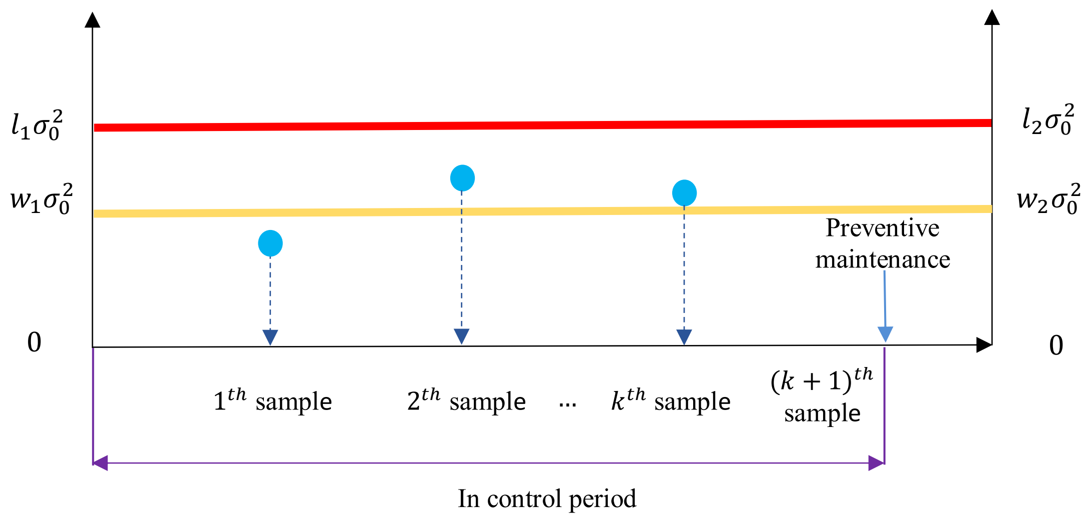


Fig 2. Graphical representation of a production cycle in condition 1

3-1-1-Inspection cost

The inspection cost includes the fixed sampling cost and variable sampling cost. In condition 1, the production process continues until $(k+1)^{th}$ sampling interval, when a planned PM activity is implemented. Since in the $(k+1)^{th}$ interval no inspection is performed, the number of sampling is equal to k. So, inspection cost in condition 1 is calculated as follows:

$$C_{S1} = [(C_F + C_V n_1)f_1 + (C_F + C_V n_2)f_2] \times k \quad (12)$$

where C_F and C_V are the fixed cost and variable cost of sampling, respectively.

3-1-2-Quality loss cost

In condition 1, the production cycle only contains the in-control period; so in this condition, the quality loss cost is equal to the costs of producing non-conforming items in the in-control state.

$$C_{Q1} = Q_1 P \times E(T_{in}|C_1) \quad (13)$$

where Q_1 is the quality loss cost per time unit when the process is in the in-control state and P represents the production rate.

3-1-3-Maintenance cost

Although in condition 1 no shift occurs and the production process remains in the in-control state throughout the cycle, the false alarm signals may issue and the workers can't often distinguish these false signals from a correct one. So, the maintenance cost in this condition includes cost of false alarm investigation and cost of preventive maintenance implementation. Based on these explanations the maintenance cost in this condition is:

$$C_{M1} = \left(\frac{kC_Y}{ARL_{01}} \times f_1 + \frac{kC_Y}{ARL_{02}} \times f_2\right) + C_P \tag{14}$$

In this equation, C_P represents the preventive maintenance cost and the first part of this Equation is the cost of investigating false alarms in which C_Y is the cost of inspection each false alarm and ARL_{01} and ARL_{02} are the average run length in the in-control state when using loose control and strict control strategies, respectively.

$$ARL_{01} = \frac{1}{\alpha_1} \tag{15}$$

$$ARL_{02} = \frac{1}{\alpha_2} \tag{16}$$

3-2-Condition 2 (C_2)

In condition 2, as shown in figure 3, the production process starts its operation in the in-control state until in a time between j^{th} and $(j+1)^{th}$ sampling interval, which an assignable cause occurs. At this time, the mean of process shifts from μ_0 to $\mu_0 + \delta\sigma_0$ and/or the standard deviation of the process changes from σ_0 to $\gamma\sigma_0$. Unfortunately, the occurrence of assignable cause can't be detected immediately in the next sampling by the control chart due to type II error. So, the process continues until $(j+i)^{th}$ sampling that a signal is alerted and at this time a reactive maintenance is performed.

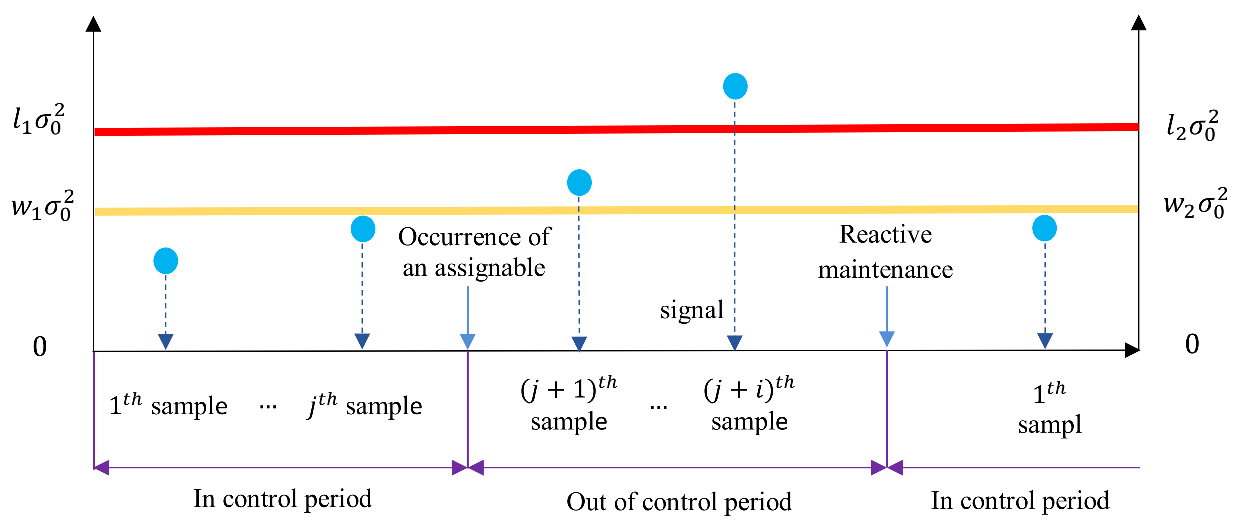


Fig 3. Graphical representation of a production cycle in condition 2

In this condition, the production cycle consists of both in-control and out-of-control periods. As mentioned before, on one hand, the time to shift follows a Weibull distribution; on the other hand, the assignable cause occurs prior the $(k+1)^{th}$ sampling interval in condition 2. Consequently, the in-control time in this condition follows a truncated Weibull distribution in the interval $[0,(k+1)h]$ according to Equation (17) and the expected in-control time can be calculated as Equation (18):

$$f(t|(k+1)h) = [(b/a)(t/a)^{b-1}e^{-(t/a)^b}]/(1-e^{-(t/a)^{(k+1)h}}) \tag{17}$$

$$E(T_{in}|C_2) = \left(\int_0^{kh_1} tf(t|(k+1)h_1)dt\right) \times p_1 + \left(\int_0^{kh_2} tf(t|(k+1)h_2)dt\right) \times p_2 \tag{18}$$

The out-of-control time in condition 2 consists of the following periods:

a) The expected time from occurring the shift in the process until detecting the assignable cause by control chart, which is calculated as $ATS_1 - \tau$.

It is reminded that τ represents the expected time between last taken samples in the in-control period until the occurrence of an assignable cause. If the loose control strategy is used for the last sample before the occurrence of an assignable cause, τ is obtained from the following Equation:

$$\tau_1 = \int_0^{(k+1)h_1} tf(t|(k+1)h_1)\ dt - h_1\left(\sum_{j=1}^{k} e^{-\left(\frac{jh_1}{a}\right)^b} - ke^{-\left(\frac{(k+1)h_1}{a}\right)^b}\right) \tag{19}$$

Similarly when using the strict control strategy, τ can be calculated as follows:

$$\tau_2 = \int_0^{(k+1)h_2} tf(t|(k+1)h_2)\ dt - h_2\left(\sum_{j=1}^{k} e^{-\left(\frac{jh_2}{a}\right)^b} - ke^{-\left(\frac{(k+1)h_2}{a}\right)^b}\right) \tag{20}$$

Hence, τ is attained according to Equation (21).

$$\tau = \tau_1 p_1 + \tau_2 p_2 \tag{21}$$

Moreover, ATS_1 is the average time to signal in the out-of-control state that is calculated as follows:

$$ATS_1 = (ARL_{11} \times h_1)p_1 + (ARL_{12} \times h_2)p_2 \tag{22}$$

where ARL_{11} and ARL_{12} are the average run length in the out-of-control state when using the loose and strict control strategies, respectively.

$$ARL_{11} = \left(\frac{1}{1-\beta_1}\right) \tag{23}$$

$$ARL_{12} = \left(\frac{1}{1-\beta_2}\right) \tag{24}$$

b) The time required to test sample and interpret the result. This time equals to $(n_1 f_1 + n_2 f_2) \times E$, where E represents the expected time to sample.

c) The expected time for searching the assignable cause that T_1.

Thus, the expected out-of-control time in condition 2 is as Equation (25).

$$E(T_{out}|C_2) = ATS_1 - \tau + (n_1 f_1 + n_2 f_2) \times E + T_1 \quad (25)$$

The probability of occurrence this condition is calculated as follows.

$$P(C_2) = [F(kh_1) \times f_1] \times P(sig_1) + [F(kh_2) \times f_2] \times P(sig_2) \quad (26)$$

where $F(.)$ is the cumulative function of Weibull distribution and $p(sig)$ represents the probability of issuing a signal by control chart.

$$P(sig_1) = 1 - {\beta_1}^k \quad (27)$$
$$P(sig_2) = 1 - {\beta_2}^k \quad (28)$$

3-2-1-Inspection Cost

In condition 2, the inspection cost is equal to summation of the cost of sampling in the in-control and out-of-control states. So, Inspection cost in this condition is calculated as follows:

$$C_{S2} = (C_F + C_V n_1)(s_1 + ARL_{11}) \times f_1 + (C_F + C_V n_2)(s_2 + ARL_{12}) \times f_2 \quad (29)$$

In this equation s_1 and s_2 are the expected number of samples before occurring the shift when using loose and strict control strategies, respectively.

$$s_1 = \sum_{j=1}^{k} e^{-(\frac{jh_1}{a})^b} - ke^{-(\frac{(k+1)h_1}{a})^b} \quad (30)$$

$$s_2 = \sum_{j=1}^{k} e^{-(\frac{jh_2}{a})^b} - ke^{-(\frac{(k+1)h_2}{a})^b} \quad (31)$$

3-2-2-Quality loss cost

In this condition, the quality loss cost is incurred to manufacture in both in-control and out-of-control periods. However, it is obvious that the quality loss cost dramatically increments when the process goes to out-of-control state because of increasing the probability of producing non-conforming items. If Q_1 and Q_2 be the expected cost per unit arisen from producing non-conforming items in the in-control and out-of-control states, respectively, the quality loss cost in condition 2 is:

$$C_{Q2} = Q_1 P \times E(T_{in}|C_2) + Q_2 P \times E(T_{out}|C_2) \quad (32)$$

3-2-3-Maintenance cost

Since in condition 2 the production process shifts to an out-of-control state before $(k+1)^{th}$ sampling interval, the reactive maintenance is performed instead of preventive maintenance. Therefore, the maintenance cost in this condition only includes the cost of implementing reactive maintenance and the cost of investigating false alarms.

$$C_{M2} = \left(\frac{s_1 C_Y}{ARL_{01}} \times f_1 + \frac{s_2 C_Y}{ARL_{02}} \times f_2\right) + C_R \quad (33)$$

The first part of equation (33) is the cost of false alarms investigation and C_R is the reactive maintenance cost.

3-3-Condition 3 (C_3)

As indicated in figure 4, the production process starts its operation in the in-control state, then at a time between j^{th} and $(j+1)^{th}$ sampling interval, an assignable cause occurs that leads to shift in either the process mean or the process variability or in both together from on-target values to off-target values. However, in this condition, it is assumed that the control chart can't detect the occurrence of the assignable cause in the next inspections due to the limitations of control charts. So, the production process continues until the end of $(k+1)^{th}$ sampling interval, when the planned PM activities must be implemented, the worker finds that the process has gone to out-of-control state. At this time, the PM activities will be replaced by RM activities.

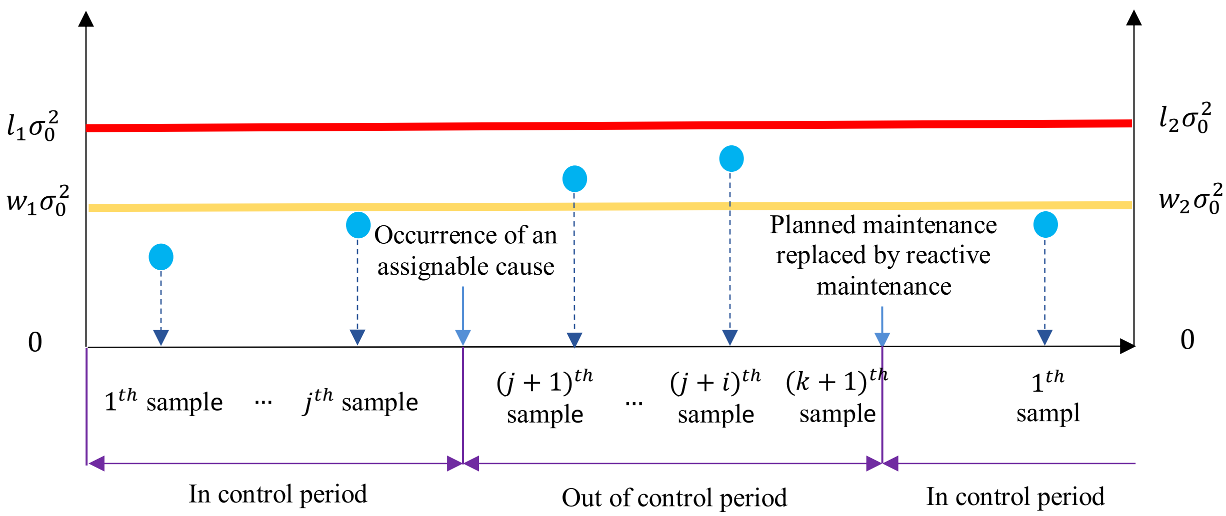


Fig 4. Graphical representation of a production cycle in condition 3

In condition 3, similar to condition 2, the in-control time follows a truncated Weibull distribution and consequently the expected in-control time is obtained as follows:

$$E(T_{in}|C_3) = \left(\int_0^{(k+1)h_1} tf(t|(k+1)h_1)dt\right) \times p_1 + \left(\int_0^{(k+1)h_2} tf(t|(k+1)h_2)dt\right) \times p_2 \quad (34)$$

The expected out-of-control time and the probability of occurrence of this condition are as equations (35) and (36), respectively.

$$E(T_{out}|C_3) = [(k+1)h_1 \times p_1 + (k+1)h_2 \times p_2] - E(T_{in}|C_3) \quad (35)$$

$$P(C_3) = [F((k+1)h_1) \times f_1 + F((k+1)h_2) \times f_2] - [F(kh_1) \times P(sig_1) \times f_1 + F(kh_2) \times P(sig_2) \times f_2] \quad (36)$$

3-3-1-Inspection cost

Since in condition 3 the production process continues until $(k+1)^{th}$ inspection interval, number of sampling points in this condition is equal to k. Hence, the cost of sampling in this condition can be defined as follows:

$$C_{S3} = [(C_F + C_V n_1) \times f_1 + (C_F + C_V n_2) \times f_2] \times k \quad (37)$$

3-3-2-Quality loss cost

In this condition similar to the condition 2, the quality loss cost is imposed to the manufacturer in both in-control and out-of-control states and given according to Equation (38).

$$C_{Q3} = Q_1 P \times E(T_{in}|C_3) + Q_2 P \times E(T_{out}|C_3) \quad (38)$$

3-3-3-Maintenance cost

In this condition, the maintenance cost consists of the false alarm cost and the reactive maintenance cost. To calculate the false alarm cost, the expected number of false alarms must be multiplied by the cost of each false alarm. Moreover, the expected number of false alarms depends on the numbers of sampling points (k) and the probability of type I error. Consequently, the maintenance cost in this condition is as equation (39).

$$C_{M3} = \left(\frac{s_1 C_Y}{ARL_{01}} \times f_1 + \frac{s_2 C_Y}{ARL_{02}} \times f_2\right) + C_R \quad (39)$$

Eventually, the expected inspection cost, quality loss cost, and maintenance cost in each production cycle are calculated as equations (40), (41) and (42), respectively.

$$E(S) = \sum_{i=1}^{3} C_{Si} \times P(C_i) \qquad \mathrm{i} = 1,2,3 \quad (40)$$

$$E(Q) = \sum_{i=1}^{3} C_{Qi} \times P(C_i) \qquad \mathrm{i} = 1,2,3 \quad (41)$$

$$E(M) = \sum_{i=1}^{3} C_{Mi} \times P(C_i) \qquad i = 1,2,3 \quad (42)$$

3-4-The optimization model

The classical EPQ model includes startup and holding costs as follows:

$$EPQ = \frac{DA}{PT} + \frac{B(P-d)T}{2} \quad (43)$$

The first and second terms of this Equation represent the expected startup cost and the expected holding cost in a production cycle. Now, the expected total cost is obtained by adding the inspection cost, quality loss cost, and maintenance cost to the EPQ model.

$$ETC = \frac{DA}{PT} + \frac{B(p-d)T}{2} + E(S) + E(Q) + E(M) \quad (44)$$

This study aims to find the control chart design parameters including n_1, n_2, h_1, h_2, w_1, w_2, l_1, l_2 and the decision related to maintenance k in a way that the ETC is minimized and some constraints are satisfied. Hence, the mathematical programming can be formulated as follows:

$$
\begin{aligned}
& Min\ \ ETC && (45)\\
s.t.\ \ & ATS_0 > ATS_l && (45.1)\\
& ATS_1 < ATS_u && (45.2)\\
& k(h_1p_1 + h_2p_2) \geq M && (45.3)\\
& 1 \leq n_i \leq n_{max} \qquad i = 1,2 && (45.4)\\
& h_i, l_i, w_i > 0 \qquad i = 1,2 && (45.5)\\
& n_i, k \in N^+ \qquad i = 1,2 && (45.6)
\end{aligned}
$$

The constraint (45.1) ensures that the time between two consecutive false alarms be greater than the pre-defined value ATS_l. This constraint is added to the optimization model for decreasing the occurrence rate of the false alarms. For quick detection of the occurred assignable cause, the ATS_1 must be less than the pre-determined value ATS_u as shown in equation (45.2). In addition, in order to guarantee the process continuity, constraint (45.3) is considered in which the time interval until implementing the planned preventive maintenance must be greater than M. Also, because of economic reasons, the sample should be taken from the process with a size less than n_{max} as shown in equation (45.4).

4-PSO algorithm for optimizing the proposed model

Since particle swarm optimization (PSO) algorithm has good performance in optimizing non-linear mathematical programming models, this paper uses this algorithm for optimizing the proposed mathematical programming. PSO as an algorithm with powerful searching ability first was proposed by Kennedy and Eberhart (1995). Hence in recent years, it has been widely used for solving optimization problems by some authors such as Clempner and Poznyak (2015), Ali Askari and Bashiri (2017), Lakhbab and Bernoussi (2016) and Alinaghian et al (2016). The optimization procedure in PSO algorithm is inspired by the social behavior of birds. In this algorithm, a swarm of particles in each iteration is generated in which each particle represents a potential solution in the feasible space. Moreover, two characteristics for each particle are defined: (1) "position" that shows location of the particle in the feasible space; and (2) "velocity" that demonstrate the moving direction of the particle.

PSO algorithm starts the search process with a swarm of particles that their positions and velocities are determined, randomly. In the next iterations, each particle moves in the direction of its velocity vector. Then, the velocity and position vectors are updated based on three factors: (1) its current velocity; (2) the best position explored by the given particle so far, which is called personal best (*pbest*); and (3) the best position explored by all of the particles, which is called global best (*gbest*). When the stopping criterion is satisfied the algorithm is ceased and the latest global best is introduced as an optimal solution.

In the mentioned iterative process, it is assumed that the $x_i(t)$ and $V_i(t)$ are the i^{th} particle position and particle velocity at iteration t, and the $P_i(t)$ represents the personal best of the i^{th}particle until iteration t.

$$x_i(t) = [x_{i1}(t), x_{i2}(t), \dots, x_{im}(t)] \tag{46}$$

$$V_i(t) = [V_{i1}(t), V_{i2}(t), \dots, V_{im}(t)] \tag{47}$$

$$P_i(t) = [P_{i1}(t), P_{i2}(t), \dots, P_{im}(t)] \tag{48}$$

As mentioned earlier, in iteration $(t+1)$, the velocity is updated based on the Equation (49).

$$V_{ij}(t+1) = \omega V_{ij}(t) + r_1C_1\left(p_{ij}(t) - x_{ij}(t)\right) + r_2C_2\left(g_j(t) - x_{ij}(t)\right) \tag{49}$$

In this equation, C_1 and C_2 are acceleration constants that called cognitive and social parameters, respectively. Moreover, r_1 and r_2 are the random variables uniformly distributed in the interval $[0,1]$;

$p_{ij}(t)$ and $g_j(t)$ represent the "*pbest*" and "*gbest*" positions, respectively. The parameter ω is called the inertia weight coefficient and is selected in the interval $(0,1)$. It is used to control the impact of the velocity vector in the previous iteration on the next one. It is usual to select a large value for the inertia weight at first iteration of the search process in order to launch a global search and then to reduce its value to obtain a better local exploration as iteration number increase (Azimifar and Payan, 2016).

Eventually, the new position of the particle is obtained as follows:

$$x_{ij}(t+1) = x_{ij}(t) + V_{ij}(t+1) \tag{50}$$

The position and velocity of each particle must be selected in the range $[X_{min}, X_{max}]$ and $[V_{min}, V_{max}]$, respectively where X_{min} and X_{max} are the lower and upper limits of particle position, V_{min} and V_{max} show the lower and upper limits of particle velocity. The first constraint guarantees that the particle remains within the feasible space and the second one improves intensification of the algorithm. In this study V_{min} and V_{max} are calculated as:

$$V_{max} = 0.1 \times (X_{max} - X_{min}) \tag{51}$$

$$V_{min} = -V_{max} \tag{52}$$

According to the mentioned explanations, the PSO algorithm can be employed for optimizing the problems with continuous decision variables, whereas in the presented model, the sample size (n) and the number of inspections until implementing preventive maintenance (k) are discrete variables. In this regard, this paper applies the following transformation to overcome this limitation.

Assume that the acceptable values for the discrete variable y_{ij} are as follow:

$$y_{ij} \in \{S, S+1, \dots, M\} \qquad y_{ij} \in Z \tag{53}$$

We consider the continuous variable y^r_{ij} in the range $(0,1)$. This continuous value is modified to the corresponding discrete value in the range defined in Equation (53) as follows:

$$y_{ij} = min\left(\lfloor S + (M - S + 1)y^r_{ij} \rfloor, M\right) \tag{54}$$

By applying this method, the PSO algorithm can be used for obtaining the optimal solution of the proposed model with both continuous and discrete decision variables. The procedure for optimizing the proposed model is shown in figure 5.

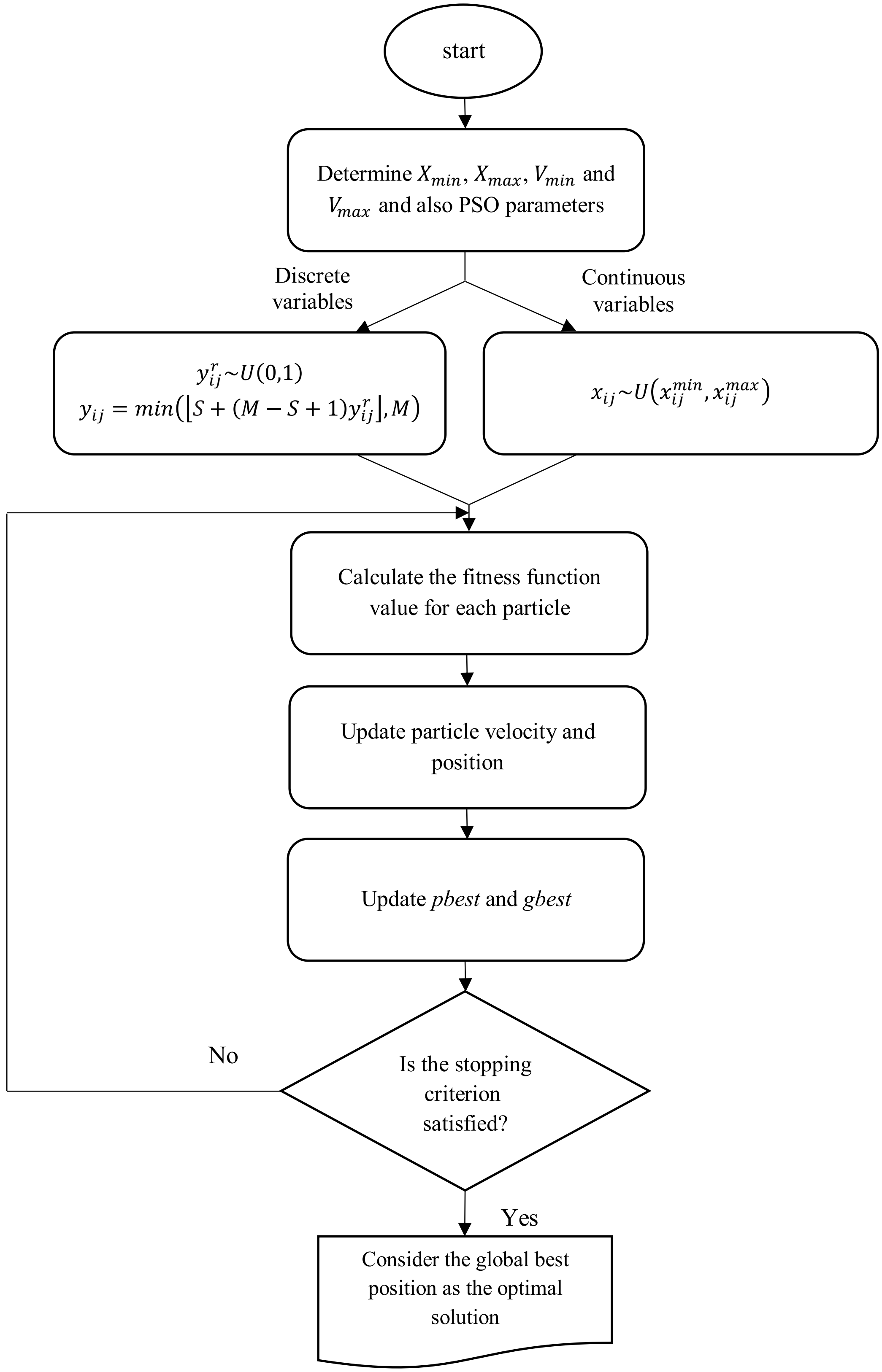


Fig 5. Optimization procedure of the proposed model

5-Numerical example

To indicate the applicability of the proposed model, an industrial example modified from Pan et al (2012) is used. In this example, the company under consideration produces a food product and sells them in packages. The VP non-central chi-square control chart is applied for monitoring the process in which samples are taken from the process with the fixed and variable sampling costs of \$10 and \$2, respectively. The needed time to sampling and plotting each observation is about 0.01 hours. Historical data indicate that the process first is in the in-control state for an average 20 hours and after that goes to an out-of-control state. In the out-of-control state, the process mean and/or process variability shift from on-target values to off-target values. The quality loss cost per unit in the in-control state is 1\$ and this cost in the out-of-control state is \$3. In the in-control state, some false alarms may be issued that investigating each of them has a cost of \$200. When the issued alarm is a true one, searching for assignable cause is performed that takes about 1 hour and then the reactive maintenance is implemented at a cost of \$5000. Also, if assignable cause does not occur during production cycle, the preventive maintenance will be taken place at the end of cycle with a cost of \$2400. The inventory holding cost per unit per year and the installation cost are \$10 and \$60, respectively. The market demand for this product is 80 units per day and 10,000 units per year and the production rate is about 100 units per day. The values of the parameters related to the mentioned industrial example are given in table 3.

Table 3. The values of the cost and process parameters in the numerical example

$C_F = 10$	$C_V = 2$	$C_P = 2400$	$C_R = 5000$
$C_Y = 200$	$Q_1 = 1$	$Q_2 = 3$	$E = 0.01$
$p = 100$	$T_1 = 1$	$a = 0.5$	$b = 1$
$B = 10$	$D = 10000$	$d = 80$	$A = 60$

This problem is solved by using the PSO algorithm for different values of shift in the process mean and process variance. In this problem, the decision variables are the sample sizes (n_1, n_2), sampling intervals (h_1, h_2), coefficients of control limits (l_1, l_2), coefficients of warning limits (w_1, w_2) and the number of inspections when any true alarm is issued until the end of production cycle (k). The values for these decision variables which minimize the ETC, while satisfying the constraints (45.1) to (45.6) are presented in table 4.

Table 4. The optimal values of ETC, ATS_0, ATS_1 and the design parameters for the different shift values (δ and γ)

γ	δ	n_1	n_2	h_1	h_2	w_1	w_2	l_1	l_2	k	d	ETC	ATS_0	ATS_1
1	0.2	18	19	1.60	0.70	4	3.35	90	8.18	54	0.65	11242.78	400.26	12.67
1	0.4	15	16	2.09	2.00	4	3.04	90	13.88	31	0.83	8058.99	400.04	2.74
1	0.6	11	12	4.00	2.43	4	3	89.33	21.20	30	1.2	8035.48	400	2.68
1	0.8	10	11	4.99	2.53	4	3	78.49	23.68	54	1.19	8032.58	401.36	2.66
1	1	9	10	4.44	2.65	4	3.01	71.16	25.77	88	1.2	8029.40	400.09	2.73
1.1	0.2	18	19	4.97	1.89	4	3.59	90	8.42	81	0.69	8055.36	400.06	2.69
1.1	0.4	12	13	2.59	2.49	4	3	90	10.13	30	1.08	8039.34	400	2.74
1.1	0.6	11	12	3.66	2.53	4	3	90	19.82	73	1.2	8035.77	401.01	2.61
1.1	0.8	10	11	4.12	2.60	4	3.38	80.48	23.28	53	1.2	8032.4	408.80	2.66
1.1	1	10	11	3.26	2.20	12.64	6.15	59.84	36.08	37	1.2	8034.03	404.68	2.71
1.2	0.2	13	14	5	2.33	4	3	90	10	30	1.05	8040.12	400	2.73
1.2	0.4	13	14	2.78	2.68	4	3	90	10	30	1.0	8038.22	400	2.73
1.2	0.6	13	14	2.63	2.53	8.4	7.58	90	29.54	90	1.17	8039.64	404.85	2.61
1.2	0.8	8	9	2.62	2.52	4.03	3.01	66.33	17.36	30	1.2	8029.99	400.12	2.64
1.2	1	7	8	2.71	2.61	4	3	57.93	17.77	86	1.2	8026.41	400	2.74
1.3	0.2	12	13	2.74	2.64	4	3	90	8	30	1.08	8036.87	400.96	2.73
1.3	0.4	11	12	3.77	2.57	4	3	90	8.18	84	1.2	8035.41	403.62	2.62
1.3	0.6	10	11	2.69	2.59	4	3	83.63	16.89	32	1.2	8033.41	413	2.62
1.3	0.8	8	9	3.81	2.57	4.01	3.06	63.72	17.29	40	1.2	8028.53	405.2	2.7
1.3	1	9	10	2.71	2.61	6.91	6.91	60.01	25.92	44	1.11	8030.2	425.43	2.74
1.4	0.2	10	11	2.58	2.48	4	3.01	83.65	8	90	1.2	8035.43	400	2.58
1.4	0.4	9	10	2.59	2.49	4	3.37	74.93	8	34	1.2	8033.52	401.29	2.58
1.4	0.6	8	9	3.12	2.51	4	3	65.08	11.65	30	1.2	8030.16	400.03	2.64
1.4	0.8	8	9	2.70	2.60	4.01	3	66.68	20.31	67	1.2	8028.69	457.24	2.69
1.4	1	7	8	2.76	2.66	4.02	3.65	57.84	16.83	53	1.2	8025.92	405.10	2.75
1.5	0.2	9	10	3.21	2.45	4	3.69	73.37	8	33	1.2	8033.11	406.19	2.6
1.5	0.4	8	9	2.59	2.49	4	3	66.39	8	30	1.2	8030.87	400.07	2.62
1.5	0.6	7	8	2.59	2.49	4	3.01	58.10	8	30	1.2	8028.63	400	2.66
1.5	0.8	11	12	2.70	2.60	9.05	5.23	78.64	29.39	49	1.18	8034.37	1206.61	2.68
1.5	1	10	11	2.89	2.57	10.82	10.82	61.30	28.08	79	1.13	8032.25	553.43	2.73

The obtained results indicate the effects of shift in the process mean and process variance on design parameters and ETC. When the magnitude of the shift in the mean and/or variance is small, the occurrence of assignable cause is detected later by the control chart. As a result, more non-conforming items are produced and so the ETC increases. Moreover, in this situation, the worker must take a sample with a large size after a short inspection interval. As the shift value increases, as shown in table 4, the optimal size of the samples decreases and the time interval between two consecutive samples increases.

5-1-Comparative study

In this Section, in order to evaluate the usefulness of the proposed model, two comparative studies are presented. In the first one, the effectiveness of integration of production planning, maintenance policy and statistical process monitoring is investigated. To do this, the proposed model is compared with two models: (1) The model in which the production run length, control chart design parameters and decision related to maintenance are optimized separately (model 1) and, (2) The model in which designing the control chart and determining the maintenance policy are integrated, but the optimal value of production run length is obtained separately (model 2). The second comparative study is conducted to indicate the superiority of the developed adaptive control chart in comparison with the control chart with the fixed parameters. For this purpose, the proposed model is compared to the same model when NCS control chart with the fixed parameters is employed, which is called "model 3" hereafter.

Model 1:

In model 1, the production run length (T) is obtained by minimizing the inventory cost (i.e. Equation (43)). Then, the control chart design parameters $(n_1, n_2, h_1, h_2, w_1, w_1, l_1, l_2)$ are attained by minimizing summation of the quality and inspection costs using PSO algorithm. According to the obtained values for these decision variables, the value of k can be calculated from the following Equation:

$$k = \frac{T}{(h_1 p_1 + h_2 p_2)} - 1 \tag{47}$$

Finally, by using the obtained values for the decision variables, the ETC is calculated. Then, the expected cost per hour (EC) is given by the ratio of the expected cost per cycle to the expected cycle time.

Model 2:

In model 2, the control chart design parameters and the decision related to maintenance are optimized, simultaneously. Also, the production run length is calculated as the average cycle time and then, we can calculate the EC by using the obtained values for the decision variables.

5-1-1-Comparison among the proposed model, model 1, and model 2

In this section, the proposed model is compared with the models 1 and 2 based on the EC value. For this purpose, the obtained decision variables and the expected cost per hour of these three models are presented in table (5).

Table 5. Numerical comparison among the proposed model, model 1, and model 2

Models	n_1	n_2	h_1	h_2	w_1	w_2	l_1	l_2	k	d	T	EC
Model 1	17	18	5.1	5	36.59	35.85	36.66	36.28	1	0.36	7.75	1171.71
Model 2	12	13	3.58	3.48	65.60	57.19	68.28	65.89	30	0.78	51.51	506.63
The proposed model	12	15	4.93	4.83	29.26	29.26	79.33	51.97	30	1.02	58.02	493.04

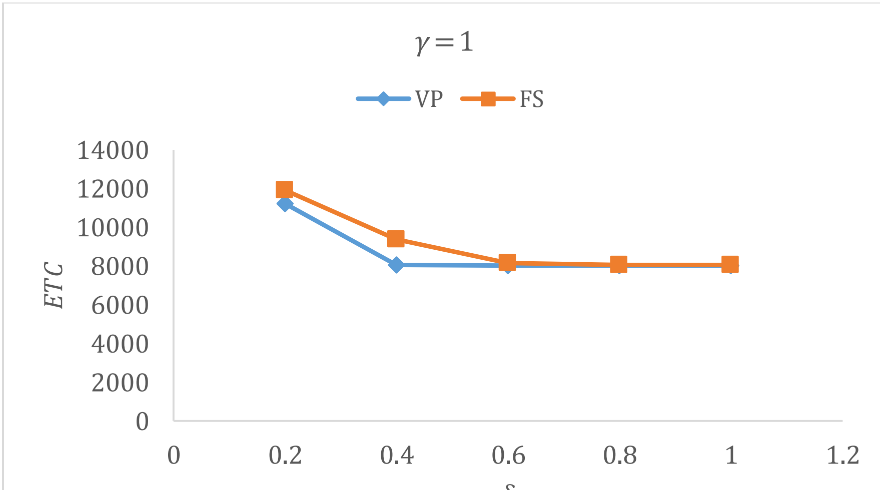
It can be observed in table 5 that the EC of model 1 is much more than the models 2 and 3. In other words, when designing the control chart and determining the maintenance policy are integrated, the costs of the model are reduced significantly. Moreover, the results confirm that integration of production planning, maintenance policy and statistical process monitoring leads to less EC in comparison with two other models. The difference between the EC of the proposed model and model 2 may seems ignorable; but when we look it at annually, it can be concluded that using the proposed scheme is more affordable than the model 2.

5-1-2-Comparison between the proposed model and model 3

In this Section, the proposed model is compared to the same model when FP NCS control chart is employed. To do this, different values for the shift in the process mean and variance are considered and then these models are compared from economic and statistical aspects. We can observe the statistical performance of the models by using the average time to signal in the in-control state (ATS_0) and in the out-of-control state (ATS_1). Therefore, for each value of shift, two models are solved by using PSO algorithm and the obtained ETC, ATS_0 and ATS_1 are presented in table 6. Table 6 also provides the percentage improvement in the cost and the average time to signal in the in-control and the out-of-control states (PI_1, PI_2 and PI_3, respectively). The results indicate that for small and moderate shifts, the use of adaptive control chart is preferred. The obtained costs of two models for different shift values in the process mean and variance are indicated in figure 6.

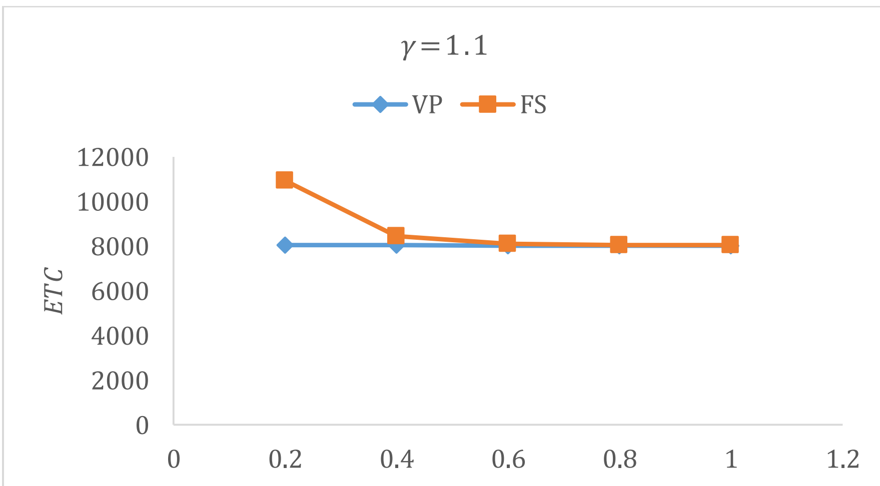
Table 6. Numerical comparison between the proposed model and model 3

γ	δ	ETC			ATS_0			ATS_1		
		VP	FP	PI_1	VP	FP	PI_2	VP	FP	PI_3
1	0.2	11242.78	11923.19	6.05	400.26	400	0.06	12.67	47.26	73.18
1	0.4	8058.99	9381.33	16.41	400.04	400	0.01	2.74	5.46	49.80
1	0.6	8035.48	8148.69	1.41	400.00	400	0	2.68	2.73	1.80
1	0.8	8032.58	8065.00	0.40	401.36	400	0.34	2.66	2.68	0.86
1	1	8029.40	8044.51	0.19	400.09	400	0.02	2.73	2.73	0.01
1.1	0.2	8055.36	10953.67	35.98	400.06	400	0.02	2.69	9.40	71.32
1.1	0.4	8039.34	8454.26	5.16	400	400	0	2.74	3.2	14.75
1.1	0.6	8035.77	8105.92	0.87	401.01	400	0.25	2.61	2.67	2.20
1.1	0.8	8032.40	8060.08	0.34	408.80	400	2.15	2.66	2.68	0.83
1.1	1	8034.03	8042.94	0.11	404.68	400	1.15	2.71	2.74	0.92
1.2	0.2	8040.12	8947.32	11.28	400	400.02	0	2.73	4.03	32.24
1.2	0.4	8038.22	8216.61	2.22	400	400.07	-0.02	2.73	2.78	2.04
1.2	0.6	8039.64	8085.75	0.57	404.85	400	1.2	2.61	2.66	0.02
1.2	0.8	8029.99	8055.82	0.32	400.12	400	0.03	2.64	2.70	2.18
1.2	1	8026.41	8041.30	0.18	400	400	0	2.74	2.74	0.19
1.3	0.2	8036.87	8267.72	2.87	400.96	400	0.24	2.73	2.80	2.30
1.3	0.4	8035.41	8124.30	1.11	403.62	400.03	0.89	2.62	2.66	1.50
1.3	0.6	8033.41	8072.92	0.49	413.00	400	3.15	2.62	2.65	1.10
1.3	0.8	8028.53	8052.02	0.29	405.20	400	1.28	2.70	2.71	0.53
1.3	1	8030.20	8039.78	0.12	425.43	400.04	5.97	2.74	2.75	0.31
1.4	0.2	8035.43	8123.76	1.10	400	400.01	0	2.58	2.66	2.94
1.4	0.4	8033.52	8086.04	0.65	401.29	400	0.32	2.58	2.65	2.85
1.4	0.6	8030.16	8063.00	0.41	400.03	400	0.01	2.64	2.68	1.66
1.4	0.8	8028.69	8048.00	0.24	457.24	400.09	12.50	2.69	2.72	1.17
1.4	1	8025.92	8038.07	0.15	405.10	400	1.26	2.75	2.75	0.13
1.5	0.2	8033.11	8081.33	0.60	406.19	400	1.52	2.60	2.65	2.06
1.5	0.4	8030.87	8068.05	0.46	400.07	400	0.02	2.62	2.68	2.23
1.5	0.6	8028.63	8054.73	0.32	400	400	0	2.66	2.70	1.64
1.5	0.8	8034.37	8043.98	0.12	1206.61	400.00	66.85	2.68	2.74	2.19
1.5	1	8032.25	8036.20	0.05	553.43	400.02	27.72	2.73	2.76	1.05



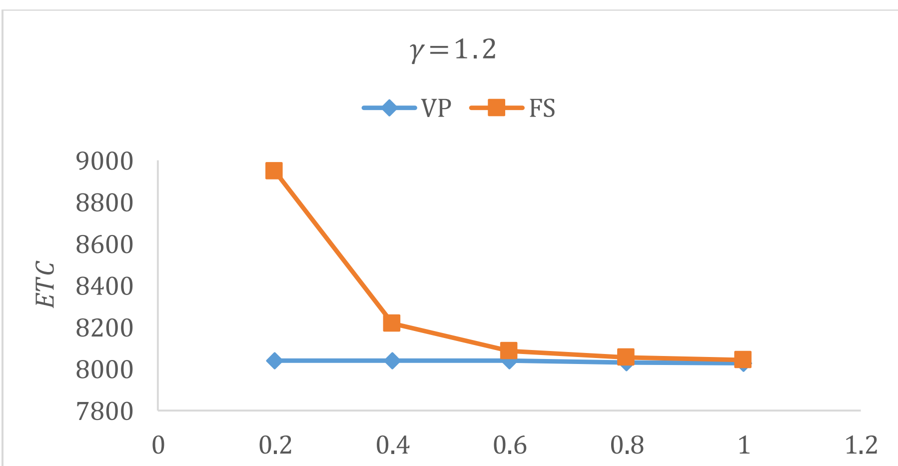
γ=1
VP
FS
14000
12000
10000
8000
6000
4000
2000
0
ETC
0
0.2
0.4
0.6
0.8
1
1.2
δ

6a



γ=1.1
VP
FS
12000
10000
8000
6000
4000
2000
0
ETC
0
0.2
0.4
0.6
0.8
1
1.2
δ

6b



γ=1.2
VP
FS
9000
8800
8600
8400
8200
8000
7800
ETC
0
0.2
0.4
0.6
0.8
1
1.2
δ

6c

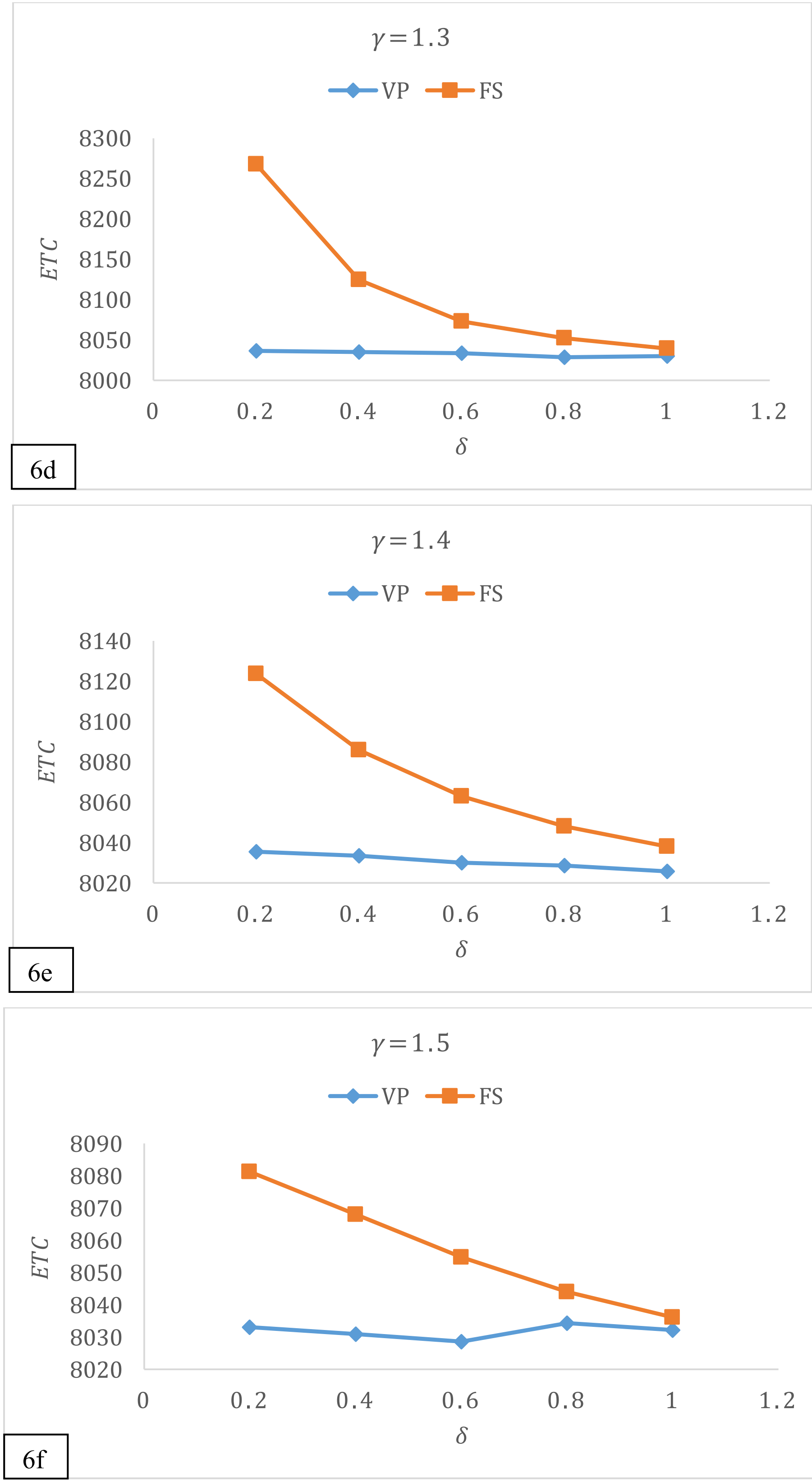
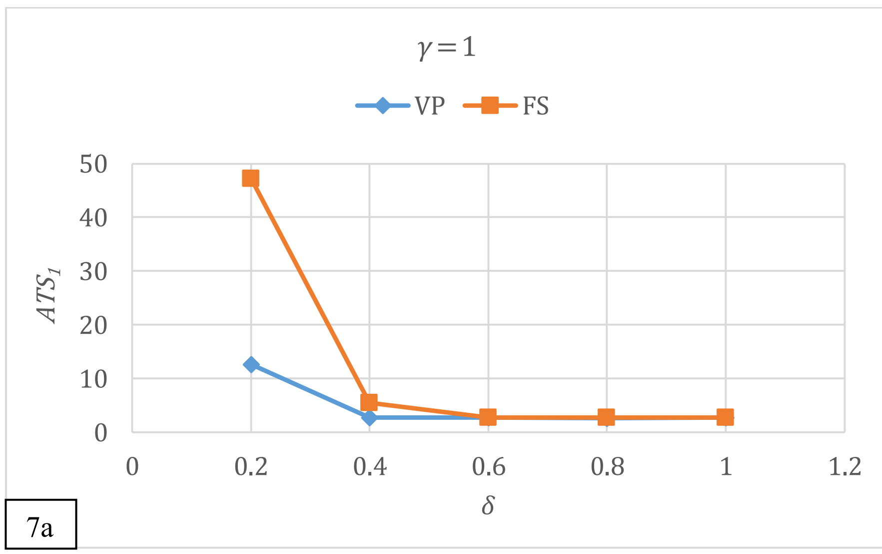


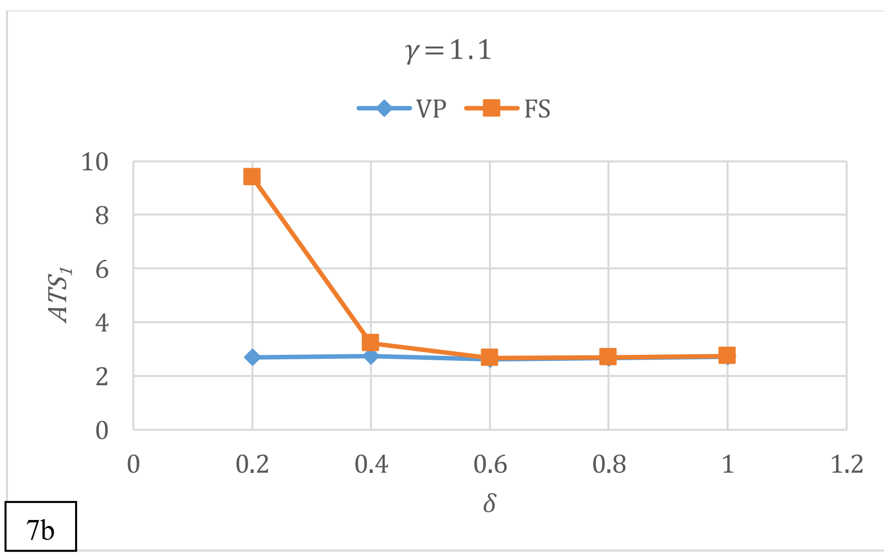
Fig 6a,6b,6c,6d,6e,6f. Comparing the ETC of the proposed model with model 3 for different values of shifts.

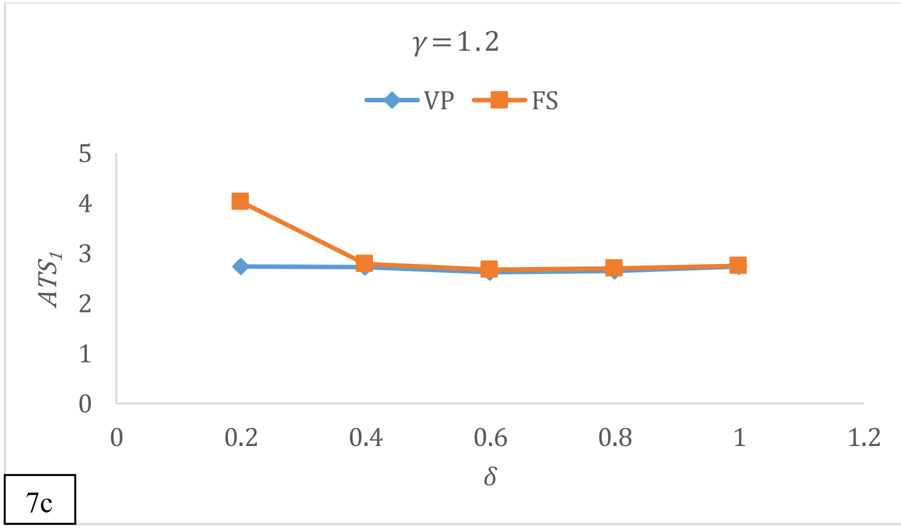
As shown in figure 6, when the magnitude of the shift is small, the cost of the integrated model with VP control chart is much less than the integrated model with FP control chart. By increasing in the values of shift, the costs in two models become closer together, so that when the shift is large, the costs are almost the same. Hence, as previously mentioned, for the small and moderate shifts using the VP control chart is more economical than employing the FP control chart

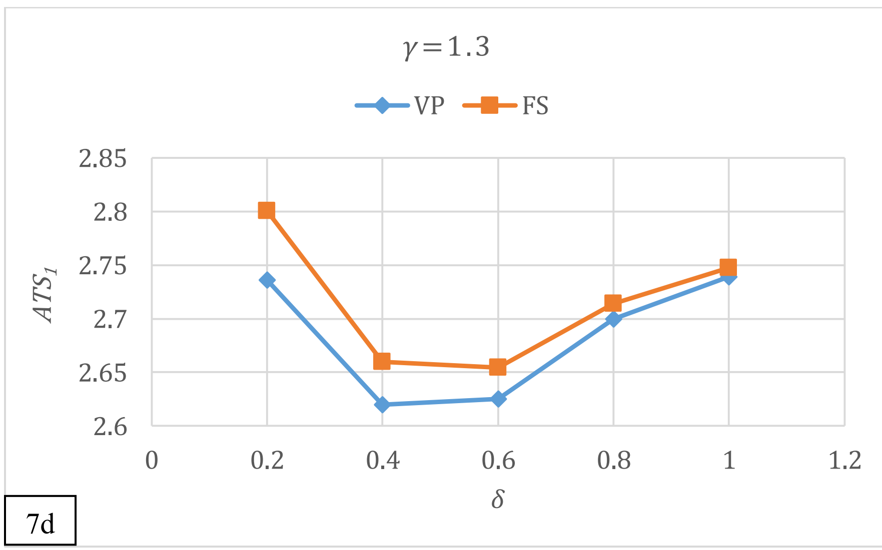
On the other hand, by observing figure 6, it is concluded that by increasing γ the difference between the costs of VP control chart and FP control chart become larger. As the result, when γ is larger, employing the FP leads to more cost saving for the system.

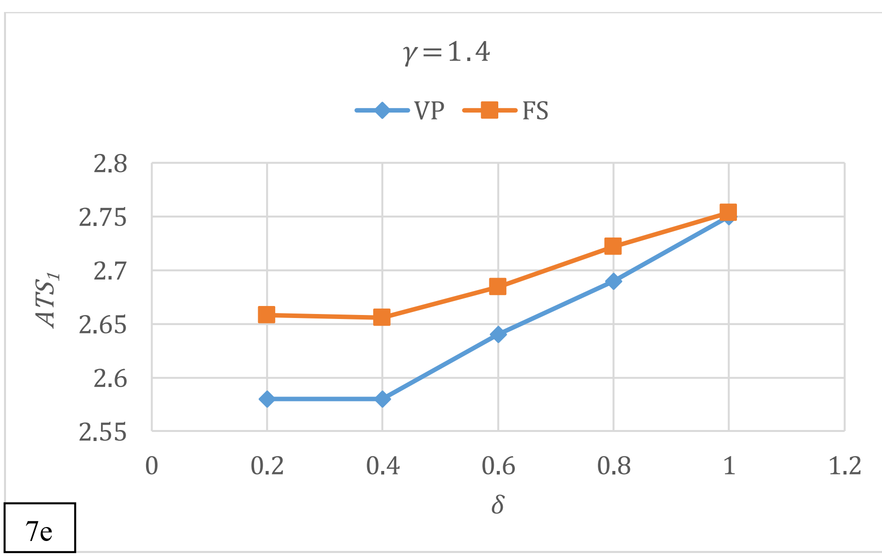
In figure 7, the two models are compared with respect to the ATS_1.






γ=1.2
VP
FS
ATS1
δ
7c


γ=1.3
VP
FS
ATS1
δ
7d


γ=1.4
VP
FS
ATS1
δ
7e

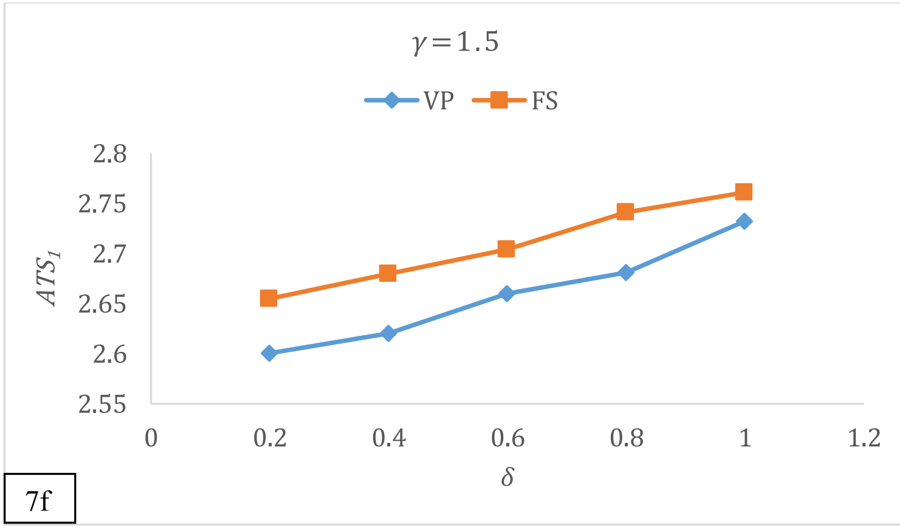


Fig 7a,7b,7c,7d,7e,7f. Comparing the ATS_1 of the proposed model with model 3 for different values of shifts.

Figure 7 confirms that the proposed model detects the occurrence of assignable cause faster than the model with FP control chart. The difference between the times required to detect assignable cause in two models for the small shifts is large, but in the large values for shift, the performance of integrated model with VP control chart is almost identical to the integrated model with FSI control chart.

It also can be seen in figure 7 that when $\gamma = \{1, 1.1, 1.2\}$, for medium and large shifts, the ATS_1 values are very close in the VP and FP charts. Conversely, for $\gamma = \{1.3, 1.4, 1.5\}$ the difference between the ATS_1 values in the mentioned control charts are significant. Consequently, when γ is bigger, the VP chart detects medium and large shifts more quickly than the FP chart, resulting in lower quality loss costs.

6-Conclusion

This study aimed to fill the gap between the real production systems and the simplified assumptions in the perfect production models. Hence, this paper relaxed three non-logical assumptions: (1) the production process is perfect that means non-conforming items never produce, (2) machine deterioration never happen, and (3) occurrence of assignable cause only leads to mean shift while process variance remains unchanged. For this purpose, this study integrated three interrelated issues of inventory control, quality control and maintenance in a unified model for an imperfect production process. Moreover, the suggested model in contrast to the most of the approaches in this field developed a VP NCS control chart to increase the cost saving and considered statistical properties in designing the control chart parameters. With respect to the high complexity of the problem, the PSO algorithm was employed to obtain the optimal values of decision variables in a way that the expected total cost per production cycle to be minimized. Finally, to validate the efficiency of the presented model, two comparative studies were conducted. The first one confirmed that integration of production planning, maintenance policy and statistical process monitoring leads to a significant increase in the cost savings. The second one indicated superiority of the developed adaptive control chart in comparison with the control chart with the fixed parameters.

As future research we suggest to extend the proposed model in two directions: first, developing a production process with multiple assignable causes to make the model more adapted to real production environments, and second, monitoring of the process with multiple quality characteristics.

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