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An extended intuitionistic fuzzy modified group complex proportional assessment approach

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Abstract

Complex proportional assessment (COPRAS) methodology is one of the well-known multiple criteria group decision-making (MCGDM) frameworks that can focus on proportional and direct dependences of the significance and utility degree of candidates under the presence of mutually conflicting criteria in real-world cases. This study elaborates a new intuitionistic fuzzy modified group complex proportional assessment (IF-MGCOPRAS) method. This group decision-making methodology makes the suitable decision by considering both concepts of the intuitionistic fuzzy positive ideal and negative ideal solutions. The performance of the candidates with respect to various criteria and corresponding criteria weights are linguistic terms that expressed as intuitionistic fuzzy numbers. Then, intuitionistic fuzzy weighted averaging (IFWA) relation is employed to aggregate individual opinions of experts. Furthermore, a new intuitionistic modified relative index is manipulated to specify the most appropriate candidate for a particular engineering application in a manufacturing industry. In this respect, an illustrative example for group decision making in an equipment selection problem is considered to demonstrate the procedure of proposed complex assessment method. The obtained results of IF-MGCOPRAS method represented that a reasonable and satisfactory assessment for equipment decision making problem is occurred. Finally, a comparative analysis and discussion with the intuitionistic fuzzy group TOPSIS method is provided.

Keywords: Multiple criteria group decision-making methodologies, intuitionistic fuzzy sets, complex proportional assessment method, equipment selection problem

1- Introduction

Multiple criteria group decision-making (MCGDM) approaches are often considered to solve complex decision-making and/or selection problems under group decision making analysis. These approaches focus on screening, prioritizing or choosing a set of candidates under usually independent, incommensurate or conflicting criteria (Hwang and Yoon 1992; Dagdeviren 1998). A group of decision makers or experts are required to prepare quantitative and/or qualitative assessments for specifying the rating of each candidate with respect to each criterion, and the evaluation criteria weights with respect to the overall objective. Therefore, the final decision is highly dependent on the opinions of the experts. In fact, vagueness and imprecision arise from a different of reasons in the group decision-making procedure, such as incomplete information, unquantifiable information, partial ignorance, and unobtainable information (Ölçer and Odabasi 2005; Vahdani et al. 2010).

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In this respect, classical MCGDM methodologies cannot effectively deal with group decision making problems with such imprecise information (Gumus 2009; Malekly et al. 2010; Mousavi et al. 2011; Vahdani and Hadipour 2010; Ye 2011). Meanwhile, in many cases the experts have vague and imprecise information about the candidates with respect to a criterion and the relative significance of criteria. To resolve these difficulties, fuzzy set theory, introduced by Zadeh (1965), has been applied and adopted for solving group decision-making and/or selection problems. In fuzzy sets theory, the linguistic terms are expressed by the experts in the group decision-making process. The concept of linguistic term is very useful to handle the situations and conditions, which are not well defined or too complex to be reasonably described in conventional quantitative expressions (Zimmermann 2001; Hwang, C. L., & Yoon, K. 2012). Hence, combination of the fuzzy set theory and MCGDM methods is a powerful tool to handle uncertain data, and these fuzzy expressions are more natural than rigid mathematical equations and rules for humans.

In the last two decades, some authors have attempted to extend and present the MCGDM approach for decision-making and/or selection problems. The main idea of the methods is to prepare a better-informed and more formalized decision-making procedure. One of the effective MCGDM methodologies to solve complex decision-making problems is the complex proportional assessment (COPRAS) method that has been proposed by Zavadskas and Kaklauskas (1996). This method focuses on proportional and direct dependences of the important and utility degree of the available candidates under the presence of mutually conflicting criteria. It regards the performance of the candidates with respect to various criteria and the corresponding criteria weights. This method makes the suitable decision by considering both the positive ideal and negative ideal solutions. The main merit of the COPRAS method is its simplicity and ability to obtain an indisputable preference order (Chatterjee et al. 2011; Turanoglu et al. 2016). Recently, the COPRAS method has been successfully applied to different engineering and management fields for solving the complex group decision making problems (for instance, selection of low-e windows in retrofit of public buildings (Kaklauskas et al. 2006), supervisor selection (Datta et al. 2009), and materials selection (Chatterjee et al. 2011)).

In the traditional COPRAS method, the ratings and weights of the criteria are known precisely. In many real-world cases, imprecise information is inadequate to model real-world situations since human opinions consisting preferences are often uncertain, and cannot estimate their preference with an exact numerical value. A more practical approach may be to use linguistic evaluation instead of numerical data, and linguistic terms defined in fuzzy numbers seem more suitable for describing those inputs in the COPRAS method. Thus, a development of the COPRAS to a fuzzy environment is a natural generalization of this methodology. In the traditional fuzzy set theory, the membership degree for an object x is $\mu(x)$ and the non-membership degree is$1-\mu(x)$, automatically. This degree of membership integrates the evidence against x and the evidence for x. In fact, the single number prepares the experts nothing about the lack of knowledge; however, in real-world conditions, information of an object belonging to a fuzzy concept may be insufficient and incomplete. In practice, the sum of the membership degree and the non-membership degree may be less than one (Li et al. 2010). There are no tools to incorporate the lack of knowledge of the membership degree in the conventional fuzzy set (Atanassov and Georgiev 1993; Atanassov et al. 2001, 2005; Büyüközkan and Güleryüz 2016). In other words, in the traditional fuzzy sets, once the preference (liking) is established using a fuzzy set, the dislike is computed by taking its complement. When the likes are described one can precisely establish the vagueness associated with the dislikes (Hernandez and Uddameri 2010). An appropriate solution is to consider the intuitionistic fuzzy set (IFS) introduced by Atanassov (1986, 1994) with ill-known membership grades, which is a generalization of the traditional fuzzy set. The IFS allows for the explicit description of both likes and dislikes which need not be complementary (Xu 2007; Büyüközkan and Göçer 2016). The reason is that, in comparison with the traditional fuzzy sets, the IFS seems to be well suited for defining a significant factor which should be considered when trying to construct really adequate models and solutions of group decision-making problems, namely hesitation of the experts (Atanassov et al. 2005; Li 2005). Hence, it is more general and capable of preparing a complete picture of the uncertainty. This flexibility makes it better suited for complex group decision-making process such as group decision-making problems in the manufacturing industry.

In recent years, many authors have investigated the IFS theory and applied it to various fields, including logic programming (Atanassov and Georgiev 1993), pattern recognition (Nguyen 2016;

Chen et al. 2016), robotic systems (Gürkan et al. 2002; Devi 2011), and mathematical programming (Li et al. 2010; Wan and Li 2015; Xu et al. 2016). Moreover, some authors have focused on the combination of IFS theory and decision-making methods. For instance, Szmidt and Kacprzyk (2002) provided some solution concepts, consisting the consensus winner and intuitionistic fuzzy core in the group decision-making procedure, and elaborated a methodology to aggregate the individual intuitionistic fuzzy preference relations into a social fuzzy preference relation. Atanassov et al. (2005) extended an intuitionistic fuzzy interpretation of multi-person MCDM. Xu and Yager (2006) proposed some geometric aggregation operators based on IFSs and implemented them to multi-attributes decision making problems. Lin et al. (2007) constructed a new model by linear programming for handling fuzzy multi-attributes decision making problems based on IFSs. Xu (2007) developed some novel intuitionistic fuzzy aggregation relations, consisting the intuitionistic fuzzy weighted averaging and intuitionistic fuzzy ordered weighted averaging relations, for aggregating intuitionistic fuzzy information. Moreover, Boran et al. (2009) prepared a multi-criterion intuitionistic fuzzy decision-making framework based on the group TOPSIS (technique for order preference by similarity to ideal solution) method for supplier selection in which the experts' knowledge was imprecise and vague.

Ye (2010) introduced an MCGDM methodology based on vector similarity measures for trapezoidal intuitionistic fuzzy numbers. Li et al. (2010) developed a linear programming methodology for solving multi-attribute group decision-making problems by using IFSs. Devi and Yadav (2013) extended an intuitionistic fuzzy elimination and choice translating reality (ELECTRE) method based on group decision analysis for solving the plant location selection problem. İNtepe (2013) presented an interval-valued intuitionistic fuzzy group decision making framework based on similarity to ideal solution for selecting the most suitable technology forecasting technique. Wu et al. (2014) prepared a novel decision framework based on intuitionistic fuzzy information and experts' judgments for wind farm project plan selection problem. Onar et al. (2015) elaborated an interval-valued intuitionistic fuzzy multi-expert approach based on hierarchical structure and new linguistic scale for prioritizing wind energy technologies. Wan et al. (2016) presented a preference relation approach based on intuitionistic fuzzy information to solve the radio frequency identification technology selection problem. Peng and Selvachandran (2017) focused on Pythagorean fuzzy set theory regarding its concepts, aggregation operators, information measures, extensions, applications, etc. In this respect, the two novel group decision making techniques as distance from average solution and COPRAS methods to represent the applicability and powerfully of the Pythagorean fuzzy set theory. In addition, Zheng et al. (2018) founded an evaluation indicator system by developing a hesitant fuzzy linguistic COPRAS method to appraise the severity of the chronic obstructive pulmonary disease.

In this study, an extension of traditional COPRAS method by a group of experts is presented to handle fuzzy decision-making and/or selection problems based on the IFS theory, where the properties of the candidates and criteria are represented by the IFS. The proposed intuitionistic fuzzy modified group COPRAS (IF-MGCOPRAS) method uses the truth and non-truth membership functions to show the satisfiability and non-satisfiability degrees for each candidate with respect to a set of criteria, respectively. In addition, it allows the experts to have the membership and non-membership degrees of the relative significance of each criterion. In the extended version of the COPRAS, linguistic terms are considered to capture fuzziness in decision information and the group decision-making procedure by means of an intuitionistic fuzzy decision matrix. In the group decision-making procedure, aggregation of the experts' opinions is very significant to appropriately perform the evaluation procedure. Thereby, intuitionistic fuzzy weighted averaging (IFWA) relation is employed to aggregate all individual experts' opinions for rating the candidates with respect to each criterion and the relative significance of the criteria. Then, a new intuitionistic fuzzy relative index is introduced, which is extended from the concept of the closeness measure to the ideal solution. It also overcomes the difficulties arising from the traditional COPRAS method in an intuitionistic fuzzy setting information. Moreover, an extended index is presented for computing the weight of each expert to achieve reliable results. However, proposed intuitionistic fuzzy group decision-making methodology can provide a useful way to help the experts to make his/her decision in a manufacturing industry. Furthermore, a comparative analysis is demonstrated with an application example in an equipment selection problem between the proposed IF-MGCOPRAS and intuitionistic fuzzy group TOPSIS method.

The remaining of this paper is organized as follows: Section 2 briefly introduces the classical COPRAS method. Section 3 illustrates IFS theory and its basic definitions and notation of the fuzzy number. Section 4 presents the procedure of proposed IF-MGCOPRAS method for solving decision-making and/or selection problems. Section 5 explains the implementation of proposed intuitionistic fuzzy group decision-making method in a real-world example in details and shows its applicability and suitability. In addition, a comparison is made between the proposed method and the intuitionistic fuzzy group TOPSIS method, and the discussion of results is given. Finally, the paper will be ended with a brief conclusion and further research suggestions in section 6.

2- Multiple criteria complex proportional assessment (COPRAS) method

The COPRAS method was first introduced by Zavadskas and Kaklauskas (1996), which is the method of multiple criteria complex proportional evaluation. It is an effective assessment approach that tries to rank each candidate described in terms of several criteria, their significance, and utility degree. In fact, this method selects the best decision by considering both the positive and negative ideal solutions. The positive ideal solution (PIS) is a solution that minimizes the cost criteria and maximizes the benefit criteria; whereas, the negative ideal solution (NIS) maximizes the cost criteria and minimizes the benefit criteria. The so-called benefit criteria are those for maximization, while the cost criteria are those for minimization. The most suitable candidate is the first one, which is closest to the positive and farthest from the negative ideal solutions, respectively. The procedure of the method of complex proportional evaluation consists of the following steps (Zavadskas and Kaklauskas 1996; Kaklauskas et al. 2006):

Step 1: Select the available set of important criteria, which describes candidates.

Step 2: Prepare the decision-making matrix (X) for an MCDM problem in which $A_1, A_2, \ldots, A_m$ are m possible candidates and $C_1, C_2, \ldots, C_n$ are n criteria.

$$X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix} \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots n. \tag{1}$$

Step 3: Determine the weights of criteria q_j .

Step 4: Normalize the decision-making matrix $\overline{X}$. The normalized values of this matrix are calculated by:

$$\overline{x}_{ij} = \frac{x_{ij}}{\sum_{i=1}^{m} x_{ij}} \qquad i = 1,2,\ldots,m;\ j = 1,2,\ldots,n. \tag{2}$$

After this step, the normalized decision-making matrix is obtained as follows:

$$\overline{X} = \begin{bmatrix} \overline{x}_{11} & \overline{x}_{12} & \cdots & \overline{x}_{1n} \\ \overline{x}_{21} & \overline{x}_{22} & \cdots & \overline{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \overline{x}_{m1} & \overline{x}_{m2} & \cdots & \overline{x}_{mn} \end{bmatrix} \quad i = 1,2,\ldots,m;\ j = 1,2,\ldots n. \tag{3}$$

Step 5: Calculate the weighted normalized decision matrix $\hat{X}$. The weighted normalized values $\hat{x}_{ij}$ are calculated by:

$$\hat{x}_{ij} = \bar{x}_{ij} \cdot q_j \qquad i = 1,2,...,m; j = 1,2,...,n. \tag{4}$$

After this step, the weighted normalized decision-making matrix is represented as below:

$$\hat{X} = \begin{bmatrix} \hat{x}_{11} & \hat{x}_{12} & \cdots & \hat{x}_{1n} \\ \hat{x}_{21} & \hat{x}_{22} & \cdots & \hat{x}_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ \hat{x}_{m1} & \hat{x}_{m2} & \cdots & \hat{x}_{mn} \end{bmatrix} \qquad i = 1,2,..., m; j = 1,2,...n. \tag{5}$$

Step 6: Calculate sums of positive criteria values (P_i) as follows:

$$P_i = \sum_{j=1}^{k} \hat{x}_{ij}. \tag{6}$$

where k is number of benefit criteria.

Step 7: Calculate sums of negative criteria values (R_i) as follows:

$$R_i = \sum_{j=k+1}^{n} \hat{x}_{ij}. \tag{7}$$

Step 8: Determine the minimum value of R_i.

$$R_{\min} = \min_i R_i \qquad i = 1,2...,m. \tag{8}$$

Step 9: Calculate the relative weight of each candidate (Q_i) by:

$$Q_i = P_i + \frac{R_{\min} \sum_{i=1}^{m} R_i}{R_i \sum_{i=1}^{m} \frac{R_{\min}}{R_i}}. \tag{9}$$

Step 10: Determine the priority of the candidates by decreasing sorting of relative weight of each candidate.

Step 11: Calculate the utility degree of each candidate.

$$N_i = \frac{Q_i}{Q_{\max}} 100\%, \tag{10}$$

where Q_i and $Q_{\max}$ are the significance of candidates that obtained from equation (9).

3. Intuitionistic Fuzzy Sets

Let X be a universe of discourse. Concept of fuzzy set introduced by Zadeh (1965):

$$F = \{(x, \mu_F(x) | x \in X)\}, \quad (11)$$

Whose basic component is only a membership degree $\mu_F(x)$ with the non-membership degree being$1 - \mu_F(x)$. However, in real-world cases when an expert is asked to define his/her judgment degree to an object, there commonly exists a hesitation or uncertainty about the degree, and there is no means to incorporate the hesitation or uncertainty in a fuzzy set (Deschrijver and Kerre 2004). To solve this issue, Atanassov (1986, 1994) by adding an uncertainty or hesitation degree lead to generalize the Zadeh's fuzzy set to IFS. IFS is defined as follows. IFS A in a finite set X can be written as:

$$A = \{(x, \mu_A(x), v_A(x) | x \in X)\} \quad (12)$$

where $\mu_A(x), v_A(x): X \to [0,1]$ are membership function and non-membership function, respectively, such that

$$0 \leq \mu_A(x) + v_A(x) \leq 1 \quad (13)$$

Third parameter of IFS is $\pi_A(x)$, known as the intuitionistic fuzzy index or hesitation degree whether x belongs to A or not:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x) \quad (14)$$

It is obviously seen that for every $x \in X$:

$$0 \leq \pi_A(x) \leq 1. \quad (15)$$

If the $\pi_A(x)$ is small, knowledge about x is more certain. If $\pi_A(x)$ is great, knowledge about x is more uncertain. Obviously, when $\mu_A(x) = 1 - v_A(x)$ for all elements of the universe, the ordinary fuzzy set concept is recovered (Shu et al. 2006).

Let A and B denote two IFSs of the universe of discourse X, where $A = \{(x, \mu_A(x), v_A(x) | x \in X)\}$, $B = \{(x, \mu_B(x), v_B(x) | x \in X)\}$. Burillo and Bustince (1996) defined the following expressions.

Definition 1. $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$ for all $x \in X$.

Definition 2. . $A \preccurlyeq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \leq v_B(x)$ for all $x \in X$. In addition, $A \geq B$ if and only if $B \leq A$; $A \succcurlyeq B$ if and only if $B \preccurlyeq A$.

Atanassov (1994) and Atanassov et al. (2001) defined addition and multiplication operations as follows.

Definition 3. Let A and B be two IFSs. Addition operation of them can be defined as:

$$A + B = \{(x, \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), v_A(x) \cdot v_B(x) | x \in X)\} \quad (16)$$

Definition 4. Let A and B be two IFSs. Multiplication operation of them can be defined as:

$$A \cdot B = \{(x, \mu_A(x) \cdot \mu_B(x), v_A(x) + v_B(x) - v_A(x) \cdot v_B(x) | x \in X)\} \quad (17)$$

Chen (2007) defined subtraction and division operations as follows.

Definition 5. Let A and B be two IFSs. Subtraction operation of them can be defined as:

$$A - B = \left\{ \left(x, \frac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)}, \frac{v_A(x)}{v_B(x)} \right) | x \in X \right\} \tag{18}$$

Condition: $A \geq B, \mu_B(x) \neq 1, v_B(x) \neq 0$ and $\mu_A(x) \cdot v_B(x) - \mu_B(x) \cdot v_A(x) \leq v_B(x) - v_A(x)$

Definition 6. Let A and B be two IFSs. Division operation of them can be defined as:

$$\frac{A}{B} = \left\{ \left(x, \frac{\mu_A(x)}{\mu_B(x)}, \frac{v_A(x) - v_B(x)}{1 - v_B(x)} \right) | x \in X \right\} \tag{19}$$

Condition: $A \leq B, \mu_B(x) \neq 0, v_B(x) \neq 1$ and $\mu_A(x) \cdot v_B(x) - \mu_B(x) \cdot v_A(x) \geq \mu_A(x) - \mu_B(x)$

Definition 7. The IFS nA for any positive integer n as follows (De et al. 2000):

$$nA = \{ (x, \mu_{nA}(x), v_{nA}(x)) | x \in X \} \tag{20}$$

where $\mu_{nA}(x) = 1 - (1 - \mu_A(x))^n$, $v_{nA}(x) = [v_A(x)]^n$.

4. Proposed Intuitionistic Fuzzy Modified Group COPRAS Method

Let A be a set of candidates and let C be a set of criteria, where

$$A = \{A_1, A_2, \dots, A_m\}, C = \{C_1, C_2, \dots, C_n\}. \tag{21}$$

Assume that the characteristics of the candidate A_i are presented by the IFS shown as follows:

$$A_i = \{(C_1, \mu_{i1}, v_{i1}, \pi_{i1}), (C_2, \mu_{i2}, v_{i2}, \pi_{i2}), \cdots (C_n, \mu_{in}, v_{in}, \pi_{in})\}, \qquad i = 1,2, \dots, m \tag{22}$$

where μ_{ij} indicates the degree to which the candidate A_i satisfies criterion C_j, v_{ij} indicates the degree to which the candidate A_i does not satisfy criterion, and π_{ij} indicates the intuitionistic fuzzy index or hesitation degree of the candidate A_i with respect to criterion C_j $((C_j, \mu_{ij}, v_{ij}, \pi_{ij}), i = 1,2, \dots, m; j = 1,2, \dots, n)$. The procedure of proposed intuitionistic fuzzy group decision-making method is depicted in figure 1 and steps of the proposed IF-MGCOPRAS are given as follows.

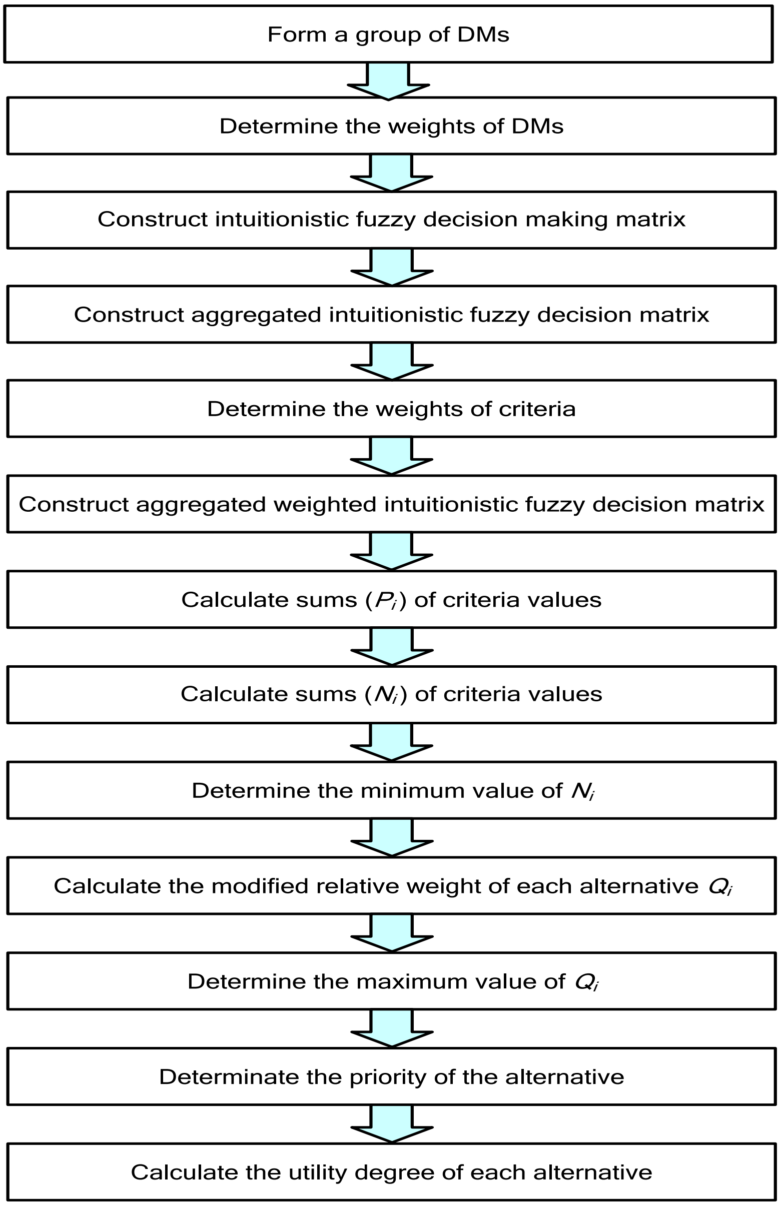

Fig. 1. Procedure of proposed IF-MGCOPRAS method for selection problems

Step 1: Determine the weights of the experts.

Assume that decision group contains l experts. The importance of the experts is regarded as linguistic terms that is expressed based on intuitionistic fuzzy numbers (Xu and Yager 2006; Xu 2007). Let $D_k = [\mu_k, v_k, \pi_k]$ be an intuitionistic fuzzy number for rating of kth expert. Then the weight of kth expert can be obtained based on following relation:

$$\varpi_k = \frac{1 - \prod_{i=1}^{m}\prod_{j=1}^{n}\mu_{ij}^{k} - 2\left(\prod_{i=1}^{m}\prod_{j=1}^{n}\mu_{ij}^{k}\right)^2}{1 - \prod_{i=1}^{m}\prod_{j=1}^{n}\mu_{ij}^{k} + \prod_{i=1}^{m}\prod_{j=1}^{n}\upsilon_{ij}^{k}} \qquad \forall k \tag{23}$$

and $\lambda_k = \dfrac{\varpi_k}{\sum_{k=1}^{l}\varpi_k}$ and $\sum_{k=1}^{l}\lambda_k = 1$.

Step 2: Construct aggregated intuitionistic fuzzy decision matrix based on the opinions of experts.

Let $R^{(k)} = \left(r_{ij}^{(k)}\right)_{m\times n}$ is an intuitionistic fuzzy decision matrix of each expert. $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_l\}$ is the weight of each expert and $\sum_{k=1}^{l} \lambda_k = 1, \lambda_k \in [0,1]$. In the decision-making process, all the individual decision opinions need to be combined into a group opinion to construct aggregated intuitionistic fuzzy decision matrix. In this sake, IFWA operator proposed by Xu (2007) is used. $R = \left(r_{ij}\right)_{m\times n}$, where

$$r_{ij} = IFWA_\lambda\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \dots, r_{ij}^{(l)}\right) = \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \dots \oplus \lambda_l r_{ij}^{(l)}$$
$$= \left[1 - \prod_{k=1}^{l}\left(1-\mu_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(v_{ij}^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(1-\mu_{ij}^{(k)}\right)^{\lambda_k} - \prod_{k=1}^{l}\left(v_{ij}^{(k)}\right)^{\lambda_k}\right] \tag{24}$$

Here, $r_{ij} = \left(\mu_{A_i}(x_j), v_{A_i}(x_j), \pi_{A_i}(x_j)\right), i = 1,2,\dots,m; j = 1,2,\dots,n.$

The aggregated intuitionistic fuzzy decision matrix can be defined as follows:

$$R = \begin{bmatrix} \left(\mu_{A_1}(x_1), v_{A_1}(x_1), \pi_{A_1}(x_1)\right) & \left(\mu_{A_1}(x_2), v_{A_1}(x_2), \pi_{A_1}(x_2)\right) & \cdots & \left(\mu_{A_1}(x_n), v_{A_1}(x_n), \pi_{A_1}(x_n)\right) \\ \left(\mu_{A_2}(x_1), v_{A_2}(x_1), \pi_{A_2}(x_1)\right) & \left(\mu_{A_2}(x_2), v_{A_2}(x_2), \pi_{A_2}(x_2)\right) & \cdots & \left(\mu_{A_2}(x_n), v_{A_2}(x_n), \pi_{A_2}(x_n)\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\mu_{A_m}(x_1), v_{A_m}(x_1), \pi_{A_m}(x_1)\right) & \left(\mu_{A_m}(x_2), v_{A_m}(x_2), \pi_{A_m}(x_2)\right) & \cdots & \left(\mu_{A_m}(x_n), v_{A_m}(x_n), \pi_{A_m}(x_n)\right) \end{bmatrix}. \tag{25}$$

Step 3: Determine the weights of criteria.

All criteria may not be assumed to be equal importance. W represents a set of grades of the importance. In order to obtain W, all the individual expert' opinions for the importance of each criterion need to be fused.

Let $w_j^{(k)} = \left[\mu_j^{(k)}, v_j^{(k)}, \pi_j^{(k)}\right]$ be an intuitionistic fuzzy number assigned to criterion C_j by the kth expert. Then the weights of the criteria are calculated by using IFWA operator:

$$w_j = IFWA_\lambda\left(w_j^{(1)}, w_j^{(2)}, \dots, w_j^{(l)}\right) = \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \dots \oplus \lambda_l w_j^{(l)}$$
$$= \left[1 - \prod_{k=1}^{l}\left(1-\mu_j^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(v_j^{(k)}\right)^{\lambda_k}, \prod_{k=1}^{l}\left(1-\mu_j^{(k)}\right)^{\lambda_k} - \prod_{k=1}^{l}\left(v_j^{(k)}\right)^{\lambda_k}\right] \tag{26}$$

where $W = \left[w_1, w_2, \dots, w_j\right]$.

Here, $w_j = \left[\mu_j, v_j, \pi_j\right],\ j = 1,2,\dots,n.$

Step 4: Construct aggregated weighted intuitionistic fuzzy decision matrix.

After determining the weights of criteria (W) and aggregated intuitionistic fuzzy decision matrix, the aggregated weighted intuitionistic fuzzy decision matrix (R') is constructed as follows:

$$R' = \begin{bmatrix} \left(\mu_{A_1W}(x_1), v_{A_1W}(x_1), \pi_{A_1W}(x_1)\right) & \left(\mu_{A_1W}(x_2), v_{A_1W}(x_2), \pi_{A_1W}(x_2)\right) & \cdots & \left(\mu_{A_1W}(x_n), v_{A_1W}(x_n), \pi_{A_1W}(x_n)\right) \\ \left(\mu_{A_2W}(x_1), v_{A_2W}(x_1), \pi_{A_2W}(x_1)\right) & \left(\mu_{A_2W}(x_2), v_{A_2W}(x_2), \pi_{A_2W}(x_2)\right) & \cdots & \left(\mu_{A_2W}(x_n), v_{A_2W}(x_n), \pi_{A_2W}(x_n)\right) \\ \vdots & \vdots & \ddots & \vdots \\ \left(\mu_{A_mW}(x_1), v_{A_mW}(x_1), \pi_{A_mW}(x_1)\right) & \left(\mu_{A_mW}(x_2), v_{A_mW}(x_2), \pi_{A_mW}(x_2)\right) & \cdots & \left(\mu_{A_mW}(x_n), v_{A_mW}(x_n), \pi_{A_mW}(x_n)\right) \end{bmatrix}. \tag{27}$$

So that $\left(\mu_{A_i}(x_j), v_{A_i}(x_j), \pi_{A_i}(x_j)\right) = r'_{ij}$.

Step 5: Calculate sums (P_i) of criteria values as follows:

$$P_i = \sum_{j=1}^{k} r'_{ij} \tag{28}$$

In Eq. (26) k is number of benefit criteria (it is assumed that in the decision-making matrix columns first of all are placed benefit criteria and ones which cost criteria are placed after).

Step 6: Calculate sums (N_i) of criteria values which smaller values are more preferable for each candidate as follows:

$$N_i = \sum_{j=k+1}^{n} r'_{ij.} \tag{29}$$

Step 7: Determine the minimum value of N_i:

$$N_{min} = \left(\left(\min_i \mu_{N_i}\right), \left(\max_i v_{N_i}\right)\right). \tag{30}$$

Step 8: Calculate the proposed modified relative weight of each candidate (Q_i) as follows:

$$Q_i = P_i + \frac{N_i \sum_{i=1}^{m} \left(\frac{N_i - N_{min}}{N_i}\right)}{N_{min} \sum_{i=1}^{m} N_i}. \tag{31}$$

The IFSs have two different concepts unlike conventional fuzzy sets: membership function and non-membership function. With utilizing the operations of the IFS, particularly minus operation through the classical COPRAS to extend a new version of this method in a fuzzy environment, the calculation of the Q_i as the relative weight of each candidate will face problems. For instance, this means that the value of R_i/R_{min} according to equation (9) in an intuitionistic fuzzy environment leads to unreasonable ranking of the candidates. In other words, the most utility is obtained for the worst candidate, which is not logical; therefore, the classical relative index in an intuitionistic fuzzy environment has not enough efficiency. Hence, in this paper a novel modified relative index is introduced under uncertainty that dissolves the aforementioned difficulties in an intuitionistic fuzzy environment and affects the ranking of candidates desirably with respect to the criteria.

Step 9: Determine the maximum value of Q_i.

$$Q_{max} = \left(\left(\max_i \mu_{Q_i}\right), \left(\min_i v_{Q_i}\right)\right). \tag{32}$$

Step 10: Rank the candidates by decreasing sorting of Q_i.

Step 11: Calculate the utility degree of each candidate.

$$U_i = \frac{Q_i}{Q_{max}} 100\% \tag{33}$$

where Q_i and Q_{max} are the significance of candidates obtained from equations (30) and (31).

5- Application example for group decision making in equipment selection

In this section, the proposed IF-MGCOPRAS method is implemented in an equipment selection problem for a manufacturing company. The management team is planning to purchase a few milling machines (potential candidates) to reduce the work in process inventory and to replace its old equipment. The high technology equipment leads significant improvements in the manufacturing processes. Also, the appropriate decisions at this phase provide the competitive advantage for this company. Selecting the most proper milling machines has great importance for the company. However, it is difficult to choose the most suitable machine that dominates other available machines by considering various characteristics under multiple criteria. To evaluate and select the best machine, the proposed method is used, which is explained in section 4.

5-1- Implementation

As an initial step, we asked to group the related personnel for the decision-making process. The team is consisting of three experts that are responsible for related activities in this manufacturing company. With a preliminary work, the decision-making team determines five possible milling machines according to the requirements of the company. The six criteria, namely price (C_1), weight (C_2), power (C_3), spindle (C_4), diameter (C_5) and stroke (C_6), are considered in the evaluation and selection process. Figure 2 depicts an overall view of the hierarchy of fuzzy decision-making problem considered in this study.

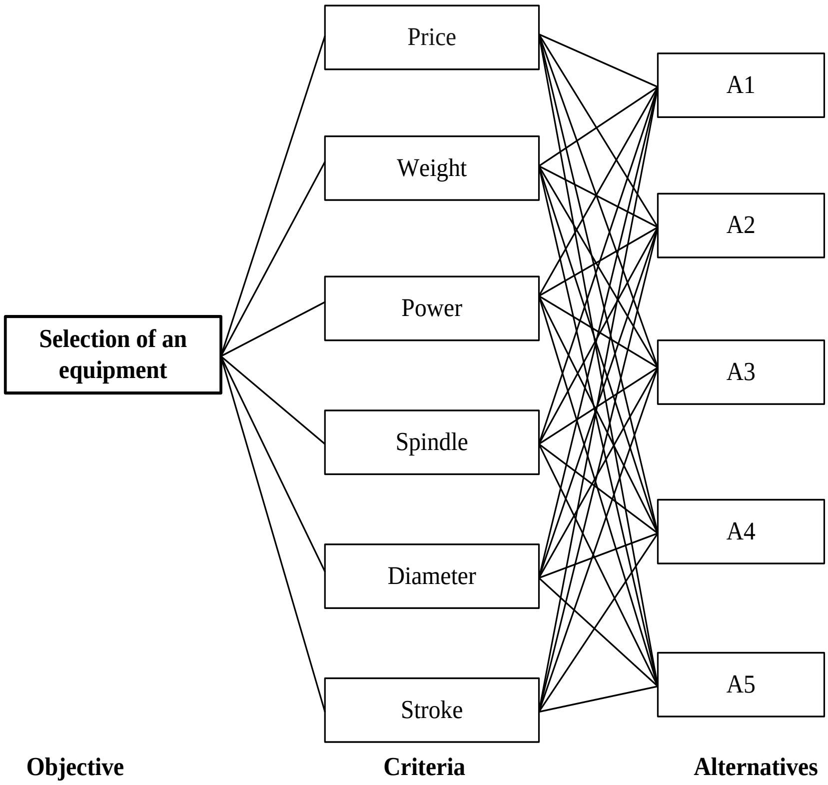

Fig. 2. Decision hierarchy of the equipment selection problem

Linguistic variables are utilized for the relative importance of selected criteria and the experts as shown in table 1. The importance degree of the experts and the weights of criteria are provided for the decision-making process in tables 2 and 3, respectively. Then three experts express the linguistic variables illustrated in Table 4 in order to assess the performance of five candidates with respect to selected criteria. Their results are presented in table 5.

Table 1. Linguistic variable for rating the importance of criteria and the decision makers

Linguistic variables	Intuitionistic fuzzy numbers
Very important (VI)	[0.90, 0.10]
Important (I)	[0.75, 0.20]
Medium (M)	[0.50, 0.45]
Unimportant (UI)	[0.35, 0.60]
Very unimportant (VUI)	[0.10, 0.90]

Table 2. The importance of decision makers

	DM_1	DM_2	DM_3
Linguistic variables	Important	Medium	Very important
Weight	0.238	0.356	0.406

Table 3. Weights of the criteria

	C_1	C_2	C_3	C_4	C_5	C_6
DM_1	UI	M	I	I	VI	M
DM_2	UI	M	I	VI	VI	M
DM_3	M	UI	M	I	I	I

Table 4. Linguistic variables for the rating of candidates

Linguistic variables	Intuitionistic fuzzy numbers
Extremely good (EG)/extremely high (EH)	[1.00, 0.00]
Very very good (VVG)/very very high (VVH)	[0.90, 0.10]
Very good (VG)/very high (VH)	[0.80, 0.10]
Good (G)/high (H)	[0.70, 0.20]
Medium good (MG)/medium high (MH)	[0.60, 0.30]
Fair (F)/medium (M)	[0.50, 0.40]
Medium bad (MB)/medium low (ML)	[0.40, 0.50]
Bad (B)/low (L)	[0.25, 0.60]
Very bad (VB)/very low (VL)	[0.10, 0.75]
Very very bad (VVB)/very very low (VVL)	[0.10, 0.90]

Table 5. Ratings of the candidates

Criteria	Candidates	Decision makers		
		DM$_1$	DM$_2$	DM$_3$
C_1	A_1	VG	VG	G
	A_2	F	F	MB
	A_3	EG	EG	VG
	A_4	MG	MG	MG
	A_5	VB	VB	MB
C_2	A_1	G	MG	G
	A_2	MB	MB	MB
	A_3	VVG	VG	EG
	A_4	VG	VG	MG
	A_5	MB	F	F
C_3	A_1	MG	MG	G
	A_2	F	MB	MB
	A_3	VG	VG	G
	A_4	MG	G	G
	A_5	B	B	VB
C_4	A_1	G	G	VG
	A_2	F	F	F
	A_3	VG	VG	VG
	A_4	G	G	G
	A_5	M	MB	M
C_5	A_1	G	G	G
	A_2	B	MB	B
	A_3	VG	EG	VG
	A_4	MG	MG	G
	A_5	VVB	VVB	B
C_6	A_1	VG	G	MG
	A_2	MG	MG	MG
	A_3	VVG	VVG	VG
	A_4	VG	VG	VG
	A_5	B	F	F

After rating each candidate with respect to each criterion by three experts, the aggregated intuitionistic fuzzy decision matrix and the weights of criteria are obtained based on the experts' judgments in table 6. The weighted intuitionistic fuzzy decision matrix is then obtained as illustrated in table 7.

Table 6. Aggregated intuitionistic fuzzy decision matrix and weight of criteria

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	[0.764,0.132,0.103]	[0.668,0.231,0.101]	[0.644,0.254,0.101]	[0.746,0.151,0.104]	[0.7,0.2,0.1]	[0.694,0.2,0.106]
A_2	[0.462,0.438,0.1]	[0.4,0.5,0.1]	[0.426,0.474,0.1]	[0.5,0.4,0.1]	[0.307,0.562,0.13]	[0.6,0.3,0.1]
A_3	[1,0,0]	[1,0,0]	[0.764,0.132,0.103]	[0.8,0.1,0.1]	[1,0,0]	[0.868,0.1,0.033]
A_4	[0.6,0.3,0.1]	[0.735,0.156,0.109]	[0.679,0.22,0.101]	[0.7,0.2,0.1]	[0.644,0.254,0.101]	[0.8,0.1,0.1]
A_5	[0.237,0.636,0.127]	[0.478,0.422,0.1]	[0.192,0.657,0.151]	[0.467,0.433,0.1]	[0.164,0.763,0.072]	[0.449,0.44,0.11]
Weight	[0.416,0.534,0.05]	[0.444,0.506,0.05]	[0.669,0.278,0.053]	[0.82,0.156,0.024]	[0.855,0.132,0.013]	[0.623,0.324,0.054]

Table 7. Weighted intuitionistic fuzzy decision matrix

	C_1	C_2	C_3	C_4	C_5	C_6
A_1	[0.318,0.596,0.087]	[0.296,0.620,0.084]	[0.431,0.462,0.108]	[0.611,0.284,0.105]	[0.599,0.306,0.096]	[0.432,0.459,0.109]
A_2	[0.192,0.738,0.07]	[0.178,0.753,0.070]	[0.285,0.620,0.095]	[0.410,0.494,0.096]	[0.263,0.620,0.117]	[0.374,0.527,0.1]
A_3	[0.416,0.534,0.05]	[0.444,0.506,0.050]	[0.511,0.374,0.115]	[0.656,0.241,0.104]	[0.855,0.132,0.013]	[0.540,0.391,0.068]
A_4	[0.249,0.674,0.077]	[0.326,0.583,0.091]	[0.454,0.437,0.109]	[0.574,0.325,0.101]	[0.551,0.353,0.096]	[0.498,0.391,0.111]
A_5	[0.098,0.83,0.071]	[0.212,0.714,0.074]	[0.129,0.752,0.119]	[0.382,0.522,0.096]	[0.140,0.795,0.065]	[0.280,0.622,0.099]

Table 8 shows the computational results by considering the P_i, N_i, the modified relative weight (Q_i), and U_i for each candidate with respect to the selected criteria. The ranking of five candidates is finally obtained based on U_i as follows: $A_3 \succ A_1 \succ A_4 \succ A_2 \succ A_5$.

Table 8. Computational results of equipment multiple criteria analysis

	P_i	N_i	Q_i	U_i	**Ranking**
A_1	[0.9495,0.0183,0.0320]	[0.7835,0.1309,0.0855]	[0.9766,0.0003,0.0229]	[0.9817,0.00020.0179]	2
A_2	[0.8050,0.0999,0.09495]	[0.7483,0.1571,0.0945]	[0.9052,0.0047,0.0899]	[0.9061,0.0047,0.0891]	4
A_3	[0.9887,0.0046,0.0065]	[0.7894,0.1258,0.0847]	[0.9948,0.00005,0.0050]	[0.9958,0.00004,0.0041]	1
A_4	[0.9475,0.0196,0.0328]	[0.7680,0.1432,0.0887]	[0.9752,0.0006,0.0241]	[0.9761,0.0006,0.0232]	3
A_5	[0.6668,0.1937,0.1393]	[0.6836,0.2168,0.0995]	[0.8233,0.0222,0.1544]	[0.8241,0.0222,0.1536]	5

According to table 8 the best candidate is A_3, and its utility degree has the highest value. It means that the needs of the experts and the company are satisfied the best. In fact, the third equipment (milling machine) assessed by using the proposed IF-MGCOPRAS method is more desirable to the experts' predefined objectives and is better than other available equipment. The top managers for this manufacturing company can also adjust decisions in accordance with their knowledge, experience and preference by taking into consideration the acquired results.

5-2- Discussion

The results illustrate that the proposed IF-MGCOPRAS method can solve the complex multi-criteria group decision problem for choosing the best equipment option. Proposed method is regarded as one applicable technique to implement within MCGDM under uncertainty. This method takes the advantages of the intuitionistic fuzzy logic and provides a systematic approach. Proposed IF-MGCOPRAS method introduces a new intuitionistic relative index based on the measure of closeness to the ideal solution, and avoids the difficulties and errors arising from the extension of classical COPRAS method in an intuitionistic fuzzy environment.

To demonstrate the validity of the proposed IF-MGCOPRAS method, a comparative analysis is performed between the proposed method and the intuitionistic fuzzy group TOPSIS method presented by Boran et al. (2009). Also, computational results of the intuitionistic fuzzy group TOPSIS are given in Table 9 according to the separation measures and relative closeness coefficient of each candidate. By considering Tables 7 and 9, it is observed that the ranking of five candidates with respected to six selected criteria are the same, where A_3 is the first rank and A_5 is the fifth rank in the equipment selection problem under multiple criteria for the manufacturing company.

Table 9. Computational results of intuitionistic fuzzy group TOPSIS method

Candidates	S^+	S^-	C_i^*	**Preference order ranking**
A_1	0.151	0.264	0.636	2
A_2	0.274	0.167	0.379	4
A_3	0.164	0.356	0.685	1
A_4	0.150	0.261	0.634	3
A_5	0.356	0.155	0.303	5

Proposed IF-MGCOPRAS and the intuitionistic fuzzy group TOPSIS method have relatively simple computations for selecting the best candidate in the complex group decision-making problems. These methods can be easily implemented by utilizing available software applications. Both methods can handle multiple conflicting criteria properly.

Two intuitionistic fuzzy group decision-making methods are based on the concept of ideal solutions. They evaluate and select the best decision by considering both the fuzzy positive and negative ideal solutions. The compromise solution obtained by these methods can help the experts to reach a final decision for a complex decision-making and/or selection problem versus conflicting criteria. The compromise solution provided by these two methods is an appropriate solution, which should satisfy the closest to the fuzzy PIS and the farthest from the fuzzy NIS. A compromise means an agreement obtained by mutual concessions. The final decision is reached to implement the multiple goal decisions by computing the relative index based on the concept of ideal solutions in the decision-making process by the experts. The candidate rankings of the two intuitionistic fuzzy group methods will be almost identical if the processes follow the same weight of the criteria.

There will be different results when the intuitionistic fuzzy group TOPSIS is designed by considering fuzzy different distances by the experts. However, the IF-MGCOPRAS method leads to an indisputable preference order through the group decision-making method of multiple criteria complex proportional evaluation. Also, the proposed fuzzy MGCOPRAS method avoids the defuzzification, and utilizes the main intuitionistic fuzzy operations (i.e., subtraction and division operations) through the evaluation and ranking process unlike the fuzzy group TOPSIS method.

6- Conclusions and suggestions

Complex group decision making by considering the multi-criteria copes with insufficient and uncertain information, and the fuzzy set theory is appropriate way to deal with these conditions. Being a generalization of the fuzzy set, the intuitionistic fuzzy set (IFS) allows decision makers or experts to prepare an additional possibility to represent imperfect knowledge. It assists the experts to use more flexible ways to simulate the real-world decision cases by utilizing the truth and non-truth membership functions to represent the satisfiability and non-satisfiability degrees, respectively. This study presents a novel multiple criteria group decision making (MCGDM) method for evaluation of the candidates in the complex decision-making and/or selection problems based on intuitionistic fuzzy modified group complex proportional assessment (IF-MGCOPRAS) method. This novel intuitionistic fuzzy method through the decision-making procedure by the experts is more adequate to deal with uncertainty than traditional approaches in the manufacturing industry. In the complex assessment procedure, the ratings of each candidate with respect to each criterion and the weights of criteria are linguistic terms as characterized by intuitionistic fuzzy numbers. First, intuitionistic fuzzy weighted averaging (IFWA) relation is used to aggregate opinions of the experts. Second, a new intuitionistic modified relative index is provided based on the incorporated fuzzy approach and concepts of positive and negative ideal solutions to solve group decision-making problems. It avoids the difficulties and errors arising from the extension of traditional COPRAS method in an intuitionistic fuzzy environment. Then, the utility degree of each candidate is obtained. The proposed method can satisfy the closest to the intuitionistic fuzzy positive ideal solution and the farthest from the intuitionistic fuzzy negative ideal solution, and finally the candidates are ranked based on the main intuitionistic fuzzy operations. Furthermore, an attempt has been made to explore the capability and applicability of proposed IF-MGCOPRAS method in the manufacturing industry.

An application example was presented through the equipment selection problem for a manufacturing company. Moreover, a comparative analysis was performed between proposed method and the intuitionistic fuzzy group TOPSIS method. It was observed that the ranking of five drilling machines with respected to six selected criteria are the same and found that the ranking of candidates in two intuitionistic fuzzy methods is almost identical if the processes follow the same weight of the criteria. The IF-MGCOPRAS method leads to an indisputable preference order through the method of multiple criteria complex proportional evaluation. However, there are different results when the intuitionistic fuzzy group TOPSIS is designed by considering fuzzy different distances by the experts. For a future research, developing a database system (DBS) to facilitate the complex group decision making and data gathering is recommended based on the proposed method for solving the group decision-making problems under uncertainty. Moreover, the Pythagorean fuzzy set as a new tool for dealing with imprecise information is more accurately and sufficiently than intuitionistic fuzzy set. Therefore, extending the proposed approach based on Pythagorean fuzzy set could enhance the presented IF-MGCOPRAS method for future direction.

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