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A two-stage robust model for portfolio selection by using goal programming

Mostafa Abdollahi Moghadam[1], Seyed Babak Ebrahimi[1*], Donya Rahmani[1]

[1]*Department of Industrial Engineering, K.N. Toosi University of Technology, Tehran, Iran*

mostafamoghadam68@gmail.com, b_ebrahimi@kntu.ac.ir, drahmani@kntu.ac.ir

Abstract

In portfolio selection models, uncertainty plays an important role. The parameter's uncertainty leads to getting away from optimal solution so it is needed to consider that in models. In this paper we presented a two-stage robust model that in first stage determines the desired percentage of investment in each industrial group by using return and risk measures from different industries. One reason of this work is that general conditions of various industries is different and according to the concepts of fundamental analysis should be chosen good groups before selection assets for investment. Another reason is that the identification of several good industries helps to diversify between several groups and reduce the risk of investment. In the second stage of the model, considering assets return, systematic risk, non-systematic risk and also first stage's result, amount of investment in each asset is determined. In both stages of the model there are uncertain parameters. To deal with uncertainty, a robust approach has been used. Since the model is a multi-objective problem, goal programming method used to solve it. The model was tested on actual data. The results showed that the portfolio formed by this model can be well-established in the conditions of high uncertainty and obtain higher returns.

Keywords: Portfolio selection, goal programming, robust Approach, parameter's uncertainty

1- Introduction

The issue of portfolio selection is one of the most important issues in the financial field. Various models and methods have been presented in this regard by various scholars. In the world of investment, investors want to get the highest expected returns from portfolios. The expected rate of return depends on the level of investors risk aversion. Portfolio selection and paper money analysis are always a crucial part of the decision. Generally speaking, only the conditions of the asset itself are considered in portfolio models. In these models, using parameters such as rate of return and variance, some assets are selected for create

*Corresponding author
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portfolios. These parameters are derived from historical data. But in the fundamental analysis concepts, it is necessary to analyze the general economic conditions of the country and various groups of stock exchanges before choosing assets. Because the conditions of the assets in terms of returns and risks depend on the industrial conditions they belong to.

The problem that matters is the uncertainty in the main parameters of these models. This uncertainty may be due to prediction errors or measurement errors. Uncertainty in economic plays an important role in financial decision-making, especially in portfolio selection issues, so it needs to develop techniques to consider this uncertainty in decision making and select a portfolio such that not to be highly sensitive to these uncertainties. Classical methods for considering uncertainty of parameters are sensitivity analysis and stochastic optimization. A robust approach is a good alternative to the previous methods because of this approach will easily give us the answer that is both feasible and optimal. More advantages of this approach will describe in the following.

The two-stage model presented in this paper acts in this way: first it determines the optimal investment percentage considering the return and risk parameters of each industry or group. As a result, a percentage of budgets that is determined for a good industry in terms of risk and return for investment are not equal to percentage of a bad industry and the share of o good industry of the total budget is higher. In previous models there are no difference between groups and may allocate a large part of the capital into an industry. Then, in the second stage, considering the constraints gained from the first stage, it selects the superior assets and creates the portfolios. But the investment climate is very uncertain and turbulent, and the parameters of this model cannot be accurately predicted. In the term, the parameters are non-deterministic; this uncertainty can affect the optimality and feasibility of the model. To solve this problem, a robust approach has been used, and with help of the method that provided by Bertsimas and sim (2003), the uncertainty of the parameters is considered in the model and the base model has become a robust model. In the robust approach, the best estimate of data is usually used in mathematical models, which are called nominal data. Data with uncertainty can be in constraints or objective function. Therefore, if the input data in the constraints take a value other than their nominal values, that constraint may be violated or the problem be unfeasible, and if the input data of the objective function are exited from their nominal value, the problem may also exit from the optimality or the optimal solution of the problem is not feasible any more.

In a robust optimization, solutions are produced under the term "robust response", which, in addition to preserving the optimality, also keep the problem feasible (Najafi and Ghahtarani, 2013). In the portfolio selection problem, some parameters have a non-deterministic nature, such as the expected returns of each asset or the systematic risk parameter, and the fluctuation of these coefficients in the portfolio selection model can also affect the answer to the problem and even its feasibility.

The rest of this paper is organized as follows: we will discuss the subject literature in Section 2 and then in Section 3, explain the two-stage basic model presented in this paper. Then, this model is converted to a robust model by using robust optimization approach. To test the robust model in Section 4, we will implement it on the actual data of the Tehran Stock Exchange, and a sensitivity analysis will be carried out on its parameters, also we will compare the results of the new model with a different robust model and in section 5 we present a conclusion of the paper and directions for further researches.

2- Literature review

In the past, a lot of research has been done on portfolio selection models, and also many innovations have been conducted. One of the most important researches in this field is Markowitz (1952) and Sharp (1963) models. Markowitz presented the primary Portfolio model, which became the basis for modern portfolio theory. The main goal of this model is to optimally allocate wealth by considering the trade-off between risk and return. He was the first person who presented the concept of portfolio and the creation of diversity formally. After Markowitz, another person named Sharpe, with the aim of reducing the computation and estimation of the Markowitz model, presented a single-index model that linked the return of each security to the returns of the stock index (Sharp, 1963). In addition to these two models, numerous models have been presented so far in the selection of asset portfolios, such as the following:

Wang et al. (2002) introduced the problem of linear interval programming and its application in the selection of asset portfolios. Chiodi et al. (2003), Fang et al. (2006) presented a semi-absolute standard deviation model for portfolio selection, which is similar to previous models, except that transaction costs are subtracted from the objective function. Wu (2012) presented a model for selecting interval asset portfolios with liquidity constraints. Jong (2012) introduced a model for optimal selection method of creating interval asset portfolios based on a satisfactory index of interval inequality function. Zhang et al. (2013) presented a multi-period portfolio selection model using interval analysis. The problem of interval asset selection was introduced by Wu et al. (2013), and this model is the generalization of the Markowitz mean-variance model. Among other cases, Rastegar and rasti barzoki(2017) presented a multi criteria project portfolio selection model with considering structural hardness and correlation between projects, Kellerer et al. (2000), Mansini et al. (2003), Papahristodoulou and Dotzauer (2004) can be pointed out.
All of the models that have been mentioned in the above researches are single-objective models for selecting portfolios. On the other hand, there are other models that consider more than one objective in choosing portfolios. One of the techniques that uses multi-objective models for optimization is the goal programming approach. This method was presented by Charnes in 1955. For the first time Lee and Lero (1973) used it in financial problems. Subsequently, in several papers, this approach was used to select portfolios, for example, Alexander and Resnick (1985), Bilbao et al. (2006), Wu et al. (2007), Li and Xu (2007), Marasovic and Babic (2011) have conducted several studies in this regard.

The main problem in these models is the lack of considering the uncertainty of the data in the selection of portfolios. One of the most important features of financial markets is uncertainty and the existence of uncontrollable variables totally influences the decision-making process of investors.

Classical methods for considering data uncertainty include sensitivity analysis and stochastic optimization. In sensitivity analysis, first, non-deterministic parameters are not considered. Then, after solving the problem, the efficiency of the answer is investigated using the sensitivity analysis. Although sensitivity analysis is a good tool for determining the efficiency rate of the response, it is not an appropriate solution for estimating the response which is robust against changes (Ghahtarani and Najafi, 2013). Mathematically, stochastic optimization is a strong model, but it also has some problems, such that the estimation of the probability distribution function of the parameters is difficult, and even if the distribution function is known, calculating their probability is still difficult, and also the change of the parameters causes the disturbance of the convexity property and consequently, the complexity of the calculation of the problem increases. Therefore, appropriate approaches which address the problems of the previous methods are needed. The robust optimization approach is one of these approaches.

In a robust approach, we are looking for near-optimal solutions that have high chance to happen, which these solutions are called robust solutions (Ghahtarani and Najafi, 2013). Robust approaches that have been proposed by researchers so far have included Soyster’s robust solution (1973), Benthal and Nemirovski (2000) robust solution and Bertsimas and Sim (2003) robust solution.

The first step in this direction was paced by Soyster (1973) in the form of a linear programming model for producing a solution feasible for all data belonging to a convex set. The model gives solutions that, for the optimality of the nominal problem, are completely conservative and the robust solution of the objective function is much worse than the solution of the nominal problem in sensitivity analysis.

In order to overcome this problem, Ben-Tal and Nemirovski (2000) presented a robust model which was capable of controlling level of conservatism. A robust model derived from this approach is a nonlinear second-order conical problem, so it's not usable for the discrete optimization problem. Due to the fact that a linear model with the Ben-Tal and Nemirovski approach becomes a nonlinear model, the complexity of the problem is high.

To solve this problem, Bertsimas and Sim (2003) presented a new method for modeling the uncertainty of data which did not have problems of the previous approaches. In this approach, a parameter Γ_i was defined which was responsible for adjusting the robustness level versus conservatism of the solution. Today, most linear optimization models using the robust methodology, implement this procedure. The most important feature of this method is that the robust counterpart of the linear problem remains linear, as well as possible assurances for feasibility of the solutions can be presented. Also, in relation to conservatism of the robust solutions, it can be noted that this methodology has the ability to control the degree of robustness of the solutions.

Among the studies conducted in the area of applying robust optimization in the portfolio selection problem are the following:

Ben-Tal et al. (2000) using the Ben-Tal and Nemirovski approach, Goldfarb and Lyengar (2003) using the Ben-Tal and Nemirovski approach, Quaranta and Zaffaroni (2008) using the Soyster approach, Chen and Tan (2009) using the Bertsimas and Sim approach and Stochastic constraints, Zhu and Fukushima (2009) using all three approaches, Sadjadi et al. (2010) the Bertsimas and Sim approach, Kawas and Thiele (2011) using the Bertsimas and Sim approach, Zymler et al. (2011) using the Benthal and Nemirovski approach, Moon and Yao (2011) using the Bertsimas and Sim approach, Ling and Xu (2012) using the approach of Benthal and Nemirovski and the elliptic uncertainty, Ghahtarani and Najafi (2013), with the approach of Bertsimas and Sim, Pinar and Pac (2014), with the approach of Bertsimas and Sim, Pachamanova .et al (2017) used robust approach to manage pension fund asset liability, Soyster and Murphy (2017) consider matrix uncertainty for robust linear programming.

In 2004, Kouchta offered a robust approach for goal programming models, in which the data uncertainty was considered in the model by Bertsimas and Sim method. In 2013, Ghahtarani and Najafi presented a new model using kouchta's (2004) model also Lee and Chesser (1980) model "which is a linear goal programming model that considers systemic risk and rate of return on portfolio selection", and the robust approach of Bertsimas and Sim.

2-1- Robust optimization

Among the robust approaches, the Bertsimas and sim (2003) approach has advantages over other methods. Unlike the Soyester model, this method has the ability to control the level of conservatism and does not consider the model to be completely conservative, and also the robust counterpart of a linear problem remains linear in this way. Due to these advantages, this paper uses Bertsimas and sim method (2003) for a robust problem. To understand this model, consider the following linear programming problem:

$$\begin{aligned} &\max \quad c'x \\ &\text{subject to} \quad Ax \le b \\ &\qquad\qquad l \le x \le u. \end{aligned} \tag{1}$$

In this case, it is assumed that only the data of the matrix A are uncertain and C does not have uncertainty in the objective function. Because this is a maximization problem, we add the constraint $z - c'x \le 0$ to the problem constraints. Consider the i-th row from matrix A. J_i is the set of uncertain coefficients in this row. Each input $a_{ij}, j \in J_i$ is a symmetric random variable such that $\tilde{a}_{ij}, j \in J_i \in \left[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}\right]$ and the variable $\eta_{ij} = \frac{\tilde{a}_{ij} - a_{ij}}{\hat{a}_{ij}}$ is a symmetric random variable with an unknown distribution in the interval $\left[-1,1\right]$.

Consider the *i*-th constraint of the nominal problem as $a_i'x \le b_i$. J_i is the set of coefficients $a_{ij}, j \in J_i$ with uncertainty. $\tilde{a}_{ij}, j \in J_i$ is based on a homogeneous distribution with the mean a_{ij}. $\tilde{a}_{ij}$ takes value for each i in the interval of $\left[a_{ij} - \hat{a}_{ij}, a_{ij} + \hat{a}_{ij}\right]$. The parameter Γ_i takes values in the range $\left[0, |J_i|\right]$ and determines how many $a_{ij}, j \in J_i$ change. In this approach, if the changes are within the limit $\lfloor \Gamma_i \rfloor$, the answer will be feasible, and if the change is more than $\lfloor \Gamma_i \rfloor$, the probably the answer will still be feasible. Note that when $\Gamma_i = 0, \beta_i(x, \Gamma_i) = 0$, the constraints will be identical to the nominal problem, and in contrast, if $\Gamma_i = |J_i|$, it will be the Soyester model. Therefore, $\Gamma_i \in [0, |J_i|]$ provides a flexible adjustment of robustness. Finally, the robust model will be as follows Bertsimas and sim (2003):

$$
\begin{aligned}
&\max \; c'x \\
&\text{subject to } \sum_j a_{ij}x_j + z_i\Gamma_i + \sum_{j \in J_i} p_{ij} \leq b_i \quad \forall i \\
&\qquad z_i + p_{ij} \geq \hat{a}_{ij} y_j \quad \forall i, j \in J_i \\
&\qquad -y_j \leq x_j \leq y_j \quad \forall j \\
&\qquad l_j \leq x_j \leq u_j \quad \forall j \\
&\qquad p_{ij} \geq 0 \quad \forall i, j \in J_i \\
&\qquad y_j \geq 0 \quad \forall j \\
&\qquad z_j \geq 0 \quad \forall i.
\end{aligned}
\tag{2}
$$

In the relations (2), z_i and p_{ij} are the auxiliary dual variables used to linearize the problem.

By examining the literature, the absence of a model which considers the fundamental analysis concepts prior to selecting the asset of the superior industries is very impressive. Also the being of a model that allocates a higher percentage of the budget to those type of industries and considers the uncertainty of the parameters, is an important requirement. In general, the innovations of this article to address the identified requirement are as follows:

- Presenting a two-stage robust model for portfolio selection and selecting the right industry prior to the asset with considering parameter's uncertainty
- Considering the standard deviation of asset return as a non-systematic risk index with beta coefficient in one model

3- Presentation of the two-stage robust goal programming model of the research

In this research, we have tried to consider a goal programming model and make some changes to it, and using the robust approach of Bertsimas and sim (2003) to find a new model. Portfolio selection with previous models has been improved by the new model.

In current economic conditions, due to some political and economic issues, the performance of different industries is not the same and some industries have relatively better and more stable performances than others. It is obvious that investors are interested in evaluating and identifying these industries and allocating a larger part of their capital to these groups. As a result, models that were previously offered to select a portfolio from a certain number of assets, are not very attractive to investors. Because these models focus only on their returns and risks from choosing an asset and do not pay attention to the group in which the asset exists. Therefore, there is a need for a model that can identify and select the most efficient and less risky industries before choosing the asset and allocate a larger part of the capital to choose the asset of these industries.

The model presented in this paper is a two-stage model which the goal programming has been used in both of its two stages. The first-phase inputs of this model are the industry's average returns and the average beta of the industry, also the other parameter that comes at this stage is w_i or the priority of each industry determined by the decision maker(investors).

In this paper priorities are determined by expert's knowledge. Because priorities numbers' are comparative amounts and investors should determine the priority of different objectives. There is no precise method for determine priorities in different conditions. The output of the first stage of this model is the optimum percentage of investment in each industry of the total budget. In fact, at this stage, more attention is paid to the industries with higher returns and less risk than the rest, and also more priority is given to the investor, and more property is invested in them.

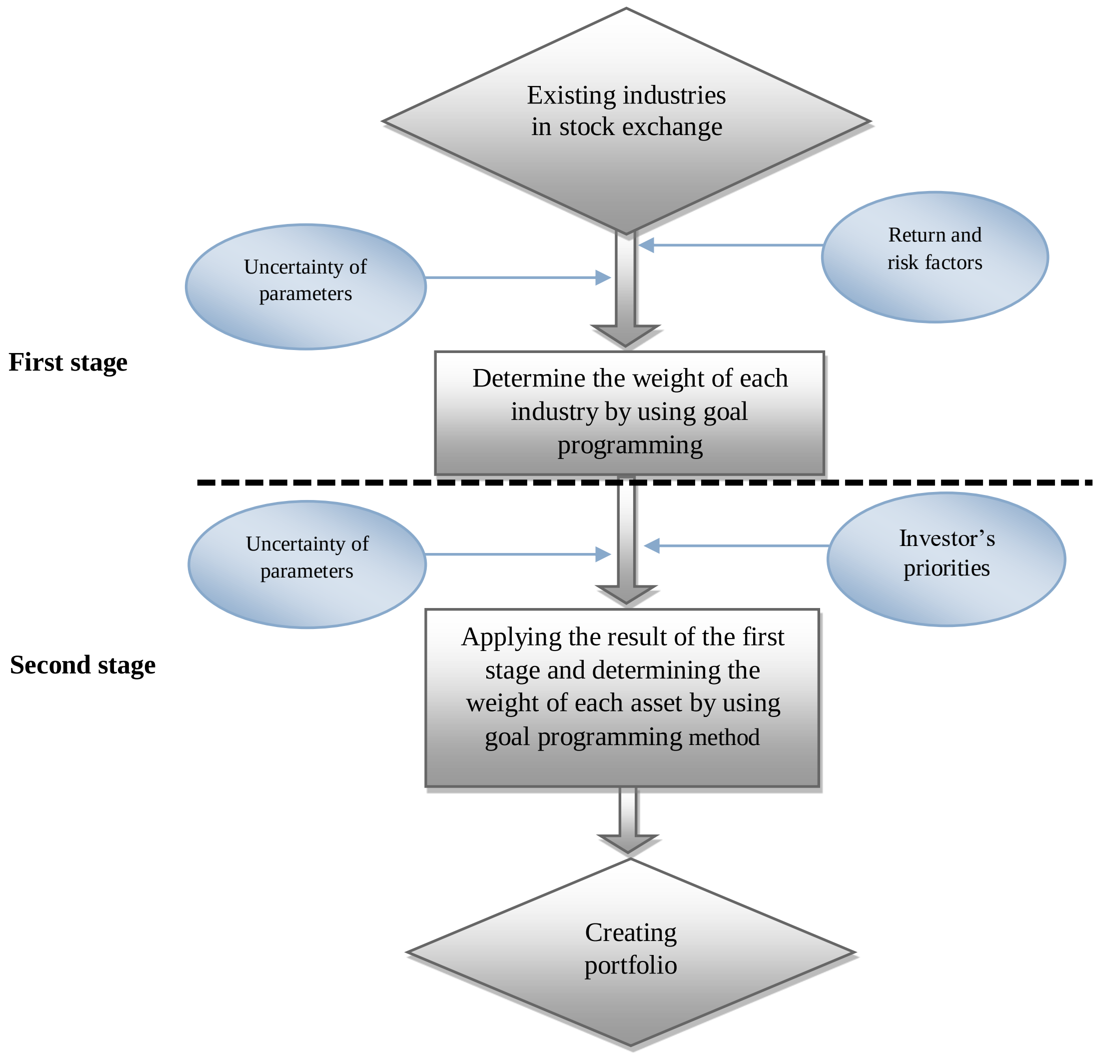

Fig1. Portfolio selection steps by new model

3-1- The first stage

The first stage of the model is as follows:

i : Industry Collection

V_i : Percentage of investment in industry i

w_i : The priority of the industry i for the investor

β_i : Average beta of industry i

r_i : Average return of industry i

β : the expected beta of the investor

R: The expected return of investment

$$\max \sum_i V_i w_i \quad (3)$$

$$\sum_i \beta_i V_i \leq \beta \quad (4)$$

$$\sum_i r_i V_i \geq R \quad (5)$$

$$\sum_i V_i = 1 \quad (6)$$

$$V_i \geq 0 \quad (7)$$

Constraint (4) related to the average beta expected by the investor, the constraint (5) expected return average; constraint (6) indicates that the sum of the coefficients must be equal to 1, and the constraint (7) indicates the sign of the decision variable.

The second phase of this model is derived from by applying changes in the model of Lee and Chesser (1980). Of these changes, is adding a constraint on the non-systematic risk index of each asset, which here is the standard deviation of the return of the asset (δ). The results of the first phase are used in the second stage in such a way that the total amount of assets of a particular industry should not be greater than the amount of the budget determined for it in the first stage.

3-2- The second stage

The second stage is as following:

W_i : Priority for each of the objectives

X_{ij} : The amount of money invested in the j-asset of the i-th industry

β_{ij} : The average beta of the j-th asset of the the i-th industry

R_{ij} : the return of asset j from the industry i

B: The expected beta of the investor

DR: The desired return of the investor

δ: The expected standard deviation of the investor (the number that investor wants standard deviation of the portfolio be less than that)

BC: Available budget for investment

M: A large positive number

$$\min\ w_1d_1^+ + w_2\left(d_2^- + d_3^-\right) + w_3d_4^+ + w_4d_5^+ + w_5\sum_{f=6}^{i+6} d_f^+ + w_6\sum_{f=n+7}^{2i+7} d_f^- \tag{8}$$

subject to:

$$\sum_i\sum_j X_{ij} + d_1^- - d_1^+ = BC \tag{9}$$

$$\sum_i\sum_j R_{ij}X_{ij} + d_2^- - d_2^+ = DR \tag{10}$$

$$\sum_i\sum_j B_{ij}X_{ij} + d_3^- - d_3^+ = B(BC) \tag{11}$$

$$\sum_i\sum_j \delta_{ij}X_{ij} + d_4^- - d_4^+ = \delta(BC) \tag{12}$$

$$BC + \sum_i\sum_j R_{ij}X_{ij} + d_5^- - d_5^+ = M \tag{13}$$

$$\sum_j X_{ij} + d_6^- - d_6^+ = V_i \qquad \forall i \tag{14}$$

$$\sum_j B_{ij} + d_7^- - d_7^+ = D_i \qquad \forall i \tag{15}$$

In the above model, the constraint (9) refers to the total available budget for investment, the constraint (10) emphasizes on the return on the portfolio, which should be greater than the DR; this amount is determined by the investor, the constraint (11) is considered to control the systematic portfolio risk, in fact, if the investor has a positive prediction of the future, increases the beta portfolio by increasing the number B, the constraint (12) focuses on the non-systematic portfolio risk, the constraint (13) seeks to maximize the total budget and return on the portfolio, and the constraints (14) and (15) specify the minimum and maximum limits of investment in each asset. The problem that has been discussed in this paper is the existence of uncertainty in some of the model's parameters, such as return, beta, and standard deviation of the asset and the way of dealing with these uncertainties. As discussed in the research background section, there are different approaches for considering the uncertainty in the parameters. The approach used here is the robust approach. In this paper, among the three robust methods, the Bertsimas and sim (2003) method is used to construct the model. The benefits of the Bertsimas method to other robust methods are the presentation of a linear model and the ability to adjust the level of robustness of the model in proportion to the level of conservatism of the solution. The final model of two-stage robust goal programming for selecting asset portfolios is as follows:

3-3- First stage of the robust model

$$\max \sum_i V_i w_i \tag{16}$$

$$-\sum_i \beta_i V_i + Z_1\Gamma_1 + \sum_i P_{1i} \le -\beta \tag{17}$$

$$-\sum_i r_i V_i + Z_2\Gamma_2 + \sum_i P_{2i} \le -R \tag{18}$$

$$Z_1 + P_{1i} \le \hat{\beta}_i y_i, \qquad \forall i \tag{19}$$

$$Z_2 + P_{2i} \le \hat{r}_i y_i, \qquad \forall i \tag{20}$$

$$-y_i \le V_i \le y_i \tag{21}$$

$$\sum_i V_i = 1 \tag{22}$$

$$V_i \ge 0, P_{1i} \ge 0, P_{2i} \ge 0, Z_1 \ge 0, Z_2 \ge 0$$

In the above model, Z and P are dual parameters of the problem that are used for linearize and solvable problems. Constraints (19) and (20) are dual constraints and $\hat{\beta}_i$ and $\hat{r}_i$ used in them, are the amount of error or fluctuation of the parameters used, which the second decimal digits of each parameter value is considered in this paper. Constraint (21) determines upper and lower bound for the problem decision variables.

3-4- Second stage of the robust model

$$\min\ w_1 d_1^+ + w_2\left(d_2^+ + d_3^+\right) + w_3 d_4^+ + w_4 d_{2n+5}^+ + w_5 \sum_{f=6}^{i+6} d_f^+ + w_6 \sum_{f=n+7}^{2i+7} d_f^- \tag{23}$$

subject to:

$$\sum_i \sum_j X_{ij} + d_1^- - d_1^+ = BC \tag{24}$$

$$-\sum_i \sum_j R_{ij} X_{ij} + Z_1 \Gamma_1 + \sum_i \sum_j P_{1ij} + d_2^- - d_2^+ = -DR \tag{25}$$

$$-\sum_i \sum_j B_{ij} X_{ij} + Z_2 \Gamma_2 + \sum_i \sum_j P_{2ij} + d_3^- - d_3^+ = -B(BC) \tag{26}$$

$$\sum_i \sum_j \delta_{ij} X_{ij} + Z_3 \Gamma_3 + \sum_i \sum_j P_{3ij} + d_4^- - d_4^+ = \delta(BC) \tag{27}$$

$$-BC - \sum_i \sum_j R_{ij} X_{ij} + Z_1 \Gamma_1 + \sum_i \sum_j P_{ij} + d_5^- - d_5^+ = -M \tag{28}$$

$$\sum_j X_{ij} + d_6^- - d_6^+ = V_i, \qquad \forall i \tag{29}$$

$$\sum_j X_{ij} + d_7^- - d_7^+ = D_i, \qquad \forall i \tag{30}$$

$$Z_1 + P_{1ij} \geq \hat{R}_{ij} y_{ij} \qquad \forall i,j \tag{31}$$

$$Z_2 + P_{2ij} \geq \hat{B}_{ij} y_{ij} \qquad \forall i,j \tag{32}$$

$$Z_3 + P_{1ij} \geq \hat{\delta}_{ij} y_{ij} \qquad \forall i,j \tag{33}$$

$$-y_{ij} \leq X_{ij} \leq y_{ij} \tag{34}$$

$$P_{ij} \geq 0, y_{ij} \geq 0, Z_1 \geq 0, Z_2 \geq 0, Z_3 \geq 0$$

N_i and D_i are the upper and lower bound for the investment in i-th industry, which are extracted from the first stage. In this model, the parameter Γ_i has the role of modulating the robustness of the proposed model against the conservatism level of the response. This parameter takes a value in the range [0,|Ji|], where J_i is the number of industries (in the first stage) and the number of assets (in the second stage). In fact, the number we consider for Γ is the number of assets we assume that their return, beta, and standard deviations are non-deterministic. Obviously, the higher number for Γ gets more conservative and the answer of the objective function gets worse value. In the above relations, the Z and P variables are dual auxiliary variables used to linearize the model and their values do not have a specific interpretation.

4- Computational results

In this section, using the real data and a two-stage robust goal programming model presented in this paper, an asset portfolio has been developed at various levels of robustness and the results have been mentioned. The data relate to six industries and twenty-three assets from these six industries, which were collected over a five-year period from 2011 to 2016. The daily price and returns of these assets are taken from the *bourseview.com* site and the beta and standard deviation values and other parameters required are calculated by Excel software. The values of parameters such as priorities (W_i), expected returns, expected beta, and expected standard deviations of portfolio are determined by expert opinion. The information used in the first stage is in table 1:

(β=0.6   R=0.35)(Expected values)

Table 1. data of 6 industries from tehran stock exchange

industries	Average of industries return	Industries Beta	Average of industries standard deviation	Industries priority
Auto manufactures	0.0536	2.48	0.2257	0.5
Sugar	0.2611	0.54	0.0201	1
Health care	0.2485	0.76	0.0195	0.75
Insurance				1.25
Diversified	0.594	0.63	0.1871	
Metals EX.Iron	0.1246	1.21	0.0211	0.75
Cement	0.0844	0.6	0.0842	0.5

By solving the model in GAMS, the results of table 2 were obtained:

Table 2. results of solving first stage

Γ(Γ1,Γ2)	(0,0)	(3,3)	(6,6)
1	0.165	0.168	0.168
2	0.1	0.1	0.1
3	0.1	0.1	0.1
4	0.435	0.432	0.432
5	0.1	0.1	0.1
6	0.1	0.1	0.1

The results are presented in three different levels for Γ_i. As you can see from the table above, as moving from the $\Gamma_i = 0$ level towards $\Gamma_i = 6$, the results will change, reducing the percentage of investment in the fourth industry and adding to the first industry. The reason for this is a better situation for the first industry in terms of risk. Of course for diversification and risk reduction according to Markowitz (1952), a minimum level has been set for each industry. Due to the small number of decision variables and the limited range of fluctuation considered for model parameters, the amount of change in results is very low. By obtaining the first stage and determining the percentage of investment in each industry, we will go to the second stage. The data used in the second stage of the model is presented in table 3.

Table 3. Data of 23 assets from 6 industries in Tehran stock exchange

Assets		**Average of annual return**	**Beta**	**Standard deviation of asset's return**
ZMYD	Auto	-0.01	3.15	0.1445
KFAN		0.02	2.44	0.1141
IKCO		0.13	2.6	0.241
SIPA		0.15	3.45	0.3108
GGAZ	Sugar	0.28	0.87	0.0203
GHND		0.38	1.74	0.0311
GHEG		0.38	-0.09	0.0157
GLOR		0.18	0.68	0.0069
DJBR	Health care	0.17	0.92	0.0188
DRZK		0.26	-0.01	0.0098
ABDI		0.59	0.16	0.0271
FTIR		0.23	1.89	0.0134
BKSZ	Insurance diversified	0.24	-0.05	0.1818
BIPZ		0.56	-0.01	0.213
BDAN		0.24	0.57	0.135
BALB		0.38	1.04	0.2704
FRVR	Metal EX. Iron	0.05	1.63	0.0259
BAHN		0.10	2.51	0.0231
SORB		0.04	0.9	0.0201
KZGZ		0.11	1.02	0.0185
SBOJ	Cement	0.05	0	0.1591
SMAZ		0.00	-0.04	0.1891
SEFH		0.11	-0.17	0.1008

Objectives' priority has been set at 1, 1.5, 0.5, 0.5, 1, and 0.3, according to the decision maker. The available budget is 1,000,000 currency units, expected returns is 300,000 units, expected beta 1, expected standard deviation of 1, a large M of 1,500,000, and different amounts for Γ_s. The results of solving the second stage in the GAMS are given in table 4:

Table 4. Result of solving second stage of robust model for different amount Γ_i

Γ(Γ1,Γ2,Γ3)	(0,0,0)	(1,1,1)	(3,3,3)	(8,8,8)	(12,12,12)	(18,18,18)	(20,20,20)	(21,21,21)	(23,23,23)
objective function	2763.51	2765.36	2770.88	2786.69	2799.11	2811.35	2824.7	2831.44	2839.21
1	0	0	0	0	0	0	0	0	0
2	0	0	0	0	0	0	0	0	0
3	60000	0	0	73632.29	74666.67	65953.65	65953.65	65953.65	65953.65
4	108000	168000	168000	94542.91	93333.33	102683.05	102683.05	102683.1	102683.1
5	0	0	0	11851.85	11851.85	27458.36	27458.36	27458.36	27458.36
6	46000	12500	12500	41481.48	41481.48	35536.45	35536.45	35536.45	35536.45
7	54000	87500	87500	46666.67	46666.67	37864.85	37864.85	37864.85	37864.85
8	0	0	0	0	0	0	0	0	0
9	0	0	0	0	0	0	0	0	0
10	0	0	0	0	0	0	0	0	0
11	100000	100000	100000	100000	100000	100000	100000	100000	100000
12	0	0	0		0	0	0	0	0
13	0	0	0	0	0	0	0	0	0
14	345178.2	264600	266651.3	285333.3	357333.3	370513.6	370513.6	370513.6	370513.6
15	421.8315	27400	25348.72	21682.3	16016.3	9658.71	9658.71	9658.71	9658.71
16	86400	140000	140000	128366.7	61686.27	55413.89	55413.89	55413.89	55413.89
17	0	0	0	0	0	0	0	0	0
18	0	0	0	0	0	0	0	0	0
19	0	0	0	0	0	0	0	0	0
20	100000	100000	100000	100000	100000	100000	100000	100000	100000
21	38285.71	39125.47	39854.18	42319.54	43696.39	50684.85	50684.85	50684.85	50684.85
22	0	0	0	0	0	0	0	0	0
23	61714.29	60874.53	60145.82	57680.46	57003.61	49315.15	49315.15	49315.15	49315.15

In the table 4, there are portfolios created on different levels of Γ. As it is clear, with increasing conservatism, the quantities assigned to the assets change and some of them are reduced and the others are increased. In fact, when the Γ_i take zero, it means that none of the values given to the model is uncertain, and the model creates portfolios with less stringency to meet the expected returns and risks, but as the robustness cost increases, the model is more conservative and reduces the value of assets that uncertainty of parameters has a greater effect on the model's goals and adds to the assets that have less effect in meeting the model's goals. Thus, when we consider the value of 23 for Γ_i, means that all its assets and parameters are uncertain and the model is in its most pessimistic state, and takes the most conservative values for portfolio creation. Obviously, with increasing conservatism, the objective function gets worse values and has less ability to meet the model's goals.

One of the changes made to this model over the previous models is adding a constraint of the standard deviation of asset returns, which is actually used as a non-systematic risk measure. The reason for this was that market conditions are risky and unpredictable, and adding another risk measure to the model would make the portfolios less risky. To show that the model is sensitive to the added constraint and this constraint improves the results in terms of risk, the sensitivity analysis is carried out on the different values of the expected standard deviations of the decision maker for the portfolio, the results of which are given in table 5:

Table 5. Sensitivity analysis on different amount of δ in second stage of robust model (Γ_i =3)

Assets	(Expected standard deviation) δ				
	0.08	0.09	0.1	0.12	0.13
1	0	0	0	0	4494.57
2	0	0	0	0	0
3	0	0	0	45831.02	72669.08
4	168000	168000	168000	122169	90836.35
5	0	0	0	0	14210.11
6	0	0	12500	38915.51	40371.71
7	189141.5	110525.2	87500	61084.49	45418.18
8	0	0	0	0	0
9	0	0	0	0	0
10	0	0	0	0	0
11	100000	100000	100000	100000	100000
12	0	0	0	0	0
13	0	0	0	0	90836.35
14	342858.5	421474.8	138446.2	69810.85	51906.49
15	0	0	153553.8	264454	216588.1
16	0	0	140000	97735.19	72669.08
17	0	0	0	0	0
18	0	0	0	0	0
19	0	0	0	0	0
20	100000	100000	100000	100000	100000
21	0	0	0	30189.15	48093.51
22	0	0	0	0	0
23	100000	100000	100000	69810.85	51906.49

It should be noted that the standard deviation of returns for different assets from different groups is very large in size and difference. For these values to be closely related and comparable to each other, all of them have been taken a natural logarithm, and have been divided by average return. As can be seen in table 5, the higher expected value for standard deviations leads to, the higher freedom of model than the selection of the criterion, and more attention is paid on the assets with higher returns and betas, also with more standard deviations. When the expected standard deviation for a portfolio is lower, it focuses more on assets with less standard deviation and does not allow large dispersion to the model. In order to show that the model has improved compared to the previous model, the returns of the portfolio created with the presented model over the out of sample data contains one year period from 2016 to 2017 at different levels of Γ compared with the returns of the portfolios formed by the Ghahtarani and Najafi's (2013) model. The data used is related to the listed assets, presented results in table 6:

Table 6. Comparing the result of this paper's model with ghahtarari and najafi's(2013) model

Γ	Portfolio's return that selected by this paper's robust model	portfolio's return that selected by Ghahtarani and Najafi's(2013) model	Differences between two models
$\Gamma=1$ (5%)	28.72	28.91	-0.19
$\Gamma=2$ (10%)	28.51	28.68	-0.17
$\Gamma=5$ (20%)	28.29	28.25	0.04
$\Gamma=12$ (50%)	27.97	27.75	0.22
$\Gamma=18$ (80%)	27.69	27.16	0.53
$\Gamma=23$ (100%)	27.56	26.7	0.86

The results of table 6 show that at low conservatism levels (low Γ), although the return difference between the two models is low, the return of the Ghahtarani and Najafi model is slightly higher, but when the model's conservatism increases, especially when the number considered For Γ_i is more than half their range, the efficiency of the model presented in this paper is increased, and as Γ increase, difference of return between two models becomes larger and new model obtained more return.

5- Conclusions and recommendations

In this paper, we presented a functional model for choosing portfolios based on the current economic conditions in which different industries are relatively spacious in terms of performance. This model can first select suitable groups for investment and then select the asset among them. Also, since the parameters used in the model are inherently non-deterministic, the robust approach of Bertsimas and sim (2003) was used and the model became a robust model that considers the uncertainty of the parameters. However, the conservatism of the model can be changed and as the conservatism (Parameter Γ) increases, the model yields more conservative values. Although, reduces the desirability of the objective function. The results show that the portfolios formed by the model presented in this paper at conservatism levels ($\Gamma > 10\%$) have always more returns than the single-stage robust model of Ghahtarani and Najafi (2013), which is somewhat similar to our model.

For future research, it is proposed to use a robust approach and goal programming in the formation of a portfolio consisting of derivative securities as well as the permissibility of borrowing sales. Also you can use the other methods of robust approach and consider parameters uncertainty in the form of different scenarios.

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