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Robust portfolio optimization based on minimax regret approach in Tehran stock exchange market

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Abstract

Portfolio optimization is one of the most important issues for effective and economic investment. There is plenty of research in the literature addressing this issue. Most of these pieces of research attempt to make the Markowitz's primary portfolio selection model more realistic or seek to solve the model for obtaining fairly optimum portfolios. An efficient frontier in the typical portfolio selection problem provides an illustrative way to express the tradeoffs between return and risk. With regard to the modern portfolio theory as introduced by Markowitz, returns are usually extracted from past data. Therefore, our purpose in this paper is to incorporate future returns scenarios in the investment decision process. In order to representative points on the efficient frontier, the minimax regret portfolio is calculated, on the basis of the aforementioned scenarios. In this way, the areas of the efficient frontier that are more robust than others are identified. The main contribution in this paper is related to the extension of the conventional minimax regret criterion formulation, in multi-objective programming problems. The validity of the proposed approach is verified through an empirical testing application on the top 75 companies of Tehran Stock Exchange Market in 2017.

Keywords: Multiple objective programming, portfolio optimization, minimax regret, robustness

1-Introduction

The basic framework proposed by Markowitz (1952) has been the most influence for the majority of financial models designed to provide a solution to the portfolio selection problem. Exclusively based on the criteria of return and risk, he minimizes the correlation between assets, which defines the risk of portfolio subjected to the given level of portfolio return value expected by the investor. The crucial assumption for this classic bi-objective approach to work is the accuracy of the estimates of return and covariance matrices. As the classic model is quite sensitive to its input parameters, the existing noise in the available estimates of risk and return will causes erroneous portfolio selection output (Hodges, 1976).

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In fact, little trust to the accuracy of past data that have been estimated by averaging them caused there is little guarantee that those estimates will match with the future values.

Researchers have always tried to make mathematical models of portfolio selection closer to reality, and help investors reach their objectives.

Also the need for more sophisticated financial tools has created space for exploring robust mathematical tools in order to protect a portfolio against input uncertainty.

Robustness is a concept of crucial importance in financial decision making. Thus, modeling processes for treating uncertainty are always necessary, when dealing with portfolio optimization problems. This feature typically leads to burdensome problem formulations, as robustness normally increases the number of constraints. It is, therefore, desired to maintain an acceptable balance between the robust modeling complexity and the overall efficiency of the results. The conventional mean-variance formulation is a quadratic programming model and its solutions provides the efficient frontier or Pareto optimal set of portfolios. On this basis, we attempt in this paper to build robust efficient frontiers, namely efficient portfolio sets that are close to optimal, under different scenarios.

More specifically, the main goal of this article is to develop a robust selection program that expands the concept of robust optimization, as it was proposed by Kouvelis and Yu (2013), to the multi objective case. Kouvelis and Yu used the concept of “regret” to identify robust solutions in optimization problems. Regret is actually the deviation of an obtained solution from the optimum solution according to a specific scenario of parameters. In other words, it can be defined as the difference between the obtained gain and the gain that we could get if we knew in advance which scenario will surely occur. Following Kouvelis and Yu ideas, we use the minimax regret criterion in order to identify robust areas in the Pareto front of multi-objective problems. We deal with input parameter uncertainty by considering time-varying alternatives for expressing a variety of market analysis horizons. In this way we are in position to identify areas of the Pareto front that are more robust than other.In this situation the specific areas of the Pareto front are characterized by the weight combination used in the objective functions. By applying our model to data from the Tehran Stock Exchange Market, we gain evidence that certain objective areas (e.g. risk) display greater robustness than others. Moreover, the calculations of the minimax regret value inform us about the amount of benefit we trade for robustness, at each choice of weights. For explanatory purposes, informative graphs and tables throughout the paper summarize all of our empirical testing results.

The remainder of this paper is organized as follows: In section 2, we review the history and applications of robust optimization models in finance with focus on robust portfolio optimization. In section 3, we present the proposed robust modeling approach and we extent the robust formulation to the multi-objective context. In Section 4, we test the proposed model with an illustrative application on the securities of the Tehran Stock Exchange in 2017. Finally, in section 5, key findings and results are presented.

2-Literature review

Recent studies in the field of portfolio theory imply that the knowledge of future returns and variances, delivered by classic point-estimation techniques based on past data, cannot be thoroughly trusted. Since risk and return are characterized by randomness, one should keep in mind that problem data could be described by a set of scenarios. Mulvey et al. (1995) were the first to work on models of mathematical optimization where data values come in sets of scenarios, while explaining the concept of robust solutions and introducing the robust model formulation. In a more financially specialized setting, Vassiliadou-Zeniou and Zenios (1996) developed robust optimization tools for managing callable bond portfolios. Kouvelis and Yu (2013) published a book on robust discrete optimization. Their book addresses multiple aspects in the robust problem formulation process, such as uncertainty handling, computational complexity results and algorithmic developments.

With regard to robust portfolio optimization, Tütüncü and Koenig (2004) described asset's risk and return using continuous uncertainty sets and developed a robust asset allocation program solved by a saddle-point algorithm. Also, Pinar and Tütüncü (2005) introduced the concept of robust profit opportunity in single-period and multi-period formulations. Also, multi-period portfolio optimization formulations with additional transactional constraints are found in Bertsimas and Pachamanova (2008). Robust optimization approach has also been included in the portfolio selection problem;

Goldfarb and Iyengar (2003) used robust optimization approach in the portfolio selection problem. Sadjadi et al. (2012) implemented robust optimization approach in Markowitz model with cardinality constraints, and solved their model with genetic algorithm. Ghahtarani and Najafi (2013) incorporated robust optimization approach and goal programming in the multi-objective portfolio selection problem. Gülpınara and Canakoglu (2017) studied portfolio selection problem under temperature uncertainty and risk references are incorporated in the suggested robust framework.

While robust optimization is intended to protect the portfolio against uncertainty, a study of robustness of optimal portfolios under stochastic dominance constraints was conducted by Dupacova and Kopa (2014). Moreover, Maillet et al. (2015) perform a worst-case minimum variance optimization with respect to alternative covariance matrix estimators. Moreover, Maillet et al. (2015) perform a worst-case minimum variance optimization with respect to alternative covariance matrix estimators.

A holistic approach of the 60-year old history of the modern portfolio optimization is attempted in Kolm et al. (2014). The 20-year old history of robust portfolio optimization is included as well as new directions are discussed. Other research articles that summarize recent history and future trends of robust portfolio optimization are those of Fabozzi et al. (2010, 2007), Mansini et al. (2014) and Scutella and Recchia (2013), where the relation between robustness and convex risk measures is also studied. A thorough inspection of both theoretical and practical research in robust optimization was made by Ben-Tal et al. (2009).

Cornuejols and Tütüncü (2006) wrote a book dedicated to optimization in finance. Within the book they go through topics of robust optimization in finance, analyzing the theory of robustness and taking a look at various types of uncertainty sets, different types of robustness (e.g. objective robustness, constraint robustness and relative robustness) and techniques such as sampling and conic optimization. They formulate robust portfolio optimization problems in single-period, multi-period and relative portfolio selection contexts. In the robust multiobjective field, an effort to characterize the location of the robust Pareto frontier with respect to the corresponding original Pareto frontier using standard multiobjective optimization techniques was made by Fliege and Werner (2014).

As mentioned, the methodological contribution of the present work is that it expands the concept of the robust solution to the multiobjective case. We incorporate future scenarios for the return and risk, which are mainly based on the perspectives of the decision maker. It is an attempt to show how this information may be exploited in order to produce robust portfolios against a variety of future scenarios. The handling of future returns scenarios are made by using the concept of the minimax regret criterion.

3-Proposed approach

It is well known where different scenarios are presented the minimax regret criterion is among the most popular criteria in decision sciences Savage (1972), along with the maximax, maximin, Hurwitz criterion etc. It actually aims at selecting the solution or alternative which is under the worst case closer to its scenario optimum. The minimax regret criterion provides less conservative solutions than the "pessimistic" approach of the maximin criterion (also used to express "robustness"). The reason is that it takes into account the regret, i.e. the deviation of each solution from the best possible solution at each scenario. The regret is not an absolute measure of performance of the solutions -as it is the case in the maximin criterion- but it is relative to the best available performance for the specific scenario. That's why it is considered to provide less conservative solutions in the sense that they have not to be "safe" according to the worst realization of parameters but according to the relevant optimum of each scenario. We can find the maximum regret for each solution across the scenarios and then comparing these regrets we can find the solution with the minimum of these maximum regrets. This minimax regret solution is considered as the robust solution. In order to explain the difference between maximin and minimax regret criterion consider the following example: Assume that we have 5 options that are evaluated in 3 scenarios: a pessimistic scenario, a most likely and an optimistic scenario. Imagine for example that we have 5 portfolios and the performances are the returns of each portfolio as shown in table 1.

Table 1. Example of the maximin criterion with 5 options and 3 scenarios

	Pessimistic	Most likely	Optimistic	Min
Portf 1	3	10	17	3
Portf 2	5	9	15	5
Portf 3	5	6	12	5
Portf 4	6	8	11	6
Portf 5	4	7	16	4

According to the maximin criterion the selected portfolio should be portfolio 4 which has the best performance in the pessimistic scenario leaving the information from the other two scenarios actually unexploited. In the minimax regret approach, we first create the regret matrix as shown below by calculating the distance from the optimum for each one of the three scenarios (table 2).

Table 2. The regret matrix and the minimax regret criterion

	Pessimistic	Most likely	Optimistic	Max
Portf 1	3	0	0	3
Portf 2	1	1	2	2
Portf 3	1	4	5	5
Portf 4	0	2	6	6
Portf 5	2	3	1	3

In this case the selected approach is the one with the minimum among the maximum regrets which is portfolio 2. With the minimax regret approach, we exploit the information from all 3 scenarios and we obtain solutions that are more balanced. Compare for example portfolios 2 and 4: The only advantage of portfolio 4 is that it outperforms in the pessimistic scenario expressing a more conservative view.

The minimax regret criterion has been also introduced in mathematical programming formulations. Specific formulations have been developed in order to express this concept in problems where there are multiple scenarios for the model's parameters. In Hauser et al. (2013) a regret function is considered as the function that measures the difference between the performance of the solution with and without the benefit of perception. If we choose x as decision vector when s is the vector of realized parameter values (scenario), then the regret associated with having chosen x rather than the optimal solution associated with scenario s (i.e. $x^*(s)$) is defined as follows (assume maximization):

$$r(x,s) = f(x^*(s),s) - f(x,s)$$

The perception is considered as the prior knowledge of the parameter scenario that will occur. Therefore, the optimal value with these parameters is the best outcome. Without this prior knowledge we can compute the minimax regret solution which is the one that has the minimum deviation from the optimal value under the worst case. Kouvelis and Yu (2013) accomplish this task for an infinite number of solutions according to the feasible region of the problem. Assume the following mathematical programming problem:

$$\begin{aligned} &Max\ z = f(x) \\ &\text{St.} \\ &x \in F \end{aligned} \tag{1}$$

According to the above formulation, the objective function to be maximized is a combination of the decision variable$x$, where x belongs in set$F$. Assume now that we have a set S of scenarios for the objective function parameters (S contains a finite number of $|S|$ scenarios), which means that the corresponding objective functions are denoted as$f^s(x)$. The minimax regret solution within the relative regret case is calculated from the following problem (see Kouvelis and Yu (2013), p. 29):

$$\begin{aligned} &Z_{MMR} = Min\ y \\ &\text{St.} \end{aligned} \tag{2}$$

$f^s(x) \geq (1-y)z^s \quad s \in S$
$x \in F$

Where z^s is the positive optimal value for the s-scenario and y is the variable that expresses the relative minimax regret.

In this work we extend the conventional formulation to the multi-objective case. Specifically, we use the weighting method in order to calculate the Pareto optimal solutions of the Pareto front. Assume that we have a problem with P objective functions:

$$Max\ (f_1(x), f_2(x), \dots, f_p(x)) \quad (3)$$
St.
$x \in F$

By using the weighting method we can calculate non-dominated points which solving the following single objective problem that has as objective function the weighting sum of the objective functions at hand (assume all objectives are for maximization):

$$Max\ z = \sum_{p=1}^{p} w_p \times f_p(x) \quad (4)$$
St.
$x \in F$

In order to be meaningful the weights and independent of the scale of the objective functions, it is better to use the normalization formulas for the objective functions as follows:

$$Max\ z = \sum_{p=1}^{p} w_p \times \frac{f_p(x) - f_{p,min}}{f_{p,max} - f_{p,min}} \quad (5)$$
St.
$x \in F$

Where$f_{p,min}$, and $f_{p,max}$ are the minimum and the maximum values of the objective functions as obtained from the payoff table (the payoff table is a $p \times p$ table that includes the individual optimization values of the objective functions). The solution of this problem corresponds to a Pareto optimal solution of the multi-objective problem. Varying the weights, we obtain a representative set of the Pareto optimal solutions of the multi-objective problem. It must be noted that with the weighting method the Pareto set is approximated. It is worth noting, that the more the weight combinations that are used makes it better in the approximation.

The concept of our proposed method is to apply the Kouvelis and Yu (2013) formulation to each combination of the weights. In this way, we obtain the minimax regret solutions that correspond to different areas of the Pareto front. Assuming that we have $|S|$ scenarios for the objective function parameters, we describe the weight space to g weight combinations and we solve the following problem:

$MMR(g) = Min\ y_g$
St.
$$\sum_{p=1}^{p} w_p^g \times \frac{f_p^s(x) - f_{p,min}^s}{f_{p,max}^s - f_{p,min}^s} \geq (1 - y_g) z_g^s \quad (6)$$
$s \in S$
$x \in F$

And we solve the above problem for every g obtaining the minimax regret solution at representative points of the Pareto front. According to the value of the minimax regret solution y_g we can draw conclusions about the areas of higher or lower robustness of the Pareto front.

Applying the above method to the portfolio optimization problem we use two objectives: the Mean Absolute Deviation (MAD), as a risk measure to be minimized and the expected portfolio return, as an objective to be maximized. For a universe of N assets and T historical periods, the objective functions are given in the formulas below:

$$Min\ z_1 = \frac{1}{T}\sum_{t=1}^{T}\left|\sum_{i=1}^{N} x_i(R_{it} - \bar{R_i})\right| \tag{7a}$$

$$Max\ z_2 = \sum_{i=1}^{N} x_i \bar{R_i} \tag{7b}$$

Where $\bar{R_i} = \frac{1}{T}\sum_{t=1}^{T} R_{it}$ and R_{it} is the return of the $i-th$ asset during the $t-th$ historical period.

For linearization of the first objective function, we operate as follows transformation (1991). On this basis, T additional positive continuous variables y_t are used for the representation of each period's absolute deviation from the mean, resulting in $2 \times T$ constraints:

$$\begin{aligned} &\sum_{i=1}^{N} x_i(R_{it} - \bar{R_i}) + y_t \geq 0 \quad t = 1,2,\dots,T \\ &\sum_{i=1}^{N} x_i(R_{it} - \bar{R_i}) - y_t \leq 0 \quad t = 1,2,\dots,T \end{aligned} \tag{8}$$

Then, the objective function is transformed to:

$$Min\ z_1 = \frac{1}{T}\sum_{t=1}^{T} y_t \tag{9}$$

Therefore, for each weight combinationg, we solve the |S| problems declared in equation (10) to identify the optimum value of the weighted sum for every scenarios.

(model 1)

$$\begin{gathered} \forall s \in S: \\ z_g^s = Max(w_1^g \frac{f_{1,max}^s - f_1^s(x)}{f_{1,max}^s - f_{1,min}^s} + w_2^g \frac{f_2^s(x) - f_{2,min}^s}{f_{2,max}^s - f_{2,min}^s}) \\ \text{St.} \\ x \in F \end{gathered} \tag{10}$$

Observe in equation (10) that the first term corresponds to the normalization of an objection function to be minimized. After the calculation of the optimal values z_g^s for the weight combinationg, we put them as parameters in the model of equation (6) in order to solve the problem that calculates the minimax regret for the specific weight combination using equation (11).

(model 2)

$$\begin{gathered} MMR(g) = Min\ y_g \\ \text{St.} \\ \left(w_1^g \frac{f_{1,max}^s - f_1^s(x)}{f_{1,max}^s - f_{1,min}^s} + w_2^g \frac{f_2^s(x) - f_{2,min}^s}{f_{2,max}^s - f_{2,min}^s}\right) \geq (1 - y_g) z_g^s \qquad s \in S \\ x \in F \end{gathered} \tag{11}$$

Subsequently, we move forward to the next weight combination and we repeat the process described with model 1 and model 2. In this manner we scan all the weight combinations calculating the minimax regret solution for each one of them. In total $(|S| + 1) \times |G|$ problems are solved. The smaller the minimax regret represents the more robustness in the corresponding efficient solution.

The flowchart of the proposed approach for portfolio optimization that uses risk and return as its objective functions is illustrated in figure 1.

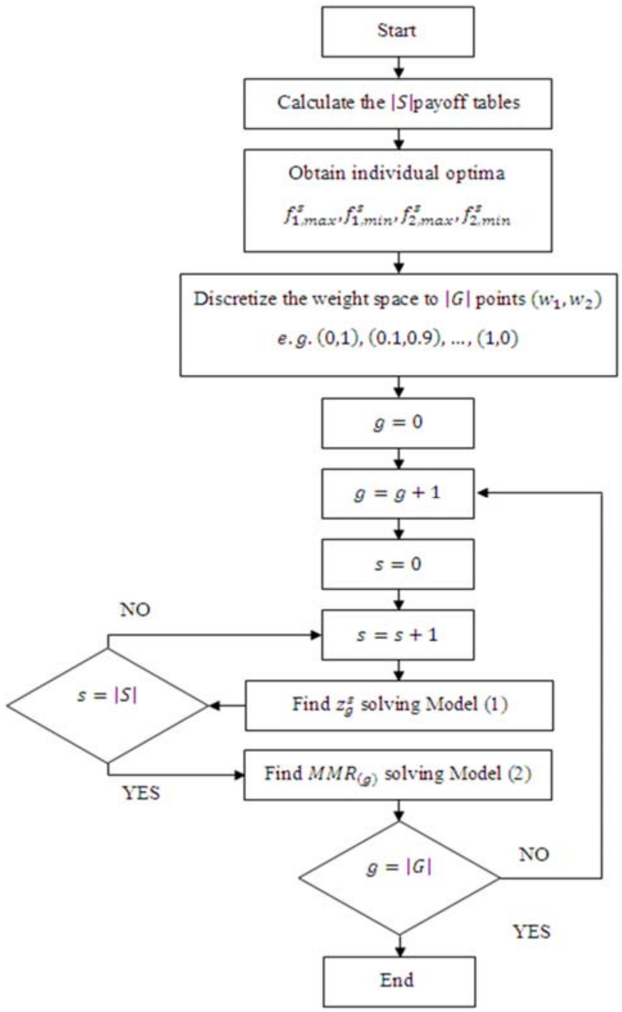

Fig 1. The flowchart of the method for the minimax regret criterion

The method is not limited to two objective problems. However, when more than two objective functions are considered the computation complexity will hardly increase. This has to do mainly with the increased number of weight combinations needed to adequately represent the multi-dimension Pareto front.

4-Empirical testing

In this section, the main purpose has been considered as applying the proposed model to Tehran Stock Exchange Market and solving the portfolio optimization problem by using data of 75 best assets as announced by Tehran Stock Exchange Market in 2017, and analyzing the performance of the proposed model and algorithms.

Therefore, we use the 75 stocks of the Tehran Stock Exchange Market, which have the best performance among existing stocks. We use five scenarios of return and risk evolution, all of which prepared in close participation with a team of portfolio managers. The 5 scenarios for the return and the risk are as follows: We used historical data of 80, 60, 40, 20, and 10 weeks, extracting the average return and MAD from the corresponding data. Therefore, Scenario 1 that corresponds to 80 weeks past horizon denotes a more long-term point of view than Scenarios 2, 3, 4 and 5 that denote a shortterm behavior. The five efficient frontiers are illustrated in figure 2 and the five payoff tables are shown in table 3.

Table 3. The payoff table in the 5 scenarios

	Scenario 1		Scenario 2		Scenario 3		Scenario 4		Scenario 5	
	MAD	Return	MAD	Return	MAD	Return	MAD	Return	MAD	Return
Min MAD	1.43	12.37	1.26	9.73	1.24	8.71	0.79	20.11	0.71	10.78
Max Return	2.51	58.93	2.14	44.92	2.44	47.82	1.98	92.46	2.33	94.54

The obtained results from using the minimax regret model are shown in table 4. We used 11 weight combinations, namely (0, 1), (0.1, 0.9), (0.2, 0.8) … (0.9, 0.1),(1, 0). The optimum of each scenario for the weight combinations (0, 1), (0.1, 0.9) and (1, 0), along with the minimax regret solution are shown in table 4, where the first objective function is the minimization of risk and the second one is the maximization of return. For each one of them we see the outputs of the 5 scenarios in terms of Wsum that represents the weighted sum of objective functions according to equation (5), and also return, MAD, the number of stocks in the portfolio and the weights of each one of them presented in this table. The minimax regret solution is presented in the last line of each scenario with bold fonts. It has to be mentioned that the minimax regret figure expresses how far we are from the individual optima of each scenario in the worst case and it is expressed as fraction from 0 to 1. The smaller the minimax regret represents the more robustness in the solution. Robust solutions are attractive because no matter which scenario will finally occur, we will be close to the optimum of the occurred scenario.

According to the figure 2, the 5 Pareto fronts correspond to each one of the considered scenarios. They are dissimilar because they correspond to different scenarios for the returns. For example, scenarios 4 and 5 correspond to higher returns than scenarios 1, 2 and 3 as it can be seen for the maximum return regions. In table 4 we can see that setting up the weights is crucial to the composition of the portfolios. As we increase the weight of "risk" we see that more securities enter to the portfolios. If we quantify the steadiness or robustness of the portfolios by the magnitude of the minimax regret figure we can identify regions of the Pareto set that are more robust which presents lower minimax regret values. It is remarkable that in the most cases the minimax regret portfolio includes more stocks than the optimal portfolios of the individual scenarios. When the weights of the objective functions are moving from max return to min risk, the stocks that have the highest return are losing weight in the optimal portfolios and they are mostly replaced by stocks that are less profitable but they are also less correlated with each other contributing to lowering the risk. Furthermore, we can see that the minimax regret solution in all the weight combinations contains more stocks in the final portfolio, than the individual scenarios optima. The minimax regret solution across the Pareto front is obtained from the minimax regret values for the specific weight combinations. Consequently, we are able to detect areas of the Pareto front that present relatively increased robustness in relation to other areas. Finally, we calculate the minimax regret solutions for the 11 weight coefficients of the relative minimax regret criterion. The results are shown in figure 3.

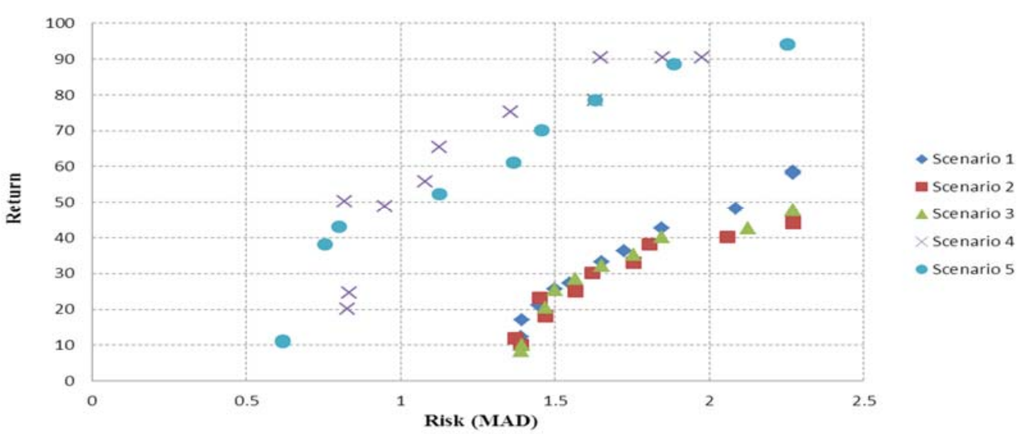

Fig 2. Representation of the 5 efficient frontiers

Table 4. Details of the obtained solutions for 4 weight coefficients

$w_1 = 0$	Scenario	W sum	MAD	Return	Stck/Portf	1	2	3	4	5	6	7	8	9	10	11	…	75
	1	0.9179	2.5102	58.931	13	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	2	0.9999	2.1450	44.922	13	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	3	0.9987	2.4434	47.804	13	0	0.1	0	0	0	0.1	0	0.1	0	0	0.1	…	0.02
	4	0.9999	1.9764	92.479	13	0	0.1	0	0	0	0	0	0	0	0	0.1	…	0
	5	0.9289	2.3353	94.542	13	0	0.1	0	0	0	0.1	0	0	0	0	0.1	…	0
	MMR=	**0.1678**			**13**	**0**	**0.1**	**0**	**0**	**0**	**0.1**	**0**	**0**	**0**	**0**	**0.1**	**…**	**0**
$w_1 = 0.1$	Scenario	W sum	MAD	Return	Stck/Portf	1	2	3	4	5	6	7	8	9	10	11	…	75
	1	0.9061	2.2974	58.089	14	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	2	0.9073	2.0772	44.100	12	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0.01
	3	0.9000	2.4434	47.821	13	0	0.1	0	0	0	0	0	0.1	0	0	0.1	…	0
	4	0.9097	1.8486	90.348	14	0	0.1	0	0	0	0	0	0	0	0	0.1	…	0
	5	0.9000	2.3353	94.542	14	0	0.1	0	0	0	0.1	0	0	0	0	0.1	…	0
	MMR=	**0.1884**			**14**	**0**	**0.1**	**0**	**0**	**0**	**0.1**	**0**	**0.08**	**0**	**0**	**0.1**	**…**	**0**
…	…		…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
$w_1 = 0.9$	Scenario	W sum	MAD	Return	Stck/Portf	1	2	3	4	5	6	7	8	9	10	11	…	75
	1	0.9078	1.4352	16.9617	15	0	0	0.05	0	0.02	0.1	0	0	0	0	0	…	0
	2	0.9014	1.2610	11.9187	12	0	0	0	0	0	0.1	0	0.1	0	0	0	…	0
	3	0.9030	1.2420	10.1132	14	0	0	0.09	0	0.01	0.1	0	0.1	0	0	0	…	0.03
	4	0.9033	0.7931	24.6355	14	0	0.06	0	0	0	0	0	0	0	0	0	…	0
	5	0.9003	0.7187	11.3294	12	0	0.1	0	0	0.02	0.1	0	0.1	0	0	0	…	0
	MMR=	**0.0896**			**14**	**0**	**0.04**	**0**	**0**	**0**	**0.1**	**0**	**0.1**	**0**	**0**	**0**	**…**	**0**
$w_1 = 1$	Scenario	W sum	MAD	Return	Stck/Portf	1	2	3	4	5	6	7	8	9	10	11	…	75
	1	0.9999	1.4528	12.3720	15	0	0	.01	0	0.01	0.1	0	0.04	0	0	0	…	0
	2	0.9999	1.3768	9.7320	14	0	0	0.04	0	0	0.1	0	0.1	0	0	0	…	0
	3	0.9999	1.3905	8.7181	15	0	0.1	0	0	0	0.1	0	0.1	0	0	0	…	0.04
	4	0.9999	0.8291	20.1060	15	0	0	0.05	0	0	0.1	0	0	0	0	0	…	0
	5	0.9999	0.7182	10.7850	15	0	0	0	0	0	0	0	0.1	0	0	0	…	0
	MMR=	**0.0848**			**15**	**0**	**0.02**	**0**	**0**	**0**	**0.1**	**0**	**0.1**	**0**	**0**	**0**	**…**	**0**

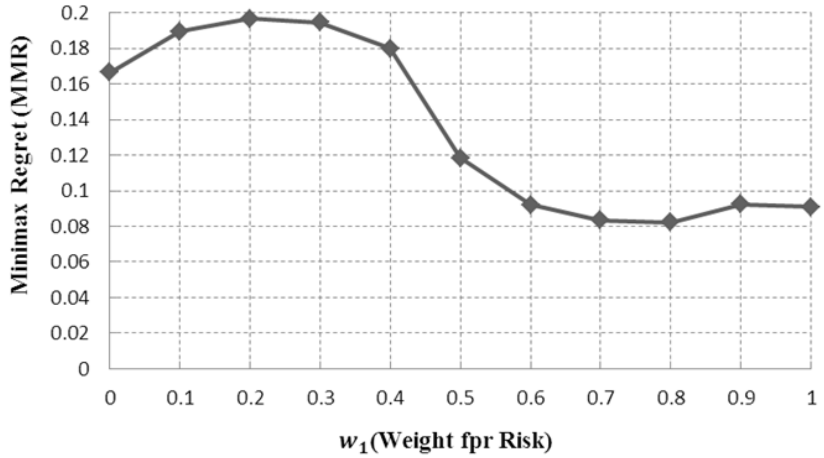

Fig 3. The MMR values across the Pareto front (relative MMR)

In figure 3, the lower the relative minimax regret is represented the more robust in the specific area of the Pareto front. Therefore, it is clear that there are areas in the Pareto front with higher robustness based on the 5 scenarios. For example, the Pareto optimal solutions that referred to weights varying from 0.1 to 0.4 are less robust than the Pareto optimal solutions that referred to weights varying from 0.6 to 1 (robust area of the Pareto front). It is obvious that when minimizing risk is weighted more the minimax regret value drops from a level of 19% to a level of 8%. Thus, the area of the Pareto front that corresponds to minimizing risk against maximizing return, provide more robust solutions in terms of the minimax regret criterion.

5-Conclusion

Investors are always trying to find an appropriate spot to invest, and they choose different ways for investment. One of these ways is investing in stock exchange markets and making portfolios. There are a lot of methods for making an appropriate portfolio; some of these methods are quantitative and some are no quantitative. The major evolution in portfolio selection was presented by Markowitz's primary. Markowitz mean-variance basic model does not include some important issues in the portfolio selection problem; these issues have been added to Markowitz primary model by researchers. In this research, we equip the multi-objective portfolio analysis tool with robust techniques. In particular, we extend the conventional formulation for the minimax regret criterion in multi-objective programming problems. Early researches highlight the growing momentum of robust portfolio optimization. Robust tools may not only be useful in theoretical research, but they also should come in hand for practical investors, as they will allow them to define uncertainty in input portfolio parameters, as they perceive it.

More specifically, we apply the proposed model in real-world data from Tehran Stock Exchange Market with 5 scenarios for the corresponding returns. The efficient frontier is approximated with 11 points each one of them corresponding to a specific weight coefficient for returns and risk. The obtained results are meaningful since they suggest the areas of the Pareto front that are more robust. The smaller the minimax regret for each weight combination represents the more robust in the specific Pareto optimal solution. In our empirical testing case that examines, it was found that the robust areas of the Pareto front are those where the weight of risk minimization is increased. Therefore, by using the weighting method for generating the Pareto optimal solutions, we can detect the robust asset selection results and the robust areas of the efficient frontier.

For the future research examining the effectiveness of the method in portfolio optimization for more objective functions and also in other multi-objective problems could be mentioned. In addition, other robustness models in the same context of the minimax regret criterion may be developed in combination with other multi-objective techniques which are appropriate for representation of the Pareto frontier.

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