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Considering different kinds of vehicles in a hierarchical hub location model

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Abstract

In this study a hierarchical hub location problem with two layers is considered. The first layer includes small hubs and the second one includes a star-shaped network central hub. The considered case is a cargo delivery network where there is hierarchy between hubs. All the hubs and links are capacitated and there are three kinds of commodities for each of which there is a special kind of vehicle. The purpose is to determine the optimal cost by locating small hubs (city hubs) and the central hub, allocation of links to the hubs, and finding the optimal number of vehicles for each hub. The model is a mixed integer nonlinear programming. A case study for this model is performed in a cargo delivery network by using published data in Iranian Statistics Organization website, and it is solved by appropriate software.

Keywords: Hierarchical hub location, capacitated hubs, multiple vehicles and commodity

1- Introduction

The purpose of this paper is considering different types of vehicles for each clustered commodity based on the mean weight in a capacitated hierarchical hub network. The focus is on the effect of different varieties of vehicle capacities that affect the number of vehicles transferring the commodities at each different levels of hierarchy. Our objective is to minimize the total cost and total number of vehicles that should be used to transport the commodities in a cargo delivery network.

Hub nodes are collection, transfer, and distribution centers which are used when direct connections between nodes are not allowed or feasible. In a hub location problem, the aim is to find the best place to locate hub facilities and allocate the demand nodes to them for sending the flows from the origins to the destinations so that the cost of collection, routing, and distribution is optimized. For more information, interested readers are referred to surveys on hub location problems (HLPs) presented by Farahani et al. (2013), Torkestani et al. (2016) and Seyedhosseini et al. (2016).

In this paper, a hub location problem in a cargo delivery network is investigated. In a cargo delivery network, commodities, from their origins, are sent to cargo delivery centers in cities. In these centers, each type of commodities with the same destinations are collected, consolidated in their origins, and prepared to be sent to their destinations.

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Each type of commodities collected in cargo delivery centers is sent towards central hubs via its special associated vehicle. That is because the dispatched commodities have different destinations, and it is not economic to send them directly to their destinations from existing urban delivery centers, hence commodities must be sent to the capital, which is considered to be the central hub, and then again be collected and consolidated with accordance to their destinations. In the central hub, if there is a direct link between the destination and the central hub, commodities are directly sent to their destinations; otherwise, they must be first sent to the delivery centers (small hubs) in the cities and then sent to their destinations.

In this network we use the hub nodes as special concentrator to act as switching, transshipment and sorting points in cargo delivery network in order to take advantages of economic scale. By the way the different variety of capacity is used to be flexible to transferring the commodities. Considering different kind of vehicles could be a practical decision for selecting various types of commodities to meet a special requirement in a network. Satisfying the variety of demands encourage us to design the network by considering special configuration with different service level as hierarchical network. So the practical aspect that designing the hub location model for the real cargo delivery network as a capacitated hierarchical system with different vehicles are the main motivation of the model.

The proposed model is applicable for all the capacitated transportation systems with multiple modes of commodity and transportation, and it can be used specially in cargo delivery industries which require a defined capacity for multiple modes of commodities which can be clustered based on the mean weight. One of the most important advantages of the proposed model is the categorization of commodities based on their mean weight and assignment of specified vehicles to each category. Therefore, a specific vehicle will be used to carry some type of commodities with the same mean weight in a network.

According to that, the main aspect distinct the present paper from previous research includes the following:

1. Considering three levels of capacities for each hub from which one is chosen as the hub capacity.
2. Considering different vehicles for each category of commodity based on the mean weight and investigation of its effects in the hierarchical hub network.
3. Investigation of the effect of limited capacity and number of vehicles used on the hierarchical hub network.
4. Developing a capacitated hierarchical hub location problem for Iran's cargo delivery network (real-world statistical dataset).

The rest of this paper is organized as follows: In Sect. “Literature review“, the relevant literature is reviewed. The problem description and optimization model are given in Section “Problem description” and “Mathematical model”, the effectiveness and usefulness of the proposed model are demonstrated by a case analysis and experimental result in Section “Experimental design and data collection (Case study)” and “Computational experiments”. “Conclusions and future research” are presented in the final section.

1-1- Literature review

Hub location has a wide logistic application. Therefore hub location problems have been studied by many researchers and from a variety of perspectives. Since the subject of this study is hierarchical hub location, the recent publications about hierarchical hub location are reviewed.

One of the most important hierarchical p-hub location models is developed by Yaman (2009) with three layers of hierarchy, in which the highest layer is a complete network which includes a central hub. The main goal is to minimize the cost of routing and locating the hub nodes.

Chen (2010) provided a heuristic method to solve a time-definite common carrier operation planning problem. Ayed (2011) studied a parcel distribution network design problem for ground shipments and proposed its optimization model. Sender and Clausen (2011) tried to create a new hub location model for wagon traffic network with three layers. The objective function is to minimize the total cost of hub operation and transportation. Chi et al. (2011) developed a hybrid hub location model with variable levels and assumption of the minimum of maximum radiuses, limited capacity and minimum number of hubs.

Davari and FazelZarandi (2012) extended the Yaman modeland studied a hierarchical hub median location problem with single allocation and fuzzy flows between nodes in three levels of hierarchy. Alumur et al. (2012) presented a single objective, three-layered model in a discrete space considering the coverage assumption and different modes of transportation. Their model consists of two types of hubs and two types of hub links (by road and by air). Yaman and Elloumi (2012) studied a two-layered star network by considering service quality considerations. Manzour-al-Ajdad et al. (2012) proposed vehicle routing and facility location issues as a main factor in logistic problems and studied hierarchical single facility routing issues with Euclidian distance. Sheu and Lin (2012) proposed a hierarchical network planning for global logistics network configuration (GLN). Hwang and Lee (2012) presented an integer programming (IP) formulation for a new hub covering model. Their model maximizes the demand covered by deadline traveling time.

Martins-de¬-Sa et al. (2013) presented tree of hubs location problem and proposed an improved Benders' decomposition algorithm to solve it. They compared the presented algorithm with modern implementations of Benders' decomposition methods. Saboury et al. (2013) considered two-layered hierarchical networks with connected accessible networks and central networks. Ryerson and Kim (2013) worked on generalization of existing models for hierarchical location problems in airline networks and considered operational metric frequency in routing as well as accessibility. Lin et al. (2013) formulated a hub location inventory model in a strategic design problem for bicycle sharing systems. They considered both total cost and service levels in design decisions.

Ahmadi et al. (2015) developed a systematic approach to make robust decisions for the single location-allocation p-hub median problem based on mean-variance theory and two stage stochastic programming. Ghaffari-Nasabet et al. (2015) considered the capacitated single and multiple allocation hub location problems with stochastic demands. They employed a robust optimization approach to model the problem and used a standard optimization package to solve it.

Kaveh et al. (2016) proposed a bi-objective hub arc location problem for a public transportation network. In this study the demand of nodes are assumed to be elastic and dependent on the utility of the hubs. Serper and Alumur (2016) studied the capacitated multi-modal hub network with different types of vehicles for a Turkish passenger network. Dukkanci and Kara (2017) presented the hierarchical multi-modal hub network with time definite delivery. Yahyaei and Bashiri (2017) studied multiple allocation hub problems under disruption risk and developed multiple cuts benders decomposition approach to solve this model.

As it can be observed in the literature, there are not a lot of studies on the involvement of vehicles in a hierarchical hub networks, and there are also a few papers which have considered different levels of capacity for vehicles which transfer multi-mode commodity in a cargo delivery system. This paper seeks to eliminate these gaps in the previous studies. Table 1 shows a summary of aforementioned data about the literature.

Table 1. A summary of the researches until 2017

Researches	Applications (TN/DS/P-DS)[a]	Type of Objective Functions (FC/TC/D/R/RM/SC/NV/B/TT/N)[b] Min	Max	Min-Max	No. of Objective Functions	No. of Levels	Number of Commodities (S/M)[c]	Constraints (CH/CA/NV/T)[d]	Real World Data (RC/RG/LP)[e]
Yaman, 2009	TN, DS	FC, TC, R	-	-	S	3	S	-	RC, RG
Chen. 2010	DS	D	-	-	S	3	S	CH	RC
Ayed, 2011	DS	FC, TC	-	-	S	2	S	CH	RG
Chi et al. 2011	DS	FC, TC	-	-	S	3	S	CH	RG
Sender & Clausen, 2011	DS	FC, TC	-	-	S	7	S	CH, CA	RC, RG
Davari & Zarandi, 2012	P-DS	TC, D	-	-	S	3	S	-	RG
Alumur et al. 2012	DS	FC, TC	-	-	S	1	S	-	RC
Sheu et al. 2012	P-DS	TC	-	RM	M	3	M	CH	RC
Manzour Al-Ajdad et al. 2012	P-DS	TC	-	-	S	3	M	CH	RG
Yaman & Elloumi, 2012	TN	TC, R	-	-	S	3	M	-	RG
Ryerson & Kim, 2013	TN	FC, TC	-	-	S	2	S	CH	RG
Martins-de-Sa et al. 2013	TN	FC, TC	-	-	S	2	S	-	RG
Saboury et al. 2013	TN	TC	-	-	S	3	M	CH	LP
Ahmadi et al. 2015	DS	SC, TC	-	RM	M	1	S	-	RC
Ghaffari-Nasab et al. 2015	DS	FC, TC	-	-	S	2	S	CH	RC
Kaveh et al. 2016	DS	TT	B	-	M	-	S	CA	RC
Serper & Alumur, 2016	DS	FC, TC	-	-	S	-	-	NV	RC
Dukkanci & Kara, 2017	DS	NA	-	-	S	3	S	T	RC
Yahyaei and Bashiri, 2017	DS	TC	-	-	S	-	S	-	RC
Our work	DS	FC, TC, NV	-	-	S	2	M	CH, CA, NV	RC

[a] TN: Telecommunications Networks, DS: Distribution Systems, P-DS: Production–Distribution Systems,
[b] FC: Fixed Cost, TC: Transportation Cost, D: Distance, R: Routing, RM: Risk Measure, SC: Setup Cost, NV: Number of Vehicles, B: Benefit, TT: Transportation Time, N: Number of Airline
[c] S: Single, M: Multi
[d] CH: Capacity of Hubs, CA: Capacity of Arcs, NV: Number of Vehicles, T: Time
[e] RC: Real Case, RG: Randomly Generated, LP: Literature Paper

2- Problem description

In this study, the hierarchical network has been designed based on the Sender and Clausen model for the p-hub median problem [Sender and Clausen, 2011], it is assumed that each origin node can only be connected to a single small hub (urban cargo deliveries), therefore the connection between the origin and the hub is a single allocation one, and each small hub must be connected to the central hub meaning the connection between the central hub and the demand nodes is a multiple allocation. Destination nodes are only allowed to be connected to the central hub, and if there is no direct connection between the destination node and the central hub, commodities must first be sent to a small hub connected to the central hub and then sent to the destination node. Accordingly, the network studied here includes two layers. First layer includes small hubs (cargo deliveries in the city), and the second layer includes the star-shaped network central hub (Hierarchical hub network). In this paper, there are 11 demand nodes, 11 cities are candidates for small hubs and one city is considered as the central hub. There is no direct link between each two origins and destinations. Therefore there should be at least one hub node between each two origins and destinations.

Three types of commodities are chosen to be sent from origin to destination nodes. First type is considered to be light commodities with a mean weight of 20 kg, second type is medium weight commodities with a mean weight of 60 kg, and the third type is heavy commodities with a mean 120 kg. Three types of trucks are considered for the delivery of commodities from origins to destinations. Each type of trucks is proportionate to the type of the commodities. Furthermore the capacity of each type of trucks on each link is different based on the hierarchy level of that link.

In brief, the main problem includes a set of origin-destination nodes (zero level of hierarchy), small hub nodes (first hierarchy level), and the central hub node (second hierarchy level) to send the demands (consisting of three commodities transported by three types of transportation vehicles) from origins to destinations in a hierarchical network for a real problem.

2-1- Assumptions

The following are basic assumptions of the model in summary:

1. The hierarchical hub network is designed as a capacitated system (All Hubs and arcs capacity are limited).
2. Based on the postal network, there are three types of commodities to be delivered. These types are categorized based on the mean weight.
3. According to the different types of commodities, different capacity for each category of commodity is needed in a network. Each hub includes three potential capacity levels from which only one level will be selected as appropriate for the costs of the problem.
4. In each hub, there are three types of vehicles with different capacities dedicated for the delivery of each three types of commodities categorized based on the mean weight.
5. The objective function is to minimize flow costs, costs of hub construction and number of vehicles in the network.

Based on the descriptions above, the cargo delivery network of this study is presented in Fig 1.

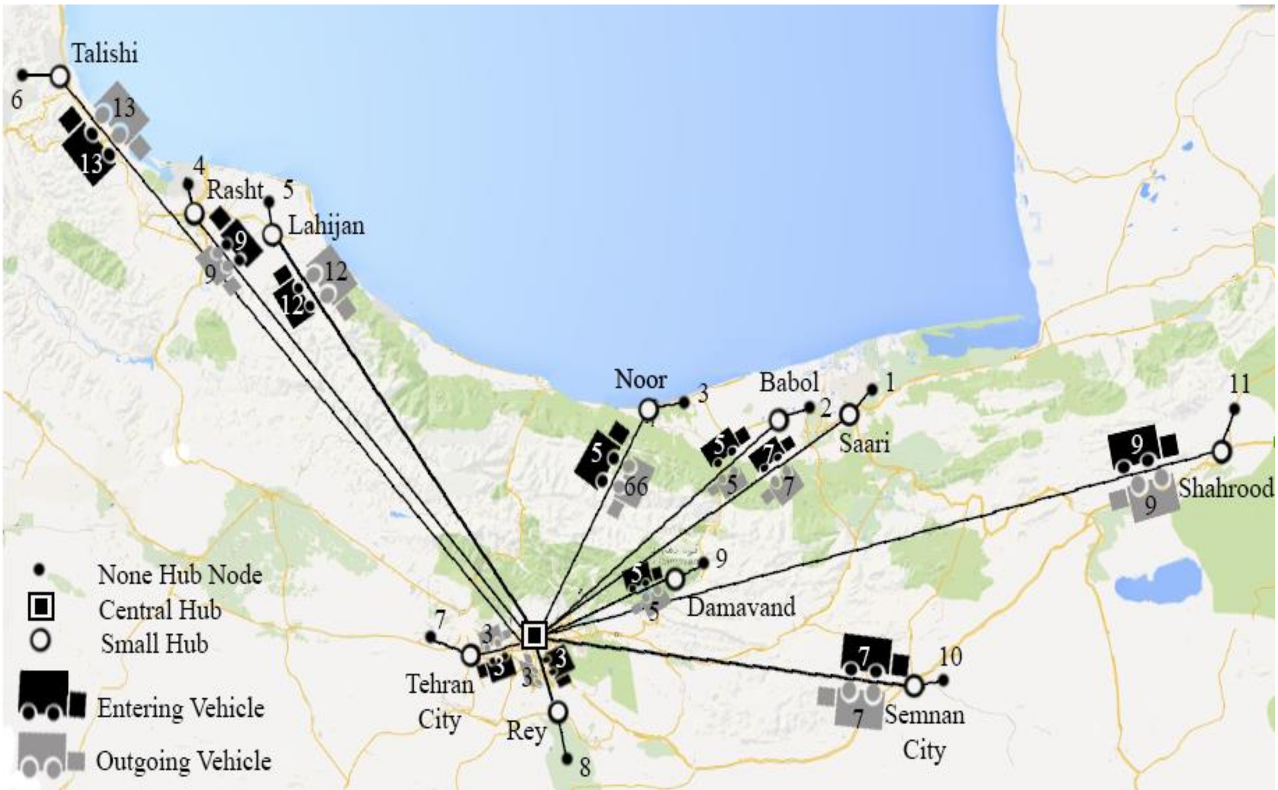

Fig 1. Overall network and the relationship between nodes and paths (11 hubs)

3- Mathematical model

In order to propose the mathematical programming, we first define some sets and indices. We are given

$V' = \{1, 2, \ldots, N\}$ a set of origin-destination nodes indexed by $i, j \in V'$; a set of small hub nodes in the collection system indexed by $s \in S$ and in the distribution system indexed by $s' \in S'$; a set of central hub nodes indexed by $l \in L$; a Set of small and central hub nodes indexed by $p \in P, P = S \cup L$, a potential hub nodes sets as V'' such that $V'' = S \cup S' \cup L$; a set of capacity levels for small hubs (central hub) indexed by $c_s \in \Gamma_s$, $(c_l \in \Gamma_l)$. Multi-mode commodities are considered by set $u \in U$, $U = \{u_1, u_2, u_3\}$. There are three different types of vehicles (denoted by set F) , $F = \{f_1, f_2, f_3\}$. Based on the small hubs and central hubs which is available to services the demand customer's, multiple allocation with nested and incoherent configuration [Torkestani et al. (2016)], the network designed as a hierarchical network. In a hub network considering different service level with the special mentioned configuration of communication called as a hierarchical hub network. The mathematical formulation of the hierarchical hub network developed as MINLP model which has defined in section (3.1). The conceptual scheme of the MINLP hierarchical network illustrate in figure 2.

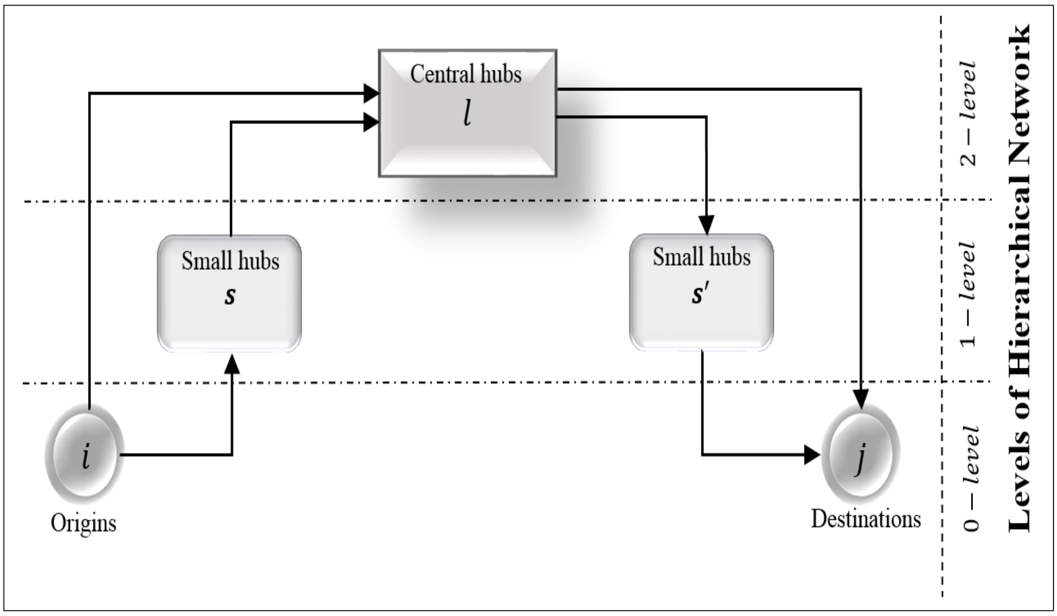

Fig 2. The conceptual scheme of the MINLP hierarchical hub network

3-1- Model formulation

In this section the capacitated hierarchical hub location problem with different types of vehicles and commodity is formulated as a mixed-integer non-linear programming (MINLP). The variables and constraints are explained by restrictions categories as follows:

Design of the hub network: we define X_{is} , X_{il} and $X_{is'}$ to be 1 if demand node (origin) $i \in V'$ is allocated to the small hub s, central hub l and small hub s', respectively; to be 0 otherwise. Similarly X_{js}, X_{jl} and $X_{js'}$ to be 1 if demand node (destination) $j \in V'$ is allocated to the small hub s, central hub l and small hub s'; to be 0 otherwise. Also M_s and M_l are 1 when small and central hubs are established at nodes $s \in S$ and $l \in L$, respectively, similarly, $M_s^{c_s}$ and $M_l^{c_l}$ are 1 when small and central hubs are established at nodes $s \in S$ and $l \in L$ with capacity levels c_s and c_l. The total number of small and central hubs are P_s and P_l. Additionally, Y_{sl} is 1 if a small hub s is assigned to central hub l, even if the small hub is the same as the central hub; and is 0 otherwise. Similarly, $Y_{s'l}$ is 1 if small hub s' is assigned to central hub l, even if the small hub is the same as central hub.

The location and allocation constraints are modeled as below:

$$\sum_{s \in S} M_s = P_s \tag{1}$$

$$\sum_{l \in L} M_l = P_l \tag{2}$$

$$\sum_{s \in S} X_{is} + \sum_{l \in L} X_{il} \leq 1 \qquad \forall\, i \in V' \tag{3}$$

$$X_{is} + X_{js'} \leq M_s \qquad \forall\, i,j \in V', s \in S,\ s' \in S' \tag{4}$$

$$X_{il} + X_{jl} \leq M_l \qquad \forall\, i,j \in V', l \in L \tag{5}$$

$$M_s + M_l \leq 1 \qquad \forall s \in S, l \in L \tag{6}$$

$$\sum_{c_s \in \Gamma_s} M_s^{c_s} = M_s \qquad \forall s \in S \tag{7}$$

$$\sum_{c_l\in\Gamma_l} M_l^{c_l} = M_l \qquad \forall l \in L \tag{8}$$

$$\sum_{s\in S} X_{is} \leq 1 \qquad \forall\, i \in V' \tag{9}$$

$$\sum_{l\in L} X_{il} \leq 1 \qquad \forall\, i \in V' \tag{10}$$

Constraints (1) and (2) represent the number of hubs. Constraint (3) guarantees that it is not possible to assign one origin node (demand) to both small hub and central hub nodes. Constraints (4) and (5) ensure that each hub node plays only one role: either as a distributer or a collector. Constraint (6) shows which kind of small or central hubs can be constructed in each nominated point. Constraints (7) and (8) show the selection of capacity level for each small or central hub. Constraints (9) and (10) represent that each node is assigned to only one hub, and each non-hub node is at most covered by one hub node.

Routing the flows: The flow of demand from node $i \in V'$ to node $j \in V'$ is denoted by W_{ij}. We define S_{is} and S_{il} as the flows that are routed between origin $i \in V'$ and small hub s and central hub l. Additionally, S_{sl}^{i}, $V_{ls'}^{i}$ and $V_{s'j}^{i}$ are defined respectively to be the flows that can be routed from small hub s to central hub l originated from origin i; from central hub l to small hub s' originated from origin i; from small hub s' to destination j originated from origin i. Flows originating from origin i and destined for destination j are denoted by O_i and D_j, respectively; $(O_i = \sum_{j\in V'} W_{ij}\ , D_j = \sum_{i\in V'} W_{ij})$

The routing flow constraints are modeled as follows:

$$\sum_{s\in S} S_{is} + \sum_{l\in L} S_{il} \leq O_i \qquad \forall\, i \in V' \tag{11}$$

$$\sum_{s'\in S'} V_{s'j}^{i} + \sum_{s'\in S'} \sum_{l\in L} V_{ls'}^{i} = W_{ij} \qquad \forall\, i,j \in V' \tag{12}$$

$$V_{s'j}^{i} \leq W_{ij} X_{js'} \qquad \forall\, i,j \in V', s' \in S' \tag{13}$$

$$V_{ls'}^{i} \leq O_i X_{is'} \qquad \forall\, i,j \in V', s' \in S', l \in L \tag{14}$$

$$S_{il} \leq O_i\, M_l \qquad \forall\, i \in V',\ l \in L \tag{15}$$

$$S_{sl}^{i} \leq O_i\, M_l \qquad \forall\, i \in V',\ s \in S, l \in L \tag{16}$$

$$V_{ls'}^{i} \leq O_i\, M_l \qquad \forall\, i \in V',\ s' \in S', l \in L \tag{17}$$

$$S_{is} + \sum_{l\in L} S_{sl}^{i} = \sum_{l\in L} V_{ls'}^{i} + \sum_{j\in V'} V_{s'j}^{i} \qquad \forall\, i \in V',\ s \in S, s' \in S' \tag{18}$$

$$S_{il} + \sum_{s\in S} S_{sl}^{i} = \sum_{s'\in S'} V_{ls'}^{i} \qquad \forall\, i \in V',\ l \in L \tag{19}$$

Constraint (11) represents that the total entering flows from an origin to a hub are smaller or equal to the total flows originating from that origin. Constraint (12) present that each origin-destination flow reaches its own appropriate destination. Constraint (13) guarantees that all transportation volume from a small hub node reaches the destination. Constraints (14), (15), (16), and (17) examine network flows when a hub node is constructed (Constraints (13-17) make sure that small and central hubs are established for each collection, transfer and distribution and may be crucial to assure feasible solutions). Constraints (18) and (19) are the flow equilibrium for each small and central hub respectively with regards to input and output flows and the kind of hierarchy among hubs (Flow balance equations for small and central hub nodes).

Capacity restrictions: The capacity of small hub s with capacity level c_s and central hub l with capacity level c_l are denoted by $B_s^{c_s}$ and $B_l^{c_l}$. The capacity of customer's commodity u between origin and small hub s and central hub l are denoted by Bc_{is}^u and Bc_{il}^u. Similarly, capacity of customer's commodity u between hubs s and l and between small hub s' to destination j, are denoted by Bc_{sl}^u and $Bc_{s'j}^u$. We define the variables b_{is}^{fu} and b_{il}^{fu} that show the capacity of transportation vehicle f used for carrying customer's commodity u between origin i and small hub s and central hub l, also the capacity of transportation vehicle f used for carrying customer's commodity u between hubs s and l and between small hub s' to destination j, respectively are denoted by b_{sl}^{fu} and $b_{s'j}^{fu}$.

The capacity constraints are formulated as follows:

$$\sum_{s\in S} S_{is} + \sum_{l\in L} S_{il} \le \sum_{u\in U}\left(\sum_{s\in S} Bc_{is}^u\, ts_{is}^u + \sum_{l\in L} Bc_{il}^u\, ts_{il}^u\right) \qquad \forall\, i \in V' \qquad (20)$$

$$\sum_{s'\in S'} V_{s'j}^i \le \sum_{u\in U}\sum_{s'\in S'} Bc_{s'j}^u\, tv_{s'j}^u \qquad \forall\, i,j \in V' \qquad (21)$$

$$\sum_{l\in L}\left(S_{sl}^i + V_{ls}^i\right) \le \sum_{u\in U}\sum_{l\in L} Bc_{s'j}^u\, t_{sl}^u \qquad \forall\, i \in V', s \in S,\ s' \in S' \qquad (22)$$

$$\sum_{i\in V'}\left(S_{is} + \sum_{l\in L} V_{ls}^i\right) \le \sum_{c_s\in \Gamma_s} B_s^{c_s}\, M_s^{c_s} \qquad \forall s \in S \qquad (23)$$

$$\sum_{i\in V'}\left(S_{il} + \sum_{s\in S} S_{sl}^i\right) \le \sum_{c_l\in \Gamma_l} B_l^{c_l}\, M_l^{c_l} \qquad \forall l \in L \qquad (24)$$

The constraints (20-22) show the upper bound arc capacity customer's commodity u at different levels of hierarchical network. The arc capacity customer's commodity u in collections routes between the origins and small hubs $i - s$ and also origins and central hubs $i - l$ guarantee in constraints (20) (figure 1 in appendix B). Constraints (21) show the arc capacity for customer's commodity u for the distribution routes between the small hubs $s - j$ and the destinations (figure 2 in appendix B). The arc capacity for customer's commodity u between the hub nodes $s - l$ (small hubs and central hubs) defined in constraints (22) (figure 3 in appendix B). Constraint (23) represents that the capacity of small-hub nodes are limited in the network. Constraint (24) represents that the capacity of central hub nodes is limited in the network.

Vehicle restrictions: Number of customer's commodity u between origin and small hub s and central hub l are denoted by ts_{is}^u and ts_{il}^u. Similarly, number of customer's commodity u between hubs s and l between small hub s' to destination j, are denoted by t_{sl}^u and $tv_{s'j}^u$. We define the variables g_{is}^{fu} and g_{il}^{fu} that show the number of transportation vehicles f used for carrying customer's commodity u between origin i and small hub s and central hub l, also the number of transportation vehicles f used for carrying customer's commodity u between hubs s and l and between small hub s' to destination j, respectively are denoted by g_{sl}^{fu} and $g_{s'j}^{fu}$.

The vehicle constraints are modeled as follows:

$$\sum_{s\in S} ts_{is}^u + \sum_{l\in L} ts_{il}^u \le \sum_{f\in F}\left(\sum_{s\in S} g_{is}^{fu}\, b_{is}^{fu} + \sum_{l\in L} g_{il}^{fu}\, b_{il}^{fu}\right) \qquad \forall\, i \in V', u \in U \qquad (25)$$

$$\sum_{l\in L} t_{sl}^u \le \sum_{f\in F}\sum_{l\in L} g_{sl}^{fu}\, b_{sl}^{fu} \qquad \forall s \in S, u \in U \qquad (26)$$

$$\sum_{j\in V'} tv_{s'j}^u \le \sum_{f\in F}\sum_{j\in V'} g_{s'j}^{fu}\, b_{s'j}^{fu} \qquad \forall\, s' \in S', u \in U \qquad (27)$$

Constraints (25), (26) and (27) with regards to the objective function determine the number of each type of vehicles and their useful capacity in the routes including: origin-small hub, small hub-central hub, and small hub-destination, respectively.

Objective function: The main goal of the model is minimizing flow costs, costs of hub construction and number of vehicles in the network. Total costs $(TotalC)$ consist of fixed costs of constructing (C_{fe}), costs of routing in the network (C_{rc}) and the costs of transportation vehicles (C_{vc}). According to the costs mentioned, we defined the fixed cost of constructing small hub node s with capacity level c_s and central hub node l with capacity level c_l to be denoted by $F_s^{c_s}$ and $F_l^{c_l}$, respectively. Similarly, C_{is}, C_{il}, C_{sl}, $C_{ls'}$ and $C_{s'j}$ are the unit routing costs between $i-s$, $i-l$, $s-l$, $l-s'$ and $s'-j$.

Additionally, CV_{is}^{fu}, CV_{il}^{fu}, CV_{sl}^{fu}, $CV_{ls'}^{fu}$ and CV_{is}^{fu} are the cost of transportation vehicles f used for carrying customer's commodity u between origin-hub $i-s$ and $i-l$, hub edge $s-l$ and $l-s'$; hub to destination $s'-j$. The Discount factor between origins and hub nodes, hub nodes, and hub nodes and destinations are defined by χ, α and δ.

The costs components have been formulated as follows:

$$C_{fe} = \sum_{s\in S}\sum_{c_s\in \Gamma_s} F_s^{c_s} M_s^{c_s} + \sum_{l\in L}\sum_{c_l\in \Gamma_l} F_l^{c_l} M_l^{c_l} \tag{28}$$

$$C_{rc} = \sum_{i\in V'}\sum_{j\in V'} W_{ij}\big(\chi \sum_{s\in S} C_{is}X_{is} + \chi \sum_{l\in L} C_{il}X_{il} + \alpha \sum_{s\in S}\sum_{l\in L} C_{sl}X_{is}\big(1 - X_{js'}\big)Y_{sl} + \alpha \sum_{s'\in S'}\sum_{l\in L} C_{ls'}X_{js'}(1 - X_{is})Y_{s'l} + \delta \sum_{s'\in S'} C_{s'j}X_{js'}\big) \tag{29}$$

$$C_{vc} = \sum_{f\in F}\sum_{u\in U}\Big(\sum_{i\in V'}\sum_{s\in S} g_{is}^{fu}CV_{is}^{fu} + \sum_{i\in V'}\sum_{l\in L} g_{il}^{fu}CV_{il}^{fu} + \sum_{s\in S}\sum_{l\in L} g_{sl}^{fu}CV_{sl}^{fu} + \sum_{l\in L}\sum_{s'\in S'} g_{ls'}^{fu}CV_{ls'}^{fu} + \sum_{j\in V'}\sum_{s'\in S'} g_{s'j}^{fu}CV_{s'j}^{fu}\Big) \tag{30}$$

The total cost used for the objective function in the proposed model contains three components C_{fe}, C_{rc} and C_{vc}; can be formulated in Eq. (31).

$$TotalC = C_{fe} + C_{rc} + C_{vc} \tag{31}$$

The capacitated hierarchical hub location can be modeled as follows:

$$\begin{aligned} &Minimize \quad TotalC \\ &S.t{:} \\ &\quad (1-27) \end{aligned}$$

The final goal is minimizing the total costs including fixed costs of constructing two kinds of hubs: small and central and the costs of routing in the network (transportation costs; collection from origins to each hub, transportation between hubs, and distribution from small hubs to destinations) and the cost of vehicles.

4- Experimental design and data collection (Case study)

The proposed model is implemented on Iran's northern parts transportation network and solved by 24.0.1 version of GAMS software using CPLEX and CONAPT solvers for the mixed integer nonlinear programming (MINLP). Then by changing the number of small hubs, discount factors, and fixed costs of constructing hubs, the sensitivity of the model to these parameters is analyzed. The model consists of eleven cities and a capital (as the central hub). Cities are the nominated nodes for establishing the small hubs. Furthermore, 11 nodes as the origins and destinations of the flows are considered.

For choosing the small hub candidates, regarding the population of cities in each province, one of the cities is chosen as a candidate of a small hub. The cities under study are: Rey, Tehran, Damavand, Semnan, Shahrood, Lahijan, Rasht, Talishi, Sari, Babol, and Noor. The data used in this model is based on real-world data and is from the Iranian Statistics Organization website, according to up to date prices. Regarding the size of the problem and assumptions, necessary changes are applied. Regarding the distances between cities and assuming an average speed of 50 Km/h for the trucks, which is rational for the roads in northern parts of Iran, the expected cost between each two demand nodes, is estimated.

In the cargo delivery network considered in this study, there are three kinds of commodities. Each kind has its own characteristics in a way that transporting them together in the same vehicle is not feasible. Hence for each kind of commodity, an appropriate vehicle is used. The first type of commodity, a light one, with an average weight of 20 kilograms, the second one with an average weight of 60 kilograms and the third one with an average weight of 120 kilograms are considered. There are three types of trucks for transportation. The first one for the first kind of commodity has the capacity of 3 tons (150 commodities) for an origin-small hub link, the capacity of 3.5 tons (175 commodities) for a small hub-central hub link, and the capacity of 4 tons (200 commodities) for a central hub-small hub link.

The second type of truck for the second kind of commodity has the capacity of 6 tons (100 commodities) for an origin-small hub link, the capacity of 6.5 tons (108 commodities) for a small hub-central hub link, and the capacity of 10 tons (166 commodities) for a central hub-small hub link.

The third type of truck, for the third kind of commodity has the capacity of 10 tons (84 commodities) for an origin-small hub link, the capacity of 10.5 tons (88 commodities) for a small hub-central hub link, and the capacity of 22 tons (183 commodities) for a large hub-small hub link. Demands are estimated according to the Iranian Statistics Organization website. The tables 7, 8 and 9 in appendix A, contain other data used in this model. According to the Iran's cargo delivery network (real-world statistical dataset) all the costs was represented by the monetary units “Rial” in Iran.

4-1-Computational experiments

The results from solving the model according to the aforementioned assumptions are presented in the Tables 2 and 3.

Table 2. Objective function value and processing time of the proposed model

Scenario	P_s	P_l	Total Hub Nodes	Discount Factors			Total Cost (Rial)	CPU Time (S)
				χ	α	δ		
(1)	11	1	12	0.9	0.8	0.89	76026200000	7.616
(2)	11	1	12	0.78	0.6	0.7	76019780000	7.89

Table 3. The number of required vehicles

Small hub position	The number of outgoing vehicles			Total number of outgoing vehicles	The number of entering vehicles			Total number of Entering vehicles
	S	M	L		S	M	L	
Shahrood	2	3	4	9	2	3	4	9
Noor	14	23	29	66	1	2	2	5
Lahijan	3	4	5	12	3	4	5	12
Rey	1	1	1	3	1	1	1	3
Babol	1	2	2	5	1	2	2	5
Rasht	2	3	4	9	2	3	4	9
Damavand	1	2	2	5	1	2	2	5
Saari	2	2	3	7	2	2	3	7
Semnan City	2	2	3	7	2	2	3	7
Tehran City	1	1	1	3	1	1	1	3
Talishi	3	5	5	13	3	5	5	13

S: Small; M: Medium; L: Large

Regarding the assumptions of the problem and the data in appendix A, the total cost of the network with 11 hub-nodes in table 2 is equal to 76026200000. Also by decreasing the discount factor, the total cost of the network decreases. After solving the problem, the number of each vehicle entering or leaving each hub node is presented in Table 3. In what follows, the effects of the main parameters of the model on the objective function and the number of vehicles are explored.

4-2- Sensitivity analysis

In this section the effects of changing key parameters of the proposed model on the total value of objective function and the number of transportation vehicles are investigated. The model is solved in 18 scenarios, and the results are presented briefly in table 2, 3, 4, 5 and 6. Discount coefficients have changed in the second scenario. The effect of demand changes has been investigated in scenarios 3, 4, 5, 6, 7, and 8. The effects of construction cost variations have been investigated in scenarios 9, 10, 11, 12, 13, 14, and the effects of the number of small hub nodes have been investigated in scenarios 15, 16, 17 and 18.

4-2-1- Changes in demand and fixed costs

The changes in the objective function according to the changes in demand and fixed costs of constructing hubs are presented in the tables 4 and 5 and figures 3, 4, 5 and 6.

Table 4. Objective function variations and the number of vehicles according to demand changes

Scenario	Demand variation	Total Cost (Rial)	Processing time (s)	Total Vehicles	Shahrood	Noor	Lahijan	Rey	Babol	Rasht	Damavand	Saari	Semnan City	Tehran City	Talishi
(1)	0%	76026200000	7.616	Outgoing	9	66	12	3	5	9	5	7	7	3	13
				Entering	9	5	12	3	5	9	5	7	7	3	13
(3)	+16%	76030590000	23.722	Outgoing	15	129	15	3	14	3	32	5	5	3	11
				Entering	3	23	3	3	14	3	32	5	3	3	11
(4)	+18%	76375690000	40.647	Outgoing	15	130	13	3	14	15	33	14	8	15	15
				Entering	3	25	10	3	14	3	33	3	3	25	3
(5)	+25%	76382640000	16.438	Outgoing	45	165	14	3	18	17	7	8	37	3	68
				Entering	45	28	14	3	17	12	7	8	37	3	16
(6)	-6%	76025490000	16.807	Outgoing	6	71	9	3	11	6	3	4	3	3	13
				Entering	6	4	9	3	11	6	3	4	4	3	9
(7)	-7%	76019630000	25.444	Outgoing	8	42	3	3	5	8	5	5	3	3	10
				Entering	8	5	3	3	5	8	5	5	5	3	10
(8)	-8%	76016260000	9.714	Outgoing	3	37	8	3	3	4	4	5	5	5	9
				Entering	3	4	8	3	3	7	4	5	5	5	9

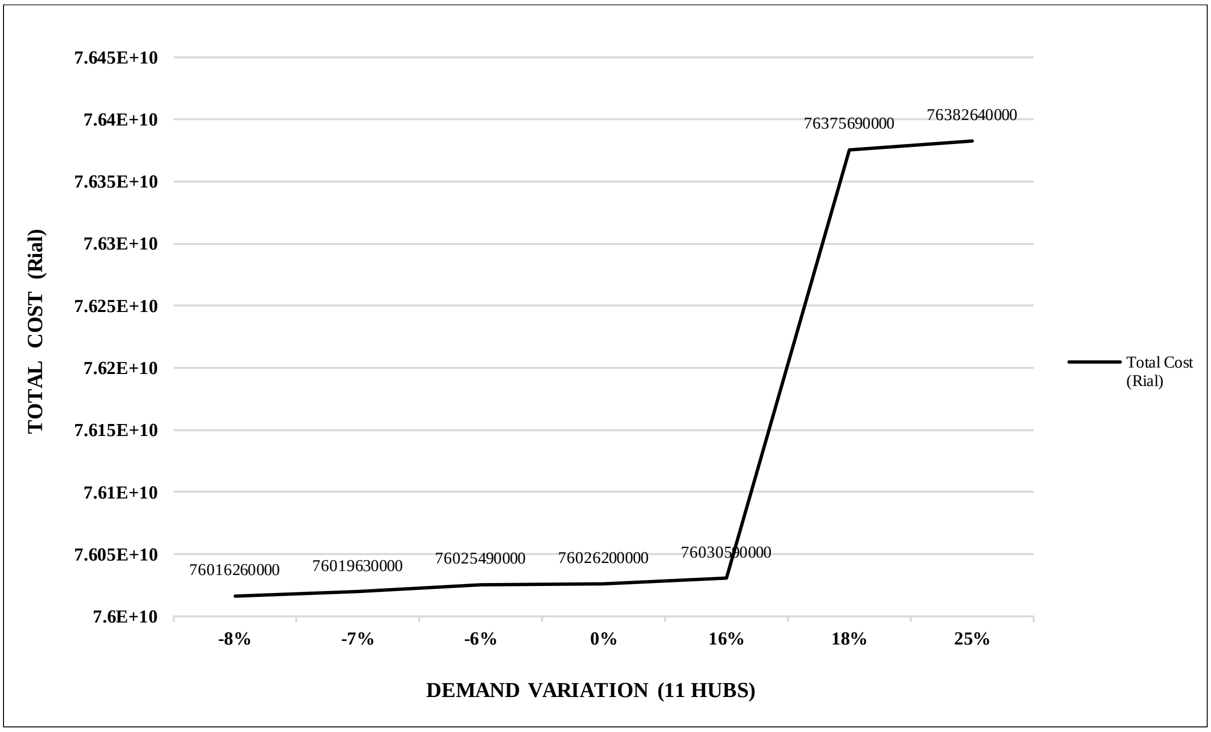

Fig 3. Sensitivity of total cost by variation the demand

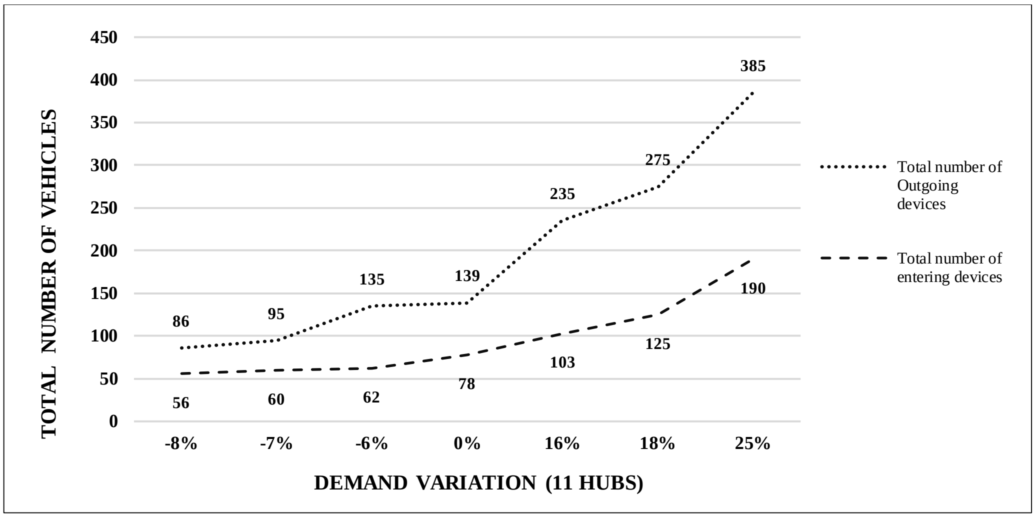

Fig 4. Sensitivity of total number of vehicles by variation the demand

As apparent in table 4 and figure 4, by increasing the amount of demand, for the network to be responsive to the current demand, the total number of vehicles should be increased. Also by increasing the total number of transportation vehicles on each link, the total cost of the system increases.

Table 5. Objective function variations and the number of vehicles according to the fixed cost of constructing hubs

Scenario	Fixed Cost variation	Total Cost (Rial)	Processing time (s)	Total Vehicles	Shahrood	Noor	Lahijan	Rey	Babol	Rasht	Damavand	Saari	Semnan City	Tehran City	Talishi
(1)	0%	76026200000	7.616	Outgoing	9	66	12	3	5	9	5	7	7	3	13
				Entering	9	5	12	3	5	9	5	7	7	3	13
(9)	+10%	83626540000	5.642	Outgoing	9	106	12	3	14	9	25	17	31	3	18
				Entering	9	5	12	3	14	9	5	17	31	3	13
(10)	+20%	91226550000	4.78	Outgoing	9	125	12	3	14	9	5	17	31	3	13
				Entering	9	5	12	3	14	9	5	17	31	3	13
(11)	+25%	95026560000	4.435	Outgoing	9	125	12	3	14	9	5	17	31	3	13
				Entering	9	5	12	3	14	9	5	17	31	3	13
(12)	-10%	68426520000	7.034	Outgoing	9	125	12	3	14	9	5	17	31	3	13
				Entering	9	5	12	3	14	9	5	17	31	3	13
(13)	-20%	60826510000	6.127	Outgoing	9	125	12	3	14	9	5	17	31	3	13
				Entering	9	5	12	3	14	9	5	17	31	3	13
(14)	-25%	57026970000	5.510	Outgoing	9	202	12	3	14	9	77	17	31	3	13
				Entering	9	5	12	3	14	9	77	17	31	3	13

By adding 10% to the fixed cost of hub construction, the objective function increases to 83626540000 monetary units and by decreasing 10% from the fixed cost of construction, the objective function decreases to 68426520000 monetary units. The changes in the objective function according to the fixed cost of constructing hubs are represented in the figures 5 and 6.

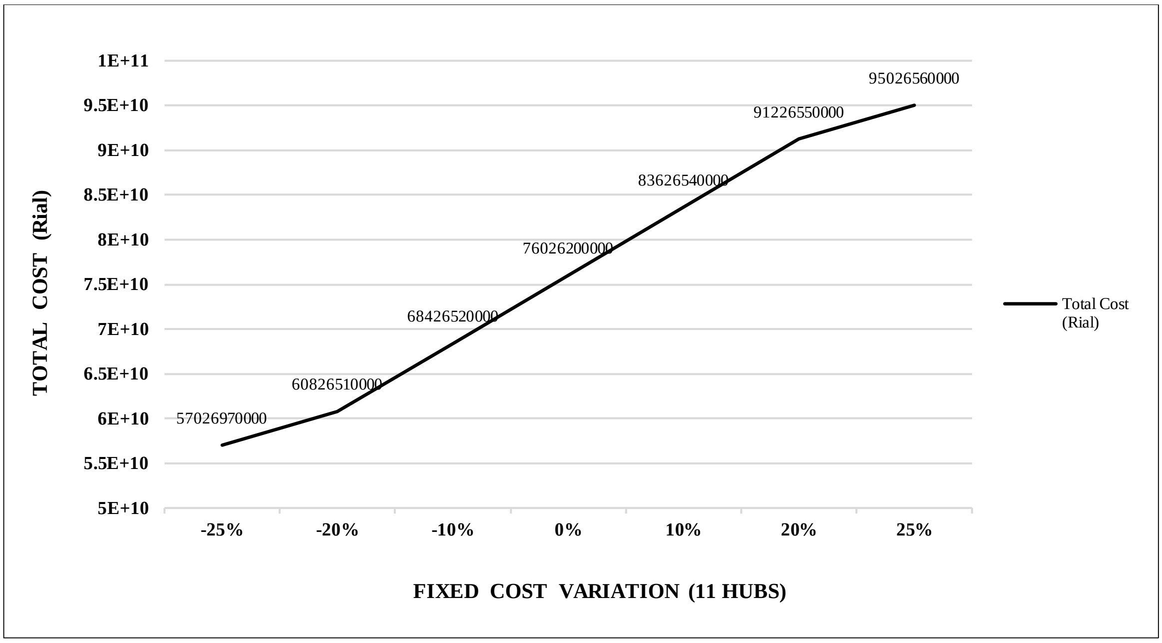

Fig 5. Sensitivity of total cost by variation the fixed cost

By decreasing the construction cost in the model, the total cost of the network decreases (table 5 and figure 5). Increase in construction cost means increase in capacity of the hub to respond to the requested commodity, so in a situation where the hub capacity increases, the hub can be responsive to more demand, and as a result the number of vehicles increases to respond to the demand. Also, from one hand decreasing the construction cost means constraining the hub capacity and hence limiting the whole network for responding. This causes the system to increase the number of vehicles to respond to the requested demand. So by increase or decrease in the construction cost the number of vehicles increases.

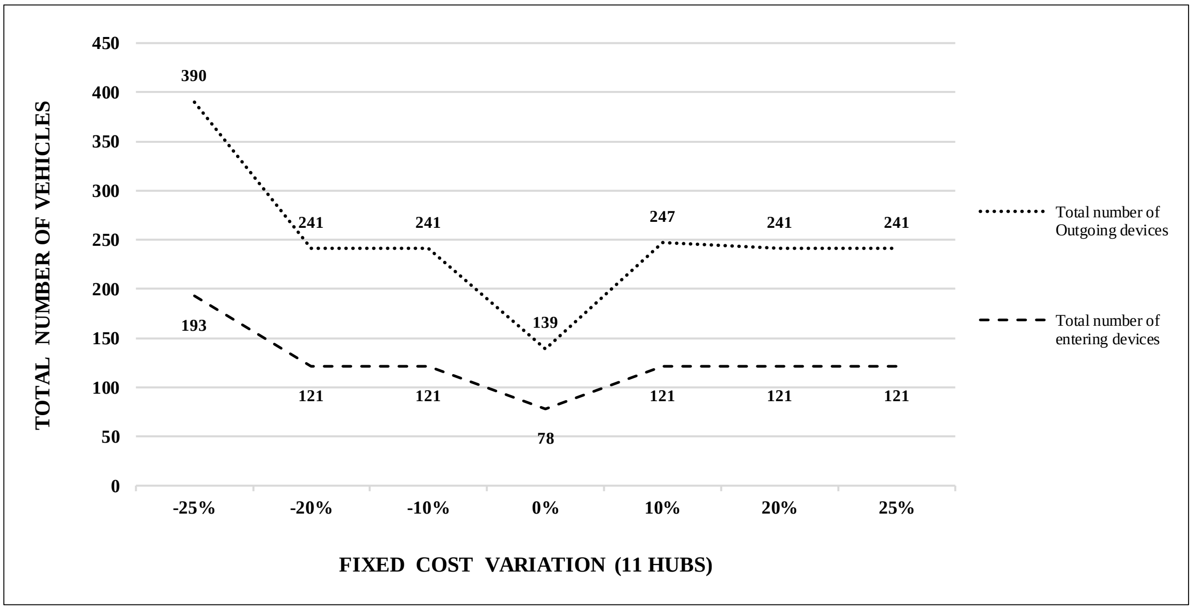

Fig 6. Sensitivity of total number of vehicles by variation the fixed cost

Due to the results by decreasing the hub capacities the number of transportation vehicles increase. The objective function tries to minimize transportation costs, and this cost is dependent on distance. The distance from Noor city to the central hub node is less than that of small capacity hubs to the central hub node, so for Noor to send the demand in the network, it should use more vehicles; otherwise, because of its low capacity, it cannot support the demand. So increasing the demand in the network also impacts the number of vehicles causing it to grow for all the nodes especially for Noor. As seen in Table 5, increase in hub construction costs results in increase in hub capacity and number of vehicles as a corollary.

4-2-2- Changes in number of small hub nodes

In this section the model is solved in several instances by changing the total number of small hub candidates, and the effects of these changes on the total value of the objective function and total number of transportation vehicles is investigated. The results are shown in Table 6, Fig. 7 and Fig. 8.

Table 6. Objective function variations and the number of vehicles according to the number of existing small hubs

Scenario	P_s	P_l	Total Hub Nodes	Total Cost (Rial)	Processing time (s)	Small hub position	The number of outgoing vehicles			Total number of outgoing vehicles	The number of Entering vehicles			Total number of entering vehicles
							S	M	L		S	M	L	
(15)	11	1	12	76026200000	7.616	Shahrood	2	3	4	9	2	3	4	9
						Noor	14	23	29	66	1	2	2	5
						Lahijan	3	4	5	12	3	4	5	12
						Rey	1	1	1	3	1	1	1	3
						Babol	1	2	2	5	1	2	2	5
						Rasht	2	3	4	9	2	3	4	9
						Damavand	1	2	2	5	1	2	2	5
						Saari	2	2	3	7	2	2	3	7
						Semnan	2	2	3	7	2	2	3	7
						Tehran	1	1	1	3	1	1	1	3
						Talishi	3	5	5	13	3	5	5	13
(16)	10	1	11	67082930000	7.765	Rey	1	2	2	5	1	1	1	3
						Noor	32	53	25	110	5	8	10	23
						Lahijan	4	7	8	19	4	7	8	19
						Damavand	1	2	2	5	1	2	2	5
						Babol	3	5	6	14	3	5	6	14
						Semnan	10	9	9	28	10	9	9	28
						Tehran	1	1	1	3	1	1	1	3
						Saari	10	9	9	28	10	9	9	28
						Shahrood	2	3	4	9	2	3	4	9
						Talishi	5	9	10	24	3	5	5	13
(17)	9	1	10	56539540000	8.239	Rey	12	18	23	53	12	19	23	54
						Noor	33	54	26	113	5	8	9	22
						Semnan	2	2	3	7	2	2	3	7
						Tehran	1	1	1	3	1	1	1	3
						Babol	3	5	6	14	3	5	6	14
						Shahrood	2	3	4	9	2	3	4	9
						Talishi	3	5	5	13	3	5	5	13
						Rey	12	19	23	54	12	19	23	54
						Lahijan	4	7	8	19	2	3	4	9
(18)	8	1	9	49097420000	17.725	Tehran	4	6	7	17	1	1	1	3
						Noor	33	54	66	153	5	8	9	22
						Shahrood	4	7	8	19	3	3	4	10
						Babol	3	5	6	14	3	5	6	14
						Rey	11	18	22	51	11	18	22	51
						Talishi	3	5	5	13	3	5	5	13
						Damavand	1	2	2	5	1	2	2	5
						Lahijan	18	30	36	84	18	30	36	84

S: Small; M: Medium; L: Large

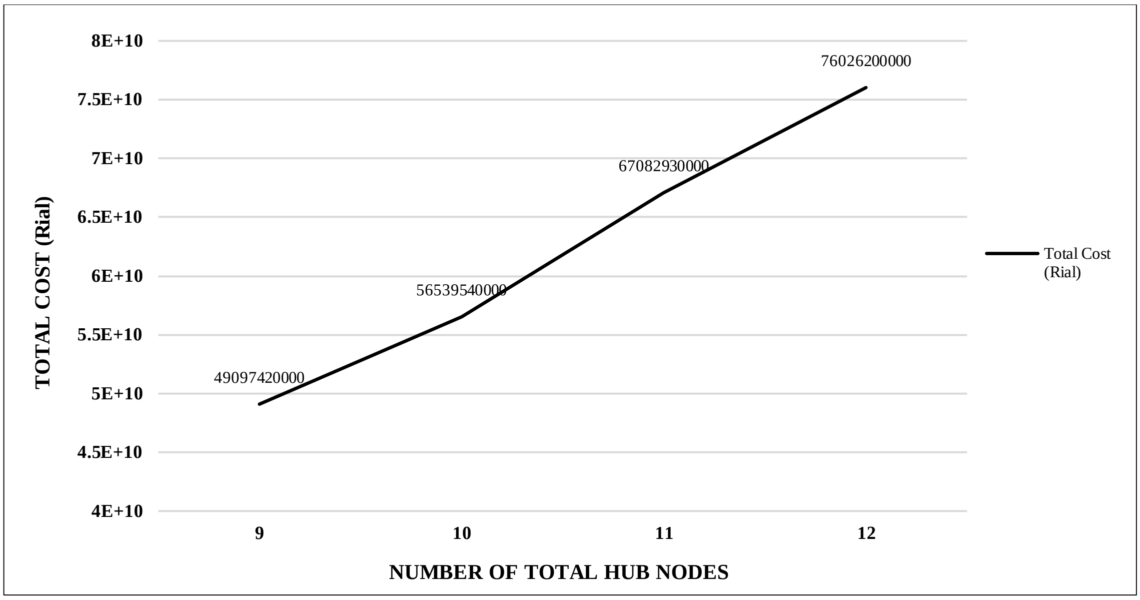

Fig 7. Sensitivity of total cost by variation the number of total hub nodes

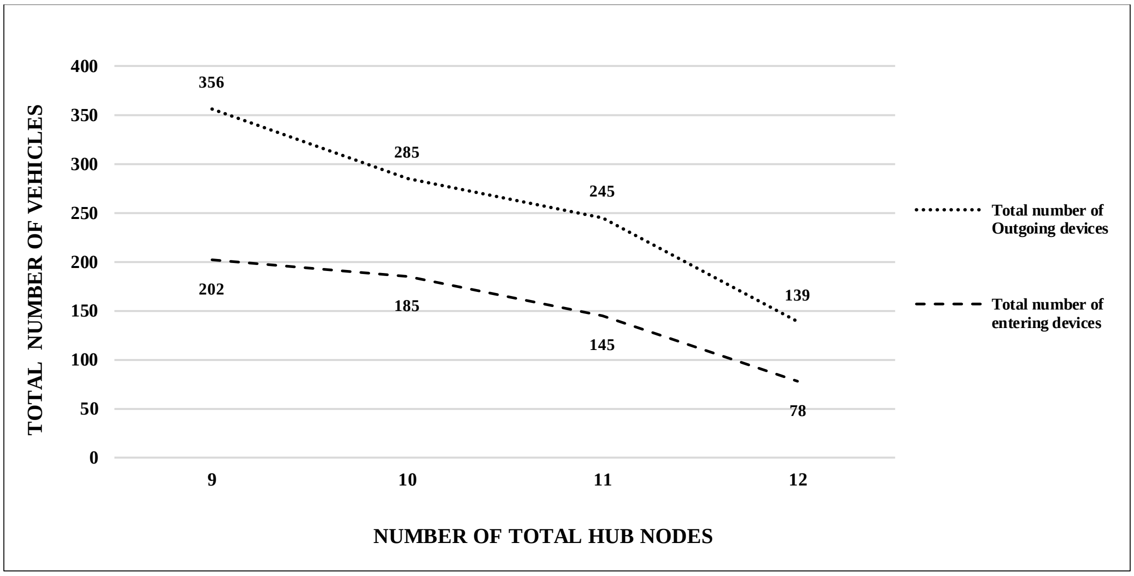

Fig 8. Sensitivity of total number of vehicles by variation the number of total hub nodes

In a constant amount of demand, for the network to be responsive to the current customers in a situation where the number of hub nodes decreases, the number of vehicles should increase so that with less hub-nodes and hub links the customer's demands can be covered. That is because by having more hub nodes the demand is spread between more hub centers, and also more hub links are constructed, so the total number of vehicles on links decreases. Hence as apparent in Fig. 8, with decrease in the number of hub nodes, the number of vehicles increases. Furthermore, a decrease in the number of hub nodes means a decrease in the construction cost in the network, and on the other hand, an increase in the number of vehicles causes a decrease in the repeated collection, transportation and distribution of the products on each hub node in the network, Hence as it can be seen in Table 6 and Fig. 7, by decreasing or eliminating the construction cost and the number of redundant links, the total cost of the network decreases.

5- Conclusions and future trends

In this study, a capacitated hierarchical hub location problem is proposed to design a network with different kinds of vehicles to transport each type of commodities categories which are based on the commodities mean weight. The model tries to minimize the total cost and total number of vehicles that should be used to transport the commodities with specific vehicles in a cargo delivery network. The scheme and the mathematical of the proposed model is developed based on the northern Iran's transportation network (Iran Postal network dataset). A variety of scenarios have been investigated to examine the effect of capacity changes on the number of vehicle used for each category of commodity in the proposed model at each levels of hierarchy. Computational results show that by increasing the discount factors, demands, construction cost and the number of constructed hubs in a network, the total number of the capacitated vehicles for each specific category of commodity and the total cost increases. Also it has been shown that hub node capacity and its distance to the central hub node hugely impacts the number of transportation vehicles. These results are obtained by comparison of the computational results of real world dataset and analysis with different scenarios. Specific issues that can be considered as a future research direction include:

1. Solving the model using heuristic or meta-heuristic algorithms for large-sized problems and comparing the results to the optimal value of the objective function solved by exact solution algorithms.
2. Considering sustainability and environmental-related factors in the model as other objective functions in addition to the total cost objective function.
3. Considering areas where physically it is not feasible to construct hub nodes in them.
4. Considering multi-objective assumption (for example optimization of both total cost and service time, etc.).

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Appendices

Part A. Data

Table 7. Transportation costs (Rial) between hubs and origin to small hub

		Small Hub											Central Hub	
		Saari	Babol	Noor	Rasht	Lahijan	Talishi	Tehran city	Rey	Damavand	Semnan city	Shahrood	To the Capital	From Capital
	Saari	0	60408	10944	61824	52700	70824	74760000	22092	6432	43460	64484	728.96	7581
	Babol	6720	0	7104	57624	70308	66768	64120000	18564	4960	43248	71440	629.8	6590
	Noor	18240	10656	0	83496	65844	62920	65800000	20076	5536	56604	68620	653.92	6831
	Rasht	58880	49392	47712	0	8680	11024	91000000	28140	10912	118932	133668	881.72	9109
	Lahijan	68000	81648	50976	11760	0	15704	14812000	30996	13600	117448	141940	142.308	1452.2
i	**Talishi**	108960	92448	58080	17808	18724	0	12068000	37296	15968	137800	156040	116.58	1195
	Tehran	42720	32976	22560	54600	65596	44824	0	33006	22004	50032	77456	26.8	5600
	Rey	40320	31824	22944	56280	45756	46176	56270010	0	2592	45368	76892	32.16	6130
	Damavand	38720	22320	16608	57288	52700	51896	55990010	6804	0	46004	67680	42.88	7200
	Semnan	32800	29376	25632	94248	68696	67600	66080000	17976	6944	0	33088	651.24	6804
	Shahrood	54880	54720	35040	119448	93620	86320	115360000	34356	11520	37312	0	112.828	1157.4

Table 8. Capacity level and hub construction costs (Rial)

	Hub Capacity	Fixed Cost (Rial)	Level of Capacity		
			c_{s1}	c_{s2}	c_{s3}
Saari	120791900	8100000000	480478	480477	480479
Babol	110124800	6300000000	399382	399383	399384
Noor	72816900	3300000000	174679	174677	174678
Rasht	103919900	9000000000	440267	440269	440268
Lahijan	76443700	6300000000	2381883	2381882	2381881
Talishi	64983400	3500000000	172218	172217	172219
Tehran city	879137800	11000000000	6210347	6210349	6210348
Rey	265485400	5500000000	563702	563703	563701
Damavand	105947300	5700000000	89860	89861	89862
Semnan city	38615600	7500000000	205927	205926	205925
Shahrood	33832600	5300000000	158098	158097	158099
	Hub Capacity	**Fixed Cost (Rial)**	**Level of Capacity**		
			c_{l1}	c_{l2}	c_{l3}
Tehran Capital	1300570500	15500000000	6701436	6701435	6701434

Table 9. Commodity capacity on each arc

		Saari	Babol	Noor	Rasht	Lahijan	Talishi	Tehran city	Rey	Damavand	Semnan city	Shahrood	Tehran Capital
	Saari	0	780	520	1510	670	5600	1510	450	170	1140	1020	1450
	Babol	790	0	470	1380	610	5100	1380	410	160	1040	930	1320
	Noor	520	470	0	910	400	3400	910	270	100	690	610	8700
	Rasht	740	670	450	0	580	4800	1300	390	150	990	870	1250
	Lahijan	550	490	330	960	0	3600	960	290	110	730	640	9200
U_1	**Talishi**	460	420	280	810	360	0	810	240	900	610	550	7800
	Tehran city	6280	5650	3770	1099	4870	40800	0	3300	1260	8320	7380	1050
	Rey	1900	1710	1140	3320	1470	12300	3320	0	380	2510	2230	3180
	Damavand	760	680	450	1320	590	4900	1320	400	0	1000	890	1270
	Semnan city	280	250	170	480	210	1800	480	140	600	0	320	4600
	Shahrood	240	220	140	420	190	1600	420	130	500	320	0	4000
	Saari	0	300	200	580	260	2200	580	170	700	440	390	5600
	Babol	300	0	180	530	240	2000	530	160	600	400	360	5100
	Noor	200	180	0	350	160	1300	350	110	400	270	240	3400
	Rasht	290	260	170	0	220	1900	500	150	600	380	340	4800
	Lahijan	210	190	130	370	0	1400	370	110	400	280	250	3600
U_2	**Talishi**	180	160	110	320	140	0	320	90	400	240	210	3000
	Tehran city	2420	2180	1450	4240	1880	15800	0	1270	480	3210	2850	4060
	Rey	730	660	440	1280	570	4800	1280	0	150	970	860	1230
	Damavand	290	260	180	510	230	1900	510	150	0	390	340	4900
	Semnan city	110	100	60	190	800	7000	190	60	200	0	130	1800
	Shahrood	90	80	60	160	700	6000	160	50	200	120	0	1500
	Saari	0	140	100	280	120	1000	280	800	300	210	190	2700
	Babol	150	0	90	260	110	1000	260	800	300	200	170	2500
	Noor	100	90	0	170	700	6000	170	500	200	130	110	1600
	Rasht	140	130	80	0	110	9000	250	700	300	190	160	2300
	Lahijan	100	90	60	180	0	7000	180	500	200	130	120	1700
U_3	**Talishi**	90	80	50	150	700	0	150	500	200	120	100	1500
	Tehran city	1170	1050	700	2050	910	7600	0	6200	230	1550	1380	1960
	Rey	350	320	210	620	270	2300	620	0	700	470	410	5900
	Damavand	140	130	80	250	110	9000	250	700	0	190	160	2300
	Semnan city	50	50	30	90	400	3000	90	300	100	0	600	9000
	Shahrood	40	40	30	80	300	3000	80	200	100	600	0	7000

Part B. Definition of the constraints (20-22)

The scheme of the constraints (20-22), the upper bound arc capacity customer's commodity u at different levels of hierarchical network defined in figures B1, B2, and B3 as follow:

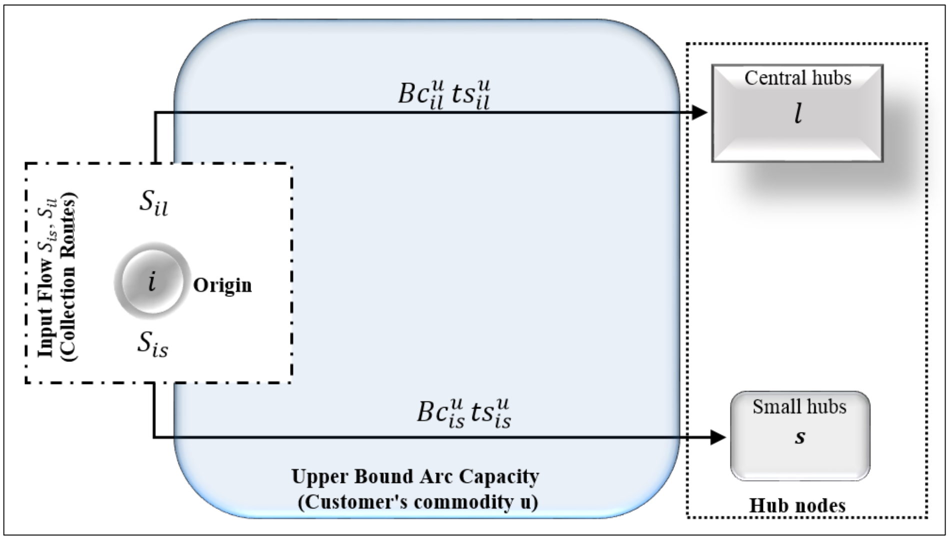

Fig B.1. The constraint (20)

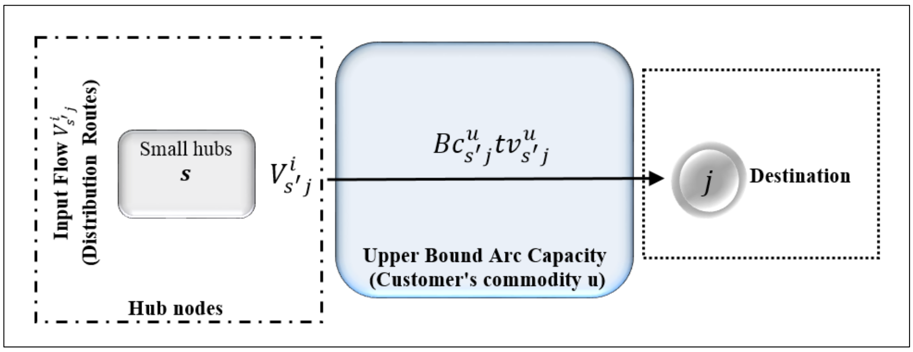

Fig B.2. The constraint (21)

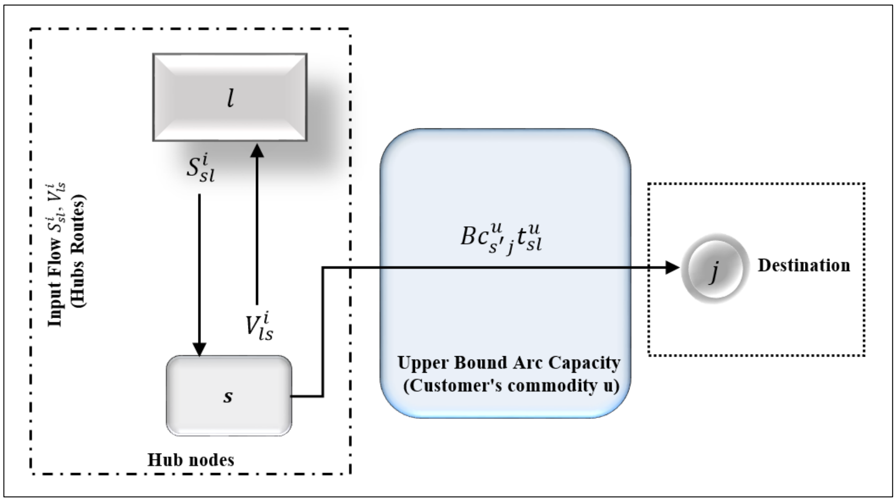

Fig B.3. The constraint (22)