Journal of Industrial and Systems Engineering
Vol. 12, Special issue on Statistical Processes and Statistical Modeling, pp. 1-14
Winter (January) 2019

Functional process capability indices for nonlinear profile

Ahmad Mohammad pour Larimi[1], Ramezan Nemati Keshteli[2*], Abdul Sattar Safaei[3]

[1]*Department of Industrial Engineering, Mazandaran Institute of Technology, Babol, Iran*
[2]*Faculty of Engineering, Eastern Guilan, Guilan University, Iran*
[3]*Department of Industrial Engineering, Babol University of Technology, Babol, Iran*

mpl.ahmad@iran.ir, r.nemati@guilan.ac.ir, s.safaei@nit.ac.ir

Abstract

A profile is a relationship between a response variable and one or more independent variables being controlled during the time. Process Capability Indices (PCI) are measured to evaluate the performance of processes in producing conforming products. Despite frequent applications of profile and a variety of available methods to monitor its different types, little researches have been carried out on determining capability indices of profile process. PCIs such as C_P and C_{PK} in profile state, are used to evaluate process capability in producing conforming profiles. This paper presents a functional approach for nonlinear profiles which usually expressed as nonlinear regression. Thus, functions such as pertaining technical specification limits, mean and natural tolerance limits are determined as nonlinear profiles and also functional limit is applied to determine Functional Capability Indices (FCI) C_P and C_{PK} of functional nonlinear profile. Easy calculation and the ability to calculate FC in each period and at each point are from the advantages of this method over the other methods.

Keywords: Profile monitoring, non-linear profile, functional process capability index, vertical density profile.

1- Introduction

Statistical process control, process performance or product quality, in some cases, are referred to the relationship between a response variable and one or multiple independent variables. Researchers introduce this relationship as "profile". In different applications, the relation may appear in various forms, such as linear, polynomial, nonlinear, and sometimes in very complicated forms. Several examples of profiles application in industries are presented by various researchers. Various methods are proposed for profile monitoring in both Phases I and II and in Phase I, the goal is checking the stability of the process as well as parameters estimation while the aim in Phase II is detecting the shifts in the profile parameters as quickly as possible (Noorossana, Saghaie and Amiri, 2011).

Young et al. (1999) indicated the relationship between vertical density of chipboard and different depths as a nonlinear profile. Walker and Wright (2002) and Williams, Woodall, and Birch (2007) presented different examples for nonlinear relationships between a response variable (quality characteristic) and one explanatory variable.

---

*Corresponding author
ISSN: 1735-8272,

Williams, Woodall, and Birch (2007) recommended the use of three different formulations of T^2 control charts based on nonlinear regression parameters in phase I. Jensen and Birch (2009) proposed a mixed-effect model to monitor nonlinear profiles in the presence of within-profiles autocorrelation in phase I. Vaghefi, Tajbakhsh, and Noorossana (2009) extended two chart control schemes based on parametric model and metrics method to monitor nonlinear profiles in phase II. Noorossana, Izadbakhsh and Nayebpour (2014) examined customer satisfaction in tourism industry during the time using logistic profile and likelihood ratio test. Noorossana, Aminnayeri and Izadbakhsh (2013) investigated on dichotomous or polytomous variables. Polytomous variable, such as ordinal variables, have various applications. An ordinal variable is a categorical variable, which its values are related to a greater/lesser sense. They proposed four methods for monitoring Ordinal Logistic Regression profile. These four methods are as follows: χ^2 statistics, Multivariate Exponentially Weighted Moving Average (MEWMA), Exponentially Weighted Moving Average (EWMA) with R statistic, and a combination χ^2 statistics and (EWMA) with R statistic, that are used to monitor OLR profiles in phase II. They evaluated the Performances of these four methods by using Average Run Length (ARL). customer satisfaction in the tourist industry and sensory measurements of an electronic nose are their case studies to demonstration of proposed methods application.

PCIs in manufacturing industries are widely used as measures of process capability and performance are employed to evaluate the performance of process in producing conforming products. They are also used as measures to evaluate suppliers and improve quality.

Hosseinifard, Abbasi and Abdollahian (2011) investigated PCIs in non-normal simple linear profiles. Their work was based upon focusing on independent variables in simple linear profile. Hosseinifard and Abbasi (2012) studied determining of PCIs in simple linear profile. They used the ratio of unconfirmed items of independent variables to determine profile PCIs. Nemati et al (2014) presented functional approach to evaluate PCIs in simple linear profiles. Wang (2014) developed two new indices for measuring the process yield for simple linear profiles with one-sided specification. It used asymptotic distribution for estimated index.

Ebadi and Amiri (2012) proposed methods to investigate the process capability in multivariate simple linear profiles based on unconformity percentage and analysis of main components. Wang (2016) evaluated the process yield in multivariate linear profiles in manufacturing processes with TS_{pkA} process yield index. The Output of This index is an exact measure for the process yield and approximate confidence interval for TS_{pkA} is constructed. Wang and Tamirat (2015) investigated the process output in multivariate linear profiles with one-way specified limit.

Wang et al (2017) proposed a new acceptance sampling plan based on the exponentially weighted moving average (EWMA) with the yield index for simple linear profiles with one-sided specifications. The EWMA model provides information about current lots' and preceding lots' quality characteristics. The plan parameters are determined according to the smoothing constant of the EWMA statistic and various risks between the producer and the customer. A practical example from wind turbine manufacturing is applied to illustrate the performance of the proposed approach. Chiang et al (2017) investigated on multivariate exponentially weighted moving average (MEWMA) control chart for detecting process shifts during the phase II monitoring of simple linear profiles (SLPs) in the presence of within-profile autocorrelation. The proposed control chart is called MEWMA-SLP. two process capability indices are proposed for evaluating the capability of in-control SLP processes, and their utilization is demonstrated through examples.

Aslam et al (2018) proposed a new multiple dependent state repetitive sampling plan based on the yield index for linear profiles. The operating characteristic function of the proposed plan is developed for linear profiles. The plan parameters of the proposed sampling plan are determined through a nonlinear optimization problem. The plan parameters are reported for various combinations of acceptable quality level and limiting quality level.

Nemati et al (2014) suggested a functional approach to evaluate process capability in a circular profile and presented a functional technique to measure PCIs in that profile in a full range of the independent variables. The functional approach uses a reference profile, specified functional limit and functional natural tolerance limit to present a functional form of a PCI.

Nonlinear profile is a specific type of profile which may be expressed in form of nonlinear regression. Abbasi Charkhi, Aminnayeri, and Amiri (2015) presented PCIs for regression logistic profiles based on S_{pmk} index. They suggested a novel index S_{pmk} to measure process capability when process quality is described by the regression logistic profiles. Wang and Sivan Guo (2014) measured the process capability of a 4-patameters logistic profile and vertical density profile. Although various approaches have been, in recent years, presented to determine PCIs in simple linear profiles, poor research has been done on determining PCIs in nonlinear profiles. Guevara and Vargas (2015) proposed two methods for measuring the capability of nonlinear profile, based on the concept of functional depth. These methods do not have distributional assumptions and extended to functional data. The Process Capability Indexes proposed by Clements (1989) to measure the capability of a process characterized by a random variable. Performance of the proposed methods is evaluated through simulation studies. An example illustrates the applicability of these methods Guevara and Vargas (2016) proposed a method to measure the capability of multivariate nonlinear profile, based on principal components for multivariate functional data and the concept of functional depth. A simulation study is conducted to assess the performance of the proposed method. An example from the sugar production illustrates the applicability of their approach. Nonlinear functions bring a kind of complexity to the mind, as today, the reduction of complexity in solving problems and the use of simple mathematical relations for many industry and manufacturing expert are very attractive. Among the proposed research, the functional process capability method is a simple and efficient method. In addition, the method has the ability to calculate process capability index in each interval of the profile. The remaining of this paper is organized as follows. Sections 2 and 3 present functional approach to evaluate profile process capability and functional approach for monitoring a nonlinear profile, respectively. Section 4 uses vertical density profile to explain the results. Discussion and conclusions are presented in Section 5.

2- Functional process capability

Traditional capability indices are calculated as Equations (1) and (2):

$$C_P = \frac{(USL - LSL)}{6\sigma} \tag{1}$$

$$C_{PK} = \min\left\{\frac{USL-\mu}{3\sigma}, \frac{\mu - LSL}{3\sigma}\right\} \tag{2}$$

Where σ is the process standard deviation, LSL is the Lower Specification Limit and USL is the upper specification limit. UNTL is the Upper Natural Tolerance Limit ($UNTL = \mu + 3\sigma$) and LNTL is the Lower Natural Tolerance Limit ($LNTL = \mu - 3\sigma$) (Kane, 1986). For non-profile single-variable qualitative specifications, each of μ, LSL, USL, LNTL and UNTL are considered as a single point. But, in the profile state, each of these parameters is expressed as a function as $\mu(x)$, $LSL(x)$, $USL(x)$, $LNTL(x)$ and $UNTL(x)$, respectively. By replacing a functional form of parameters by equations of C_P and C_{PK} above functional forms of C_P and C_{PK} are obtained as $C_P(x)$ and $C_{PK}(x)$. Considering the functional limit presented for simple linear profile, FCIs of simple linear profile are determined as equation (3) and equation (4) (Nemati et al, 2014).

$$C_P(x) = \frac{USL_y(x) - LSL_y(x)}{UNTL_y(x) - LNTL_y(x)} \text{ , } x \in [x_l, x_u] \tag{3}$$

$$C_{PK}(x) = \min\left\{\frac{USL_y(x) - \mu_y(x)}{UNTL_y(x) - \mu_y(x)}, \frac{\mu_y(x) - LSL_y(x)}{\mu_y(x) - LNTL_y(x)}\right\} \text{ , } x \in [x_l, x_u] \tag{4}$$

Nemati et al (2014) describes that, $C_P(x)$ and $C_{PK}(x)$ represent process capability in generating a part of the profile at a point x of dependent variable. To calculate a number as the amount of process capability in profile generation, numerator and denominator of $C_P(x)$ and $C_{PK}(x)$ must be integrated into the defined range. The constrained marked area between curves is used as the result of these integrals. $C_P(profile)$ and $C_{PK}(profile)$ represent process capability in generating relative profile.

$$C_P(profile) = \frac{\int_{x_l}^{x_u}\left[USL_y(x) - LSL_y(x)\right]dx}{\int_{x_l}^{x_u}\left[UNTL_y\left(x\right) - LNTL_y(x)\right]dx} \tag{5}$$

$$C_{PK}\left(profile\right) = \min\left\{\frac{\int_{x_l}^{x_u}\left[USL_y\left(x\right) - \mu_y\left(x\right)\right]dx}{\int_{x_l}^{x_u}\left[UNTL_y\left(x\right) - \mu_y\left(x\right)\right]dx}, \frac{\int_{x_l}^{x_u}\left[\mu_y\left(x\right) - LSL_y\left(x\right)\right]dx}{\int_{x_l}^{x_u}\left[\mu_y\left(x\right) - LNTL_y\left(x\right)\right]dx}\right\} \tag{6}$$

3- Developing Functional process capability for nonlinear profile

A new method is proposed to develop functional process capability (FCI) for nonlinear profile and we considered vertical density profile as Illustrative example.

3-1- Nonlinear profile

In some profiles, there is a non-linear relationship between a response variable and one or more independent variables. A nonlinear profile is usually defined by a nonlinear regression model. The existence of nonlinear relationship in a regression means that if derivation is performed relative to parameters, the parameter still remains in the relationship. The overall equation of nonlinear regression is modeled as:

$$y_{ij} = f\,(x_{ij}, \beta_i) + \varepsilon_{ij} \tag{7}$$

That β_i is a $p \times 1$ vector of model parameters in i^{th} instance and its vector is as follows:

$$\beta_i = \left[\beta_{1i}, \beta_{2i} \ldots \beta_{pi}\right] \tag{8}$$

Moreover, x_{ij} is value of the independent variable in i^{th} vector instance which its vector is written as $x_i = \left[x_{1i}, x_{i2}, \ldots, x_{in}\right]$. y_{ij} is the value corresponding to x_{ij} and ε_{ij} is the value of number i of independent random variable ε_j with a normal distribution having the average of zero and variance of σ_j^2.

A nonlinear profile investigated by Williams, Woodall and Birch (2007) for woodboard vertical density for each $i = 1, \ldots, m$ and $j = 1, \ldots, n$ is shown in Eq. (9). Wang and Guo (2014) proposed the specification limits of the response at each level of the independent variable as Table 1.

$$f\,(x_{ij}, \beta) = \begin{cases} a_1(x_{ij} - c)^{b_1} + d \ , \ x_j > c \\ a_2(c - x_{ij})^{b_2} + d \ , \ x_j \le c \end{cases} \tag{9}$$

Table1. Specification limits for each level of independent variable

j	1-51	52-263	264-314
x	$0.002\times(j-1)$	$0.002\times(j-1)$	$0.002\times(j-1)$
USL_j	$65-0.23\times(j-1)$	$65-0.23\times50$	$53.5-0.23\times(j-263)$
LSL_j	$51-0.23\times(j-1)$	$51-0.23\times50$	$39.5-0.23\times(j-263)$

3-2- Proposed procedure

We propose a step-by-step procedure to evaluate process capability in nonlinear profile that has been shown in figure 1.

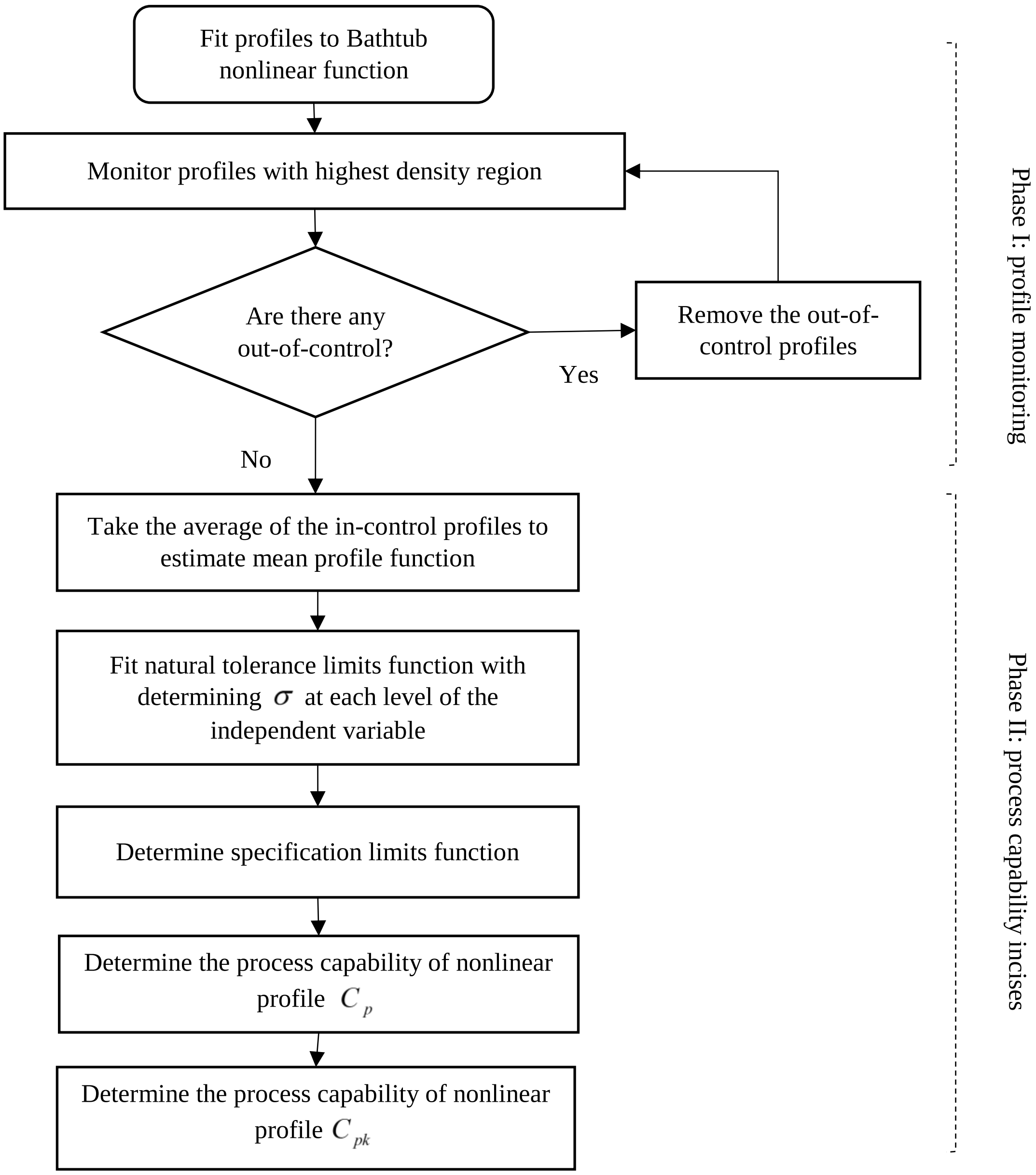

Fig.1. Flow chart of the procedure to evaluate process capability nonlinear profile

Hyndman (1996) proposed the highest density region (HDR) boxplot that can be used to detect any outliers for the profiles of data. In this paper $\mu_y(x)$, $LNTL(x)$ and $UNTL(x)$ fitted from local mean of y_i, $\mu-3\sigma$ and $\mu+3\sigma$ in each design point of x_i when process is in statistical control. Functional μ, UNTL and LNTL of y are given by:

$$\mu_y(x)=\begin{cases}a_1(x-c_1)^{b_1}+d_1\ ,\ x>c_1\\ a_2(c_1-x)^{b_2}+d_1\ ,\ x\le c_1\end{cases} \tag{10}$$

$$UNTL_y(x)=\begin{cases}a_3(x-c_2)^{b_3}+d_2\ ,\ x>c_2\\ a_4(c_2-x)^{b_4}+d_2\ ,\ x\le c_2\end{cases} \tag{11}$$

$$LNTL_y(x)=\begin{cases}a_5(x-c_3)^{b_5}+d_3\ ,\ x>c_3\\ a_6(c_3-x)^{b_6}+d_3\ ,\ x\le c_3\end{cases} \tag{12}$$

The specified functional limit of y is assumed as:

$$USL_y(x)=\begin{cases}a_7(x-c_4)^{b_7}+d_4\ ,x>c_4\\ a_8(c_4-x)^{b_8}+d_4\ ,x\le c_4\end{cases} \tag{13}$$

$$LSL_y(x)=\begin{cases}a_9(x-c_5)^{b_9}+d_5\ ,x>c_5\\ a_{10}(c_5-x)^{b_{10}}+d_5\ ,x\le c_5\end{cases} \tag{14}$$

$C_P(profile)$ and $C_{PK}(profile)$ of the nonlinear profile are calculated using the area constrained to, *LNTL(x)* , *UNTL(x), LSL(x),* and *USL(x)* (functional specification limit) . In general, $C_{PU}(profile)$ and $C_{PL}(profile)$ are obtained by equation (22) and equation (23).

$$C_P=\frac{\int_{x_l}^{x_u}\left[USL_y(x)-LSL_y(x)\right]dx}{\int_{x_l}^{x_u}\left[UNTL_y(x)-LNTL_y(x)\right]dx}=\frac{\left(\begin{array}{l}\int_{x_l}^{c_4}\left[a_8(c_4-x)^{b_8}+d_4\right]dx+\int_{c_4}^{x_u}\left[a_7(x-c_4)^{b_7}+d_4\right]dx\\ -\int_{x_l}^{c_5}\left[a_{10}(c_5-x)^{b_{10}}+d_5\right]dx-\int_{c_5}^{x_u}\left[a_9(x-c_5)^{b_9}+d_5\right]dx\end{array}\right)}{\left(\begin{array}{l}\int_{x_l}^{c_2}\left[a_4(c_2-x)^{b_4}+d_2\right]dx+\int_{c_2}^{x_u}\left[a_3(x-c_2)^{b_3}+d_2\right]dx\\ -\int_{x_l}^{c_3}\left[a_6(c_3-x)^{b_6}+d_3\right]dx-\int_{c_3}^{x_u}\left[a_5(x-c_3)^{b_5}+d_3\right]dx\end{array}\right)} \tag{15}$$

$$C_{PK}=\min\left\{\frac{\int_{x_l}^{x_u}\left[USL_y(x)-\mu_y(x)\right]dx}{\int_{x_l}^{x_u}\left[UNTL_y(x)-\mu_y(x)\right]dx},\frac{\int_{x_l}^{x_u}\left[\mu_y(x)-LSL_y(x)\right]dx}{\int_{x_l}^{x_u}\left[\mu_y(x)-LNTL_y(x)\right]dx}\right\}=\min\{C_{PU}(profile),C_{PL}(profile)\} \tag{16}$$

$$C_{PU}(profile)=\frac{\int_{x_l}^{x_u}\left[USL_y(x)-\mu_y(x)\right]dx}{\int_{x_l}^{x_u}\left[UNTL_y(x)-\mu_y(x)\right]dx}=\frac{\left(\begin{array}{l}\int_{x_l}^{c_4}\left[a_8(c_4-x)^{b_8}+d_4\right]dx+\int_{c_4}^{x_u}\left[a_7(x-c_4)^{b_7}+d_4\right]dx\\ -\int_{x_l}^{c_2}\left[a_2(c_1-x)^{b_2}+d_1\right]dx-\int_{c_1}^{x_u}\left[a_1(x-c_1)^{b_1}+d_1\right]dx\end{array}\right)}{\left(\begin{array}{l}\int_{x_l}^{c_3}\left[a_6(c_3-x)^{b_6}+d_3\right]dx+\int_{c_3}^{x_u}\left[a_5(x-c_3)^{b_5}+d_3\right]dx\\ -\int_{x_l}^{c_1}\left[a_2(c_1-x)^{b_2}+d_1\right]dx-\int_{c_1}^{x_u}\left[a_1(x-c_1)^{b_1}+d_1\right]dx\end{array}\right)} \tag{17}$$

$$C_{PL}(profile)=\frac{\int_{x_l}^{x_u}\left[\mu_y(x)-LSL_y(x)\right]dx}{\int_{x_l}^{x_u}\left[\mu_y(x)-LNTL_y(x)\right]dx}=\frac{\left(\begin{array}{l}\int_{x_l}^{c_2}\left[a_2(c_1-x)^{b_2}+d_1\right]dx+\int_{c_1}^{x_u}\left[a_1(x-c_1)^{b_1}+d_1\right]dx\\ -\int_{x_l}^{c_5}\left[a_{10}(c_5-x)^{b_{10}}+d_5\right]dx-\int_{c_5}^{x_u}\left[a_9(x-c_5)^{b_9}+d_5\right]dx\end{array}\right)}{\left(\begin{array}{l}\int_{x_l}^{c_2}\left[a_2(c_1-x)^{b_2}+d_1\right]dx+\int_{c_1}^{x_u}\left[a_1(x-c_1)^{b_1}+d_1\right]dx\\ -\int_{x_l}^{c_3}\left[a_6(c_3-x)^{b_6}+d_3\right]dx-\int_{c_3}^{x_u}\left[a_5(x-c_3)^{b_5}+d_3\right]dx\end{array}\right)} \tag{18}$$

4- Case example

Young et al (1999) proposed the relationship between the vertical density of woodboard and different depths as a nonlinear profile. A profile meter (benefiting from Laser technology) is used for standard sampling of vertical density. The device scans different depths and measures their densities. In this example, it is called x_{ij} the depth of i^{th} location for j^{th} sheet and showed it with the vector x_{ij}.

Values of x_{ij} were obtained for 314 depths. In fact, values of independent variable x_{ij} are specified and equal to $0.002j$ mm for every $j = 1,2,...,314$. In other words, for every $0.002j$ from depth toward inside the sheet, vertical density is recorded and the values of (x_{ij}, y_{ij}) will form a nonlinear fitness curve.

using HDR method profiles 6 indicate the outliers. So, profile 6 is omitted from the calculation of nonlinear profile process capability and it is bolted in figure 2.

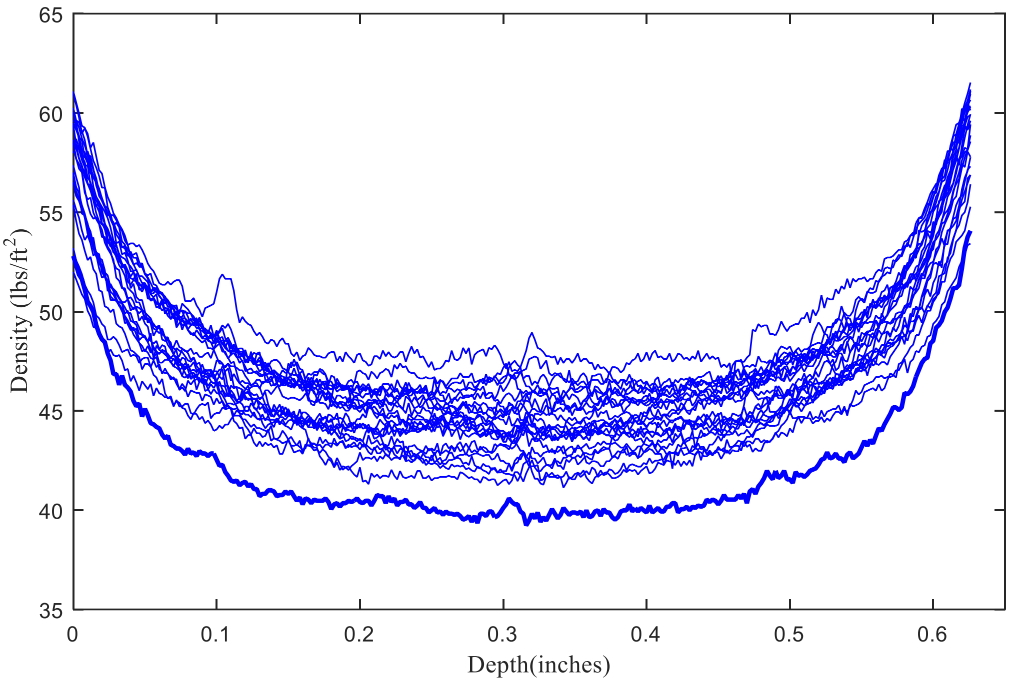

Fig.2. Profiles and outlier in bold (6)

Mean function $\mu(x)$ is fitted by local mean of y_i in each level of independent variable (Figure 3). In the same way we will estimate $LNTL(x)$ and $UNTL(x)$ from $\mu - 3\sigma$ and $\mu + 3\sigma$ in each design point x_i when process is in statistical control (figure 4 and figure 5). Respectively equation of $\mu(x)$, $UNTL(x)$ and $LNTL(x)$ are as follows:

$$\mu_y(x) = \begin{cases} 2600.67(x - 0.316)^{4.49354} + 44.6719 & , x > 0.316 \\ 2270.82(0.316 - x)^{4.5276} + 44.6719 & , x \le 0.316 \end{cases}$$

$$UNTL_y(x) = \begin{cases} 6399.03(x - 0.322)^{5} + 49.1281 & , x > 0.322 \\ 5971.09(0.322 - x)^{5.26102} + 49.1281 & , x \le 0.322 \end{cases}$$

$$LNTL_y(x) = \begin{cases} 636.288(x - 0.314)^{3.47967} + 40 & , x > 0.314 \\ 421(0.314 - x)^{3.29976} + 40 & , x \le 0.314 \end{cases}$$

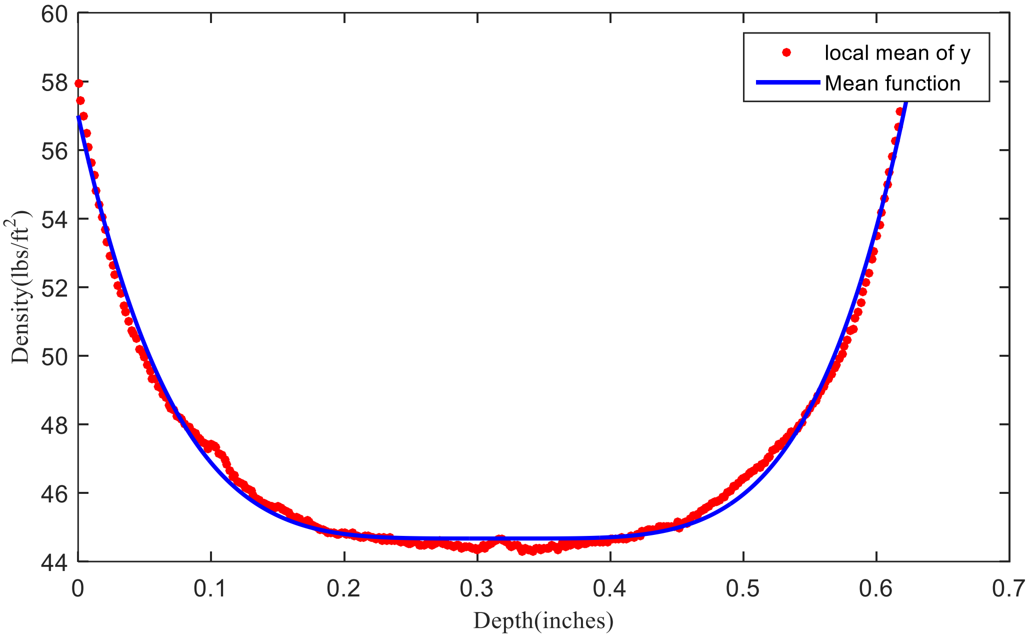

Fig.3. Mean function estimation from local Mean

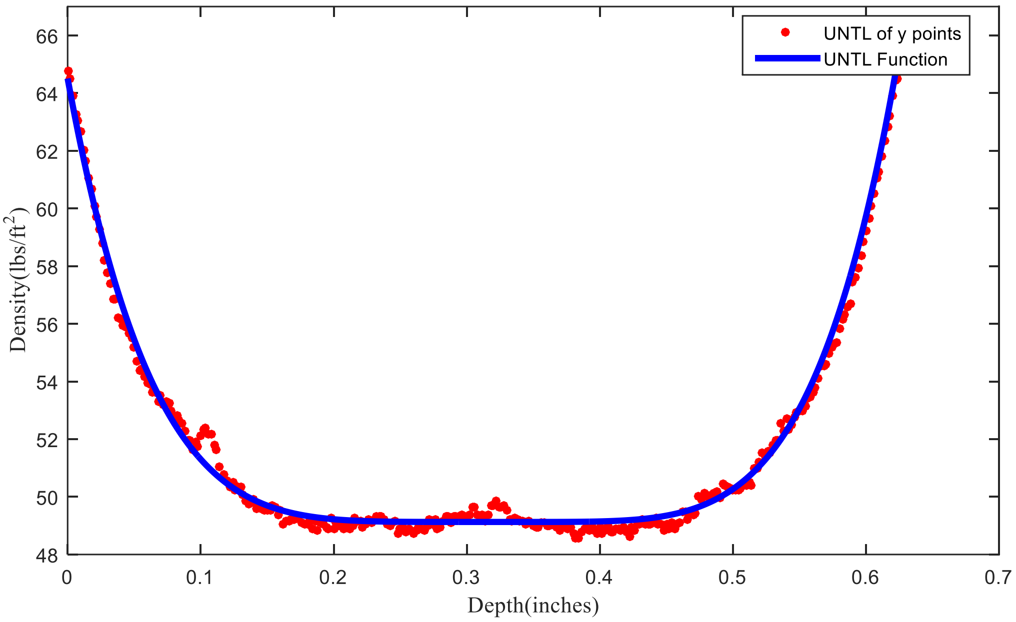

Fig.4. Functional UNTL fitted by $\mu+3\sigma$ in each design point x_i

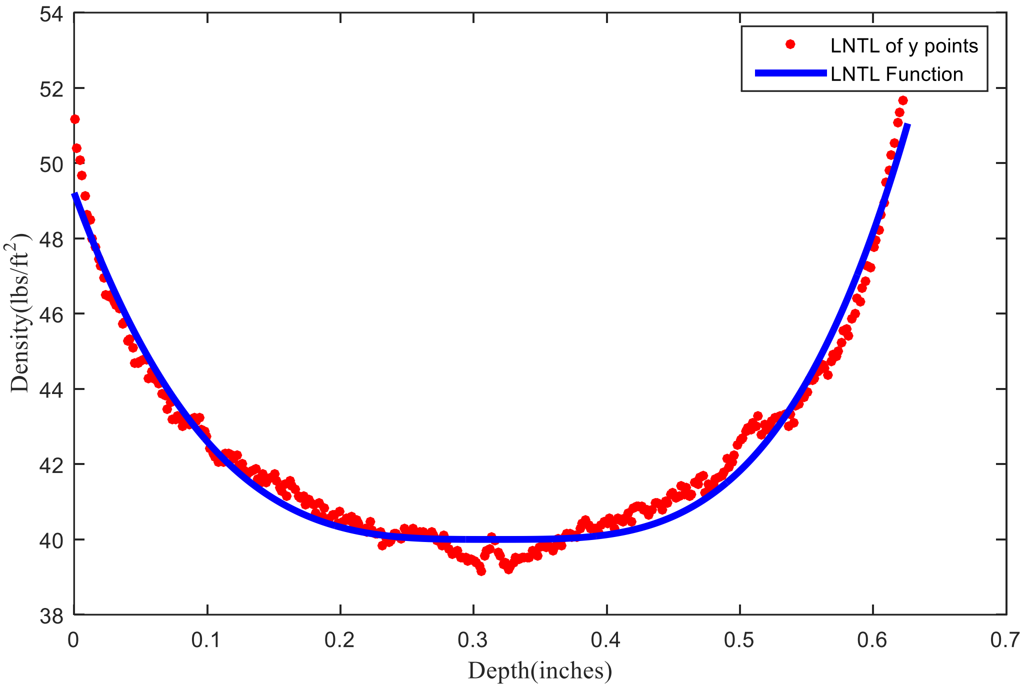

Fig.5. $LNTL(x)$ estimated from each level of independent variable

Specification limits estimated from specification limits for each level of independent variable proposed by Wang and Sivan Guo (2014). The functional upper specification limit of vertical density profile is obtained as follows for every $a_7 = 5835.94$, $a_8 = 6573.02$, $b_7 = 5.236$, $b_8 = 5.3548$, $d_4 = 53.2695$, $c_4 = 0.313$ (figure 6).

$$USL_y(x) = \begin{cases} 5835.94(x-0.313)^{5.236} + 53.2695 & , x > 0.313 \\ 6573.02(0.313-x)^{5.3548} + 53.2695 & , x \le 0.313 \end{cases}$$

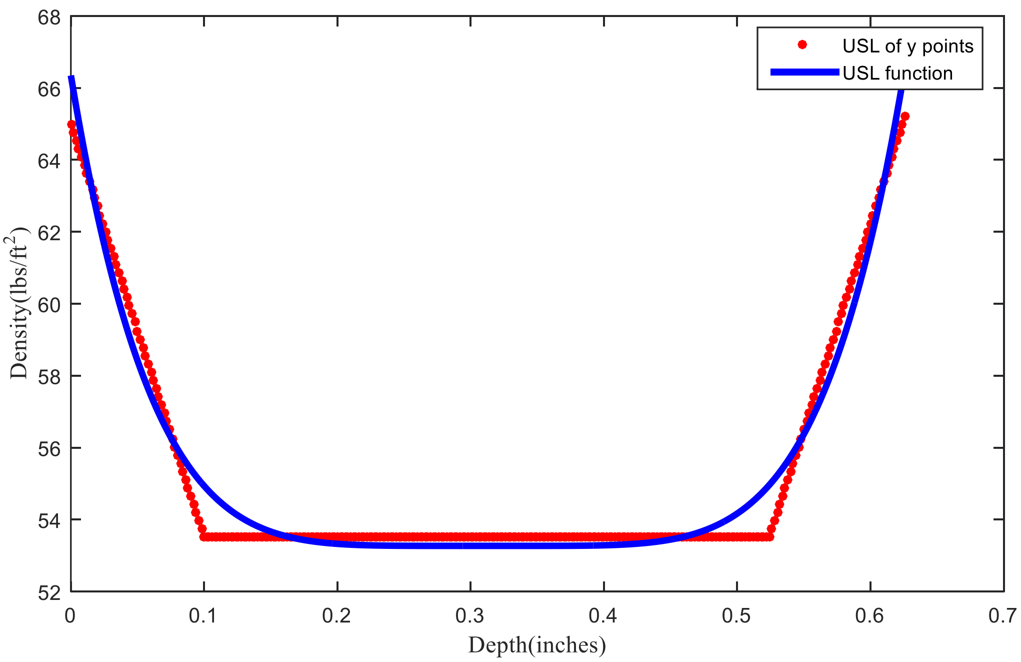

Fig.6. USL Functional fitted from USL for each level of independent variable

Using equation (14) and despite $a_9 = 5907$, $a_{10} = 6656.83$, $b_9 = 5.24685$, $b_{10} = 5.3661$, $d_5 = 39.2793$ and $c_5 = 0.313$, The functional lower specification limit of vertical density as follows:

$$LSL_y(x) = \begin{cases} 5907(x - 0.313)^{5.24685} + 39.2793 & , x > 0.313 \\ 6656.83(0.313 - x)^{5.3661} + 39.2793 & , x \leq 0.313 \end{cases}$$

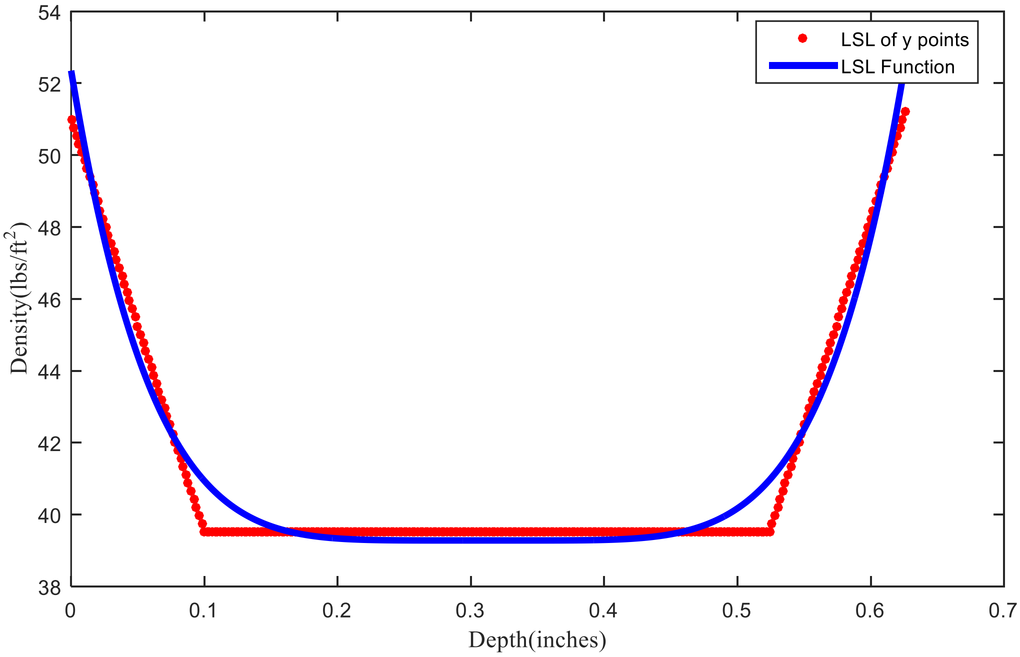

Fig.7. $LSL(x)$ estimated in design point x_i

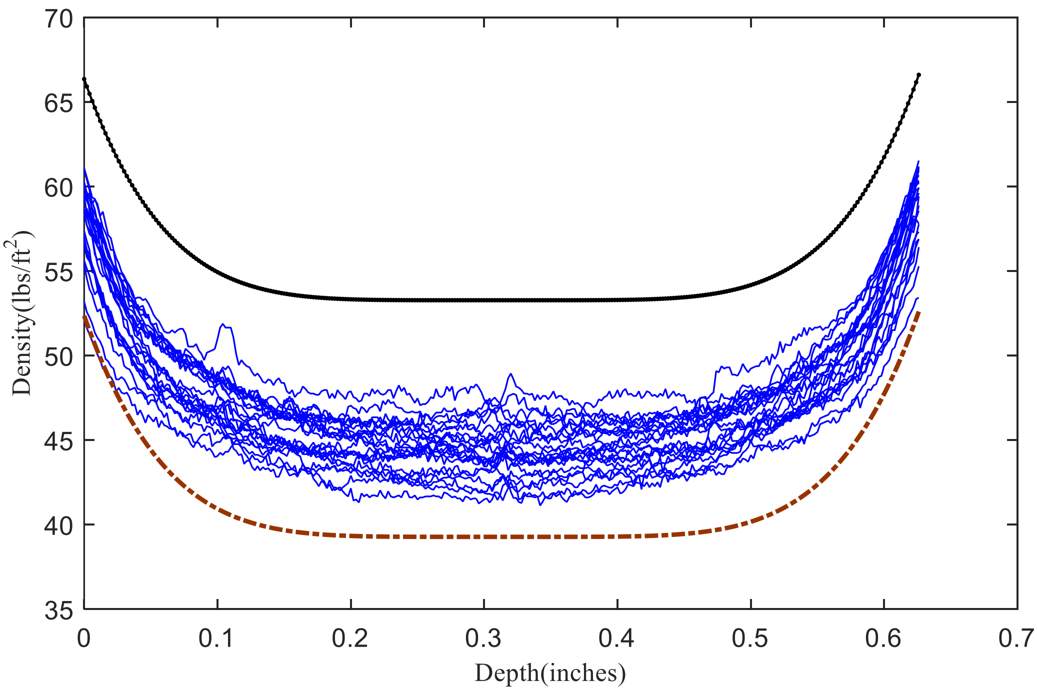

Fig.8. profiles between $LSL(x)$ and $USL(x)$

Figure 8 shows the number of remained profiles between two functional specification limits. This graph shows that no profile has been taken out of this range.

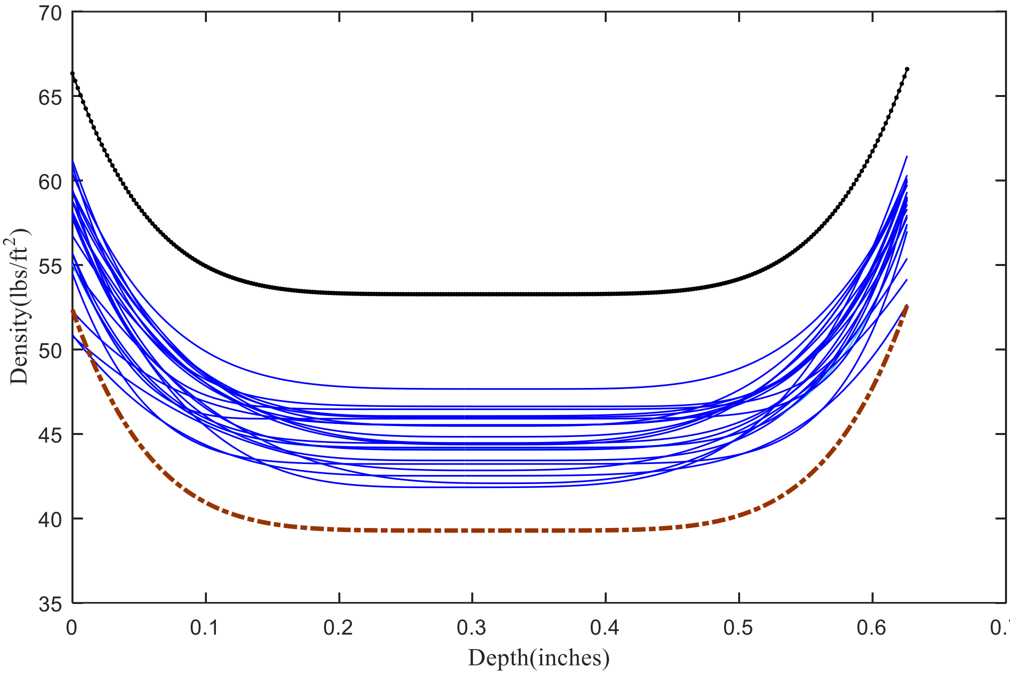

Fig.9. 23 remained fitted VDPs in enclosed area of functional LSL and USL

As shown in figure 9, the remained profiles are approximated as functional profile. This figure indicates that these profiles are under control, between *LSL(x)* and *USL(x)*.

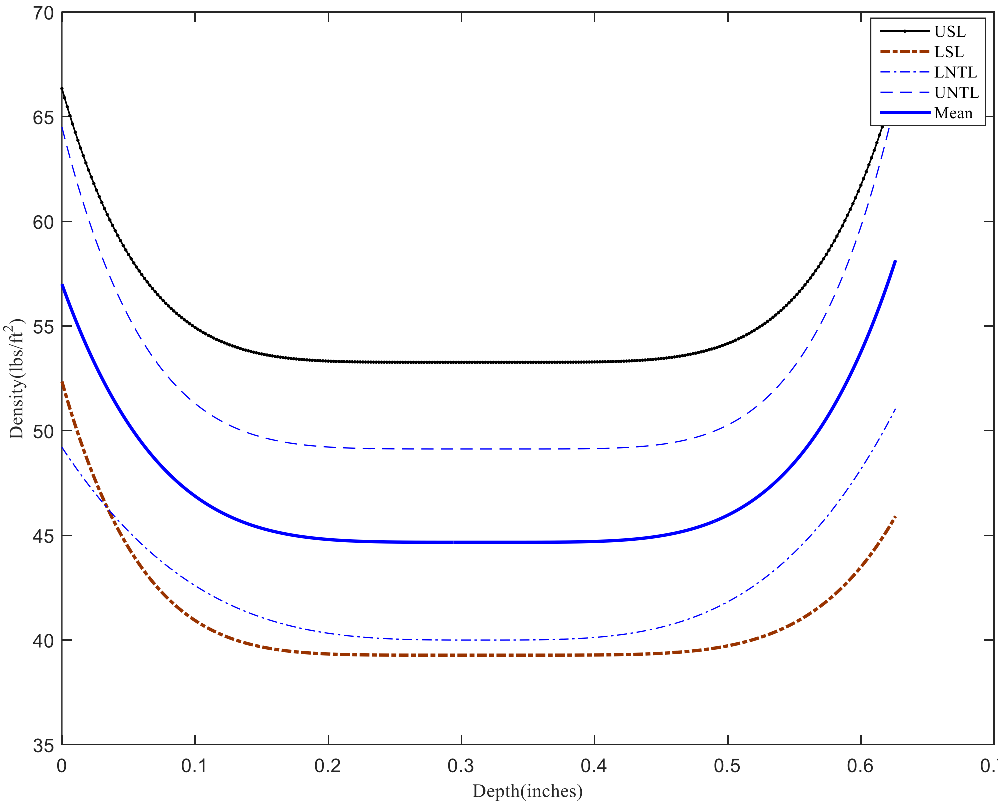

Fig.10. Functional LSL, USL, μ, LNTL and UNTL

Considering presented functional limits for the nonlinear profile and using the marked area in figure10, capability indices are obtained by as follows:

$$C_P\left(profile\right)=\frac{\begin{pmatrix}\int_0^{0.313}\left[(6573.02(0.313-x)^{5.3548}+53.2695)\right]dx+\int_{0.313}^{0.626}\left[5835.94(x-0.313)^{5.236}+53.2695\right]dx\\-\int_0^{0.313}\left[6656.83(0.313-x)^{5.3661}+39.2793\right]dx-\int_{0.313}^{0.626}\left[5907(x-0.313)^{5.24685}+39.2793\right]dx\end{pmatrix}}{\begin{pmatrix}\int_0^{0.322}\left[(5971.09(0.322-x)^{5.26102}+49.1281)\right]dx+\int_{0.322}^{0.626}\left[6399.03(x-0.322)^{5}+49.1281\right]dx\\-\int_0^{0.314}\left[421(0.314-x)^{3.29976}+40\right]dx-\int_{0.314}^{0.626}\left[636.288(x-0.314)^{3.47967}+40\right]dx\end{pmatrix}}=1.46$$

$$C_{PU}\left(profile\right)=\frac{\begin{pmatrix}\int_0^{0.313}\left[(6573.02(0.313-x)^{5.3548}+53.2695)\right]dx+\int_{0.313}^{0.626}\left[5835.94(x-0.313)^{5.236}+53.2695\right]dx\\-\int_0^{0.316}\left[2270.82(0.316-x)^{4.5276}+44.6719\right]dx-\int_{0.316}^{0.626}\left[2600.67(0.316-x)^{4.49354}+44.6719\right]dx\end{pmatrix}}{\begin{pmatrix}\int_0^{0.322}\left[(5971.09(0.322-x)^{5.26102}+49.1281)\right]dx+\int_{0.322}^{0.626}\left[6399.03(x-0.322)^{5}+49.1281\right]dx\\-\int_0^{0.316}\left[2270.82(0.316-x)^{4.5276}+44.6719\right]dx-\int_{0.316}^{0.626}\left[2600.67(0.316-x)^{4.49354}+44.6719\right]dx\end{pmatrix}}=1.514$$

$$C_{PL}\left(profile\right)=\frac{\begin{pmatrix}\int_0^{0.316}\left[2270.82(0.316-x)^{4.5276}+44.6719\right]dx+\int_{0.316}^{0.626}\left[2600.67(0.316-x)^{4.49354}+44.6719\right]dx\\-\int_0^{0.313}\left[6656.83(0.313-x)^{5.3661}+39.2793\right]dx-\int_{0.313}^{0.626}\left[5907(x-0.313)^{5.24685}+39.2793\right]dx\end{pmatrix}}{\begin{pmatrix}\int_0^{0.316}\left[2270.82(0.316-x)^{4.5276}+44.6719\right]dx+\int_{0.316}^{0.626}\left[2600.67(0.316-x)^{4.49354}+44.6719\right]dx\\-\int_0^{0.314}\left[421(0.314-x)^{3.29976}+40\right]dx-\int_{0.314}^{0.626}\left[636.288(x-0.314)^{3.47967}+40\right]dx\end{pmatrix}}=1.16$$

$$C_{pk}=\min\left\{C_{PU}(profile),C_{PL}(profile)\right\}=\min\left\{1.514,1.16\right\}=1.16$$

The computational results obtained by this method are very similar to the previous methods. Wang and Guo (2014) have shown that capability index of this process in production of coinciding products is located between 1.0605 to 1.0940 after removing the Profile 6. Similar to the traditional univariate case *C_P(profile)* shows the potential capability of process. *C_P(profile)* is greater than 1 if difference of *UNTL(x)*, and *LNTL(x)*is less than difference of *USL(x)* and *LSL(x)*. In this case C_P and C_{PK} greater than 1. Accordingly, we can conclude that process has a high capability to producing woodboard. in general, its good process at generating nonlinear profiles between depth and vertical.

5- Conclusion

This paper developed an FCI (Functional Capability Index) for nonlinear profiles. Using this FCI, it is possible to estimate process capability in producing mentioned profile for every value of the independent variable. Furthermore, the manner of determining a figure as the process capability in generating a nonlinear profile in the whole range of independent variable is presented here. A case example on density of woodboard was reviewed in which depth and density were independent and dependent variables, respectively. The results indicated that the considered process performs well in producing intact nonlinear profiles between depth and density.

It is suggested to develop the functional approach for capability evaluation of other types of profiles including multivariate profiles and geometric profiles. For future research, it is recommended to evaluating the capability of the profile generator process when parameters or independent variable are uncertain.

References

Aslam, M., Wang, F. K., Khan, N., & Jun, C. H., 2018. A multiple dependent state repetitive sampling plan for linear profiles. *Journal of the Operational Research Society, 69*(3), pp. 467-473.

Chiang, J.Y., Lio, Y.L. and Tsai, T.R., 2017. MEWMA Control Chart and Process Capability Indices for Simple Linear Profiles with Within-profile Autocorrelation. Quality and Reliability Engineering International, 33(5), pp.1083-1094.

Clements, J.A., 1989. Process capability calculations for non-normal distributions. *Quality progress*, 22, pp.95-100.

Ebadi, M. and Amiri, A., 2012. Evaluation of process capability in multivariate simple linear profiles. *Scientia Iranica*, *19*(6), pp.1960-1968.

Guevara, R.D. and Vargas, J.A., 2015. Process capability analysis for nonlinear profiles using depth functions. *Quality and Reliability Engineering International*, *31*(3), pp.465-487.

Guevara, R.D. and Vargas, J.A., 2016. Evaluation of process capability in multivariate nonlinear profiles .*Journal of Statistical Computation and Simulation*,*86(12)* ,pp.2411-2428.

Hosseinifard, S.Z., Abbasi, B. and Abdollahian, M., 2011, April. Performance analysis in non-normal linear profiles using gamma distribution. *In Information Technology: New Generations (ITNG), 2011 Eighth International Conference on* (pp. 603-607). IEEE.

Hyndman, R.J., 1996. Computing and graphing highest density regions. *The American Statistician, 50*(2), pp.120-126.

Jensen, W.A. and Birch, J.B., 2009. Profile monitoring via nonlinear mixed models. *Journal of Quality Technology*, *41*(1), p.18.

Kane, V.E., 1986. Process capability indices. *Journal of quality technology*, *18*(1), pp.41-52.

Keshteli, R.N., Kazemzadeh, R.B., Amiri, A. and Noorossana, R., 2014. Developing functional process capability indices for simple linear profile. *Scientia Iranica. Transaction E, Industrial Engineering*, *21*(3), p.1044.

Keshteli, R. N., Kazemzadeh, R.B., Amiri, A. and Noorossana, R., 2014. Functional process capability indices for circular profile. *Quality and Reliability Engineering International*, *30*(5), pp.633-644.

Noorossana, R., Aminnayeri, M. and Izadbakhsh, H., 2013. Statistical monitoring of polytomous logistic profiles in phase II. *Scientia Iranica*, *20*(3), pp.958-966.

Noorossana, R., Izadbakhsh, H. and Nayebpour, M.R., 2014. A Likelihood Ratio Test Approach to Profile Monitoring in Tourism Industry. *Applications & Applied Mathematics*, *9*(2).

Noorossana, R., Saghaei, A. and Amiri, A., 2011. *Statistical analysis of profile monitoring (Vol. 865)*. John Wiley & Sons.

Rezaye Abbasi Charkhi, M., Aminnayeri, M. and Amiri, A., 2016. Process capability indices for logistic regression profile. *Quality and Reliability Engineering International*, *32*(5), pp.1655-1661.

Vaghefi, A., Tajbakhsh, S.D. and Noorossana, R., 2009. Phase II monitoring of nonlinear profiles. *Communications in Statistics—Theory and Methods*, *38*(11), pp.1834-1851.

Walker, E. and Wright, S.P., 2002. Comparing curves using additive models. *Journal of Quality Technology*, *34*(1), p.118.

Wang, F.K., 2014. Measuring the process yield for simple linear profiles with one-sided specification. *Quality and Reliability Engineering International*, *30*(8), pp.1145-1151.

Wang, F.K. and Guo, Y.C., 2014. Measuring process yield for nonlinear profiles. *Quality and Reliability Engineering International, 30*(8), pp.1333-1339.

Wang, F.K., 2016. Process yield analysis for multivariate linear profiles. *Quality Technology & Quantitative Management, 13*(2), pp.124-138.

Wang, F.K. and Tamirat, Y., 2016. Process Yield for Multivariate Linear Profiles with One-sided Specification Limits. *Quality and Reliability Engineering International, 32*(4), pp.1281-1293.

Wang, F.K., Huang, C.Y. and Tamirat, Y., 2017. Implementing EWMA Yield Index for Simple Linear Profiles with One-sided Specifications in Product Acceptance Determination. *Quality and Reliability Engineering International*, *33*(2), pp.401-412.

Williams, J.D., Woodall, W.H. and Birch, J.B., 2007. Statistical monitoring of nonlinear product and process quality profiles. *Quality and Reliability Engineering International, 23*(8), pp.925-941.

Young, T.M., Winistorfer, P.M. and Wang, S., 1999. Multivariate control charts of MDF and OSB vertical density profile attributes. *Forest Products Journal, 49*(5), p.79.

Zahra Hosseinifard, S. and Abbasi, B., 2012. Evaluation of process capability indices of linear profiles. *International Journal of Quality & Reliability Management*, *29*(2), pp.162-176.