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Approximating the step change point of the process fraction nonconforming using genetic algorithm to optimize the likelihood function

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Abstract

Control charts are standard statistical process control (SPC) tools for detecting assignable causes. These charts trigger a signal when a process gets out of control but they do not indicate when the process change has begun. Identifying the real time of the change in the process, called the change point, is very important for eliminating the source(s) of the change. Knowing when a process has begun to change simplifies the identification of the special cause and consequently saves time and expenditure. This study uses genetic algorithms (GA) with optimum search features for approximately optimizing the likelihood function of the process fraction nonconforming. Extensive simulation results show that the proposed estimator outperforms the Maximum Likelihood Estimator (MLE) designed for step change regarding to speed and variance.

Keywords: Quality control, Statistical process control, Change point, Genetic algorithm, np chart.

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1. Introduction and literature review

Statistical process control (SPC) has played an important role in industry for many years. The control chart is a powerful SPC tool that monitors changes and discovers variation in a process in order to distinguish between special and common causes of variation. In SPC, upper and lower control limits can be defined based on the probability distribution of the product's quality characteristics. When the sample observations of the process are within the control limits, we conclude that the process is in control. However, if the sample observations fall outside the control limits, an out-of-control signal is received. The time when a special cause manifests itself into a process is referred to as change point. Once a change is detected, process engineers begin their search for the special cause disturbing the process. Upon signaling, control charts do not provide specific information regarding the cause of process change nor when the process changed. They only suggest that a change has occurred.

The process parameter(s) are usually be affected by changes in the process. These changes may be classified into single step change, multiple step changes, drift, and monotonic change (Amiri, Allahyari, 2012). They have provided a thorough overview of change point estimation problems in different types of control charts, and classified published articles according to different criteria such as the types of changes and the applied estimation approaches.

Samuel and Pignatiello (1998) analyzed a step change in the rate parameter for a Poisson process. Samuel, Pignatiello and Calvin (1998a and 1998b) considered step change in a normal process mean and normal process variance. Samuel and Pignatiello (2001) proposed an MLE for the process fraction nonconforming change point by applying the step change likelihood function. They evaluated the performance of their proposed estimator when an np chart signals out of control point and concluded that their estimator has reasonably good accuracy and precise performance (Pignatiello and Samuel, 2001). Perry, Pignatiello and Simpson (2007) developed a change-point estimator from the change likelihood function for a binomial random variable without assuming any change type. The only assumption in their research is that the predicted change type is monotonic. They also compared the performance of their estimator with the one suggested by Samuel and Pignatiello (2001).

Perry and Pignatiello (2006) proposed the MLE for the change point of a normal process mean when a linear trend disturbance is present. The performance of the proposed estimator was studied and compared with the performance of MLE designed for step changes. Perry, Pignatiello and Simpson (2006) compared the performance of the MLE for the time of drift in a Poisson rate parameter designed for linear trends with the MLE of the process change point designed for step changes when a linear trend disturbance is present. They showed that the MLE of the process change point designed for linear trends outperforms the MLE designed for step changes and the CUSUM control chart estimator. Noorossana and Shademan (2009) proposed a MLE for the change point of a normal process mean that does not require the knowledge of the exact change type but assumed that it is monotonic (isotonic or antitonic). Zandi et al. (2011) introduced MLE for the change point of process fraction nonconforming when the process was subjected to a linear trend disturbance.

In the context of SPC, the fuzzy set theory has been used to model fuzzy data, particularly for constructing attribute control charts based on linguistic data. Zarandi, Alaeddini and turksen (2008) combined fuzzified sensitivity criteria and fuzzy adaptive sampling rules to make more sensitive and proactive control charts. Their hybrid method keeps the rate of false alarms reasonably low. Ghazanfari et al. (2008) used data clustering to estimate the change point in Shewhart control charts. Their approach is `applicable to both phase I and phase II of normal and non-normal processes. Alaeddini, Ghazanfari and Nayeri (2009) developed a hybrid fuzzy clustering and statistical approach for change point estimation. Their approach can effectively estimate the change point in processes with either fixed or variable sampling strategies. Zarandi and Alaeddini (2010) extended the fuzzy statistical clustering (FSC) to a general form so as to estimate the change point in a wide range of processes. Kazemi, Bazargan and Yaghoobi (2013) extend the FSC approach to estimate the process change point in the presence of the linear trend disturbance. Their approach provides an accurate estimate of the process change point in different control charts.

Genetic algorithm was first developed by Holland (1975). It uses computer programs to simulate the evolutionary process with the chromosome as the solution to the solved problem. Based on the environmental adaptation of chromosomes, researchers identified a fitness value such that a researcher could determine whether a chromosome would survive until the next generation. The evolutionary process continues until the target has been met. By self-adaptation and an iteration threshold, the algorithm has the ability to evolve to the optimum solution for a problem.

In this research, first the step change-point problem of a process fraction nonconforming is introduced and a new method for obtaining an approximate MLE for the step change point of the process fraction nonconforming is proposed. The method uses a genetic algorithm for optimizing the likelihood function. The proposed estimator can be used for the detection of a change point when either p or np chart has shown a signal. Next, the obtained estimator is compared with the MLE of the process fraction nonconforming change point.

2. Process step change model

The binomial distribution is often used to model the number of successes in n trials. Often in an industrial quality control setting, the binomial distribution is used to model the number of defective items in a sample of size n. In other words the probability that there are x defectives in a random sample of n items is

$$P(X = x) = \binom{n}{x} p^x (1-p)^{n-x} \tag{1}$$

Where $0 \le x \le n$ and $0 \le p \le 1$ denotes the process fraction nonconforming. We will assume that the process is initially in-control and the observations come from a binomial distribution with $p = p_0$, a known parameter value and at an unknown point in time τ (known as the process change point), p changes to $p = p_1 = \delta p_0$, where $p_0 \neq p_1$ and δ is the unknown magnitude of the change. Values of $\delta > 1$ represent an increase in p while $\delta < 1$ represents a decrease or improvement in p.

Let D_i denote the number of nonconforming units in the i th subgroup and n_i be the size of the i th subgroup, Then $\hat{p}_i = {D_i}/{n_i}$ represents the subgroup fraction nonconforming. We assume that the first signal of a change in p occurs at subgroup number T. Hence either $D_T < LCL_D$ or $D_T > UCL_D$ but for $< T$, $LCL_D \le Di \le UCL_D$. We further assume that the control chart signal is not a false alarm. Thus, $D_1, D_2, \dots, D_\tau$ are the numbers of nonconforming units from the in-control process while $D_{\tau+1}, D_{\tau+2}, \dots, D_T$ are from the changed process. We now focus on τ identifying the last subgroup from the in-control process or equivalently, $\tau + 1$, the first subgroup from the changed process.

Samuel and Pignatiello (2001) consider the derivation of the maximum likelihood estimator (MLE) of τ the process fraction nonconforming change point. Maximum likelihood estimation techniques are discussed in Casella and Berger (2002). Samuel and Pignatiello first compute the value of p_1 that maximizes the likelihood function, or equivalently its logarithm and then propose the maximum likelihood estimate of the change point τ.

Suppose that $n_1 = n_2 = \cdots = n_T = n$, The likelihood function is:

$$L(\tau, p_1|D) = \prod_{i=1}^{\tau} \binom{n}{D_i} {p_0}^{D_i}(1-p_0)^{n-D_i} \times \prod_{i=\tau+1}^{T} \binom{n}{D_i} {p_1}^{D_i}(1-p_1)^{n-D_i} \quad (2)$$

In this paper, we propose an estimator using GA for step change point model from binomial process. We consider both τ and p_1 together for optimizing likelihood function.

3. Genetic algorithm

Genetic algorithm is random search method for global optimization. GA is a method triggered by the basic structure of organism evolution and was first proposed by John Holland in 1975. It combines Charles Darwin's principle of "natural selection" and "survival of the fittest" with the computer-constructed evolution mechanism to select better species from the original population. The information is exchanged randomly, in the hope of a superior offspring. The genetic algorithm uses a population of strings to encode the initial candidate solutions and then employs genetic operators (selection, mutation, crossover) to generate new populations based on the initial population, and gradually evolves towards the best solution. The convergence speed of GA is closely related to the procedure and parameters of the genetic operators such as selection, mutation and crossover.

The genetic algorithm not only avoids the trap of local optimization, but also reduces much computational time to find the optimum. Therefore, it is quite capable of solving optimization problems. Owing to its diverse characteristics, the genetic algorithm has diverse applications such as engineering, social sciences, and medicine.

3.1. Simulation study on the estimation of the change point with GA

This section performs a simulation study to estimate the change point. The estimation is based on using GA to optimize the likelihood function of a binomial step change model. The GA procedure for identifying change point of a binomial model is as follows:

Step1: Initialize algorithm parameters p_c(crossover) and p_m(mutation), iteration and number of chromosomes, these parameters will be fixed during the entire optimization process.
Step2: Initialize a random population.
Step3: For each chromosome, calculate its fitness value.
Step4: Based on the Roulette wheel selection, select chromosomes.
Step5: Generate the new generation, based on crossover and mutation.
Step6: If the stopping criterion is satisfied, then stop and return the best chromosome, otherwise, go to step 3.

We know that the values of parameters $p_1$and τ are in intervals [0, 1] and [1, T] respectively. The following steps illustrate the whole searching process.

(1) Randomly generate a population with each chromosome containing 2 digits. Then process the reproduction of the new generation fitness.
(2) Apply convex crossover and use 0.8 as the crossover rate and 0.05 as the mutation rate, too.

For the simulation study, we first generate random observations. For this purpose we compute our estimators using an np chart. In other words, we estimate the change point after an np chart has signaled that a process change has occurred.

The observations are randomly generated in subgroups of size $n = 150$ from a binomial distribution with parameter $p_0 = 0.1$ for subgroups 1, 2, ..., 100. If any of these subgroups produced a D_i which exceeded a control limit, all data from that subgroup are discarded and another subgroup is generated to replace it. By continuing, if needed, so that no false alarms are observed during the in-control phase of the simulations. Then, starting with subgroup 101, observations are randomly generated from a binomial distribution with parameter $p_1 = \delta p_0$, until the np chart issued a signal. At this point, the estimators $\hat{\tau}_G$ and $\hat{\tau}_{sc}$ where computed using GA and the method proposed by Samuel and Pignatiello (2001) respectively.

This procedure is repeated for a total of 10000 independent simulation ones for each of several values of δ. The average of these 10000 change point estimates is determined along with their mean squared errors (MSE). The results for $\delta > 1$ and $\delta < 1$ are shown in Tables 1 and 2. $E(T)$ is the expected time at which the control chart first signals a disturbance in the process fraction nonconforming. The estimation of the probability that the change-point estimated value, falls within a certain interval around true change point ($\tau = 100$) for different values of δ is reported in Tables 3 and 4.

The results in Table 1 and Table 2 show that, except for $\delta = 1.5$ and $\delta = 0.5$ the MSE_G is smaller than MSE_{sc} . This shows that the estimator based on genetic algorithm performs at least as well as the estimator proposed by Sumuel and Pignatiello. We note that, as the δ increases to 3 or decreases to 0.5, the mean squared error for both estimators, decreases. However, more accurate estimates are obtained using the proposed method in almost all cases. Thus, it can be concluded that the proposed estimator outperforms the previous estimator and provides a more accurate estimate of the true process change point when a step change in process fraction nonconforming is present.
We next consider the frequency with which the change point estimation is within a distance m from the true change point, for $m = 1, 2, \ldots, 10, 15$. The results are reported in Table 3 and Table 4 for different δ values. Table 3 shows that $\bar{\hat{\tau}}_G$ is more precise for all values of δ. Naturally, as the δ increases, the precision of the two estimators improves. The estimated change point and precision of the estimates for two different change point estimators are plotted in Figures 1-4. These figures show that the precision provided by the proposed estimator in most cases is better than that of the other estimators. Moreover, the precision of both estimators improves are improved value of δ increases.

Table 1. Accuracy performances for two estimators of the change point (for increases in fraction nonconforming) when used with an np chart.

δ	$E(T)$	$\bar{\hat{\tau}}_G$	$\bar{\hat{\tau}}_{sc}$	MSE_G	MSE_{sc}
1.1	237.987	108.823	110.759	31.9752	32.7066
1.2	148.449	101.141	102.39	10.7549	10.8331
1.3	120.497	100.054	101.366	5.8681	6.778
1.4	110.05	99.376	100.97	4.4174	5.1533
1.5	105.578	99.836	99.685	4.2114	4.2102
2	101.299	99.882	99.81	1.6508	1.6515
3	101	99.981	99.981	0.15705	0.15705

Table 2. Accuracy performances for two estimators of the change point (for decreases in fraction nonconforming) when used with an np chart.

$\boldsymbol{\delta}$	$\boldsymbol{E(T)}$	$\hat{\bar{\boldsymbol{\tau}}}_{\boldsymbol{G}}$	$\hat{\bar{\boldsymbol{\tau}}}_{\boldsymbol{sc}}$	$\boldsymbol{MSE}_{\boldsymbol{G}}$	$\boldsymbol{MSE}_{\boldsymbol{sc}}$
0.9	510.64	101.453	102.624	29.2154	30.1246
0.8	423.304	99.877	100.972	7.15199	7.3436
0.7	264.235	100.0975	100.2247	3.169197	3.3099
0.6	152.82	99.9539	99.9479	1.69655	1.8562
0.5	118.56	99.98	99.99	0.997796	0.97132

Table 3. Precision of estimators (for increases in fraction nonconforming) when used with an np chart, in-control process fraction nonconforming ($p_0 = 0.1$, $n = 150$, $\tau = 100$)

$\boldsymbol{\delta}$	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**	**2**	**3**
$\hat{p}(\hat{\bar{\tau}}_G = \tau)$	0.056	0.193	0.327	0.444	0.603	0.8890.	0.984
$\hat{p}(\hat{\bar{\tau}}_{sc} = \tau)$	0.036	0.070	0.100	0.103	0.585	857	0.984
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 1)$	0.128	0.343	0.553	0.711	0.815	0.960	0.997
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 1)$	0.111	0.297	0.479	0.580	0.796	0.960	0.997
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 2)$	0.186	0.454	0.681	0.818	0.897	0.980	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 2)$	0.170	0.415	0.63	0.772	0.895	0.977	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 3)$	0.239	0.540	0.767	0.884	0.947	0.9820.	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 3)$	0.204	0.502	0.728	0.866	0.94	982	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 4)$	0.282	0.608	0.804	0.924	0.961	0.984	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 4)$	0.256	0.572	0.797	0.91	0.962	0.984	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 5)$	0.319	0.667	0.880	0.944	0.974	0.9870.	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 5)$	0.292	0.636	0.851	0.945	0.974	987	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 6)$	0.353	0.713	0.90	0.962	0.977	0.9890.	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 6)$	0.333	0.692	0.884	0.961	0.977	989	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 7)$	0.383	0.748	0.917	0.973	0.982	0.990	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 7)$	0.378	0.729	0.905	0.975	0.982	0.990	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 8)$	0.416	0.783	0.937	0.978	0.985	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 8)$	0.404	0.757	0.922	0.978	0.985	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 9)$	0.441	0.8040.	0.948	0.979	0.987	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 9)$	0.431	791	0.938	0.98	0.987	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 10)$	0.465	0.825	0.956	0.981	0.991	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 10)$	0.458	0.811	0.953	0.983	0.991	1	1
$\boldsymbol{\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \leq 15)}$	**0.566**	**0.8940.**	**0.981**	**0.988**	**0.994**	**1**	**1**
$\boldsymbol{\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \leq 15)}$	**0.543**	**908**	**0.982**	**0.988**	**0.994**	**1**	**1**

Table 4. Precision of estimators (for decreases in fraction nonconforming) when used with an np chart, in-control process fraction nonconforming ($p_0 = 0.1$, $n = 150$, $\tau = 100$)

δ	0.9	0.8	0.7	0.6	0.5
$\hat{p}(\hat{\bar{\tau}}_G = \tau)$	0.074	0.220	0.401	0.5664	0.73185
$\hat{p}(\hat{\bar{\tau}}_{sc} = \tau)$	0.047	0.414	0.132	0.5530	0.72149
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 1)$	0.181	0.39397	0.64193	0.79639	0.92137
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 1)$	0.158	0.388	0.58894	0.71127	0.8224
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 2)$	0.25	0.542	0.76229	0.90471	0.97177
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 2)$	0.241	0.533	0.75477	0.87324	0.94349
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 4)$	0.332	0.707	0.8997	0.97384	0.99194
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 4)$	0.329	0.699	0.8794	0.9659	0.99194
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 5)$	0.362	0.73568	0.9338	0.98295	0.99597
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 5)$	0.359	0.73266	0.90754	0.98195	0.97679
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 6)$	0.413	0.78191	0.95286	0.99498	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 6)$	0.398	0.7799	0.9407	0.9829	0.99698
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 7)$	0.452	0.80905	0.96289	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 7)$	0.441	0.80704	0.95578	0.99599	0.99899
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 8)$	0.482	0.83216	0.97192	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 8)$	0.468	0.83015	0.96281	0.999	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 7)$	0.452	0.80905	0.96289	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 7)$	0.441	0.80704	0.95578	0.99599	0.99899
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 9)$	0.513	0.85226	0.97894	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 9)$	0.499	0.85025	0.97085	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 10)$	0.546	0.912	0.98195	1	1
$\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 10)$	0.532	0.910	0.9799	1	1
$\mathbf{\hat{p}(\lvert\hat{\bar{\tau}}_G - \tau\rvert \le 15)}$	**0.648**	**0.954**	**0.99198**	**1**	**1**
$\mathbf{\hat{p}(\lvert\hat{\bar{\tau}}_{sc} - \tau\rvert \le 15)}$	**0.642**	**0.95**	**0.99097**	**1**	**1**

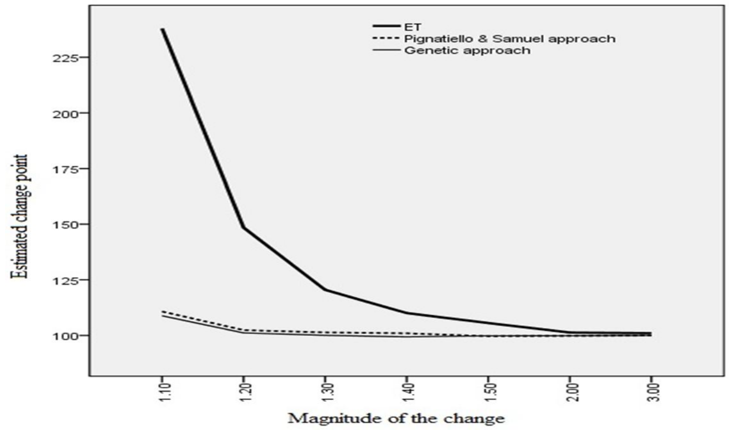

Figure 1. Estimated change point for two different change point estimators and expected time of the first genuine alarm from np chart (ET) with different magnitude of change.

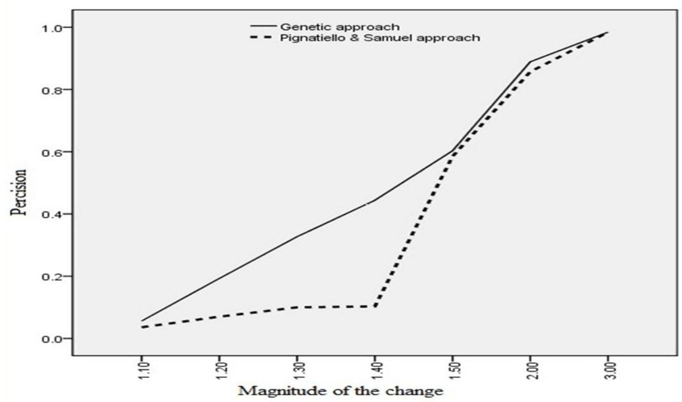

Figure 2. Precision of estimators for the estimated accurate change point $\hat{p}(\hat{\tau} = \tau)$.

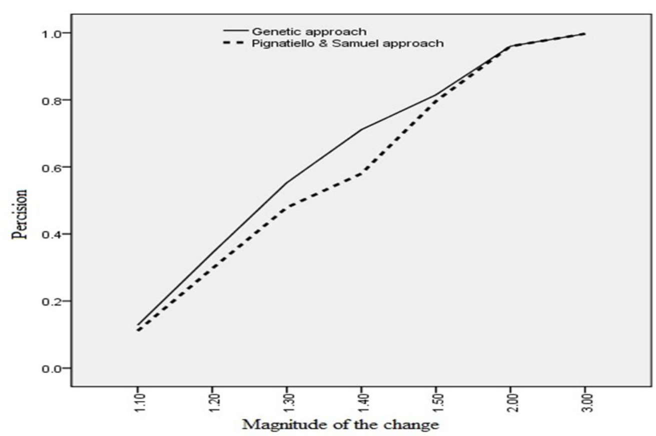

Figure 3. Precision of estimators for tolerance 1 subgroup $\hat{p}(|\hat{\tau} - \tau| \leq 1)$.

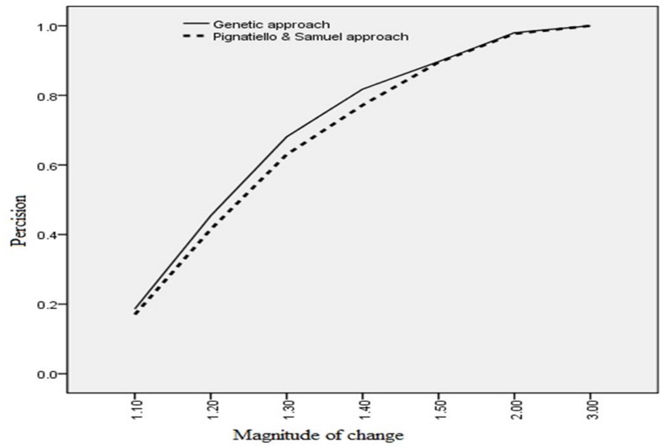

Figure 4. Precision of estimators for tolerance 2 subgroup $\hat{p}(|\hat{\tau} - \tau| \leq 2)$

4. Conclusion

When a control chart signals an out-of-control condition, a search begins to identify and to eliminate the source(s) of the special cause. Change point detection techniques provide useful starting points in the procedure of searching for a special cause following a control chart signal. An estimated change point accompanied by confidence intervals on the process change point would provide an appropriate starting point in the search for a special cause following a control chart signal. Estimation of the genuine time and the real source of the disturbance cause(s) in the process fraction nonconforming is valuable for process engineers and technicians who would like to gain easier and quicker identification of the variables and/or procedures that might cause a change in their processes.

The most difficult aspect in estimation the change point of the processes is the identification and finding of the procedure used to estimate nuisance parameters (like p_1), while we are only interested to find the time τ (known as the process change point). Since the genetic algorithm (GA) procedure can simultaneously select an appropriate subset of the parameters in the likelihood function, it reduces the huge searching time compared to the traditional estimation methods. Rather than relying on the gradient information, it searches the optimal solution by simulating the natural evolution process. GA has proven to be a suitable method for solving large scale optimization problems which are nonlinear, non-convex and non-continuous. It has several significant advantages, such as strong robustness, convergence to global optimum and parallel search capability.

In this article, a new estimator based on optimizing the likelihood function with GA was proposed that helps to identify the change point when a disturbance of step shifts happened in the process fraction nonconforming. The performance of the proposed method was compared with a previous estimator developed by Samuel and Pignatiello (2001) in the presence of step change type. The performance of the two estimators was compared using an np chart with different values for p_1 . The results show that the proposed approach performs better than the previous approach. The results also indicate that our estimator is particularly effective for estimating the time of a decrease in the process fraction nonconforming. The estimation of the time of the change in the process fraction nonconforming that is obtained using our estimator will be useful to process engineers who will be able to more easily and quickly identify variables and procedures for might cause a change in their processes.

The following problems may be considered for future research:

(1) In this study, we used GA and its application for optimizing the likelihood function to find the real time of change in binomial process. Employing GA for other kinds of changes, for example, monotonic or linear trend disturbance for binomial or other processes such as normal, Poisson can be a research topic for future.

(2) Because of the ability of GA in optimization of functions with many variables, using GA for estimating multiple change points in the processes can be regarded as a future research.

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