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The Resource-Constraint Project Scheduling Problem of the project subcontractors in a cooperative environment: Highway construction case study

Mojtaba Moradi [1], Ashkan Hafezalkotob [1*], Vahidreza Ghezavati [1]

[1]*Department of Industrial Engineering, Islamic Azad University, South Tehran Branch, Tehran, Iran*

st_m_moradi@azad.ac.ir, A_hafez@azad.ac.ir, V_ghezavati@azad.ac.ir

Abstract

Large-scale projects often have several activities which are performed by subcontractors with limited multi-resources. Project scheduling with limited resources is one of the most famous problems in the research operations and optimization cases. The resource-constraint project scheduling problem (RCPSP) is a NP-hard problem in which the activities of a project must be scheduled to reduce the project duration. Therefore, subcontractors of the construction projects join together to decrease the project time and finally increase the project profit. This is an incentive for the subcontractors to form coalitions. This study presents a model based on the resource leveling problem. Results of the proposed model show that the subcontractors can earn more profit by the cooperation rather than working individually. Moreover, it is demonstrated that techniques such as the Shapley Value, Max-Min Core, and Equal Profit Method are able to fairly allocate extra profit of the cooperation among the subcontractors.

Keyword: RCPSP, cooperative game theory, imputation

1- Introduction

Large-scale projects are usually performed by some subcontractors with several multi-resources such as human resources, material, budget, equipment and etc. These projects employ required subcontractors to perform the subprojects in a specified scheduling (Fernandez 2012). The number and proficiency of the subcontractors in a project depend on the project characteristics (Kang 2001). Generally, the chosen subcontractors divide a project into some subprojects and the dividing decision can be cost effective for them or not. (Kumaraswamy et al. 2000; Perng et al. 2005). Project planning is scheduling set of activities that form the subprojects. In fact, dependency of the activities is related to their precedency of implementation; namely, implementation of an activity depends on completion of the others' tasks. Therefore, this project has precedence constraints among the activities. Furthermore, inadequacy of resources in a project may be called as resources constraints. Both the precedence and resources constraints must be harmonic in the project planning. Objective of the activities scheduling is optimum allocation of the limited resources during the whole project. In fact, activities determination and definition of their order, which must be done in a specific time, are the activities scheduling and sequencing, respectively. Those project planning problems which have limitations in resources and these limitations are considered in all of the planning are called as the resource-constraint project scheduling problems (RCPSP). In other words, RCPSP is the problem of scheduling the activities of a project satisfying precedency and resource limitations in order to reduce

*Corresponding author
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the project time. For performing activities of a subproject, some resources such as time, budget, human power and equipment are required. These resources are divided into two groups: renewable resources like human power and non-renewable resources such as material. Each activity can be performed in several modes (such as manually, semi-mechanized and mechanized) with different types and variable amount of resources (Drexl et al, 1993).

In RCPSP, activity i requires r_i^k unit of resource, $k = 1,...,m$, for each unit of execution time (d_i). In addition, resource r_i^k has R^k constraint per unit of time. The parameters (d_i, r_i^k, R^k) are non-negative and specified. Objective of this model is determining the start time and the performance mode of each activity in order to reduce the project time. It is clear that the RCPSP solution must provide precedency of the activities and limitation of the resources (Herroelen et al, 1998).

$$Min\, S_{n+1} \tag{1}$$

S. t:

$$\sum_t \sum_i r_i^k(t) \leq R^k \quad , \quad \forall\, k = 1, 2, \dots, m \tag{2}$$

$$S_i + d_{ij} \leq S_j \quad , \quad \forall (i,j) = 1, 2, \dots, n \tag{3}$$

$$S_i \geq 0 \quad , \quad \forall\, i = 1, 2, \dots, n \tag{4}$$

Zheng et al. (2015) proposed application of the multi-agent optimization algorithm (MAOA) for solving the RCPSP in detail, and the impact of MAOA parameters is studied based on the Taguchi method. Lin (2015) introduced the RCPSP problems which originate from a relocation project. He also proposed the optional recycling operations which are applied only when necessary. Tofighian et al. (2015) considered the integrated bi-objective problem of scheduling to improve total expected profit and resource application fluctuation as well. Okubo et al. (2015) proposed the RCPSP/πRC which considers limitation of real energy such as power constraint during the peak hours and setting up operations. Nadjafi et al. (2015) scheduled subproject activities to reduce the makespan due to the precedency and resources limitation, by using evolutionary algorithms such as differential evolution (DE). Bibiks et al. (2015) presented a new metaheuristic algorithm called Discrete Flower Pollination Algorithm (DFPA) to solve the RCPSP problems. Yun-Chia Liang et al. (2004) proposed an ant colony optimization (ACO) algorithm for the RCPSP which the activity on network (AON) is discussed and the forward-parallel method is applied for choosing the activity. Tereso, Anabela Pereira et al. (2009) introduced an allocation method of multiple resources in order to reduce the waste arising from their hidden unemployment on their utilization in the similar activity. Virginie Andre et al. (2013) studied the method to generate a simulation model for the RCPSP with an uncertain demand of resource under transport limitations. Liu et al. (2015) proposed a building information modeling (BIM) for construction projects under limitation of resources by achieving BIM generation models with work package information, process simulations and optimization algorithms. Kellenbrink et al. (2015) studied the variable structures in scheduling the projects. They introduced the concepts of coercive and voluntary selections, voluntary activities and interdependent activities. These points enhance the variability of modeling real-world RCPSP. Vanhoucke et al. (2015) proposed a novel approach to solve the RCPSP by considering three logical constraints. These constraints expand the communication of activities and also define the RCPSP different from the traditional ones in the literature. Shou et al. (2015) proposed a hybrid particle swarm optimization (PSO) method to solve the preemptive RCPSP.

Delivering the subprojects is the most important issue in successful projects due to the specified cost and determined duration of the subprojects. If the subprojects are not delivered on time, the project budget will increase and the economic justification of the project will be lost. In addition, if the project is finished with overplus budget, its beneficiaries have to use it more expensively. This leads to increased inflation in the society and hence makes it an important issue which affects the governmental projects. Therefore, in some countries, the government fines the contractors of the public projects when they fail to deliver the project on time. These contractors cooperate with each other and form coalitions to reduce and avoid fines (Fernandez 2012). Rahman et al. (2004) present collaboration among the contractors and the possibility to avoid opportunism, and encourage cooperation to obtain the main objectives.

The relationships among the subcontractors of a coalition are increased when their cooperation completes the agreements and solves the problems. This cooperation can improve the efficiency of the construction projects (Humphreys et al. 2003). Case studies display that the cooperation results in better project execution such as less costs, reduced delivery delays and improved customer satisfaction (Barlow 2000; DeVilbiss et al. 2000). As mentioned, large-scale projects are done by multiple subcontractors which have various resources such as human resources, budget, equipment and material. In some countries, hiring workers for short periods may be costly due to the work rules. This is also true for utilizing some equipment (Mattila et al. 1998). The subcontractors can form a coalition and share their resources in order to enhance total profit of the project. Collaboration of the subcontractors in a coalition often leads to more profit for them and for the client as well. Therefore, the clients and the subcontractors should focus on impartial allocation of the cooperation extra profit that provides motivation for consolidating and saving relationships (Memon, 2014). The grand coalition has at least the sum of the profits of single players, independently. In other words, status of the subcontractors in a coalition must be better than each one, independently (Barron, 2013). Sometimes, subcontractors follow their objectives through cooperation and forming coalitions (Just et al. 2004; Madani 2011). They do not always try to maximize their gains through competition (Parrachino et al. 2006; Madani et al. 2012). Game theory methods have been shown as a helpful framework to enlarge dimensions of the construction projects (Lazar 2000). Therefore, application of the game theory methods has been increased in these projects over the last two decades. Based on these methods, Perng et al. (2005) found that the subcontractors in a coalition may obtain more profits which reduces the project costs. By using the game theory methods, Eriksson (2007) explains that the collaboration in buyer-supplier relationships of the construction projects is low; however, the long-term contracting provides collaboration incentives. Cooperative game theory methods define some solutions for sharing the extra cooperation profit in a coalition with transferable resources such as equipment (Drechsel, et al. 2010). In addition, Hsueh et al. (2011) applied the cooperative game theory to allocate the joint profits among the coalition subcontractors with respect to their contributions.

Generally, the cooperative game theory presents efficient understandings of the resources trading and sharing among the subcontractors which may lead to some plans for applying the allocated resources (Parrachino et al. 2006). Ho et al. (2004) represented a decision model based on the game theory methods for investigating the subcontractor demands and inquiring the opportunism entity. Nasiri et al. (2016) proposed a mathematical programming model to specify the price and the maximum flow in the supply chain management, synchronously. They used this model for each player in singular and cooperative modes, respectively, and compared the results to find the maximum benefit. They found that the price will decrease and the sale amount will increase when two or more players collaborate and form a coalition. Zibaei et al. (2013) presented a new vehicle routing problem (VRP) for decreasing the transportation costs when there are some players. For a multi-depot VRP, they proposed allocated vehicles in the collaboration among the depots players. Gharehbolagh et al. (2016) represented a mathematical programming model considering utilization of the triangular reliability function in the decentralized networks for cooperation of the owners to save a satisfying flow in the network. This paper studies and analyzes the subcontractors' collaboration with the cooperative game theory methods such as the Shapley Value, Max-Min Core, and Equal Profit Method.

1-1-Research gap

Three topics are presented in this paper that hasn't been considered in previous studies:

- Numerous methods are illustrated for solving RCPSP in previous researches, but in no research, project time and resource availability are considered simultaneously.

- The prior studies just introduced several methods to assign the profit of the cooperation mode, but satisfaction level for each subcontractor, super-additivity and stability of coalitions haven't been considered.

- The cost of idle manpower and unemployment equipment are investigated in both the cooperation mode and singular mode.

This paper is organized in six sections. Prerequisites and assumptions are studied in section 2. Section 3 describes the model formulation. The cooperative game theory is presented in section 4. In section 5, the proposed model is applied in a case study and finally, the conclusions are discussed in section 6.

2- Prerequisites and assumptions

The resource-constraint project scheduling problem (RCPSP) is a NP-hard problem in which the activities of a project must be scheduled to decrease the project time. Therefore, subcontractors of the construction projects form coalitions to reduce the project duration and finally increase its profit.

2-1- Assumptions

The following assumptions determine the scope of this work for the model formulation:

1- There are $|R|$ constraint resources such as budget, materials, human resources, equipment that the subcontractors can transfer to others in a coalition.
2- There are $|K|$ subcontractors involved in a project that have logical behavior. The subcontractors can perform their activities cooperatively as a coalition.
3- A decision variable is the implementation time of each activity for each subcontractor in several modes. All activities are compressible; hence, each time has a determined cost.
4- The main assumption of the cooperative game theory is the transferable utility (TU) games, so the utilities obtained from the resource-constraint project scheduling problem model, i.e., penalties or rewards, are considered transferable.
5- Objective function is maximizing net present value of the project including three functions: net present value of the total payment, cost of the activities and the overtime cost using renewable resources, and the reward (the penalty) of early (delayed) delivery of the project.
6- All payments are received at the end of the project and implementation costs of the activities are paid at the end of each activity.
7- Sum of the used renewable resources should not be more than their capacity in any period of the project.
8- If a project is selected to be performed by a subcontractor, the model must be such that a subcontractor would perform all the project activities.

Before describing the main model, the indices, parameters and decision variables will be explained. $|.|$ denotes the number of members in a set.

2-2- Indices and sets

i	The set of all activities of the project, $i=\{1, 2, \ldots, i, j, \ldots, \lvert n\rvert\}$
k	The set of subcontractors of a project, $k=\{1, 2, \ldots, \lvert K\rvert\}$
m	The set of modes of the activities, $m=\{1, 2, \ldots, \lvert M\rvert\}$
q	The set of nonrenewable resources of the project, $q=\{1, 2, \ldots, \lvert Q\rvert\}$
r	The set of renewable resources of the project, $r=\{1, 2, \ldots, \lvert R\rvert\}$
t	The start time of the activity when the project applied, $t=\{1, 2, \ldots, \lvert T\rvert\}$

2-3- Input parameters

TP_k	Total payments to the subcontractor for the subcontractor *k*
C_{kim}	The cost of the activity *i* in the mode *m* for the subcontractor *k*
CO_r	The cost of overtime for resource *r*
CU_r	The cost of unemployment for resource *r*
D_{kimq}	The amount of nonrenewable resource *q* for activity *i* in the mode *m* for the subcontractor *k*
D_{kimr}	The amount of renewable resource *r* for activity *i* in the mode *m* for the subcontractor *k*
R_q	The amount of available nonrenewable resource *q*
R_r	The amount of available renewable resource *r*
AR_r	The capacity of available renewable resource *r* without any overtime
DL_k	Deadline of project for the subcontractor *k*
rew_k	Reward rate of project for the subcontractor *k*
pen_k	Penalty rate of project for the subcontractor *k*
β	Discount rate

2-4- Decision variables

S_{kimt}	If the activity *i* starts in the mode *m* for the subcontractor *k*, is one otherwise zero
x_k	If the subcontractor *k* starts the project, is one otherwise zero
a_k	If the subcontractor *k* completes the project after the deadline, is one otherwise zero
O_{rt}	The amount of overtime for renewable resource *r* in period *t*
U_{rt}	The amount of unemployment for resource *r* in period *t*

3- Model formulation

Nonlinear programming model selects subcontractors or coalition of the subcontractors for maximizing net present value of the project as follows:

$$Max\ z = \sum_{k\in K} TP_k * \frac{x_k}{(1+\beta)^{\sum_{m\in M_n}\sum_{t=1}^{T} S_{knmt}}} - \sum_{k\in K}\sum_{i\in n_k}\sum_{m\in M_i}\sum_{t=1}^{T} C_{kim} * \frac{S_{kimt}}{(1+\beta)^{(t+d_{kim})*S_{kimt}}}$$
$$- \sum_{r\in R_r}\sum_{t=1}^{T} CO_r * \frac{O_{rt}}{(1+\beta)^t} - \sum_{r\in R_r}\sum_{t=1}^{T} CU_r * \frac{U_{rt}}{(1+\beta)^t}$$
$$+ \sum_{k} x_k * (rew_k(1-a_k) + pen_k * a_k) * \left(DL_k - \sum_{m\in M_n}\sum_{t=1}^{T} S_{knmt}\right)$$
$$/(1+\beta)^{\sum_{m\in M_n}\sum_{t=1}^{T} S_{knmt}} \quad (5)$$

s.t:

$$\sum_{m\in M_j}\sum_{t=1}^{T} t * S_{kjmt} \geq \sum_{m\in M_j}\sum_{t=1}^{T} (t+d_{kim}) * S_{kimt} \qquad \forall k\in K, \forall(i,j) \quad (6)$$

$$\sum_{k\in K}\sum_{i\in n_k}\sum_{m\in M_i}\sum_{t=1}^{T} D_{kimq} * S_{kimt} \leq R_q \qquad \forall q\in Q \quad (7)$$

$$\sum_{k\in K}\sum_{i\in n_k}\sum_{m\in M_i}\sum_{u=t-d_{kim}+1}^{t} D_{kimr} * S_{kimu} \leq R_r \qquad \forall r\in R, \forall t \in \{1,\dots,T\} \quad (8)$$

$$\sum_{m\in M_i}\sum_{t=1}^{T} S_{kimt} = x_k \qquad \forall k\in K, \forall i \quad (9)$$

$$\sum_{k\in K}\sum_{i\in n_k}\sum_{m\in M_i}\sum_{u=t-d_{kim}+1}^{t} D_{kimr} * S_{kimu} + u_{rt} - o_{rt} = AR_r \qquad \forall r\in R, \forall t \in \{1,\dots,T\} \quad (10)$$

$$u_{rt} \geq 0, \qquad o_{rt} \geq 0 \qquad \forall r\in R, \forall t \in \{1,\dots,T\} \quad (11)$$

$$\sum_{m\in M_n}\sum_{t=1}^{T} S_{knmt} - DL_k \leq M * a_k \qquad \forall k\in K \quad (12)$$

$$DL_k - \sum_{m\in M_n}\sum_{t=1}^{T} S_{knmt} \leq M * (1-a_k) \qquad \forall k\in K \quad (13)$$

$S_{kimt}, a_k, x_k = \{0,1\}$ (14)

Equation (5) presents objective function which contains: positive cash flow, implementing cost of the activities, overtime cost of the resources, cost of unused resources and delayed completion penalty (early completion reward) of the project, respectively. Precedence constraints among the activities are observed in equation (6) and renewable/nonrenewable resources constraints are displayed in equations (7) and (8). In the solution process, if one project is selected and implemented, then the whole of its activities must be run in the model and vice versa. These conditions are shown in equation (9). Equation (10) represents constraint of the overtime resources in the period t. Furthermore, u_{rt} and o_{rt} define non-negative decision variables in equation (11) which have converse relationship with each other. Deadline of the project for the subcontractor k (DL_k) has already been determined and the delayed completion penalty (early completion reward) of the project measured by this index are represented in equations (12) and (13).

Cooperation of the subcontractors may reduce the project makespan project and increase the client payment. Difference between the coalitional reward and sum of the individual rewards would be extra profit of the cooperation.

4- Cooperative game theory

The main goal of the cooperative game theory methods is to specify fair allocation of the extra cooperation profits. There are some methods for sharing these extra profits. This paper will present the Shapley Value, the Equal Profit Method, the Max-Min Core, and the Utopia Payoffs.

4-1- Imputation

Let P be a game in the characteristic function form with a set of players (subcontractors) in a project. An imputation in an n-person game is an n-tuple $\vec{z}$ of real numbers such that:

Individual Rationality:

(i) $z_k \geq P(k)$, *i= 1, 2, … , n* (15)

Collective Rationality:

(ii) $\sum_{i=1}^{|K|} z_i = P(V_n)$ (16)

The imputation $\vec{z}$ is also called a payoff vector or an allocation.

In the next section, this paper represents the cooperative game theory methods which can be considered to allocate a fair share (imputation) of the client's payment to the subcontractors.

4-1-1- Core

The core of an n-person game is a set of feasible imputations that cannot be improved by any coalition. In other words, the core of a coalitional game is directly related to stability of the grand coalition. Because of the super-additivity in a coalitional game (K, P), the players have an incentive to form the grand coalition k. Therefore, the core of a game is a set of imputations which guarantee that no coalition of players has an incentive to leave k in order to form another coalition $V \subset K$. This leads to the following definition.

Definition. A vector $\vec{z} \in R^n$ is the core allocation of a cooperative game, if $\vec{z}$ satisfies the efficiency requirements as below:

$\sum_{k \in k} z_k = P(K)$, *(17)*

and for every coalition $V \subset N$

$\sum_{k \in V} z_k \geq P(V_n)$, $\forall V \subset K$ *(18)*

Therefore, core of a game is a subset of imputations in which each coalition obtains at least the reward of cooperation with that coalition.

4-1-2- Max-Min core

Since the core offers a set of solutions (a space of solutions), the Max-Min Core method can be used to present a single solution for the profit allocation. This method shirks simultaneously the solution space from each side of the boundary until a single solution is achieved. The Max-Min Core method is shown in the following linear programming model:

$$Min\ \varepsilon \tag{19}$$

subject to:

$$e(V,\vec{z}) = P(V_n) - \sum_{k\in V} z_k \leq \varepsilon,\ \forall\, V \subset K,\ V \neq K \tag{20}$$

$$\sum_{\forall_{k\epsilon K}} z_k = P(K) \tag{21}$$

In the objective function, ε presents the maximum dissatisfaction of all coalitions among the players. Therefore, model (19)-(21) minimize the maximum dissatisfaction level of all coalitions.

4-1-3- Shapley value

The Shapley value is obtained by averaging the marginal contributions of agents over joining orders of the coalition. An imputation z_i represents the Shapley Value if:

$$z_i = \sum_{\substack{V\subset N \\ k\in V}} \frac{(|V|-1)!(|N|-|V|)!}{|N|!}\big(P(V_n) - P(V_n - [k])\big) \tag{22}$$

Shapley method allocates unique imputation to the subcontractors depending on their role or efficacy in possible coalitions.

4-1-4- Utopia payoffs

The Utopia Payoff will be a pair (M_k,m_k), if *A* is an infinite assignment problem; then player $k \in K$ takes the maximum value M_k and minimum value m_k in which the members of (m_k ,M_k) may take the value infinity. The Utopia Payoff of player k is given by $M_k = P(N) - P\left({}^{N}\!/_{\{k\}}\right)$, M_k presents the marginal contribution of player k in the grand coalition. If matrix of the allocation problems is bounded, all utopia payoffs are limited. The minimum right for the player k is shown by $m_k = \max\limits_{V:k'\in V}\left\{P(V) - \sum_{k'\in V\setminus\{k\}} M_{k'}\right\}$.

4-1-5- Equal profit method

Equal Profit Method (EPM) is an allocation method that minimizes the maximum variances in the pairwise relative profit of players. This allocation is shown as the following LP model:

$$Min\ f \tag{23}$$

subject to:

$$f\ \geq \frac{z_k}{P(\{k\})} - \frac{z_{k'}}{P(\{k'\})}\ ,\ \forall (k,k') \in K, \tag{24}$$

$$\sum_{k\in V_n} z_k \geq P(V_n), for\ all\ V_n \subseteq K \tag{25}$$

$$\sum_{k\in K} z_k = P(K). \tag{26}$$

The constraint set (24) measures the variance among the relative profit of the players. In the objective function, f denotes the largest variance that must be minimized. The constraints (25) and (26) ensure stability condition of the allocation.

5- Numerical example

In order to test the proposed model, this paper studies a construction project in Iran which has several subcontractors. The municipality of Tehran is client of the Yadegar Emam highway project which must be implemented by some civil subcontractors. Yadegar Emam is a main north-south and east-west highway in the northwest and west of Tehran. It starts from Seoul street in the north of Tehran and crosses the Chamran highway, Evin district, Saadat Abad district, Niayesh highway, Farahzad district, Gharb town, Hemmat highway, Kashani street, Zhandarmery town, Aryashahr district, Sattarkhan street, Tarasht district, Sheykh Fazlollah highway, Zanjan and Azadi streets and finally finishes at Hashemi street. The western rout of this highway is under construction and it continues to the Fath square which is the path to the Azadegan highway.

This research studied the western rout of the Yadegar Emam highway that it is under construction. This part is performed by some subcontractors because the project size is too large and none of the subcontractors has enough budget, materials, human resources and equipment to deliver the project at a specified time. This paper considers three following phases of this part of the project which is under construction by three contractors (see figure 1).

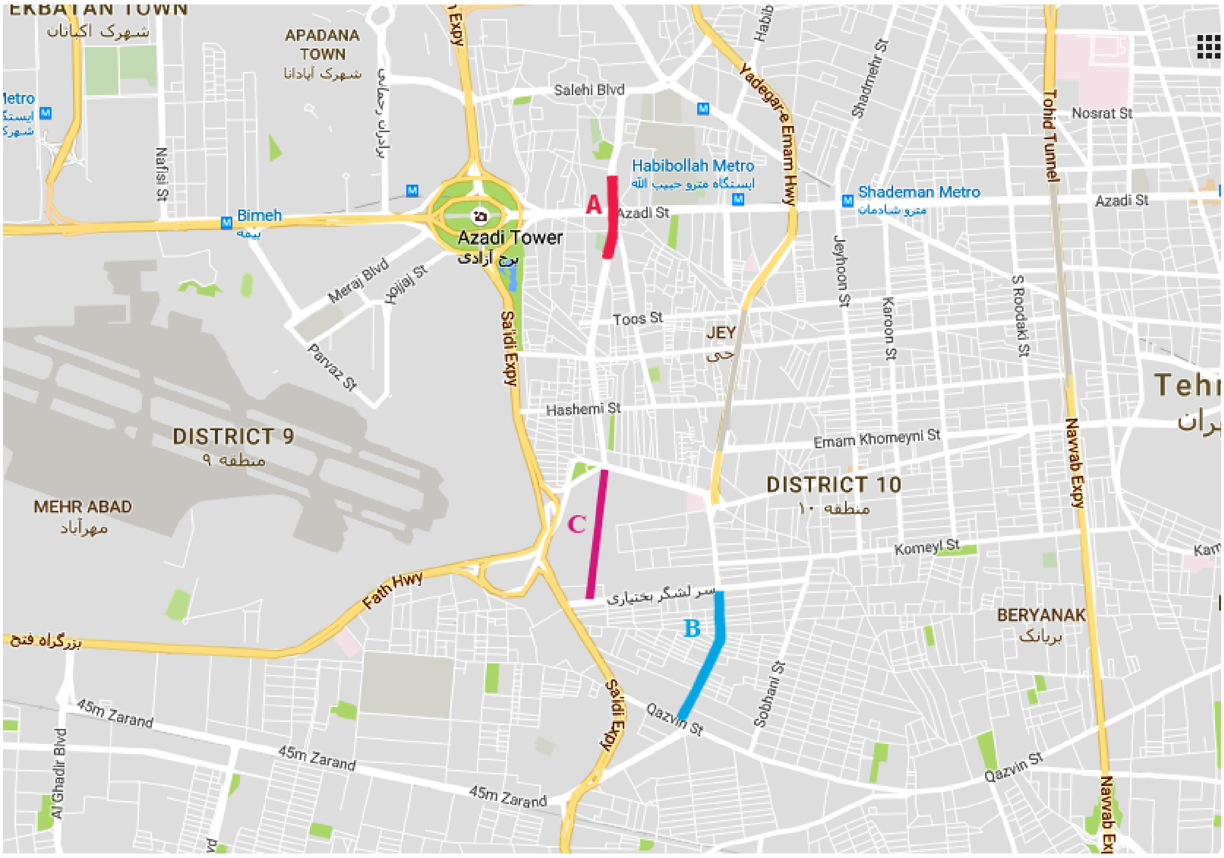

Fig.1 Yadegare Emam highway project that are considered (Yadegare Emam Hwy – Google Maps).

- Phase A is approximately 522 meters and starts from south of the Akbari street and crosses the Azadi street to north of the Ostad Moeen street. This phase must be implemented by contractor A in 38 months.

- Phase B is approximately 5.5 kilometers and starts from the Sarlashgar Bakhtiari (30metri Jay) street to the Ghazvin street. This phase must be implemented by contractor B in 52 months.

- Phase C is approximately 1.7 kilometers and starts from south of the Ostad Moeen street to the Sarlashgar Bakhtiari (30metri Jay) street. This phase must be implemented by contractor C in 45 months.

Due to strict deadline of the project and constraint resources of the subcontractors, they are interested to form a coalition to reduce the activities time. Therefore, this research applies the model (5)-(14) and solves the resource constraint project scheduling problems for coalitional implementation of the project by the contactors. For the project delivery, each subcontractor has a specific predefined standard time. Standard time of the coalition is the greatest standard time among its members. Therefore, it makes a motivation for the subcontractors to obtain more profits in a coalition.

Subcontractors A and C perform three main activities determined as (1) Substructure, (2) Superstructure and (3) Installing appropriate traffic signs and beautifying the highway. Subcontractor B, in addition to the previous activities, has the destruction responsibility because there are houses and buildings on the way as shown in figure1. These subcontractors can perform their activities individually or cooperatively as coalitions.

Table 1 shows the renewable resources, acceptable renewable resources and nonrenewable resources of the project subcontractors. The renewable resources of each subcontractor are the human resources and equipment. The nonrenewable resources are the budget and materials.

Table 1. Classification of the subcontractors' resources

Renewable resources		**Acceptable resources**		**Nonrenewable resources**	
Human	equipment	Human	equipment	budget	material
250	60	150	18	87000	79

DL_k is the deadline specified for each subcontractor, separately. Meanwhile, value of the rewards (or penalties) for every subcontractor is calculated based on the comparison between the related deadline (DL_k) and the finished time (S_{knmt}). Subcontractor B must perform its tasks in 52 months and subcontractor A and subcontractor C must complete their own in 38 and 45 months. Therefore, they can reduce these times by cooperating and sharing their resources.

In the numerical example, unemployment and overtime costs of the resource r in the objective function are 38 and 150 $\$$. The reward and the penalty rate in the objective function are 100 and 120, respectively. Table 2 shows the objective function value or the net present value of the project for coalitions of the subcontractors.

Table 2. Values of the objective function for coalitions of the subcontractors

Coalition / **Value of characteristic function**	{A}	{B}	{C}	{A, B}	{A, C}	{B, C}	{A, B, C}
TP(k)	3878.2	86076.8	94858.5	426421.7	437908.1	519040.4	711865.3

As table 2 shows, there are 711865.336 units of net present value of the project which are achieved from coalition of the subcontractors {A, B, C}. In the grand coalition, profit of the subcontractors has 527051.8 units more than profit of subcontractors in the singular mode. In order to stable the grand coalition and increase the satisfaction level of subcontractors, surplus profits should be allocated fairly among coalition members. Thus, some methods of the cooperative game theory like; the Shapely Value, Max-Min Core and EPM, are used to allocate fairly surplus profit among the subcontractors.

The problems of Max-Min Core (29)-(21) and EPM (23)-(26) are solved by using Lingo 11 package. It seems that various methods result in different allocations.

Table 3. Allocations of the project's net present value for the grand coalition based on different TU game methods for three subcontractors

	Cooperation Mechanisms					
	Utopia Payoffs		Shapley value	Max-min Core	EPM	τ-value
	M	m				
Subcontractor A	192820	152460	179470	152464.4	152464.4	179370
Subcontractor B	273960	233600	261130	273957.3	273957.3	260500
Subcontractor C	285440	245080	271270	285443.6	285443.6	271990

As table 3 shows, allocation of the Max-Min Core and EPM methods to each subcontractor are similar and the highest shares belong to the subcontractor C. Moreover, net present value of the subcontractor B is more than the subcontractor A. As can be seen, any collaboration system may generate different results. Therefore, it is important to specify the profit allocation type of the project subcontractors. Fig.2 represents the core space of three subcontractors of the Yadegar Emam Highway construction.

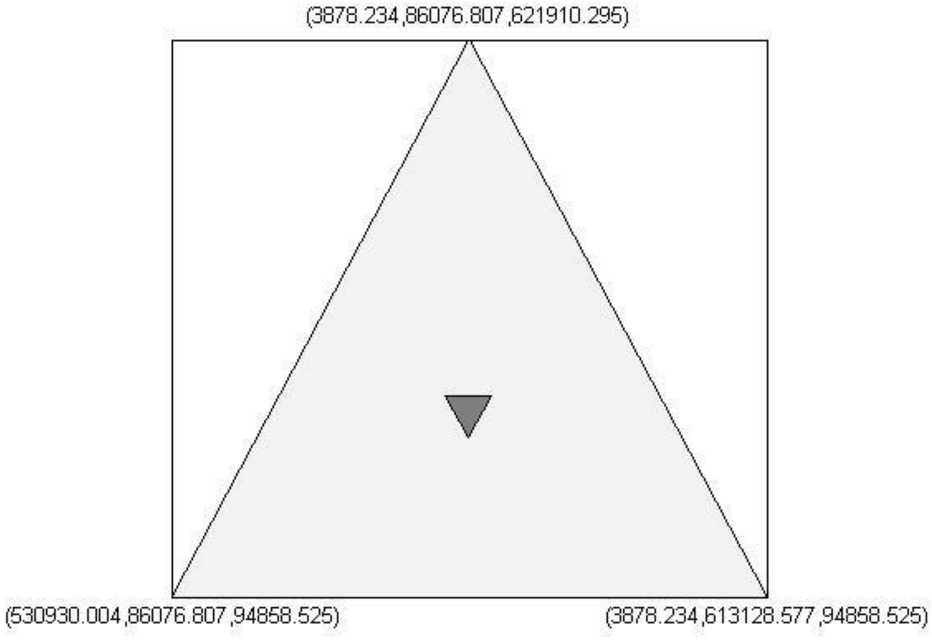

Fig.2. Core space for three subcontractors of Yadegare Emam Highway construction

Table 4 shows sensitivity of the Shapley values versus the reward and penalty rates of the project. In several conditions, sum of the Shapley values of the subcontractors in the grand coalition is more than the sum of the Shapley values of the subcontractors in the individual mode.

Table 4. Sensitivity of Shapley values versus the reward and the penalty rates of the project

	Reward				
	100	**200**	**300**	**400**	**500**
A	179470	180940	182160	181630	182500
B	261130	259010	258410	258470	257320
C	271270	272360	271740	271540	272180

	Penalty				
	120	**220**	**320**	**420**	**520**
A	179470	180000	180390	179390	182820
B	261130	260260	260610	258950	256710
C	271270	271610	270790	273950	272150

Satisfaction of a coalition V_n from the imputation $\vec{z} = (z_1, z_2, \dots, z_n)$, is difference of the profit allocated to the each subcontractor of the coalition V_n and the same subcontractor when acting independently, i.e., $F_{V_n}(P, \vec{z}) = \sum_{i \in V_n} z_i - P(V_n)$. Table 5 presents that in case of increasing the subcontractors in a coalition, the obtained profit decreases due to adding new subcontractors.
The Max-Min Core method maximizes the minimum satisfaction of the coalitions. This method prepares the largest minimum satisfaction (i.e., zero) and applies fairness by maximizing the minimum satisfaction of the whole coalitions of subcontractors.

Table 5. Coalition satisfactions for various methods of CGT

Coalition	**Shapley**	**EPM**	**Min-Max core**	**τ-value**
C_1={A}	175591.766	148586.166	148586.166	175491.766
C_2={B}	175053.193	187880.493	187880.493	174423.193
C_3={C}	176411.475	190585.075	190585.075	177131.475
C_4={A, B}	14178.244	-0.056	-0.056	13448.244
C_5={A, C}	12831.993	-0.007	-0.007	13451.993
C_6={B, C}	13359.599	40360.499	40360.499	13449.599
$\boldsymbol{Min}_{F_{V_n}(P,\vec{z})}$	12831.993	-0.056	-0.056	13448.244
$\boldsymbol{Max}_{F_{V_n}(P,\vec{z})}$	176411.475	190585.075	190585.075	177131.475
$\boldsymbol{Sum}_{F_{V_n}(P,\vec{z})}$	567426.27	567412.17	567412.17	567396.27

The cooperative game theory methods share the extra cooperation profit and give understanding for selection of the best allocation which result in maximization of the minimum satisfaction. Therefore, it makes a great motivation for the subcontractors to collaborate and keep their cooperation.

5-1-Managerial implications

The employer and subcontractors of the Yadegar Emam Highway construction project can improve their decisions by considering the important results of the study as follows:

- The surplus profit obtained in full cooperation of subcontractors is considerable. As displayed in table 2, profit of the subcontractors in full cooperation has 527051.8 units more than profit of subcontractors in separate mode. Therefore, it can motivate the subcontractors to form grand coalition.
- Since the cooperation of the subcontractors is various in the coalitions, so the impact of each subcontractor is different in the coalition's profit. For example, subcontractor A has several profit in coalitions {A, B}, {A, C} and {A, B, C}.
- The subcontractors of a coalition don't need to provide new equipment or employee new manpower, because the subcontractors have applied idle manpower and equipment of other subcontractors.

6- Conclusion

Many construction projects contain specific subprojects which are performed by some subcontractors. Resource-constraint project scheduling problem (RCPSP) among the subcontractors in these subprojects is very important and effective in total time and cost of the projects. This paper presented a nonlinear programming model for these problems when the subcontractors collaborate with each other by sharing their budgets, material, human resources and equipment. Then, some cooperative game theory methods were presented for fair allocation of net present value of the cooperation.

As it is seen in the previous section, the amount of net present value achieved in the grand coalition is more than another coalition and the single mode. Therefore, forming a coalition is important to obtain more net present value of the project; moreover, forming the grand coalition maximized net present value of the project. It is represented that the potential of more net present value of the coalitions and the supper additive property are feasible. For future studies, the factors which were assumed certain here (e.g., time, cost, etc.), can be considered in uncertain conditions.

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