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A comprehensive model for concurrent optimization of product family and its supply chain network design considering reverse logistic

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Abstract

The study of product family and its design as well as issues related to supply chain is as fascinating discussion, and its modeling and optimization consider as a challenge for industries and businesses. In this paper, using a consolidated approach, a comprehensive model in the Mixed Integer Linear Program (MILP) dominant is proposed to concurrent optimization of product family and its supply chain network design by considering reverse logistics. In the proposed model, different levels of bill of material, including components, sub-assemblies, sub-sub-assemblies and finished products are considered while there are the possibility of substitution at all levels. The supply chain network, includes 5 levels consist of suppliers, factories, distribution centers, customers and recycling centers. To solve low complexity instances in the view of products design and supply chain network structure, CPLEX solver has been applied. To solve high complexity instances, a heuristic method based on linear programming rounding has been developed, which caused a considerable reduction in solving time with an acceptable gap.

Keywords: Supply chain network, product family, closed-loop network, mixed integer linear programming, LP rounding based method

1- Introduction

Recently, increasing demand heterogeneity and shorter product life cycles have created a rigid competition among most manufacturing and service companies (Fixson, 2005). In today's competitive market, paying attention to the diversity of customer demand is inevitable (Porter, 2008). This diversity of demand has been reported in various industries (Bonev et al., 2015). Since it is believed that increasing the variety of products potentially leads to greater profitability (Young, 2005), the focus of the companies on increasing the product diversity and agility of their product line could increase their market share. On the other hand, this increase in product diversity will potentially lead to increased complexity in the delivery, production and distribution program (Ruijter et al., 2011). In dealing such a dilemma, companies are faced with a difficult decision making. Design and development of product families recognized as an effective means to achieve economics scale for more variety of products in diverse markets (Meyer and Utterback, 1993).

A product family is a set of individual products that share common technology and address a related set of market application (Meyer and Lehnerd, 1997).

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Product family design is a difficult task that involves the complexity of product design with challenging multi-product design synchronization (Simpson et al., 2012). By sharing components and production processes across a platform of products, companies can develop differentiated products efficiently, increase the flexibility and responsiveness of their manufacturing processes, and take market share away from competitors that develop only one product at a time (Robertson and Ulrich, 1998).

Any changes in the product structure subsequently affect the supply chain and, consequently, the quality and cost of the product, which is a major factor in the competitive market (Rezapour et al., 2015). It has been proven that 85% of logistics costs are based on design choices (Laurentie et al., 2006) and more than 70% of product costs are determined by decision making at this stage (H‘mida and Martin, 2007). On the other hand, the cost of design changes increases with the passing of the design phase of the product life cycle and the entry into the production phase, collaboration with supply chain partners in the design phase of the product life cycle can make a lot of profit (Gokhan et al., 2010). The design of a supply chain has a major impact on the organization of a family of products (Hsuan Mikkola and Skjøtt-Larsen, 2004). Peterson et al. argue that the creation of an integrated stream between suppliers and the development of products has a direct impact on process design decisions and, consequently, on the configuration of the supply chain network (Petersen et al., 2005). The combination of products design and the supply chain decisions surely will reduce additional cost (Nepal et al., 2012).

So far, many studies have been done on product family and supply chain scope. But it is mainly studied in this area as a problem of optimization considering the priority and the future for these two concepts (e.g. Yu and Huang, 2010 and Mansoornejad et al., 2010). Sometimes it is considered as a bi-level problem or a Stakelberg game (e.g. Du et al., 2014, de Weck et al., 2003 and Wang et al., 2016). Lavigne et al., (2016) by comparing simultaneous and sequential mode, showed that when the product family and its supply chain network are optimized simultaneously, 1-25 percent would be effective in reducing costs (Baud-Lavigne et al., 2016).

There are also different incentives for creation of reverse flow of products in the design of the supply chain network and returning products to the production cycle at the end of their life cycle e.g. increased profitability, ethical responsibility, legislation, secured spare part supply, increased market share and brand protection(Seitz and Peattie, 2004). In recent decades, many companies like HP, Xerox and Kodak have benefited from this approach (Üster et al., 2007).

In this paper, we present a comprehensive model for concurrent optimization of a product family and its closed-loop supply chain network is presented in a closed considering multiple time periods which have created dynamism in our model. Also, due to the increasing complexity of the samples, it was not possible to solve them in a polynomial time, an approximate method based on linear programming rounding has been proposed, which resulted in a significant reduction in sample solving time. It should be noted that this reduction in solving time resulted in a gap of less than 2% of the exact solution.

Table 1 presents a number of recent papers published based on product family and supply chain characteristics and decisions such as single product or product family design optimization, simultaneous or sequential optimization, type of supply chain network, single-period or multi-periodic model.

Table 1. Some of recent papers in product/product family and supply chain network

Article	Design of		Supply chain network		Optimization		Period		Modeling and Solution Approach
	Single product	Product family	Typical	Closed-loop	Asynchronous	Concurrent	Single	Multiple	
Lamothe et al. (2006)		✓	✓		✓			✓	MILP
Labbi et al. (2015)	✓			✓		✓		✓	MILP
Wang et al. (2016)		✓	✓		✓		✓		NLP-GA
Mostafavi (2014)	✓		✓			✓	✓		MILP
Chiu and Okudan (2014)	✓		✓		✓		✓		NLP
Rezapour et al. (2015)		✓	✓			✓	✓		Bi-objective
Khajavirad et al. (2009)		✓	✓		✓		✓		Multi objective-GA
Zhu and He (2017)	✓			✓	✓		✓		Game-theoretic approach
Yang et al. (2015)		✓	✓		✓		✓		NLP-NGA
Stefansdottir and Grunow (2018)	✓		✓			✓		✓	SMILP
Lavigne et al. (2016)		✓	✓			✓	✓		MILP
This paper (2018)		✓		✓		✓		✓	MILP-LPR

The main contributions of this paper are as follows:

- Considering reverse flow of final products at the end of their life cycle from customers to recycling centers
- Supposing possibility of substitution at all levels of BOMs. In some instances, the products structure has broken up to 5 levels.
- Consideration of multiple periods in mathematical model, which makes it dynamic.
- Developing a heuristic method based on linear programming rounding to solve high complexity instances in the view of products and supply chain structure, which significantly reduces the problems solving time in acceptable gap.

In the next section, after the problem description, the proposed mathematical model will be presented. Section 3, involves the problem-solving approach includes exact solution for first-level instances and proposed heuristic method for solving second-level problems. In section 4, the results of the implementation of the model and the application of the proposed solving algorithm are presented. In section 5, the results of the paper and suggestions for future studies are presented.

2- Model formulation

2-1- Problem Description

In the context of simultaneous optimization of the product family and supply chain, two concepts of supply chain network and product family design are discussed. In this paper, the supply chain is considered as a 5-level public network consisting of suppliers, factories, distribution centers, customers, and recycling centers, as shown below in figure 1.

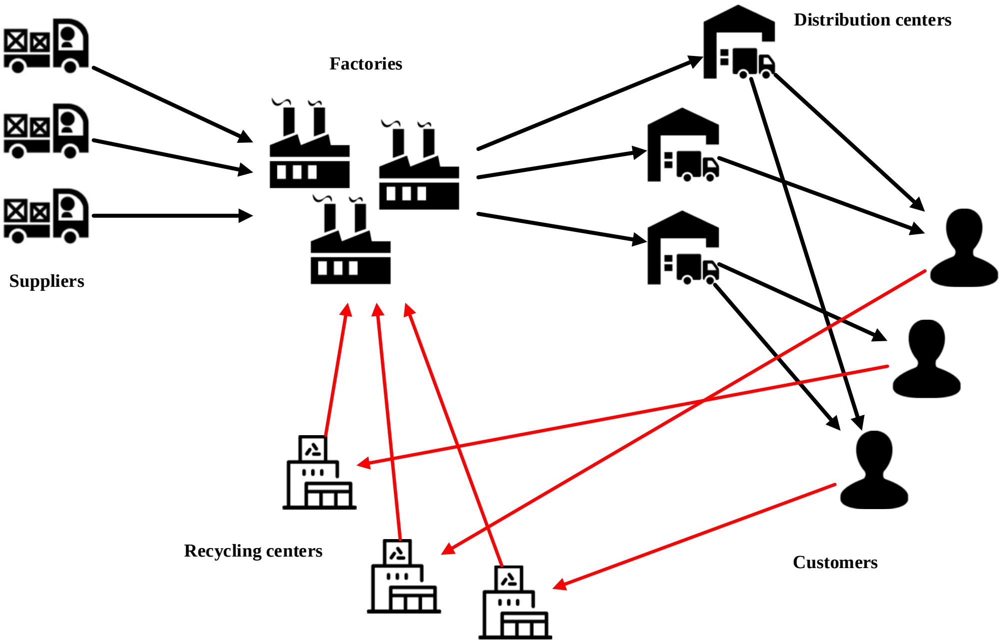

Fig 1. Considered supply chain network structure

Components and sub-assemblies can be moved between factories. The finished products are shipped between factories and distribution centers and customers. Products will be delivered to recycling centers at the end of their lifetime.

The objectives of optimizing this set are to determine the optimal design of the products, optimal allocation of production and assembling of products to factories, and to determine the optimal flow between the network nodes.

In design of the product family, we seek to determine the optimal structure of all the finished products. One of the methods used to deal with this issue is the definition of the BOM as a decision variable. Using of this approach will usually results in the formation of quadratic equations in the mathematical model (Chen, 2010). In this paper, we consider the concept of substitution through product transformation presented in (Baud-Lavigne et al., 2016). When part X can be replaced by part Y, a virtual process can transform X into Y. Then, a mixture is created in a plant containing an amount of X that is made up of the actual X parts and the alternatives that have been transformed in to Y. This modeling allows substitution, while keeping the formulation light. In fact, the number of additional variables is exactly equal to the number of substitution possibilities(Baud-Lavigne et al., 2016).

The problem is formulated as a multi-period mixed integer linear programming model (MILP).

2-2- Model notations

The notations used to demonstrate the proposed model are defined below:

Network nodes	
s	Suppliers (SU)
i	Factories (PL)
d	Distribution centers (DI)
c	Customers (CU)
e	Recycling centers (RE)
Sets	
$p, q \in P$	Products
$C, U, F \subset r$	Production level in BOM

Parameters

g^{PC}	Quantity of component *C* in *p*. *g* represents the BOM
h^{PU}	Quantity of subset *U* in *p*
de_{ct}^{p}	Demand of product *p* by customer *i* in period *t*
RT^{p}	The proportion of product *p* returned to recycling centers
RJ^{pr}	A proportion of product *p* that is returned to the factories after being recycled as *r* (component or subset)

Decision variable (Cost)

Continuous variables:

A_{it}^{pr}	Quantity of *p* produced at plant *i* in period *t* (α_{it}^{pr})
ST_{it}^{qpr}	Quantity of *q* that substitute for *p* in level *r* at plant *i* in period *t* (σ_{it}^{qpr})
FS_{sit}^{pr}	Flow of *r* (component or subset) product *p* from supplier *s* to plant *i* in period *t* (ϕ_{sit}^{pr})
FP_{idt}^{p}	Flow of finished product *p* from plant *i* to distribution center *d* in period *t* (μ_{idt}^{p})
FD_{dct}^{p}	Flow of finished product *p* from distribution center *d* to customer *c* in period *t* (ε_{dct}^{p})
FC_{cet}^{p}	Flow of product *p* from customer *c* to recycling center *e* (λ_{cet}^{p})
FE_{eit}^{pr}	Flow of *r* (component or subset) product *p* from recycling center *e* to plant *i* in period *t* (ν_{eit}^{pr})

Binary variables:

X_{it}^{pr}	Production of *p* at *i* in period *t* (β_{it}^{pr})
M_{sit}^{pr}	Use of flow of *p* in level *r* from *s* to *i* in period *t* (ω_{sit}^{pr})
O_{idt}^{p}	Use of flow of finished product *p* from *i* to *d* in period *t* (υ_{idt}^{p})
Y_{dct}^{p}	Use of flow of finished product *p* from *d* to *c* in period *t* (π_{dct}^{p})
W_{cet}^{p}	Use of flow of finished product *p* from *c* to *e* in period *t* (ζ_{cet}^{p})
T_{eit}^{pr}	Use of flow of *p* in level *r* from e to *i* in period *t* (τ_{eit}^{pr})

2-3- Model assumptions

Each model is always presented with specific assumptions that are determined according to the complexity of the problem. For example, in this model, the flow capacity between all nodes is assumed to be limited, which will be quite different from the time when this assumption does not exist in the model. The assumptions of this paper are as follows:

- The capacity of all network nodes is limited.
- Customer demand in each period should be provided in the same period.
- There are substitution possibility at all levels of the BOM, including components, sub-assemblies and the final product.
- Substitution of products only can be done in factories.

2-4- Mathematical formulation

In this section, a mathematical model is proposed to optimize concurrent design of the product family and its supply chain network considering reverse logistic. There are different ways to formulate a real-world problem, and there are various ways to optimize each of them. This paper attempts to propose a model with minimal complexity.

$$Z = \min \sum_{i \in PL} \sum_{p \in P} \sum_{r \in U \cup F} \sum_{t} (A_{it}^{pr} \alpha_{i}^{pr} + B_{it}^{pr} \beta_{i}^{pr}) + \sum_{i \in PL} \sum_{p \in P} \sum_{q \in P \backslash \{p\}} \sum_{t} ST_{it}^{qpr} \sigma_{i}^{qpr} +$$

$$\sum_{i \in PL} \sum_{s \in SU} \sum_{p \in P} \sum_{r \in R \cup S} \sum_{t} (RF_{sit}^{pr} \phi_{si}^{pr} + M_{sit}^{pr} \omega_{ij}^{p}) + \sum_{i \in PL} \sum_{d \in DI} \sum_{p \in P} \sum_{t} (\mathrm{FD}_{idt}^{p} \mu_{id}^{p} + O_{idt}^{p} \upsilon_{id}^{p}) +$$

$$\sum_{d \in DI} \sum_{c \in CU} \sum_{p \in P} \sum_{t} (HF_{dct}^{p} \varepsilon_{dc}^{p} + QF_{dct}^{p} \pi_{dc}^{p}) + \sum_{c \in CU} \sum_{e \in RE} \sum_{p \in P} \sum_{t} (K_{cet}^{p} \lambda_{ce}^{p} + FE_{cet}^{p} \psi_{ce}^{p}) +$$

$$\sum_{e \in RE} \sum_{i \in PL} \sum_{p \in P} \sum_{r \in R \cup S} \sum_{t} (L_{eit}^{pr} \theta_{ei}^{pr} + T_{eit}^{pr} \tau_{ei}^{pr})$$

$$\sum_{s \in SU} RF_{sit}^{PR} + \sum_{e \in RE} L_{ei(t-1)}^{PR} = g^{PR} * A_{it}^{PU} \qquad \forall i \in PL, \forall p \in P, \forall t \qquad (1)$$

$$\sum_{s \in SU} RF_{sit}^{PU} + \sum_{e \in RE} L_{ei(t-1)}^{PU} + A_{it}^{PU} + \sum_{q \in P \backslash \{p\}} ST_{it}^{qpU} = h^{PU} * A_{it}^{PF} + \sum_{q \in P \backslash \{p\}} ST_{it}^{pqU} \qquad \forall i \in PL, \forall p \in P, \forall t \qquad (2)$$

$$\sum_{d \in DI} FD_{idt}^{p} = A_{it}^{PF} + \sum_{q \in P \backslash \{p\}} ST_{it}^{qpF} \qquad \forall i \in PL, \forall p \in P, \forall t \qquad (3)$$

$$\sum_{c \in CU} HF_{dct}^{p} = \sum_{i \in PL} FD_{idt}^{p} \qquad \forall d \in DI, \forall p \in P, \forall t \qquad (4)$$

$$\sum_{d \in DI} HF_{dct}^{p} = de_{ct}^{p} \qquad \forall c \in CU, \forall p \in P, \forall t \qquad (5)$$

$$\sum_{c \in CU} \sum_{e \in RE} K_{cet}^{p} = \mathrm{RT}^{p} * \sum_{c \in CU} de_{ct}^{p} \qquad \forall p \in P, \forall t \qquad (6)$$

$$\sum_{c \in CU} K_{cet}^{p} * \mathrm{RJ}^{pr} = \sum_{i \in PL} L_{eit}^{pr} \qquad \forall r \in RS, \forall e \in RE, \forall p \in P, \forall t \qquad (7)$$

$$A_{it}^{pr} \leq B_{it}^{pr} * A_{it\,max}^{pr} \qquad \forall i \in PL, \forall p \in P, \forall r \in SF, \forall t \qquad (8)$$

$$RF_{sit}^{pr} \leq M_{sit}^{pr} * RF_{sit\,max}^{pr} \qquad \forall s \in SU, \forall i \in PL, \forall p \in P, \forall r \in RS, \forall t \qquad (9)$$

$$FD_{idt}^{p} \leq O_{idt}^{p} * FD_{idt\,max}^{p} \qquad \forall i \in PL, \forall d \in DI, \forall p \in P, \forall t \qquad (10)$$

$$HF_{dct}^{p} \leq QF_{dct}^{p} * HF_{dct\,max}^{p} \qquad \forall d \in DI, \forall c \in CU, \forall p \in P, \forall t \qquad (11)$$

$$K_{cet}^{p} \leq FE_{cet}^{p} * K_{cet\,max}^{p} \qquad \forall c \in CU, \forall e \in RE, \forall p \in P, \forall t \qquad (12)$$

$$L_{eit}^{pr} \leq T_{eit}^{pr} * L_{eit\,max}^{pr} \qquad \forall c \in CU, \forall e \in RE, \forall p \in P, \forall r \in RS, \forall t \tag{13}$$

$$\sum_{q \in P \setminus \{p\}} ST_{it}^{pqr} \leq A_{it}^{pr} \qquad \forall i \in PL, \forall p \in P, \forall r \in SF, \forall t \tag{14}$$

The objective function is to minimize production costs, replacement cost, variable costs, and direct and indirect flows between nodes throughout the network.

Constraints (1) and (2) control the flow of input and output in each factory. (3) and (4) control the flow of finished products from factories until they are received by distribution centers and exit from them. Customer satisfaction is controlled in (5). Constraints (6) and (7) correspond to the reverse flow of products in the network. Constraints (8) means that if a product is produced at each of the factories, its fixed cost is considered. (9) to (13) guarantee that the fixed cost of the model should be considered if the flow is established on each of the network axes. Constraints (14) control the maximum conceivable replacement of products and components.

The developed model for the problem in this paper includes continuous and discrete variables and is formulated as a MILP model. These problems are NP-Hard (Cornuéjols, 2007). On the other hand, the proposed model includes the knapsack and facility location problem that they are in the NP-Hard category problems too (Magazine and Chern, 1984) , (Charikar et al., 2001). Hence, the simultaneous optimization of the product family and supply chain network is surely NP-Hard. We have used heuristic methods to solve complex instances.

3- Solution procedure

3-1- Definition of main factors of the problem

In order to evaluate the model, five set of problems have been considered. As shown in the table 2, from Problem 1 to 5, level of the BOM and consequently the complexity of issues increases. In the first and second problems, the structure of the products is two-level and consists only of the components and finished product. The product structure gradually has been broken from 2 up to 4 levels. This trend is also repeated for the number of products. The structure of the supply chain network has been expanded with increasing number of nodes in each sector, and has grown as suppliers, factories, customers, distribution and recycling centers. Also, for the implementation of the developed model, a generic BOM is used. This GBOM consists of a three-level structure, including the finished product, sub-assemblies, and components, as shown in figure 2.

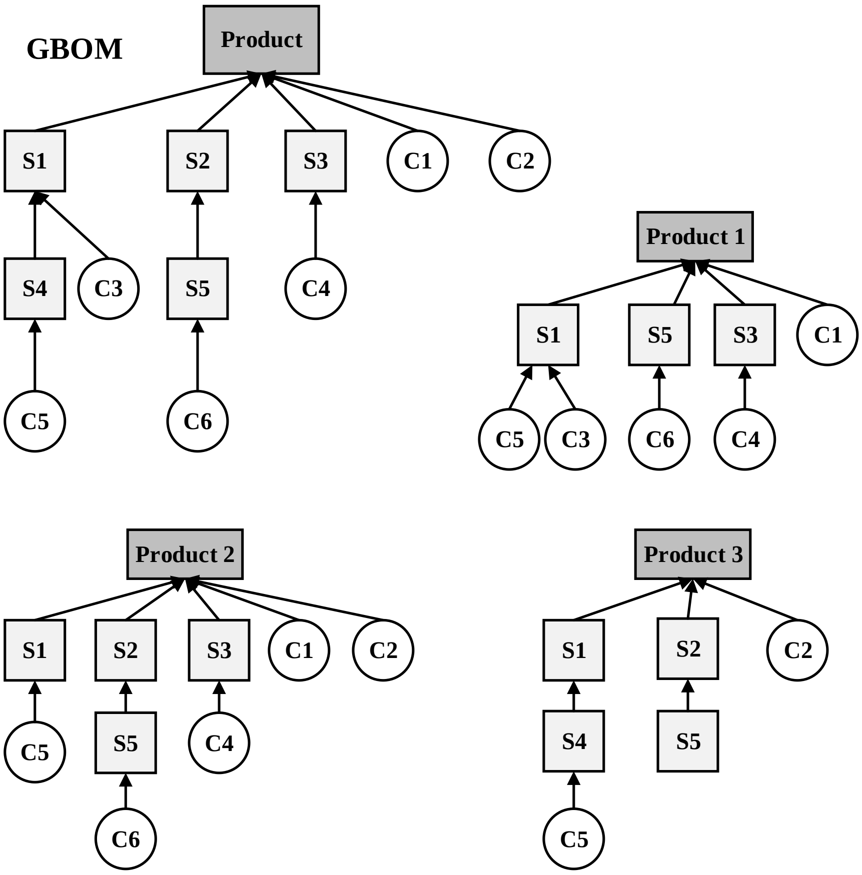

Fig 2. The structure of considered supply chain network

To solve the developed model, GAMS software version 24.5.6 and a 64-bit Intel Core i5 2.40GHz system with 4GB of memory are used. As will be shown below, with the increase in the dimensions of the problem, the time to solve it increases. They are not solve in a polynomial time. One of the most common ways to reduce the problem solving time is to use heuristic methods.

Table 2. Instances parameters

Number of problems elements	Prob. 1	Prob. 2	Prob. 3	Prob. 4	Prob. 5
BOM height	2	2	3	3	4
Products	5	8	10	15	20
Suppliers	15	20	30	40	80
Factories	3	8	10	13	15
Distribution centers	1	2	5	10	20
Customers	100	100	100	500	1000
Recycling centers	1	3	5	8	15
Max demand	50	100	500	750	1000
Periods	1	1	2	2	3

3-2- Identifying the factor of increasing the complexity of the problem

In order to provide a heuristic method, the first attempt has been made to identify a part of the problem that causes increase in the solving time. For this purpose, the presented problems 1 to 5 are evaluated in terms of different conditions: eliminating the possibility of substitution or fixed taking binary production variables or ignoring the reverse flow. The results are presented in table 3. For this purpose, the binary variables were fixed at different stages in their optimal amount and evaluated. The purpose of this work is to determine the effectiveness of each of these variables on the final results as well as the problems solving time. To compare different situations, $t_c = \frac{t_2}{t_1}$ is used as a time index.

In which, t_c indicates the relative solving time of the problem after fixing the binary variables to the initial conditions, t_1 as the reference time, and t_2 represents the solving time after applying the assumptions. Obviously, after fixing the variables, it always takes less time than the original problem to solve. Hence, the fraction presented is always a positive number between 0 and 1. As this number is closer to 0, it shows that the related variable is more effective in the problem solving time.

Table 3. The first level problems solving by consideration certain assumptions

Assumption	Prob. 1	Prob. 2	Prob. 3	Prob. 4	Prob. 5
Without substitution	0.47	0.41	0.44	0.52	0.82
Fixed variables:					
X_{it}^{pr}	0.53	0.15	0.21	0.03	0.08
M_{sit}^{pr}	0.90	0.78	0.95	0.96	0.84
T_{eit}^{pr}	0.94	0.81	0.86	0.88	0.96

As it is clear from the results, eliminating the possibility of substitution, the solving time has been reduced. However, with the increase in the dimensions of the problem (fourth and fifth problem), the reduction in the solution time to the initial state is decreasing and the gap in the solution time decreases with consideration of substitution and without it. Therefore, simplifying the problem with the use of heuristic methods in this section will not help to reduce solving time issue.

What certain is that the existence of binary variables in the optimization problems increases the complexity of their solution procedure. In the next step, identifying the problem factor during the problem solving, the binary variables were investigated.

Initially, the flow of material between suppliers and factories M_{sit}^{pr} was fixed in optimal quantities obtained from the exact solution. The next variable, which has been analyzed in the same way, is X_{it}^{pr}, which indicates the production or non-production of p in the factory i. The last section that has been evaluated relates to reverse logistics and flow from recycling centers to factories. As can be seen in the table 3, the problem solving time has dropped significantly in all cases. The results lead the research to find a method to simplify the model and solve it in the variable integer part of the integer decision, and in the section of determining the value of X_{it}^{pr}. On the other hand, we know that the solving of relaxed linear programming problems is easily available by branch and bound technique. By doing this, instead of the binary variables, continuous values will be available after the solution. But in order to determine the status of these variables, we can use the approximation algorithms based on linear programming relaxation. However, with this method, the best answer is not possible after the model is solved, but by limiting and determining the status of some of the binary variables, the solution is simpler and the results will be less time-consuming. This method has been used in many articles and has had significant success (see Gabow et al., (2009), T. Melo et al., (2009) and Byrka et al., (2010)). Melo et al. (2006) that achieved the average gap of LP relaxation about 2% of the optimal solution (M. T. Melo et al., 2006).

3-3- Approximate heuristic algorithm

In order to achieve a heuristic method, at the first, the model was relaxed of binary variables and the results were extracted. The binary variables are divided into three general categories. The first group of variables whose values are smaller than or equal to 0.1, the second group of variables with values between 0.1 and 0.9, and the third category of variables with values greater than or equal to 0.9. In the next step, the variables with high adhesion to zero are fixed on zero. Third category variables, variables with adhesion toward one, will take one. Since in this way some of the variables are imposed arbitrarily at a given value, it is possible to get out of the feasible area and become problematic. In the proposed method has tried to overcome this issue. When it is infeasible, variables that have been fixed at zero or one are released, and solver will re-launch the model. The proposed algorithm is as follows:

```
solve LPRelaxation
for every production of products in plants fractional variable $X_{it}^{pr}$ do
  if $X_{it}^{pr} \le 0.1$ then
    $X_{it}^{pr} \to 0$
  end if
  if $X_{it}^{pr} \ge 0.9$ then
    $X_{it}^{pr} \to 1$
  end if
end for
solve LP
  if LP is infisible then
  Relax all $X_{it}^{pr}$ that were fixed at 0
  end if
Solve LP
```

In order to compare the results of the proposed algorithm, it is necessary to introduce the indexes of this comparison. Since the application of heuristic methods usually simplifies the problem, it is therefore expected to always spend less time compared to the main problem solving time. The second indicator is the distance from the optimal response, which can be measured by this formula:

$$Gap = \frac{HSC - ESC}{ESC}$$

In that, HSC represents the total cost of the problem by applying the heuristic method. ESC represents the overall cost of the problem under initial conditions and its exact resolution. The more this fraction is closer to zero, the proposed approach will be more reasonable and worthwhile.

4- Computational experiments

4-1- Exact solution

The results of the problems solving are shown in the table 4, which includes the minimum, maximum and average time of solving 15 samples generated from each of the problems. As the complexity of the problem increases, its solution time is increasing, so that in the fifth problem, the average solving time is about 90 minutes. In some instances, this time has been more than an hour. The use of branch and bound technique to solve low complexity instances is very convenient and efficient, but it is clear that, with increasing dimensions and complexity of the instances, this will not solve them at a logical time.

Table 4. Solving time of first-level problems with exact solution

	Prob. 1	Prob. 2	Prob. 3	Prob. 4	Prob. 5
Mean time (s)	2.160	3.480	21.960	432.264	2328.488
Minimum	0.013	0.133	5.720	87.702	607.824
Maximum	4.237	9.742	29.528	1597.778	5091.084

4-1-1- Optimal structure of the BOMs in a product family

One of the goals of product family and its supply chain network optimization is to determine the optimal composition of production. In this paper, using the concept of substitution, that’s determined by the production or non-production of each of the sub-assemblies or components of the GBOM.

In this section, the output of one of the instances of problem 5 has been reviewed. In this instance, the product structure has been broken down to 4 levels including the finished product, sub-assemblies, sub-sub-assemblies and components. Given the concept of substitution, the production optimal combination of each level of BOM is determined based on table 5 information. Fifth problem contains 20 types of products from a product family, but in order to prevent prolongation of the word, only the structure of 5 products has been shown here.

In this table, cells that do not contain any value of zero or one, represent the absence of that sub-assembly or component in the product's BOM. In cases where the value is zero, it shows that as a result of solving the model, its production is not economical and the required amount is supplied using the concept of substitution and from other manufactured products.

Table 5. Status of production of components, sub-assemblies and finished products at all levels of BOMs

$\sum_{t}\sum_{i} X_{it}^{pr}$ Product	r → P	r → S1	r → S2	r → S3	r → S4	r → S5	r → C1	r → C2	r → C3	r → C4	r → C5	r → C6
Product1	0	1	-	0	-	0	-	-	1	1	1	1
Product2	0	0	1	1	-	1	1	1	-	0	0	1
Product3	1	1	1	-	1	1	-	1	-	-	0	-
Product4	0	1	0	-	1	-	-	-	1	-	0	0
Product5	1	0	-	-	-	1	-	0	0	-	0	-

4-2- Impact of LP rounding on the model function

In the following, solving problems 1 through 10 by the proposed algorithm, the solving time, and the value of the relative objective function compared with the exact solution of the issues discussed. The results of the problems solving are presented in table 6. For each of them, 15 samples have been solved by maintaining network and product structure by changing the cost of production and networking.

Table 6. Function of the proposed method in solving first level problems

Index	Prob. 1	Prob. 2	Prob. 3	Prob. 4	Prob. 5
t_{new} (s)	0.874	1.430	0.137	0.003	0.867
t_{comp} (s)	0.424	0.458	0.170	0.005	0.002
Gap (%)	0.000	0.236	0.018	0.336	0.218

The results of the table above are depicted in the following graph. As is evident, in general, with the progression from problem 1 to problem 5, the solution time in the innovative way, compared with the exact solution, is better. Though due to the increasing complexity of the issues, such results were not unexpected.

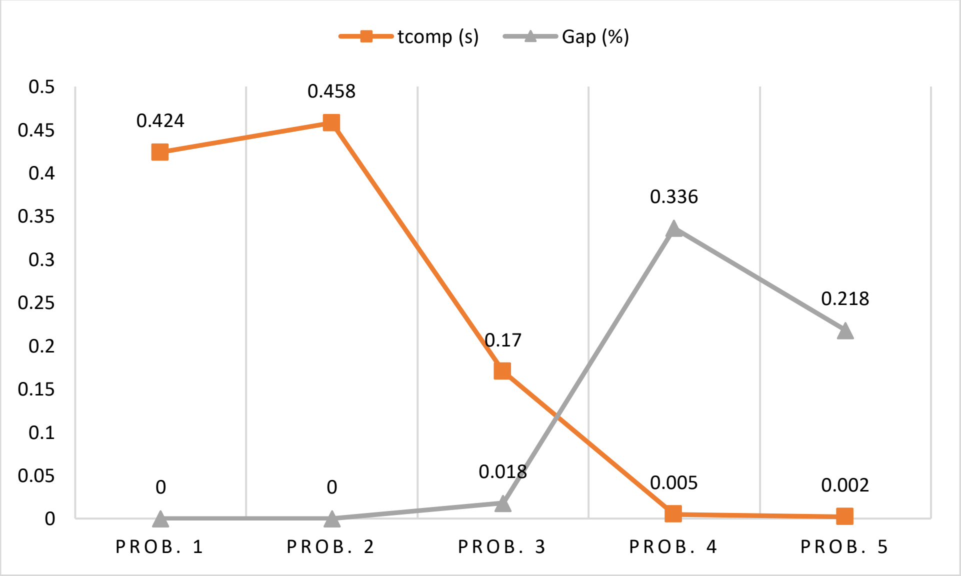

Fig 3. Performance of the proposed method versus exact solution

Also, according to the literature of the subject, the gap between the amount of the solution in the exact method and the proposed method in this research is at an acceptable level. The process of changing this distance in percentages is shown in figure 3.

4-3- Second level problems (high complexity)

In order to measure the performance of the proposed method, a series of problems 6 to 10 are presented as second level in table 7. In these problems, from 6 to 10, the complexity of that has increased with increasing network elements as well as product structure in the form of BOM levels and the possibility of substitution at different product levels.

Table 7. Second level problems parameters

Number of problems elements	Prob. 6	Prob. 7	Prob. 8	Prob. 9	Prob. 10
BOM height	4	4	4	5	5
Products	25	30	30	35	50
Suppliers	85	90	100	110	120
Factories	18	25	30	35	40
Distribution centers	25	28	32	35	35
Customers	1000	1200	1500	1800	2000
Recycling centers	15	18	20	23	25
Max demand	1000	1000	1500	1700	2000
Periods	3	3	4	4	5

The function of typical branch and bound method and the heuristic method for second level problems has been shown in table 8. It should be noted that in cases where calculation of the amount of Gap has not been possible (Prob. 6 to Prob. 10), the value of the objective function obtained by heuristic method is compared with the best upper bound obtained in 15 samples of the problem.

Table 8. Second level problems solving with the proposed method

Index	Prob. 6	Prob. 7	Prob. 8	Prob. 9	Prob. 10
t_{new} (s)	8.776	11.013	12.595	15.383	15.636
Gap (%)	0.025	0.686	1.437	2.081	1.808

As shown in table 8, the optimality gap obtained by solving the problems by proposed heuristic method, with a little ignore, is less than 2%. The time to solve the problems of the second level has increased with increasing structural complexity, but this process has a very slow pace and is fully justified and acceptable.

5- Conclusion and future research

This paper addresses an important issue concerning the integration of product family and its required supply chain network. The main purpose of this paper is to provide a comprehensive model for optimizing the design of a family of products and its supply chain, taking into account the reverse flow of products at the end of its life cycle which is presented in a mixed integer linear program (MILP) model. The proposed model has been formulated multi-periodically, which makes it dynamic.

The supply chain network, includes 5 level consist of suppliers, factories, distribution centers, customers and recycling centers. To define the product family in the developed model, the concept of substitution is used. As the components, sub-assemblies and the final products, by doing a partial activity, can become turned into another in the same family of the products.

The small and medium-scale view of the structure of the network and products and reverse logistics problems, is solved by employing GAMS software. Since problem solving is not possible in the polynomials time, an approximate heuristic approach is developed based on LP-rounding algorithm which in some instances resulted in a reduction of the problem-solving time to 0.002 times the exact solution. Since this reduction in time, with a gap less than 2% of the optimal value of the problem is obtained, the developed method is at an acceptable level in the subject literature.

The developed model allows business owners to identify optimal product combinations compatible with their potential supply chains using a product family-based approach at minimum cost.

The development of exact solution methods for high complexity instances in the comprehensive developed model can be considered as the subject of future research.

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