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A cross entropy algorithm for continuous covering location problem

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Abstract

Covering problem tries to locate the least number of facilities and each demand has at least one facility located within a specific distance. This paper considers a cross entropy algorithm for solving the mixed integer nonlinear programming (MINLP) for covering location model. The model is solved to determine the best covering value. Also, this paper proposes a Cross Entropy (CE) algorithm considering multivariate normal density function for solving large scale problems. For showing capabilities of the proposed algorithm, it is compared with *GAMS*. Finally, a numerical example and a case study are expressed to illustrate the proposed model. For case study, Tehran's special drugstores consider and determine how to locate 7 more drugstores to cover all 22 districts in Tehran.
Keywords: Continuous covering location problem (CCLP), uncertainty, cross entropy (CE)

1-Introduction

Covering problem is to locate a set of new facilities such that customers can receive service by each facility if the distance between the customer and the facility is equal or less than a predefined number. This critical value is called coverage. Church and ReVelle (1974) modeled the maximization covering problem. Covering problems are divided into two problems; Total covering and partial covering problems, based on covering all or some demand points. The total covering problem is modeled by Toregas (1971). Up to the present time many developments have occurred about total covering and partial covering problems in solution technique and assumptions. Covering problem has many applications such as: designing of switching circuits, data retrieving, assembly line balancing, airline staff scheduling, locating defend networks, distributing products, warehouse locating, location emergency service facility (Francis et al. 1992). Some researchers investigated network covering problems such as Church and ReVelle (1974), Schilling et al. (1993), Owen and Daskin (1998) and Drezner and Wesolowsky (1999).

By reviewing literature of the covering location models, it be seen that these problems have been investigated in discrete space, only. While there are some cases in real world which may be occurred in continuous space, we introduce a continuous covering location problem in this paper; we are interested in finding the location of k facilities in continuous space in order to serve customers at n demand points so that the total cost of installation facilities sites and uncovered customers are minimized.

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The continuous covering location problem could be used for electronic service facilities like BTS towers or Wi-Fi centers, Aircraft refueling problems and robotics areas (Plaster , 1995).

The reminder of the paper is organized as follows; in section2 a literature review about uncertainty in covering location model are provided. In section 3 we present the mathematical model. Solution approach proposing *CE* algorithm and based on α- cut method is provided in section4. In section5, a numerical example and case study are given to illustrate the usability of proposed model. Finally, Section6 draws the conclusions and future works.

2-Literature review

In this section, review literature of covering problems and uncertain location models are provided. Liu et al. (2010) presented a location model that assigns online demands to the capacitated regional warehouses currently serving in-store demands in a multi-channel supply chain. The model explicitly considered the trade-off between the risk pooling effect and the transportation cost in a two-echelon inventory/logistics system. They formulated the assignment problem as a non-linear integer programming model. A strategic supply chain management problem was studied by Peng et al. (2011) to design reliable networks that perform as well as possible under normal conditions, while also performing relatively well when disruptions strike. They presented a mixed-integer programming model whose objective was to minimize the nominal cost while reducing the uncertainty using the *p*-robustness criterion which bounds the cost in disruption scenarios. Chen et al. (2011) presented a multi-criteria decision analysis for environmental uncertainty assessment with regard to avoiding and eliminating damages and loss under natural disasters in international airport projects. They used the ANP to demonstrate one of its utility modes in decision making support to location selection problems, which aims at an evaluation of different projects from different locations. A corresponding framework for value-based performance and uncertainty optimization in a single-stage supply chain problem was developed by Hahn and Kuhn (2012). They applied Economic Value Added as a prevalent metric of value-based performance to mid-term sales and operations planning. Due to the uncertainty of future events in a scenario based problem, they also used robust optimization methods to deal with operational risks in physical and financial supply chain management. Nickel S. et al (2012) provided a multi-period supply chain network design problem. In this problem, uncertainty was assumed for demand and interest rates, which was described by a set of scenarios. Accordingly, the problem was formulated as a multi-stage stochastic mixed-integer linear programming problem.

Hosseininezhad et al. (2013) proposed a continuous covering location model with risk consideration. Because the model considered uncertain covering radius, fuzzy concept introduced for customer satisfaction degree of covering. The model is solved by a fuzzy method named a-cut. After solving the model, the zones with the largest possibilities are determined for locating new facilities. Mohammadi et al. (2013) developed multi-objective multi-mode transportation model for hub covering location problem under uncertainty. In this model, uncertain parameter is the transportation time between each pair of nodes and it influence by a risk factor in the network. Akgun et al (2014) developed a model that minimizes the risk of demand point that is not supported by the located facilities. The goal is to choose the locations to support the demand points is constructed. In this paper, the uncertainty of demand point is calculated as the multiplication of the threat, the vulnerability of the demand point and consequence. Zhang et al. (2016) investigated a facility location model that considered the disruptions of facilities and the cost savings from the inventory risk-pooling effect and economies of scale. Facilities might have heterogeneous disruption probabilities. When a facility failed, its customers may be re-assigned to other ones that survive, to hedge against lost-sales costs. Puga and Tancrez (2016) studied on a location-inventory problem for the design of large networks with uncertain demand. They defined a non-linear formulation that integrates location, allocation and inventory decisions, and also includes the costs of transportation, cycle inventory, safety stock, ordering and facility opening. Berman et al. (2016) studied the effect of a decision maker's risk attitude on the median and center problems, with uncertain demand in the mean variance framework. They provided a mathematical formulation for both types in the form of quadratic programming. Lutter et al. (2017) introduced robust optimization and set covering problem by combining robust and probabilistic optimization. They defined new constraint and for highlighting their new approach, a case study for the location of emergency services was introduced.

In the rest of this section, aforementioned articles are classified based on location model, uncertainty and space as shown in *Table 1* in order to help the reader appreciate the symmetry associated with the facility location problems.

Table 1. Comparison between the works

Author(s)	*Location model*	*Uncertainty type*	*Space*
Liu et al. (2010)	*Two-echelon inventory/logistics*	*Stochastic demand*	*Discrete*
Peng et al. (2011)	*Reliable logistics network design*	*Disruption*	*Discrete*
Chen et al. (2011)	*Location selection*	*Disaster*	*Discrete*
Hahn and Kuhn (2012)	*Single-stage supply chain*	*Scenario Based*	*Discrete*
Nickel S. et al (2012)	*Multi-stage supply chain*	*Scenario Based*	*Discrete*
Mohammadi et al(2013)	*Hub location*	*chance constraint*	*Discrete*
Akgun et al (2014)	*P-center location*	*Disaster*	*Discrete*
Zhang et al (2015)	*Facility location*	*risk pooling*	*Discrete*
Puga and tancrez (2016)	*Location-inventory model*	*Uncertain demand risk-pooling*	*Discrete*
Berman et al (2016)	*p-median and center location*	*Scenario based*	*Discrete*
This research	*Continues covering location*	*Uncertain covering radius*	*Continuous*

In the next section, a continuous covering model with uncertainty consideration is introduced. The model is solved to determine the best covering value. Also, this paper proposes a *CE* algorithm considering multivariate normal density function for solving large scale problems.

3-Continuous covering location problem (CCLP)

In this section, the continuous covering model based on the hosseininezhad et al. (2013) is considered. For this model, the space is divided into n zones as shown in **Error! Reference source not found.**. Two new variables z_{ji} and u_{ji} are also introduced in this model. z_{ji} shows whether facility j is located in zone i or not and u_{ji} shows whether customer i is covered by facility j or not. It is assumed that distance between each customer and the facility is Euclidean. Accordingly, the mixed integer nonlinear programming model P_2 is as follows:

$$min \sum_{j=1}^{k}\sum_{i=1}^{n} z_{ji}f_i + M\sum_{i=1}^{n}\left(\frac{C_i}{C}\right)q_i \qquad (5)$$

S.t.

$$\left(\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2}\right) \le R + L(1 - u_{ji}), \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k \qquad (6)$$

$$\left(\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2}\right) \ge R - Lu_{ji}, \qquad \forall i = 1,2,\dots,n, \quad \forall j = 1,2,\dots,k \qquad (7)$$

$$\sum_{j=1}^{k} u_{ji} + q_i \ge 1, \qquad i = 1,2,\dots,n \qquad (8)$$

$$\sum_{i=1}^{n} z_{ji}\left(\sqrt{\left(x_j - a_i\right)^2 + \left(y_j - b_i\right)^2}\right) \le D, \qquad \forall j = 1,2,\dots,k \qquad (9)$$

$$\sum_{i=1}^{n} z_{ji} = 1, \qquad \forall j = 1,2,\dots,k \qquad (10)$$

$$\sum_{j=1}^{k} z_{ji} \le 1, \qquad \forall i = 1,2,\dots,n \qquad (11)$$

$$\sum_{j=1}^{k}\sum_{i=1}^{n} z_{ji} \le P, \tag{12}$$

Notations of the model are as follows,

Notations		Description
Sets/Indices		
	N	*set of zones(customer) in a continuous space indexed by i , {i=1,2,…,n}*
	K	*set of new facilities to be located indexed by j, {j=1,2,…,k}*
Parameters		
	a_i	*x coordinate of customer i*
	b_i	*y coordinate of customer i*
	f_i	*Installation cost of each facility in zone i*
	D	*Maximum distance a facility could be located from center of a zone for belonging to the zone*
	R	*Maximum distance a customer could be located from a facility to be covered by the facility (covering radius)*
	C_i	*importance of customer i*
	C	*Overall importance of customers*
	M	*Penalty of uncovered customers which is a large value*
	L	*a large value*
	P	*Number of facilities that can be open*
Decision variable		
	x_j	*x coordinate of facility j*
	y_j	*y coordinate of facility j*
	z_{ji}	*Binary variable; equal to1 if facility j is located in zone i; equal to 0 otherwise*
	u_{ji}	*Binary variable; equal to 1 if customer i is covered by facility j; equal to 0 otherwise;*
	q_i	*Binary variable; equal to 1 if customer i is not covered; equal to 0 otherwise*

Equation (5) is objective function of the model P_2 and constitutes of two terms; the first term is installation cost and the second term is risk cost which is cost of uncovered customers based on importance of each customer. Constraint sets (6), (7) are covering constraints; Guarantee that each customer can be covered by a facility if distance between them is smaller than R; u_{ji}'s ($\forall i = 1,2,\dots,n$ and $\forall j = 1,2,\dots,k$) constitute covering matrix. If distance between customer i and facility j is greater than R then $u_{ji} = 0$ and $u_{ji} = 1$ otherwise; since L is a large value Constraint sets (6), (7) will be satisfied, simultaneously. Constraint set (8) indicates the demand constraint which guarantees that q_i is *1* if u_{ji} is *zero*, it means that customer i is not covered. Constraint set (9) guarantees if distance between facility j and zone i is greater than D, $z_{ji} = 0$, so facility j does not belong to zone i and facility j will be located in zone i, if distance between facility j and zone i is smaller than *D*, Constraint set (10) Guarantees that facility j is installed only in one zone. Constraint set (11) Guarantees that at most one facility could be located in zone i. constraint set (12) defines number of facilities that should be open.

4-Solution method

In this section a solution method based on *Cross Entropy (CE)* algorithm is proposed.

4-1- Cross entropy (CE) algorithm

The main idea of *CE* algorithm, which was introduced by Rubinstein (1997), is related to the design of an effective learning mechanism throughout the search. It associates an estimation problem to the original combinatorial optimization problem, called the associated stochastic problem, characterized by a density function ϕ. The stochastic problem is solved by identifying the optimal importance sampling density ϕ^*, which is the one that minimizes the Kullback-Leibler distance with respect to the original density ϕ. This distance is also called the *CE* between ϕ andϕ^*. The minimization of the *CE* leads to the definition of optimal updating rules for the density functions, and consequently to the generation of improved feasible vectors. The method terminates when convergence to a point in the feasible region is achieved. The most important features of *CE* algorithm have been thoroughly exposed by de Boer et al. (2005). Chepuri and Homem-de-Mello (2005) considered CE algorithm to solve the vehicle routing problem with stochastic demands. Sebaa et al (2014) solved the location and tuning problems with cross entropy approach and compared the performance of CE algorithm with genetic algorithm. In this paper, we present a *CE* algorithm to solve *CCLP*. Since we want to generate vectors to identify the location of each facility, a density functions is considered. Consequently, let us define a family of density function ϕ on *X*, and use *2*-dimensional multivariate normal density function for locating facilities under the following probability distribution function:

$$\phi(X,\mu,\Sigma) = e^{-(X-\mu)\Sigma^{-1}(X-\mu)^T/2} \Big/ \frac{1}{\Sigma^{1/2}2\pi} \qquad (4)$$

Where *X*=(*x*,*y*) and $\mu = (\mu_x, \mu_y)$ are *1-by-2* vectors *x* , *y* coordinates of facility locations and mean of feasible space and Σ is a 2-by-2 symmetric positive definite matrix covariance. We estimate *X* via *Monte Carlo* simulation. In this regard, *X* could be estimated by drawing a random sample $X_1, \dots, X_{P-size}$ from $\phi(X,\mu,\Sigma)$, where *P-size* is CE population size, after generation each one of iterations, the best solutions (elites) are selected to constitute new space solution. *Figure 1* shows the procedure of the algorithm.

In the following, we provide the CE procedure for CCLP as pseudo-code. As illustrated in *step13*, the algorithm terminates when either the variance of multivariate normal density function converges to a small value (*Min error*) or when a pre-fixed maximum number of iteration (*max iteration*) is reached.

Algorithm: Cross Entropy (CE) sodo code for Continuous Covering Location Problem (CCLP)

1: *Determine population size and elite size, initial error, Min error and max iteration*
2: *Generate initial multivariate normal distribution parameters (Mean and covariance) based on x and y coordinates of zones,*
3: *Draw a simple population* $X_1, \dots, X_{P-size} \sim MV\ Normal(X, \mu_x^t, \mu_y^t, \Sigma^t)$
4: ***For*** *each random vector* X_i
5:   *Define continuous covering location problem (CCLP)*
6:   *Reconsider feasibility if needed*
7:   *Solve (CCLP)*
8: ***End for***
9: *Sort sample population in ascending order based on the objective function value*
10: *Select best solution of sample population(elites)*
11: *Compute* μ_x, μ_y *and* Σ*:*
12:
$$\mu_x^{t+1} = \bar{x}_{elite}^t$$
$$\mu_y^{t+1} = \bar{y}_{elite}^t$$
$$\Sigma^{t+1} = covariance(\bar{x}_{elite}^t, \bar{y}_{elite}^t)$$
$$e_j^{t+1} = variance(\bar{x}_{elite}^t, \bar{y}_{elite}^t)_j,\ j = 1, \dots, k\ for\ each\ new facility\ j$$
$$error^{\,t+1} = \bar{e}^{t+1}$$

13: ***If*** $error^{\,t+1} \le Min\ error \vee$ *t= max iteration*
14:   *Stop*
15: ***Else***
16:   *t ← t + 1 and go back to step 2*
17: ***End if***

Fig 1. Procedure of the proposed algorithm

4-2- Computational results

In this section, we present results of applying the *CE* algorithm on some instances of the continuous covering location problem. The algorithm has been implemented in *Matlab*, and run on a *Corei5 at 2.53 GHz* with *3GB* of *RAM* memory. The *CE* parameters are as follows; the population size is *250* and the elite size is *25*. We compare obtained CPU time and objective value of the proposed algorithm with *GAMS* software which is solved by *SBB* (*Simple Branch & Bound*) solver (baron solver-12). These results are expressed as a percent deviation from the best-known solutions by *GAMS*. The deviation is computed as follows:

$$dev. = \frac{F^{best} - F^{*}}{F^{*}} \times 100 \tag{15}$$

Where F^{best} is the total cost found by the proposed algorithm, and F^* refers to the best found by *GAMS*. We ran the algorithms ten times for each instance and report the following statistics based on the best solutions. As shown in table 2 the proposed method provides better solution and smaller CPU Time. For all of the problems, the gap for gams is 0.01. Note CPU time for *GAMS* to provide a feasible solution is very high for *n>50* & *k>5*. Then the algorithm has been solved for large scale cases until *n=1000* and *k*=100. Figure 2 and figure 3 show objective function vs. iteration and location of facilities for *n=500* and *k=50*. Table 3 provides results for large scale cases.

Table 2. Comparision between results of the Proposed Algorithm(CE) and GAMS solution

			GAMS (Baron solver)		Proposed Algorithm(CE)			dev.
n	**k**	**R**	**Time(Sec.)**	**Objective**	μ_{t^*}**(Sec.)**	σ_t^{**}**(Sec.)**	**Best**	
20	2	2	12	1216.892	0.803	0.51	1216.892	0.000
	3	1.8	85	1757.892	0.82836	0.53	1848.892	0.052
	4	1.6	69	2435	0.7716	0.05	2454.704	0.008
30	2	2.4	48	1191.325	1.15214	0.61	1207.402	0.013
	3	2.2	331	1708.17	1.93876	0.12	1738.17	0.018
	4	2	563	2294.17	2.46228	0.15	2346.772	0.023
	5	1.8	1291	2865.17	2.82048	0.05	2934.264	0.024
40	2	2.8	38	1107.03	1.06392	0.21	1107.03	0.000
	3	2.4	193	1638.519	1.88916	0.15	1648.607	0.006
	4	2	1320	2162.519	2.50224	0.18	2215.113	0.024
	5	1.8	5880	2864.401	2.96244	0.15	2791.125	0.000
50	2	3	100	1197.136	1.00464	0.12	1198.11	0.001
	3	3	190	1587.193	1.64268	0.05	1595.579	0.005
	4	2.2	1030	2302.258	2.01396	0.11	2254.568	0.000
	5	2	1533	2807.883	2.42268	0.11	2876.426	0.024
	6	1.8	4620	3466.426	2.94528	0.34	3404.09	0.000

*Average Time over 10 runs

** Time Standard Deviation over 10 runs

Table 3. Results of the proposed algorithm (CE) for large scale problems

#	n	M	Time(Sec.)				F^{best}				GAMS(Baron solver)			dev. (%)	
			Best	Avr.	Std.	C.V.	Best	Avr.	Std.	C.V.	Time(Sec.)	LB	UB	Best	Avr.
1	100	5	4.134	5.187	0.619	11.941	3143.748	3157.677	11.927	0.378	18000.000	2582.000	5909.000	-46.797	-46.562
2	100	10	18.798	30.379	10.549	34.726	5811.589	5877.440	45.800	0.779	18000.000	5305.000	10778.000	-46.079	-45.468
4	150	5	4.555	5.524	0.531	9.605	4798.448	4958.553	112.290	2.265	18000.000	2571.000	5980.000	-19.758	-17.081
5	150	10	12.496	14.179	1.074	7.576	6417.408	6665.189	168.988	2.535	18000.000	5264.000	10920.000	-41.233	-38.963
6	150	15	24.274	26.573	1.874	7.052	9022.489	9190.628	115.032	1.252	18000.000	8060.000	15800.000	-42.896	-41.831
7	200	5	5.725	7.110	1.063	14.945	3021.215	3144.107	48.359	1.538	18000.000	2537.000	6280.000	-51.891	-49.935
8	200	10	16.661	23.273	10.782	46.329	5623.519	5751.599	97.277	1.691	18000.000	5129.000	11221.000	-49.884	-48.743
9	200	15	37.830	65.779	16.035	24.378	8433.145	8550.979	115.393	1.349	18000.000	7809.000	16112.000	-47.659	-46.928
10	200	20	95.722	98.575	0.963	0.977	11372.650	11479.670	78.770	0.686	18000.000	10579.000	20943.000	-45.697	-45.186
11	300	5	6.162	9.706	6.758	69.620	2827.619	2942.572	68.154	2.316	18000.000	2528.000	5986.000	-52.763	-50.842
12	300	10	17.129	29.991	15.193	50.658	5461.964	5585.892	83.149	1.489	18000.000	5082.000	10936.000	-50.055	-48.922
13	300	15	73.321	76.426	2.735	3.579	8300.199	8428.803	88.111	1.045	18000.000	7693.000	15838.000	-47.593	-46.781
14	300	20	98.624	99.446	0.826	0.831	11160.670	11285.864	100.778	0.893	18000.000	10342.000	20696.000	-46.073	-45.468
15	300	30	146.516	149.156	1.651	1.107	17232.920	17542.449	223.164	1.272	18000.000	15778.000	30174.000	-42.888	-41.862
16	500	5	11.326	14.506	2.232	15.386	3333.048	3370.796	20.904	0.620	18000.000	2528.000	5991.000	-44.366	-43.736
17	500	10	31.590	45.786	12.106	26.440	5863.962	5940.348	54.154	0.912	18000.000	5076.000	10965.000	-46.521	-45.824
18	500	15	70.778	110.393	19.837	17.970	8518.412	8583.694	52.623	0.613	18000.000	7647.000	15914.000	-46.472	-46.062
19	500	20	96.580	153.011	20.213	13.210	11174.810	11326.646	76.033	0.671	18000.000	10259.000	20833.000	-46.360	-45.631
20	500	30	224.158	230.833	3.859	1.672	16665.630	16980.993	221.816	1.306	18000.000	15550.000	30590.000	-45.519	-44.488
21	500	50	384.511	387.017	2.788	0.720	28754.280	29243.232	238.276	0.815	18000.000	NF	NF	*	*
22	1000	5	25.584	29.600	3.718	12.562	3420.706	3452.719	25.105	0.727	18000.000	2519.000	5999.000	-42.979	-42.445
23	1000	10	57.096	65.311	5.921	9.065	5973.489	6103.208	82.572	1.353	18000.000	5050.000	10991.000	-45.651	-44.471
24	1000	15	95.753	121.007	19.779	16.346	8625.463	8747.928	82.899	0.948	18000.000	7595.000	15973.000	-46.000	-45.233
25	1000	20	131.400	172.810	40.630	23.511	11327.830	11418.793	82.323	0.721	18000.000	10149.000	20947.000	-45.921	-45.487
26	1000	30	291.784	357.567	43.905	12.279	16818.600	17022.362	132.108	0.776	18000.000	NF	NF	*	*
27	1000	50	548.359	588.955	17.156	2.913	28394.870	28561.002	160.145	0.561	18000.000	NF	NF	*	*
28	1000	100	1179.383	1195.364	10.094	0.844	61121.460	62177.151	510.462	0.821	18000.000	NF	NF	*	*

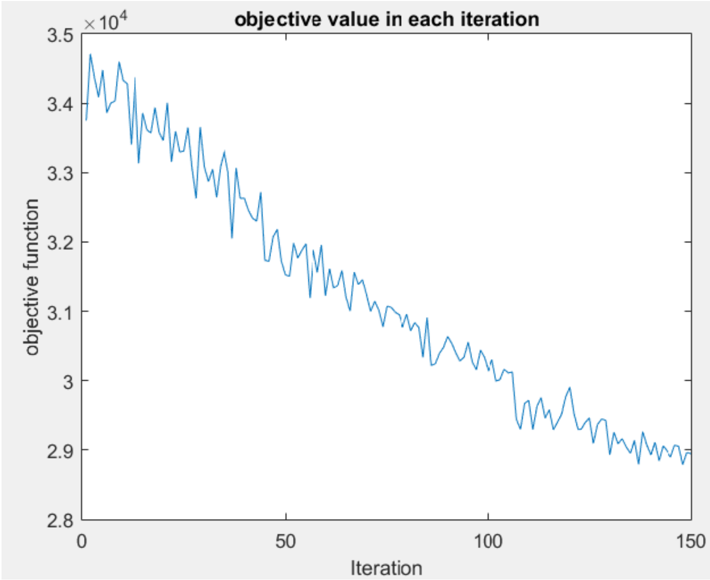

Fig 2. Objective function vs. iteration for n=500 and k=50

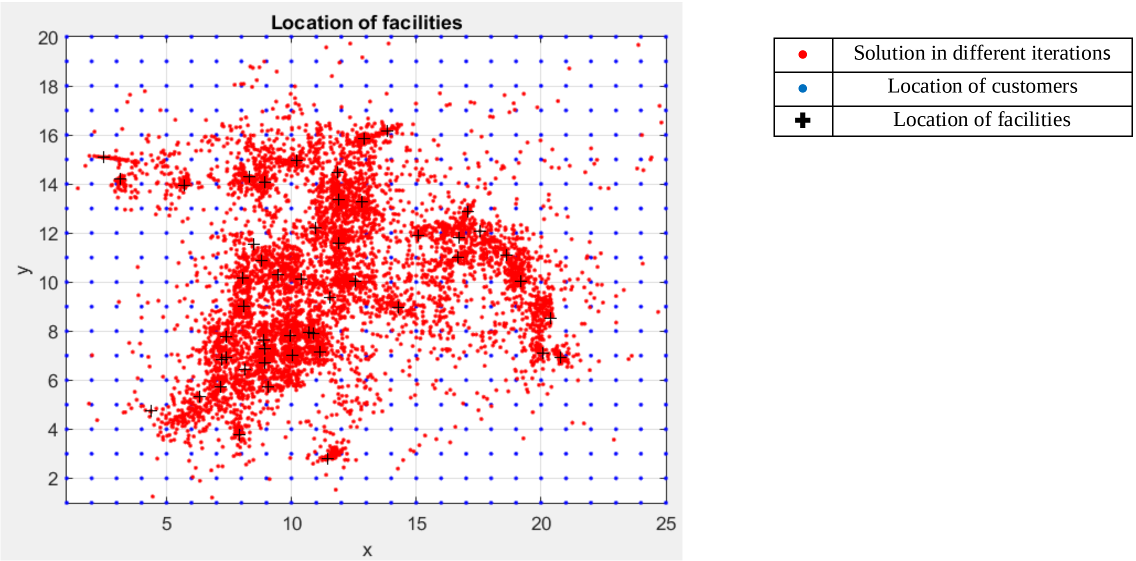

Fig 3. Location of facilities for n=500 and k=50

Sensitivity analysis for parameter M and L are shown in figure 4 and figure 5.

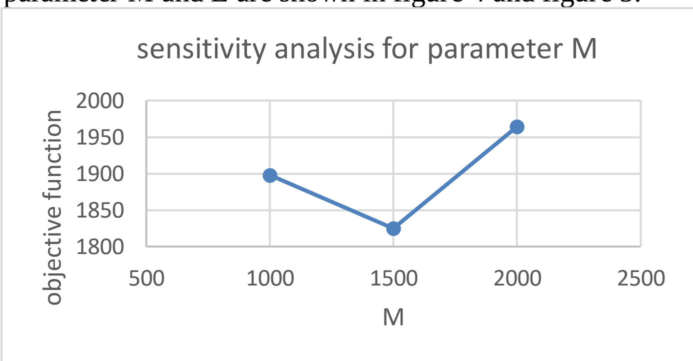

Fig 4. sensitivity analysis of parameter M

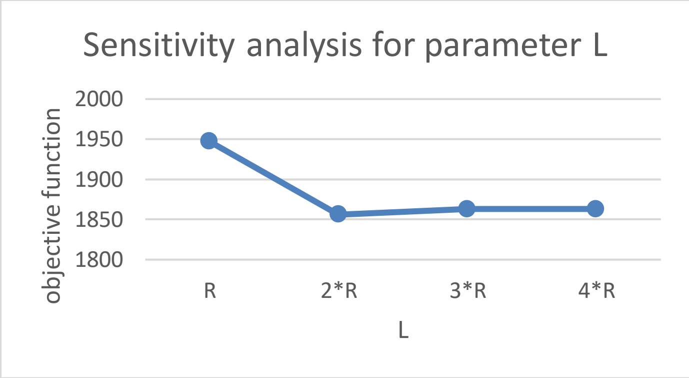

Fig 5. Sensitivity analysis of parameter L

5-Numerical example and case study

In this section, a numerical example is expressed to illustrate the introduced model**.**

5-1- Example

Suppose we want to locate *25* new facilities in a region including *256* zones (customers) as shown in figure 6. Numbers in each region express importance of each customer from *1* to *5*. Fixed cost in green, white, yellow and red zones are 900, 1000, 1100 and 1500, respectively.

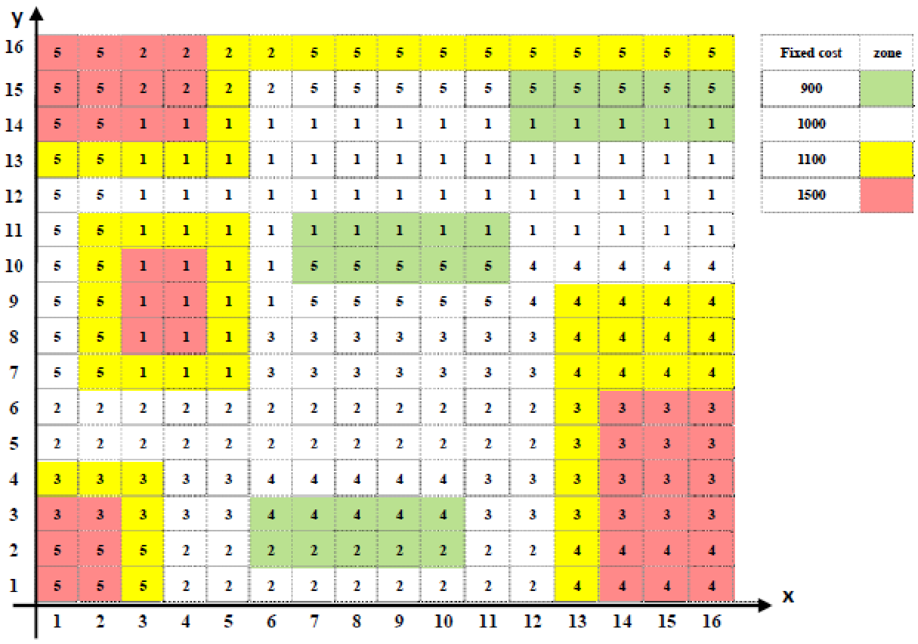

Fig 6. Example with 256 zones

In this example, some neighborhood zones with the same fixed cost are selected as a zone set. For example zone *25* and *38* are selected as zone set {25, 38}.

Coordinates of new facilities based on possibility values applying (19) and (20), and cost values for the numerical example 2 are shown in table 4.

Table 4. Coordinate of new facilities based on possibility values for Example

Facility	Coordinate x	Coordinate y
1	*2.70*	*2.10*
2	*5.98*	*2.35*
3	*11.85*	*5.52*
4	*15.03*	*6.91*
5	*5.99*	*7.83*
6	*15.12*	*14.99*
7	*2.40*	*5.36*
8	*9.70*	*1.88*
9	*5.78*	*14.77*
10	*1.89*	*12.40*
11	*11.95*	*8.67*
12	*14.71*	*9.95*
13	*11.89*	*15.06*
14	*1.34*	*8.43*
15	*13.33*	*2.29*
16	*4.59*	*5.26*
17	*8.90*	*11.33*
18	*8.14*	*5.76*
19	*10.73*	*10.84*
20	*9.38*	*15.05*
21	*9.55*	*3.27*
22	*9.11*	*7.19*
23	*7.75*	*14.84*
24	*6.59*	*10.98*
25	*7.41*	*3.14*

The final solution is shown in figure 7.

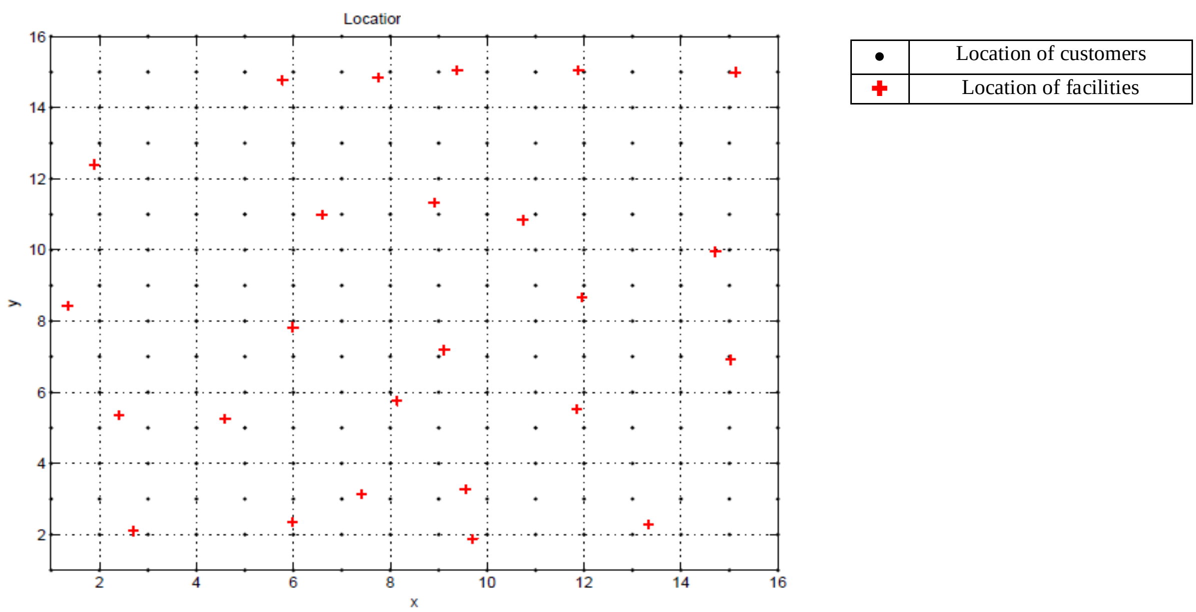

Fig 7. Results of the numerical example

5-2- Case study

In Tehran city, that contains 22 districts, there are 15 special drugstores which service people for their special kinds of medicines. Now, one problem is that these drugstores could be able to service more than one district and cover more districts. The goal is to service quickly, so, we locate 7 new drugstores in Tehran with continuous covering location model with fixed cost.

Tehran contains 22 districts and we want to locate 7 new drugstores in these districts. For solving problem, we divided space to 2-dimensional horizontal and vertical. The vertical dimension consider from 0 to 41 and horizontal dimension consider from 0 to 55. Now location of every district is available. In this problem, importance of every district determine with condition and number of service office in that district. Location and importance of every district is shown in *table 5*. The problem is solved by cross entropy

Table 5. Importance and location of every districts

district	*importance*	*x*	*Y*	*district*	*importance*	*x*	*y*
1	*9*	*35.5*	*36.5*	*12*	*7*	*33*	*17.5*
2	*11*	*28*	*32.5*	*13*	*4*	*39*	*20.5*
3	*7*	*37*	*28.5*	*14*	*6*	*37.5*	*18*
4	*11*	*42*	*27*	*15*	*6*	*41*	*10.5*
5	*11*	*22.5*	*25*	*16*	*6*	*32.5*	*10.5*
6	*7*	*32.5*	*26.5*	*17*	*3*	*27.5*	*13.5*
7	*6*	*36*	*26*	*18*	*4*	*25.5*	*12*
8	*3*	*42*	*25*	*19*	*4*	*29.5*	*9.5*
9	*2*	*25.5*	*20*	*20*	*5*	*35*	*2*
10	*3*	*27.5*	*20*	*21*	*3*	*15.5*	*21*
11	*5*	*30.5*	*16*	*22*	*5*	*8.5*	*29*

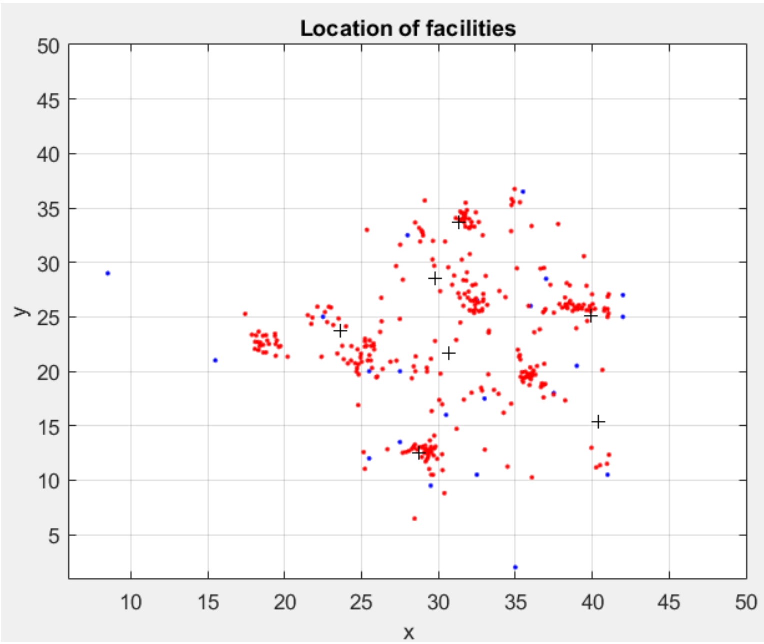

Fig 8. Location of new 7 drugstores

Figure 8 shows one of solutions. Location of every drug store is shown in table 6.

Table 6. *possibility of districts and service offices*

Selected location	*y*	*x*	*districts*	*Services office*
14	*15.3*	*40.3*	*14,15*	*1*
17	*12.5*	*28.7*	*11,16,17,18,19*	*2*
10	*21.62*	*30.6*	*10,12*	*3*
2	*33.73*	*31.3*	*1,2*	*4*
5	*23.7*	*23.6*	*5,9*	*5*
8	*25*	*39.8*	*3,4,7,8,13*	*6*
6	*28.5*	*29.7*	*2,6*	*7*

With considering the final solution districts of 20, 21 and 22 is not covering and:
Drug store that located in district 14, cover a districts 14 and 15.
Drug store that located in district 17, cover a districts 11, 16, 17, 18, 19.
Drug store that located in district 10, cover districts 10 and12
Drug store that located in district 2, cover districts 1 and 2.
Drug store that located in district 5, cover districts 5 and 9.
Drug store that located in district 8, cover districts 3, 4, 7, 8, 13.
Drug store that located in district 6, cover districts 2 and 6.
For analysis of case study, 3 parameters consider. The number of facilities, covering radius and Maximum distance a facility could be located from the center of a zone. Figure 9 and 10 show how changing each of these parameters, influence the objective function of the model. As shown in figure 7, considering more facilities reduces the objective function. For the parameter of covering radius, by increasing the covering radius, the objective function decreases and this is shown in figure 8.

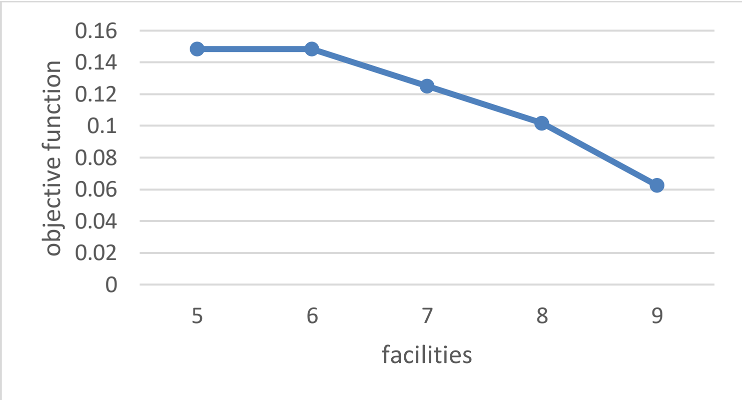

Fig 9. sensitivity analysis of parameter number of facilities

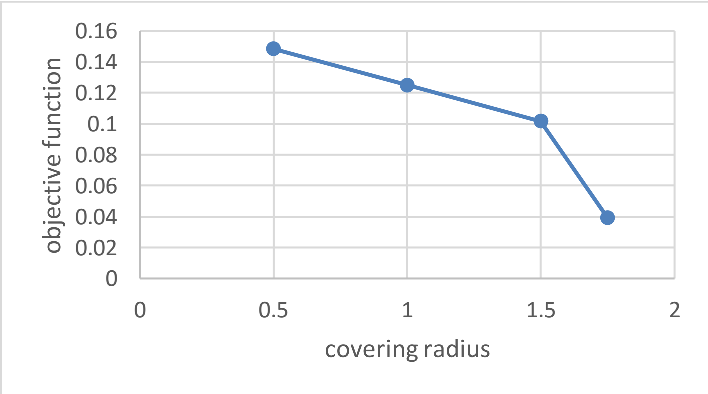

Fig 10. sensitivity analysis of parameter covering radius

6-Conclusion

This paper considered the cross entropy algorithm for solving the covering location model. The presented model's advantage over the traditional covering location ones was consideration of continuous space for the covering problems. Providing robust uncertainty location model is another usability of the proposed model. Comparing with the *GAMS*, the proposed algorithm based on *CE* provided more acceptable results in CPU time and the objective value, especially in large scale problems. The case study showed how to cover demand of 22 districts in Tehran with more 7 new service offices. Also, the location of these facilities is demonstrated by the model. Extension of the model as a continuous covering location allocation model and providing a heuristic method are two research issues which we think may need future investigations.

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