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Coordination of a green supply chain with one manufacturer and two duopolistic retailers through an environmental and social cost sharing contract

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Abstract

In this paper, an incentive contract is proposed to coordinate the environmental and social decisions in a manufacturer-duopolistic retailers green supply chain. The manufacturer invests in new technology to enhance the green level of the products and two retailers invest in the corporate social responsibility (CSR). The investigated supply chain is modeled under three decision-making structures. In the decentralized model, a manufacturer-Stackelberg game under the two different game behaviors of retailers (Cournot and Collusion) is investigated. Afterward, the centralized model as a benchmark is established. Then, an environmental and social cost sharing contract is developed to encourage the supply chain members to participate in the coordination model. Under the coordination model, the surplus profit is shared among the members based on the members' bargaining power. Results demonstrate that the proposed coordination contract not only improves the profitability of entire supply chain and members, but also enhances the green quality and CSR investment compared with the decentralized model. Therefore, the proposed coordination model is of high importance from environmental consideration.

Keywords: Green supply chain, channel coordination, competing retailers, corporate social responsibility (CSR), cost sharing contract

1- Introduction

In the recent decade, with the development of industries, environmental concerns have increased considerably across the world. Green supply chain management attracts significant attention from companies. The green supply chain goals are to minimize the environmental degradation and maximize resource efficiency. For the sake of protecting the environment, a manufacturer can invest in new technology to improve the green quality of the products. For example, in order to protect the environment, Adidas applies MMVEA and Eco-Grip technologies to diminish harmful materials which use in the production process (Song and Gao, 2018). On the other hand, by increasing the environmental awareness, the customers' purchasing behavior has changed and they prefer the green products to the traditional ones. According to the BBMG Conscious Consumer Report, 67% of Americans agree that purchasing products with environmental quality is importance (Zhang et al., 2015). Therefore, the green quality of the products increases the market demand. Consequently, investing in the green quality not only positively affects the profitability of the manufacturer, but also boosts the retailer's market demand.

On the other hand, with the increasing trend of globalization, most companies increasingly pay attention to corporate social responsibility (CSR). Various definitions of CSR are provided in the literature.

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CSR can be defined as a company's activities that impact on human rights, environmental protection, greenhouse emissions control and social groups (Ni et al., 2010).

In the current business environment, the CSR investment made by a company can be considered as a marketing means that impacts on the customers' purchasing behavior. In practice, the CSR is applied by various firms and supply chains such as WalMart, Adidas, GAP and Nike (Panda, 2014). Investing in the CSR activities by companies improves risk management, their reputation, customer loyalty and respond to non-governmental structure (Hsueh, 2014). CSR is not just an inner issue for a member of the supply chain. Like the green quality, the effect of CSR efforts extends to other supply chain members. Therefore, a coordination mechanism should be designed to optimize the profitability of the supply chain.

Coordination mechanisms are an effectual way to prevent supply chain from double marginalization which is one of the important sources of supply chain inefficiencies. Under the traditional (decentralized) structure, each supply chain member decides on his/her decisions without considering the other supply chain members. However, each member's decisions impact on the other supply chain members and consequently the profitability of the entire supply chain is not necessarily optimal under the decentralized model. In the centralized model, a central decision-maker optimizes the profitability of the entire supply chain. However, the profitability of members may decrease under the centralized model in comparison with the decentralized model. Therefore, they don't participate in the centralized model. Under the coordination model, the supply chain members are motivated to change the locally optimal decisions into globally optimal decisions using an incentive mechanism. Therefore, coordinating the members' decisions improves the profitability of entire supply chain as well as the profitability of members in comparison to the decentralized model. Various mechanisms are applied in the literature such as quantity discount, delay in payment, buyback, revenue sharing, cost sharing. In this paper, we develop an environmental and social cost sharing contract.

In this paper, a green supply chain consisting of one manufacturer and two duopolistic retailers is studied. In the proposed supply chain, the demand depends on the green quality and the CSR investment. The manufacturer decides on the green quality (s) and two duopolistic retailers determine the CSR investment. The supply chain is investigated in three different models: (1) decentralized model, (2) centralized model, (3) coordination model. Under the decentralized model, a Stackelberg game is used which in the manufacturer acts as the leader and the retailers act as the followers. The two duopolistic retailers compete on the CSR investment and two retailers' behaviors, Cournot and Collusion, are investigated. In the proposed supply chain, each member's decisions impact on its profitability and the profitability of the other members. Therefore, three scenarios may occur in the supply chain under the centralized model: (1) the retailer benefits in the centralized model while the manufacturer incurs a loss, (2) the manufacturer benefits in the centralized model while two retailers incur a loss, (3) the manufacturer and retailers benefit in the centralized model. All contracts in the literature can coordinate one of the above scenarios while the environmental and social cost sharing contract can coordinate the supply chain under all above scenarios.

The rest of this paper is as follows. A review of literature is represented in section 2. The problem definition and assumptions are prepared in section 3. The model formulation including the decentralized, centralized and coordinated models are developed in section 4. The numerical examples and sensitivity analysis are provided in section 5. Finally, section 6 concludes conclusion and further research directions.

2- Literature review

In this section, a brief review of related literature is presented. Coordination of the supply chain with respect to corporate social responsibility and environmental indicators is one of the new subjects. In order to coordinate the supply chain, various contracts are used in the literature such as quantity discount (Li and Liu, 2006; Li et al., 2016; Johari et al., 2017), delay in payment (Heydari, 2015; Duan et al., 2012; Aljazzar et al., 2016; Hojati et al., 2017), buyback (Hou et al., 2010; Zhao et al., 2014), revenue sharing (Linh and Hong, 2009; Kanda and Deshmukh, 2009; Yang et al., 2017) and collaborative decision-making (Nematollahi et al., 2017). One of the other contracts that is used as an incentive mechanism to coordinate the supply chain is cost sharing contract. The cost sharing contract is commonly applied in different industries such as pharmaceuticals, agricultural, manufacturing, etc. (Yang and Chen, 2017). According to the cost sharing contract, the member who benefits under the centralized model shares in the cost of the other member who incurs a loss under the centralized model. Xu et al., (2017) considered a sustainable supply chain under the cap-and-trade regulation and coordinated the supply chain by wholesale price and cost sharing contract. Dai et al., (2017) compared cartelization and cost sharing contract in a two-echelon green supply chain. Yang and Chen, (2017) investigated the effect of revenue sharing and cost sharing on the carbon emissions abatement and the profitability. In the following, we review the relevant literature in the two main groups: (1) Supply chain coordination and green efforts, (2) Supply chain coordination and corporate social responsibility.

The green supply chain is a hot subject that investigated by the researchers. Swami and Shah, (2013) considered a two-echelon supply chain consisting of one manufacturer and one retailer which in both members invest in the green efforts. Their proposed supply chain coordinated with the two-part tariff contract. Zhang and Liu, (2013) coordinated a three-echelon supply chain using a revenue sharing contract and the manufacturer produced the green products. Zhang et al., (2015) proposed a manufacturer-retailer supply chain that the manufacturer produced two kinds of the products, the traditional and environmental products. A return contract was used to coordinate the supply chain. Basiri and Heydari, (2017) developed a collaboration model to coordinate the manufacturer-retailer supply chain which in the manufacturer produced both green and non-green products. Song and Gao, (2018) investigated a green supply chain consisting of a manufacturer and a retailer that the manufacturer invested to improve the green level of products. The supply chain coordinated with a revenue sharing contract.

There are some studies that coordinate the supply chain by considering the CSR investment such as Ni et al., (2010) considered the CSR investment in a two-echelon supply chain. The cost of the CSR was shared between the upstream and downstream by the wholesale price contract. Panda, (2014) investigated a two-echelon supply chain by considering two cases, CSR manufacturer and CSR retailer. In order to coordinate the supply chain a revenue sharing contract is used. Hsueh, (2014) coordinated a two-echelon supply chain regarding CSR investment via a revenue sharing contract. The market demand affected by the upstream’s CSR. Panda and Modak, (2016) explored the effect of CSR on the coordination of a manufacturer-retailer supply chain. Modak et al., (2016) proposed a three-echelon supply chain consisting of a manufacturer, multiple distributors and multiple retailers. The demand was dependent on the retail price and the manufacturer’s suggested retail price. Nematollahi et al., (2017) developed a collaboration model to coordinate the social responsibility and order quantity in a supplier-retailer supply chain.

According to the reviewed literature and table 1, simultaneous coordination of green quality and CSR investment has not been investigated. Moreover, the competition on the CSR investment is ignored in these papers. To fill the gap, we simultaneously coordinate the environmental and social decisions in a two-echelon competitive supply chain which in the manufacturer invests to increase the green quality of products and two duopolistic retailers invest in the CSR. The two retailers compete on the corporate social responsibility because by growing the consumers’ awareness, the consumers prefer to purchase from the retailer with the high CSR investment. The retailers’ behaviors, Cournot and Collusion, are investigated and explore the effect of them on the decision variables, and the profit of members and entire supply chain. In the proposed supply chain, three scenarios may occur. All the contracts in the literature can coordinate one of the three scenarios. In the current paper, the proposed environmental and social cost sharing contract can coordinate the supply chain under all three scenarios.

Table 1. Comparing the related studies and current paper

Reference	Decisions	Supply chain structure	Demand function	Competition	Coordination mechanism
Swami and Shah, (2013)	Green efforts, wholesale price, retail price	A manufacturer-a retailer	Green efforts and retail price dependent	----	Two-part tariff contract
Basiri and Heydari, (2017)	Green quality, sales effort, retail price	A manufacturer-a retailer	Green quality, sales effort and retail price dependent	----	Collaboration model
Zhu and He (2017)	Green level of products, wholesale price, price	A manufacturer-a retailer in two supply chain	Green level of products, price dependent	Competition on green level and price	Wholesale price, cost sharing contract
Song and Gao, (2018)	Green level of products, wholesale price, retail price	A manufacturer-a retailer	Green level of products and price dependent	----	Revenue sharing contract
Ni et al. (2010)	CSR investment, impact of CSR on the wholesale price, price	A supplier- a retailer	CSR and retail price dependent	----	Wholesale price contract
Hsueh, (2014)	CSR investment, order quantity	A manufacturer-a retailer	Stochastic, CSR investment	----	Revenue sharing contract
Panda, (2014)	Wholesale price, retail price (consumer surplus)	A manufacturer-a retailer	Retail price dependent	----	Revenue sharing contract
Panda and Modak, (2016)	Retail price (consumer surplus)	A manufacturer-a retailer	Retail price dependent	----	Transfer pricing policies
Modak et al., (2016)	Retail price (consumer surplus)	A manufacturer, multiple distributors, multiple retailers	Retail price dependent	----	Revenue sharing contract
Nematollahi et al. (2017)	CSR investment, order quantity	A supplier-a retailer	Stochastic, CSR dependent	----	Economic and social collaboration models
Proposed model	Green quality, CSR investment	A manufacturer-two retailers	Green quality and CSR dependent	Competition on the CSR investment	Environmental and social cost sharing contract

3- Problem definition

In this paper, a two-echelon green supply chain consisting of one manufacturer and two duopolistic retailers is investigated. One kind of product is produced by the manufacturer and is sold via duopolistic competing retailers. Since consumers have paid more attention to protect the environment, the green products are preferred by the consumers. Therefore, the manufacturer attempts to invest in new technology in order to increase the green quality of the products. On the other hand, two duopolistic retailers invest in CSR. The two retailers compete on the CSR investment and the consumers prefer to purchase products from the retailer with more CSR investment. The market demand of products depends on the green quality and two retailers' CSR investment. The manufacturer's decision variable is the green quality of the products and the retailers decide on the CSR investment. In this paper, we investigate the simultaneous coordination of green quality and CSR investment decisions. The proposed supply chain structure is shown in figure 1.

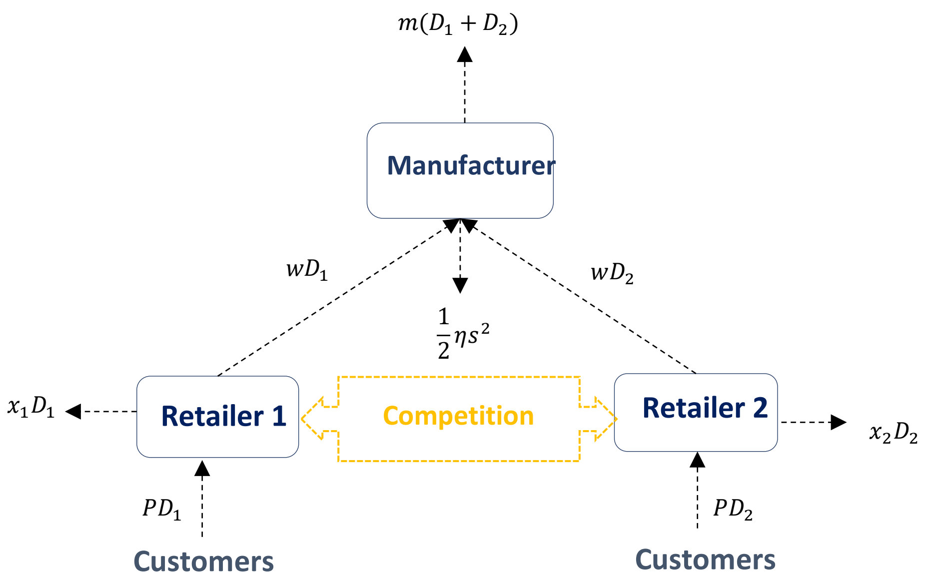


Fig. 1. The investigated supply chain model

3-1- Notation

The parameters and variables that are used in this paper are expressed in table 2.

Table 2: The major parameters and variables that are used in this paper

Parameters	Description
d	Initial demand
τ	Sensitively coefficient of the green quality on the demand, $\tau > 0$
μ	Sensitively coefficient of the CSR investment on the demand, $\mu > 0$
γ	The competitor's sensitively coefficient of the CSR investment on the demand, $\gamma > 0$
η	Cost coefficient of the green quality, $\eta > 0$
p	Retail price
w	Wholesale price
e	Manufacture's unit purchase price
α	Manufacturer's bargaining power
$\Pi_{\mathrm{r}i}$	The profit of ith retailer
Π_m	The manufacturer's profit
Π_{sc}	The whole supply chain profit
Decisions variables	
x_i	Retailer's CSR investment
s	Manufacturer's green quality

Note: the superscripts S-CT, S-CN, CEN, CO-CT and CO-CN in each profit function mark the Stackelberg-Cournot, Stackelberg-Collusion, centralized, coordinated Cournot and coordinated Collusion models, respectively.

3-2- Assumption

The following assumptions are considered in the current paper:

1. The market demand of retailer i, $(i = 1, 2)$ is formulated as $D_i = d + \tau s + \mu x_i - \gamma x_j$. The market demand is a linearly increasing function of the manufacturer's green quality and the CSR investment of retailer i and a linearly decreasing function of the competitor's CSR investment. In the real world, the effect of CSR investment of retailer i is more than the effect of CSR investment of rival retailer on the demand. Therefore, we assume $\mu > \eta$.
2. The quadratic function is considered for the manufacturer's green quality investment. This form of the cost function is applied in the literature such as Basiri and Heydari, (2017); Swami and Shah, (2013).

3. The retailers invest x_i for each unit of product in the CSR investment such as Nematollahi et al., (2017).
4. Under the decentralized model, a Stackelberg game is considered, in which the manufacturer acts as the leader and the two retailers act as the followers. In addition, two duopolistic retailers may follow Cournot or Collusion behaviors.
5. Under the coordination model, the surplus profit will be divided based on the members' bargaining power such as Heydari et al., (2016); Hojati et al., (2017).

4-Mathematical model

In this section, the proposed supply chain is investigated in three structures: (1) decentralized model, (2) centralized model, (3) coordination model. In each model, the optimal value of the decision variables and the profit of supply chain members as well as the profitability of whole supply chain are calculated.

4-1- Decentralized model

Under the decentralized model, each member takes decisions individually to optimize its own profit without considering the other supply chain members. The decisions not only impact on his/her profitability, but also influence the profitability of the other members and the whole supply chain. In this model, a Stackelberg game is used where the manufacturer acts as the leader and two retailers are the followers. First, the manufacturer decides on the green quality, then the retailers determine the CSR investment based on the manufacturer's green effort decision. The two retailers may behave in two ways, (1) Cournot, i.e., two retailers compete with each other, (2) collusion, i.e., two retailers jointly make decisions. In the following, the optimal decisions are obtained under these two retailers' behaviors.

4-1-1- Stackelberg-Cournot model

Under the Stackelberg-Cournot model, the manufacturer acts as the leader and two retailers act as the followers and the duopolistic retailers make decisions independently. In the Stackelberg-Cournot model, the profit function of retailer i is calculated as follows:

$$\Pi_{ri}^{S-CT}(x_i) = (p-w)(d+\tau s+\mu x_i-\gamma x_j) - x_i(d+\tau s+\mu x_i-\gamma x_j) \quad (1)$$

Where the first term demonstrates the retailer's revenue and the second term is the CSR investment cost.
The demand of manufacturer is the sum of the demand of retailers and it is formulated as $D = D_1 + D_2 = 2d + 2\tau s + \big((\mu-\gamma)(x_1+x_2)\big)$. The unit production cost for the manufacturer is e and the manufacturer sells the products at price w to the retailers. In addition, the manufacturer invests in the green quality. According to Basiri and Heydari, (2017), the cost of green quality can be calculated as $\frac{1}{2}\eta s^2$. Therefore, the profit function of the manufacturer is calculated as follows:

$$\Pi_{m}^{S-CT}(s) = (w-e)(2d+2\tau s+(\mu-\gamma)(x_1+x_2)) - \frac{1}{2}\eta s^2 \quad (2)$$

Where the first term is the manufacturer's revenue of selling products to the two retailers. The second term shows the green quality investment.
Proposition 1: In the Stackelberg-Cournot model, the profit function of retailers and the manufacturer are concave with respect to x_i, s and the optimal values are as follow:

$$s^{S\_CT} = \frac{2\tau\mu(w-e)}{\eta(2\mu-\gamma)} \quad (3)$$

$$x_i{}^{S\_CT} = \frac{\eta(2\mu-\gamma)\big(-d+\mu(p-w)\big) - 2\tau^2\mu(w-e)}{\eta(2\mu-\gamma)^2} \quad (4)$$

Proof. *"See Appendix A".*

4-1-2- Stackelberg-Collusion model

Under the Stackelberg-Collusion model, the manufacturer acts as the leader and two retailers act as the followers and the duopolistic retailers jointly make decisions. In the Stackelberg-Collusion model, the profit function of retailers is calculated as follows:

$$\Pi_r^{S-CN}(x_1,x_2) = \Pi_{r1} + \Pi_{r2} \\ = (p-w-x_1)(d+\tau s+\mu x_1-\gamma x_2) + (p-w-x_2)(d+\tau s+\mu x_2-\gamma x_1) \quad (5)$$

Proposition 2: Under Stackelberg-Collusion structure, the profit function of the retailers and the manufacturer are concave with respect to x_1, x_2 and s, and the optimal values are:

$$s^{S-CN} = \frac{\tau(w-e)}{\eta} \quad (6)$$

$$x_1{}^{S-CN} = \frac{\eta\big(-d+(\mu-\gamma)(p-w)\big)-\tau^2(w-e)}{\eta(2\mu-2\gamma)} \quad (7)$$

$$x_2{}^{S-CN} = \frac{\eta\big(-d+(\mu-\gamma)(p-w)\big)-\tau^2(w-e)}{\eta(2\mu-2\gamma)} \quad (8)$$

Proof. *"See Appendix B".*

4.2. Centralized model

Under the centralized model, a decision maker takes decisions in order to optimize the whole supply chain profit. In this model, the decision variables are determined from the whole supply chain viewpoint. In the centralized model, the profit function of whole supply chain is the sum of retailers' and manufacturer's profit function. The profit of whole supply chain is calculated as follows:

$$\Pi_{sc}^{CEN}(x_1,x_2,s) = \Pi_{r1} + \Pi_{r2} + \Pi_m \\ = (p-e)(2d+2\tau s+(\mu-\gamma)(x_1+x_2)) - x_1(d+\tau s+\mu x_1-\gamma x_2) - x_2(d \\ + \tau s+\mu x_2-\gamma x_1) - \frac{1}{2}\eta s^2 \quad (9)$$

Proposition 3: Under the centralized model, the profit function of the supply chain is concave with respect to x_1, x_2 and s and the optimal values are:

$$s^{CEN} = \frac{2\tau[(p-e)(\mu-\gamma)+d]}{(2\mu-2\gamma)\eta-2\tau^2} \quad (10)$$

$$x_1{}^{CEN} = \frac{[\eta(2\mu-2\gamma)-2\tau^2][-d+(\mu-\gamma)(p-e)]-2\tau^2[(p-e)(\mu-\gamma)+d]}{\eta(2\mu-2\gamma)^2-2\tau^2(2\mu-2\gamma)} \quad (11)$$

$$x_2{}^{CEN} = \frac{[\eta(2\mu-2\gamma)-2\tau^2][-d+(\mu-\gamma)(p-e)]-2\tau^2[(p-e)(\mu-\gamma)+d]}{\eta(2\mu-2\gamma)^2-2\tau^2(2\mu-2\gamma)} \quad (12)$$

Proof. *"See Appendix C".*

Under the centralized model, the profit of supply chain optimizes while the profit of members may decrease in this model. Therefore, the member who loses profit doesn't participate in the centralized model.

4-3- Coordinated model

In the coordination model, the profit of supply chain and profit of each member improve in comparison with the decentralized model using an incentive contract. In this section, we propose an incentive contract in order to coordinate the investigated supply chain, either under the Cournot or Collusion behaviors. Under the coordination model, the decision variables are equal to that of the centralized structure. In the proposed supply chain, not only the manufacturer's decision impact on the retailers' performance, but also the retailers' decisions impact on the manufacturer's performance. Under such a case, three scenarios may occur: (1) two duopolistic retailers benefit from the centralized model while the manufacturer incurs a loss, (2) the manufacturer benefits from the centralized model while two duopolistic retailers incur a loss, (3) the manufacturer and two duopolistic retailers benefit from the centralized model. In this paper, an environmental and social cost sharing contract is developed in order to coordinate the supply chain under all three scenarios.

Scenario 1: According to the environmental and social cost sharing contract, if the manufacturer benefits from the centralized model while two duopolistic retailers incur a loss, the manufacturer pays a fraction of the retailers' CSR investment to entice two duopolistic retailers to participate in the coordination model. In this scenario, the profit function of retailer i is as follows:

$$\Pi_{ri}^{CO}(x_i) = (p-w)\big(d+\tau s+\mu x_i-\gamma x_j\big) - (1-\varphi_1)x_i\big(d+\tau s+\mu x_i-\gamma x_j\big) \quad (13)$$

The retailer participates in the coordination model if and only if his/her profit increases in comparison with the decentralized model. Therefore, the retailer's condition for participation in the coordination model is:

$$\Pi_{ri}^{Co}({x_1}^{CEN}, {x_2}^{CEN}, s^{CEN}, \varphi_1) \geq \Pi_{ri}^{Y}({x_1}^{Y}, {x_2}^{Y}, s^{Y}) \tag{14}$$

where Y denotes the retailers' behavior, Cournot or Collusion. The minimum acceptable fraction φ_1 from retailer point of view is:

$${\varphi_1}^{min} = 1 + [\frac{\theta_1 - p + w}{{x_1}^{CEN}}] \tag{15}$$

where$\theta_1 = [(p - w - {x_1}^{Y})(d + \tau s^{Y} + \mu {x_1}^{Y} - \gamma {x_2}^{Y})]/[d + \tau s^{CEN} + \mu {x_1}^{CEN} - \gamma {x_2}^{CEN}]$.

Under this scenario, the manufacturer's profit function is as follows:

$$\Pi_{m}^{Co}(s) = (w - e)(2d + 2\tau s + (\mu - \gamma)(x_1 + x_2)) - \frac{1}{2}\eta s^2 - \varphi_1[x_1(d + \tau s + \mu x_1 - \gamma x_2) + x_2(d + \tau s + \mu x_2 - \gamma x_1)] \tag{16}$$

The manufacturer accepts the developed contract if and only if his/her profit increases in comparison with the decentralized model. Therefore, the manufacturer's condition for participation in the coordination model is:

$$\Pi_{m}^{Co}({x_1}^{CEN}, {x_2}^{CEN}, s^{CEN}, \varphi_1) \geq \Pi_{m}^{Y}({x_1}^{Y}, {x_2}^{Y}, s^{Y}) \tag{17}$$

The maximum acceptable fraction φ_1 from the manufacturer point of view is:

$${\varphi_1}^{max} = \frac{\theta_2}{\theta_3} \tag{18}$$

Where $\theta_2 = (w - e)[(2\tau s^{CEN} + (\mu - \gamma)({x_1}^{CEN} + {x_2}^{CEN})) - (2\tau s^{Y} + (\mu - \gamma)({x_1}^{Y} + {x_2}^{Y}))] + \frac{1}{2}\eta({s^{Y}}^2 - {s^{CEN}}^2)$ and $\theta_3 = {x_1}^{CEN}(d + \tau s^{CEN} + \mu {x_1}^{CEN} - \gamma {x_2}^{CEN}) + {x_2}^{CEN}(d + \tau s^{CEN} + \mu {x_2}^{CEN} - \gamma {x_1}^{CEN})$.

The supply chain can coordinate, if the interval $[{\varphi_1}^{min}, {\varphi_1}^{max}]$ is not empty. Each value of φ_1 in the $[{\varphi_1}^{min}, {\varphi_1}^{max}]$ interval coordinate the supply chain. If φ_1 is equal with ${\varphi_1}^{min}$, the surplus profit is gained by the manufacturer and if φ_1 is equal with ${\varphi_1}^{max}$, the surplus profit is gained by the retailers. The manufacturer's bargaining power against the retailers is α, therefore each retailers' bargaining power is $(1 - \alpha)/2$. In order to calculate an appropriate value of φ_1, a linear function based on the bargaining power is applied as follows:

$$\varphi_1 = \alpha {\varphi_1}^{min} + (1 - \alpha){\varphi_1}^{max} \tag{19}$$

The manufacturer prefers ${\varphi_1}^{min}$ while the two retailers would like to implement ${\varphi_1}^{max}$.

Scenario 2: If the two duopolistic retailers benefit from the centralized model while the manufacturer loses some profits, the two retailers pay a fraction of the manufacturer's green quality investment to entice the manufacturer to participate in the coordination model. Under this scenario, the manufacturer's profit function is as follows:

$$\Pi_{m}^{CO}(s) = (w - e)(2d + 2\tau s + (\mu - \gamma)(x_1 + x_2)) - \frac{1}{2}(1 - 2\varphi_2)\eta s^2 \tag{20}$$

The manufacturer participates in the coordination model if and only if his/her profit increases in comparison with the decentralized model. Therefore, the manufacturer's condition for participation in the coordination model is:

$$\Pi_{m}^{Co}({x_1}^{CEN}, {x_2}^{CEN}, s^{CEN}, \varphi_2) \geq \Pi_{m}^{Y}({x_1}^{Y}, {x_2}^{Y}, s^{Y}) \tag{21}$$

The minimum acceptable fraction φ_2 from manufacturer point of view is:

$${\varphi_2}^{min} = \frac{\theta_4}{{s^{CEN}}^2} + \frac{1}{2} \tag{22}$$

where $\theta_4 = (w - e)[(2\tau s^{Y} + (\mu - \gamma)({x_1}^{Y} + {x_2}^{Y})) - (2\tau s^{CEN} + (\mu - \gamma)({x_1}^{CEN} + {x_2}^{CEN}))] - \frac{1}{2}\eta {s^{Y}}^2$.

In the scenario 2, the profit of retailer i function is calculated as follows:

$$\Pi_{ri}^{CO}(x_i) = (p - w - x_i)(d + \tau s + \mu x_i - \gamma x_j) - \frac{1}{2}\varphi_2 \eta s^2 \tag{23}$$

The coordination model is acceptable for the retailer if and only if his/her profit increases in comparison with the decentralized model. Therefore, the retailer's condition for participation in the coordination model is:

$$\Pi_{ri}^{Co}({x_1}^{CEN}, {x_2}^{CEN}, s^{CEN}, \varphi_2) \geq \Pi_{ri}^{Y}({x_1}^{Y}, {x_2}^{Y}, s^{Y}) \tag{24}$$

The maximum acceptable fraction φ_2 from retailer point of view is:

$${\varphi_2}^{max} = \frac{-2\theta_5}{\eta {s^{CEN}}^2} \tag{25}$$

where $\theta_5 = ((p - w - {x_1}^{Y})(d + \tau s^{Y} + \mu {x_1}^{Y} - \gamma {x_2}^{Y})) - ((p - w - {x_2}^{CEN})(d + \tau s^{CEN} + \mu {x_2}^{CEN} - \gamma {x_1}^{CEN}))$.

If the interval $[{\varphi_2}^{min}, {\varphi_2}^{max}]$ is not empty, the supply chain can coordinate. Each value of φ_2 in the $[{\varphi_2}^{min}, {\varphi_2}^{max}]$ interval coordinate the supply chain. If φ_2 is equal with ${\varphi_2}^{min}$, the surplus profit is gained by the two retailers and if φ_2 is equal with ${\varphi_2}^{max}$, the surplus profit is gained by the manufacturer. The manufacturer's bargaining power is α, therefore each retailers' bargaining power is $(1-\alpha)/2$. In order to calculate an appropriate value of φ_2, a linear function based on the bargaining power is used as follows:

$$\varphi_2 = \alpha {\varphi_2}^{max} + (1-\alpha){\varphi_2}^{min} \tag{26}$$

The manufacturer prefers ${\varphi_2}^{max}$ while the two retailers prefer to implement ${\varphi_2}^{min}$.

Scenario 3: The manufacturer and two duopolistic retailers benefit from the centralized model while the surplus profit may not be shared fairly between the members and the centralized model is not accepted. Under such case, the surplus profit will be divided among the members based on the bargaining power. Under this contract, the surplus profit is obtained in the coordination is:

$$\Delta TP_{sc} = {\Pi_{sc}}^{CEN}({x_1}^{CEN}, {x_2}^{CEN}, s^{CEN}) - {\Pi_{sc}}^{Y}({x_1}^{Y}, {x_2}^{Y}, s^{Y}) \tag{27}$$

It assumed that the manufacture's bargaining power is α and the each retailers' bargaining power is $(1-\alpha)/2$. The profit of each supply chain member improves in proportion to its bargaining power. In the coordination model, the manufacturer's profit share from the surplus profit can be determined as:

$$\alpha\Delta TP_{sc} = \alpha\left({\Pi_{sc}}^{CEN}({x_1}^{CEN}, {x_2}^{CEN}, s^{CEN}) - {\Pi_{sc}}^{Y}({x_1}^{Y}, {x_2}^{Y}, s^{Y})\right) \tag{28}$$

In this scenario, the profit of the manufacturer is calculated as follows:

$${\Pi_m}^{CO} = {\Pi_m}^{Y} + \alpha\Delta TP_{sc} \tag{29}$$

Under the coordination model, the manufacturer's profit is the sum of the manufacturer's profit in the decentralized model (${\Pi_m}^{Y}$) and his profit share from the surplus profit ($\alpha\Delta TP_{sc}$). Under the coordination model, the retailer's profit share from the surplus profit can be calculated as:

$$\frac{(1-\alpha)}{2}\Delta TP_{sc} = \frac{(1-\alpha)}{2}\left({\Pi_{sc}}^{CEN}({x_1}^{CEN}, {x_2}^{CEN}, s^{CEN}) - {\Pi_{sc}}^{Y}({x_1}^{Y}, {x_2}^{Y}, s^{Y})\right) \tag{30}$$

Under the proposed contract, the retailers' profit is calculated as:

$${\Pi_{ri}}^{CO} = {\Pi_{ri}}^{Y} + \frac{(1-\alpha)}{2}\Delta TP_{sc} \tag{31}$$

Under the proposed contract, the retailer's profit is the sum of the retailer's profit in the decentralized model (${\Pi_{ri}}^{Y}$) and his profit share from the surplus profit ($\frac{(1-\alpha)}{2}\Delta TP_{sc}$).

5- Numerical experiments

To demonstrate the proposed model performance, three test problems are investigated. The data of the three test problems are denoted in table 3.

Table 3. Parameters of three investigated test problems

Parameters	Test problem1	Test problem 2	Test problem 3
d	100	150	200
p	30	37	45
w	15	23	25
e	5	10	12
τ	10	8	9
μ	18	22	25
γ	5	7	10
η	45	60	70
α	0.5	0.4	0.6

By running the model, the profit functions and decision variables in the decentralized, centralized and coordination models are computed. As illustrated in table 4, three scenarios may occur. In the test problem 1, under the centralized model, the profit of the retailers improves in comparison with the Stackelberg-Cournot model while the manufacturer incurs a loss. Therefore, the manufacturer's green quality cost is shared by the retailers in order to encourage the manufacturer to participate in the joint decision-making. On the other hand, in the centralized model, the profit of all three members increases in comparison with the Stackelberg-Collusion model. In this case, the surplus profit might not be fairly divided among the supply chain members and they would reject the centralized model. Therefore, in the coordination model, the surplus profit is shared based on the members' bargaining power. In the test problem 2, under the centralized model, the profit of manufacturer increases while two retailers incur a loss in comparison with the Stackelberg-Cournot and Stackelberg-Collusion models. In such a case, the manufacturer pays a fraction of the retailers' CSR investment to entice two retailers to accept the coordination model. In the test problem 3, under the centralized model, the profit of all three members increases in comparison with Stackelberg-Cournot and Stackelberg-Collusion models but the surplus profit might not be shared between the members fairly and they don't accept the coordination model. So, under the coordination model, the surplus profit is divided based on the bargaining power among the members of supply chain.

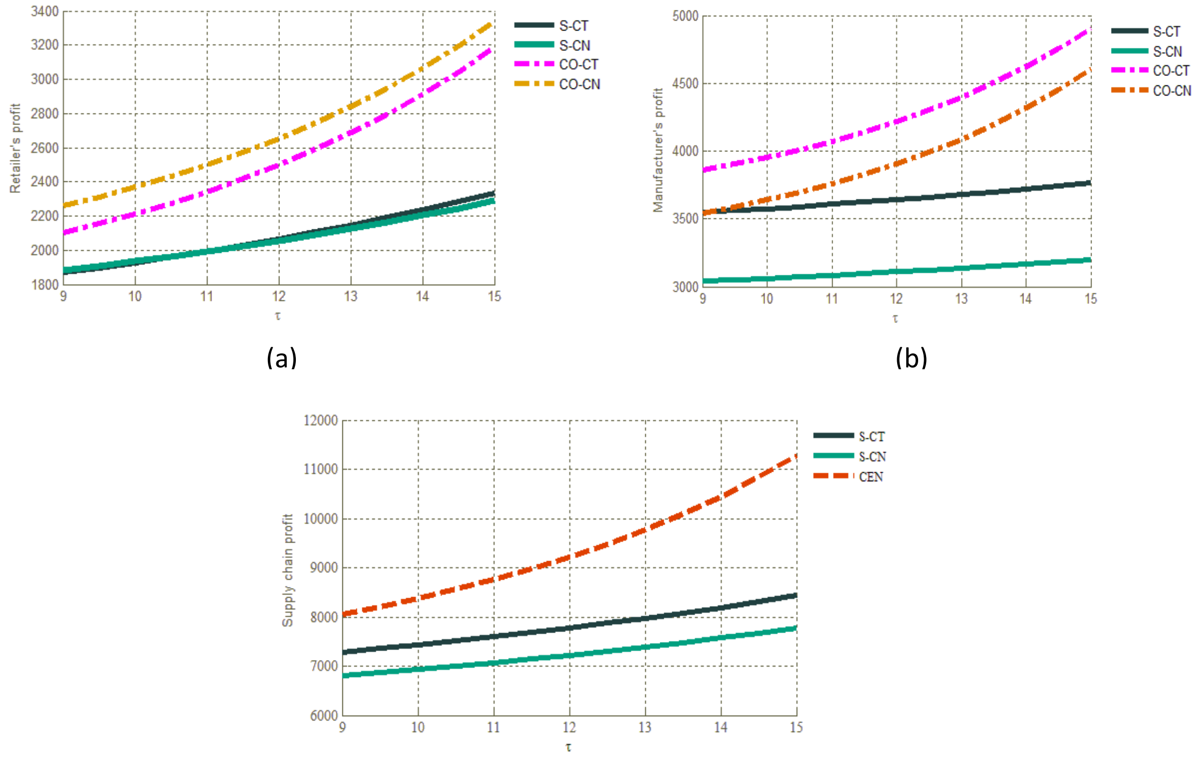
According to table 4, the following observations are obtained:

1. As demonstrated in table 4, under the centralized model, the CSR investment of retailers (x_i) and the green quality of manufacturer (s) increase compared to the Stackelberg-Cournot and Stackelberg-Collusion models in all three test problems. It means that the joint decision-making increases the green level of the entire supply chain. On the other hand, in the Stackelberg-Collusion, the CSR investment of retailers (x_i) and the green quality of manufacturer (s) decrease in comparison with the Stackelberg-Cournot. Therefore, the retailers' cooperation decreases the green level of the supply chain in comparison with the Stackelberg-Cournot model.
2. By comparing the results of the Stackelberg-Cournot and Stackelberg-Collusion models, the profit of retailers improves when two duopolistic retailers cooperate with each other for making decisions on the CSR investment. However, the manufacturer's profit decreases under the retailers' cooperation compared to the Stackelberg-Cournot model.
3. In all test problems, the centralized model improves the profitability of the entire supply chain in comparison with the decentralized model. However, the profit of all members may not improve by shifting from the decentralized to the centralized model. Therefore, they don't accept to participate in the joint decision-making. In such case, an incentive contract can entice members of the supply chain to participate in the joint decision-making and provide a win-win situation for all three members.
4. The developed environmental and social cost sharing contract not only increases the profit of entire supply chain, but also maximizes the profit of all three members compared to the decentralized model. It is obvious that the environmental and social cost sharing contract can coordinate the supply chain under all three scenarios. Under this contract, the surplus profit is divided among the members based on the bargaining power of the members.

Table 4. The result of the running model in the decentralized, centralized and coordination models

	Test problem 1	Test problem 2	Test problem 3
Stackelberg-Cournot			
x_i	4.65	3.82	7.02
s	2.58	2.06	2.08
Π_{ri}	1927.68	2277.85	4205.58
Π_m	3575.65	5692.86	8277.77
Π_{sc}	7431.01	10248.58	16688.93
Stackelberg-Collusion			
x_i	2.79	1.53	2.83
s	2.22	1.73	1.67
Π_{ri}	1935.19	2329.60	4421.15
Π_m	3061.11	4770.13	6597.77
Π_{sc}	6931.49	9429.34	15440.08
Centralized			
x_i	5.28	7.08	7.89
s	8.76	5.31	6.45
Π_{ri}	2490.47	2066.19	4557.43
Π_m	3398.55	6921.14	8331.85
Π_{sc}	8379.51	11053.52	17446.72
Coordination-S-CT			
φ_{min}	0.05	0.10	----
φ_{max}	0.32	0.29	----
φ	0.18	0.21	----
Π_{ri}	2164.80	2519.34	4357.13
Π_m	4049.89	6014.84	8732.44
Π_{sc}	8379.51	11053.52	17446.72
Coordination-S-CT			
φ_{min}	----	0.12	----
φ_{max}	----	0.50	----
φ	----	0.35	----
Π_{ri}	2297.19	2816.86	4822.48
Π_m	3785.11	5419.80	7801.76
Π_{sc}	8379.51	11053.52	17446.72

In the following, a set of sensitivity analysis on the key parameters is examined. The data of test problem 1 is taken for sensitivity analyses. Figure 2 demonstrates the changes in the profitability of retailers, manufacturer and entire supply chain as τ changes. As illustrated in figure 2(a) and 2(b), the profitability of the retailers and the manufacturer increase by increasing τ in all models. Under all values of τ, the retailers' and manufacturer's profit in the coordinated Cournot model are more than the Stackelberg-Cournot and also the retailers' and manufacturer's profit in the coordinated Collusion are more than the Stackelberg-Collusion. Moreover, under the high values of τ, the difference between the retailers' profit (and the manufacturer's profit) in the coordination model and decentralized model increases. Therefore, the developed contract is applicable from retailers' and manufacturer's viewpoint. According to figure 2(c), the profit of entire supply chain improves by increasing τ. Under the centralized model, the profit of supply chain is more than the Stackelberg-Cournot and Stackelberg-Collusion models, and also under the high level of τ, the difference between the centralized and decentralized model increases. Therefore, the centralized model is profitable from the entire supply chain.



(c)

Fig. 2. Changes the retailer's profit, manufacturer's profit, and supply chain profit by increasing the sensitively coefficient of the green quality (τ) in structures

The changes in the profitability of retailers, manufacturer and supply chain by changing μ, γ are shown in figure 3. According to figure 3(a) and 3(b), under all values of μ, γ, in the coordinated Cournot model the profitability of retailers and manufacturer are greater than the Stackelberg-Cournot model and also in the coordinated Collusion model the profitability of retailers and manufacturer are greater than the Stackelberg-Collusion model. It reveals the applicability of the proposed contract from retailers' and manufacturer's viewpoint. As demonstrated in figure 3(c), in all values of μ, γ, the profit of entire supply chain in the centralized model is more than the Stackelberg-Cournot and Stackelberg-Collusion models. By increasing μ (sensitively coefficient of the CSR investment on the demand) and decreasing γ (the competitor's sensitively coefficient of the CSR investment on the demand), the profit of the supply chain improves.

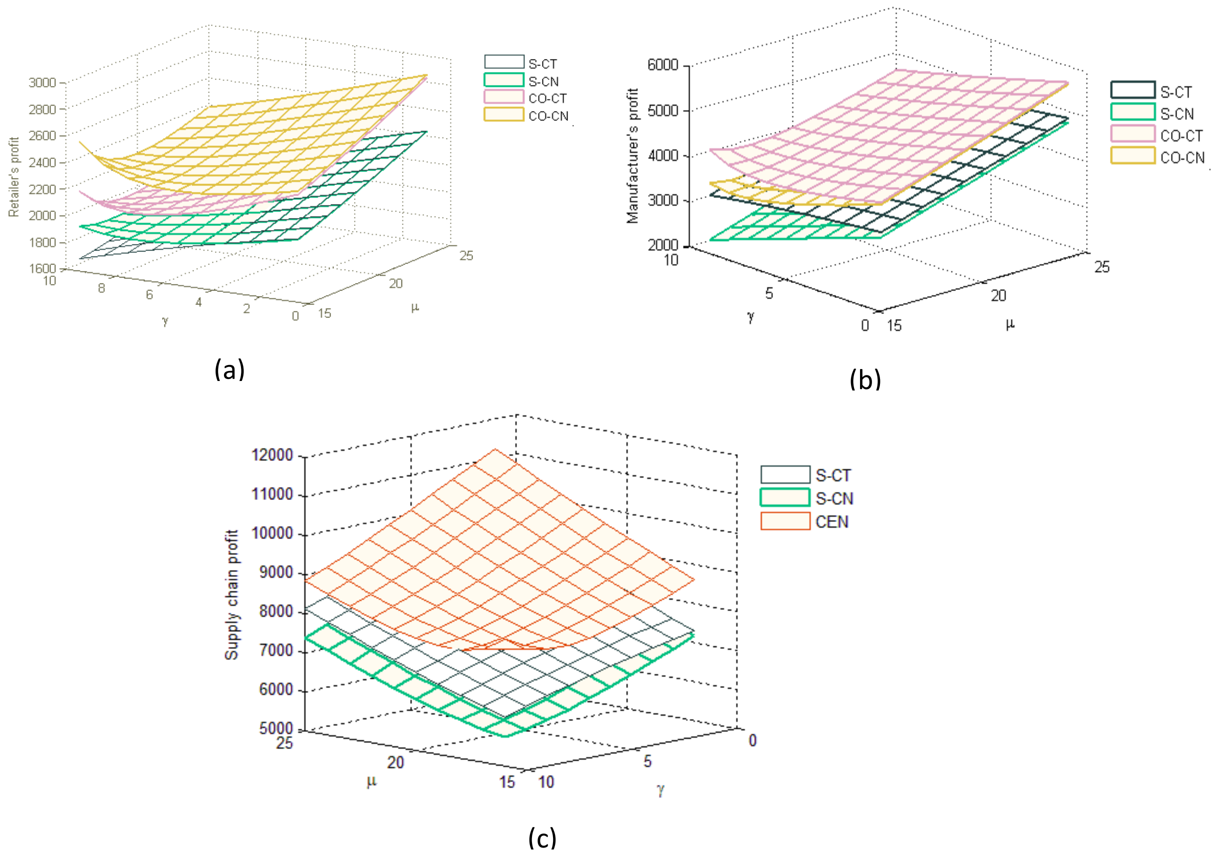


Fig. 3. Changes the retailer's profit, manufacturer's profit, and supply chain profit by changes in the sensitively coefficient of the CSR and sensitively coefficient of the competitor's CSR

Figure 4 denotes the changes of supply chain profit by increasing η. By increasing η, the profit of entire supply chain decreases in the centralized and decentralized models. However, in the centralized model, the profit of supply chain is greater than the Stackelberg-Cournot and Stackelberg-Collusion. Under all values of η, the joint decision-making is more profitable for the supply chain in comparison with the Stackelberg-Cournot and Stackelberg-Collusion.

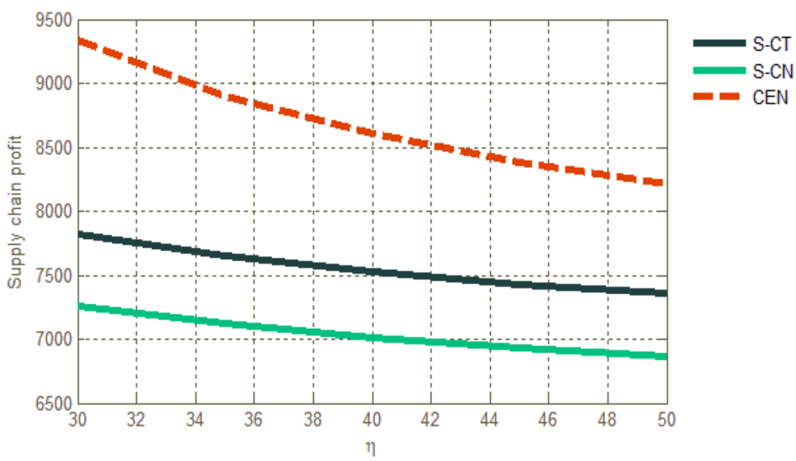


Fig. 4. Changes the retailer's profit, manufacturer's profit, and supply chain profit by increasing the cost of green quality

Changes the green quality by increasing η is shown in figure 5. According to figure 5, the green quality decreases in all models while the amount of green quality in the centralized model is more than the Stackelberg-Cournot and Stackelberg-Collusion models. Therefore, by increasing η, the green level of supply chain decreases in all models while the joint decision-making model creates a greener supply chain in comparison with the Stackelberg-Cournot and Stackelberg-Collusion models. The joint decision-making is capable of improving the green level of the supply chain.

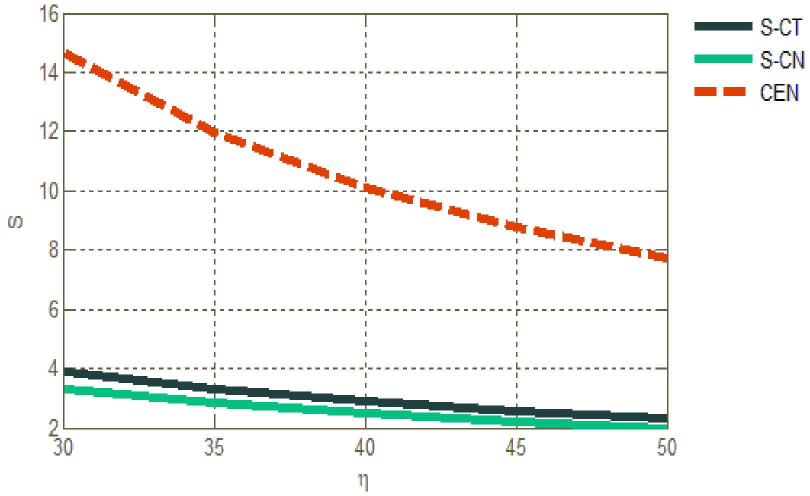


Fig. 5. Changes the green quality by increasing the cost of green quality

Under the coordination model, the surplus profit is shared among the members based on the members' bargaining power. The changes of members' surplus profit by changes the bargaining power are demonstrated in figure 6. By increasing the manufacturer's bargaining power, the manufacturer's share of surplus profit increases and the retailers' share of surplus profit decrease. If the manufacturer's bargaining power is equal to zero, the surplus profit is divided between the two retailers and the profit of the manufacturer is equal to the decentralized model. If the manufacturer's bargaining power is equal to one, the manufacturer gains the surplus profit and the retailers' profit are equal to the decentralized model.

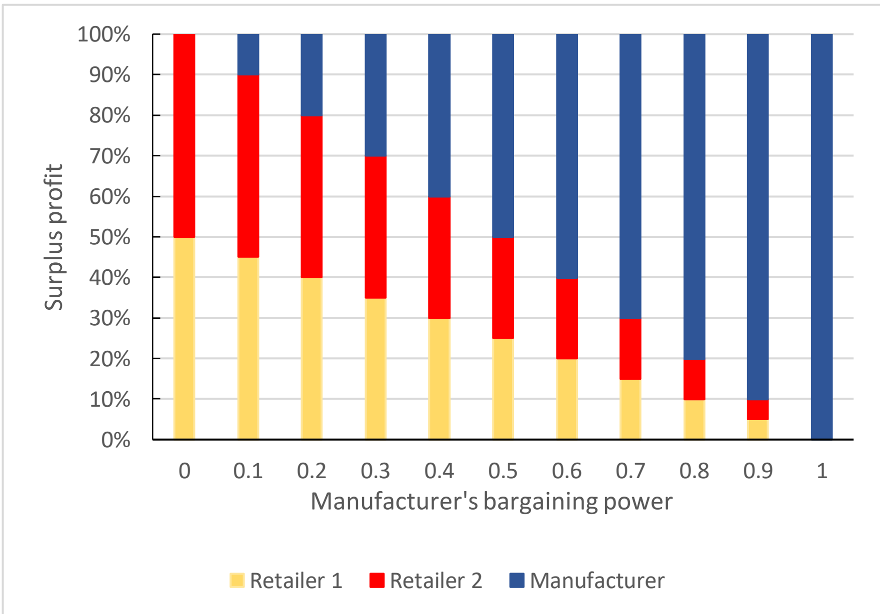


Fig. 6. Changes the retailer's surplus profit, manufacturer's surplus profit, and supply chain surplus profit by changes the manufacturer's bargaining power

6- Conclusion

In this paper, an incentive contract was developed to coordinate the environmental and social decisions in a manufacturer-duopolistic retailers green supply chain. In the proposed supply chain, the manufacturer invested in the technology to increase the green level of the products. On the other hand, two retailers invested in the CSR and compete on it with each other. The market demand was dependent on the green quality of manufacturer and the CSR of the retailer and the CSR of the rival retailer. The supply chain investigated in three different models: (1) decentralized, (2) centralized, (3) coordination models. In the decentralized model, the two Stackelberg-Cournot and Stackelberg-Collusion models are formulated. Under the centralized model, the profitability of entire supply chain is improved compared to the decentralized model while the profitability of some members reduced in comparison to the decentralized model. Therefore, the centralized model wasn’t acceptable for them. An environmental and social cost sharing contract was proposed in order to coordinate the supply chain and guarantee the participation of the members. Under the coordination model, the surplus profit was divided based on the members’ bargaining power. In the proposed supply chain, three different scenarios occurred. The numerical examples illustrated that the developed contract could coordinate the supply chain in all three scenarios. The results demonstrated that: (1) the higher green quality and CSR investment gained in the centralized model. Moreover, in the Stackelberg-Cournot model, the green quality and CSR investment were more than the Stackelberg-Collusion. (2) From the retailers’ viewpoints, the Stackelberg-Collusion was more profitable than the Stackelberg-Cournot. Furthermore, the coordinated Collusion was superior to the coordinated Cournot. (3) From the manufacturer’s viewpoints, the Stackelberg-Cournot was more profitable than the Stackelberg-Collusion. Furthermore, the coordinated Cournot was superior to the coordinated Collusion. (4) Under the coordination model not only the profitability of the members and entire supply chain improved compared to the decentralized model, but also the environmental and social level of the supply chain enhanced. For future research, the current study can be extended by considering multiple supply chains that compete with each other on the environmental issues. Furthermore, in this paper, the demand was considered deterministic while the market demand can be considered stochastic. In addition, a cost sharing contract was used to coordinate the investigated supply chain. Other incentive mechanisms can be implied to coordinate the green quality and the CSR investment decisions. In the current paper, a green supply chain with complete information was investigated. This study can be extended by considering a green supply chain under incomplete information.

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Appendix

Appendix A.

To prove concavity of the retailer's profit function, the first and second derivatives with respect to x_i is taken.

$$\frac{\partial \Pi_{ri}}{\partial x_i} = -d - \tau s - 2\mu x_i + \gamma x_j + \mu(p - w) \tag{A-1}$$

$$\frac{\partial^2 \Pi_{ri}}{\partial {x_i}^2} = -2\mu < 0 \tag{A-2}$$

Since the second order derivative is always negative, therefore the retailer's profit function is concave and the optimal value is calculated as follows:

$$x_1 = \frac{-d - \tau s + \gamma x_2 + \mu(p - w)}{2\mu} \tag{A-3}$$

$$x_2 = \frac{-d - \tau s + \gamma x_1 + \mu(p - w)}{2\mu} \tag{A-4}$$

Thus, by substituting x_1 into Eq. (A-4) and x_2 into Eq. (A-3), we obtain:

$${x_1}^{S\_CT} = \frac{(2\mu + \gamma)(-d - \tau s + \mu(p - w))}{4\mu^2 - \gamma^2} \tag{A-5}$$

$${x_2}^{S\_CT} = \frac{(2\mu + \gamma)(-d - \tau s + \mu(p - w))}{4\mu^2 - \gamma^2} \tag{A-6}$$

By substituting the optimal variables of the retailers in the manufacturer's profit function, the manufacturer profit function is:

$$\Pi_m^{S-CT}(s) = (w - e)\left[2d + 2\tau s + 2(\mu - \gamma)\left(\frac{(2\mu + \gamma)\big(-d - \tau s + \mu(p - w)\big)}{4\mu^2 - \gamma^2}\right)\right] - \frac{1}{2}\eta s^2 \tag{A-7}$$

To prove concavity of the manufacturer's profit function, the first and second derivatives with respect to s is computed.

$$\frac{\partial \Pi_m}{\partial s} = 2\tau(w - e) - \frac{2(\mu - \gamma)\tau}{2\mu - \gamma}(w - e) - \eta s \tag{A-8}$$

$$\frac{\partial^2 \Pi_m}{\partial s^2} = -\eta < 0 \tag{A-9}$$

Since the second order derivative is always negative, therefore the manufacturer's profit function is concave and the optimal value is calculated as follows:

$$s^{S\_CT} = \frac{2\tau\mu(w - e)}{\eta(2\mu - \gamma)} \tag{A-10}$$

By substituting Eq. (A-7) in Eq. (A-3), the optimal value of x_i can be calculated as:

$${x_i}^{S\_CT} = \frac{\eta(2\mu - \gamma)\big(-d + \mu(p - w)\big) - 2\tau^2\mu(w - e)}{\eta(2\mu - \gamma)^2} \tag{A-11}$$

Appendix B.

To prove the concavity of the retailers' profit function with respect to x_1, x_2, the Hessian matrix is calculated.

$$H(\Pi_r) = \begin{bmatrix} \frac{\partial^2 \Pi_r}{\partial {x_1}^2} & \frac{\partial^2 \Pi_r}{\partial x_1 \partial x_2} \\ \frac{\partial^2 \Pi_r}{\partial x_2 \partial x_1} & \frac{\partial^2 \Pi_r}{\partial {x_2}^2} \end{bmatrix}$$

$$\frac{\partial \Pi_r}{\partial x_1} = -d - \tau s - 2\mu x_1 + 2\gamma x_2 + (\mu - \gamma)(p - w) \tag{B-1}$$

$$H_{11} = \frac{\partial^2 \Pi_r}{\partial {x_1}^2} = -2\mu < 0 \tag{B-2}$$

$$\frac{\partial \Pi_r}{\partial x_2} = -d - \tau s - 2\mu x_2 + 2\gamma x_1 + (\mu - \gamma)(p - w) \tag{B-3}$$

$$\frac{\partial^2 \Pi_r}{\partial {x_2}^2} = -2\mu \tag{B-4}$$

$$\frac{\partial^2 \Pi_r}{\partial x_1 \partial x_2} = \frac{\partial^2 \Pi_r}{\partial x_2 \partial x_1} = 2\gamma \tag{B-5}$$

$$H_{22} = 4(\mu^2 - \gamma^2) > 0 \tag{B-6}$$

The first principal minor of Hessian matrix (H_{11}) has a negative value. The second principal minor (H_{22}) is always positive because the effect of CSR investment of retailer i is more than the effect of CSR investment of rival retailer on the demand $(\mu > \eta)$. Therefore, the retailers' profit function is concave and the optimal values are as follows:

$${x_1}^{S-CN} = \frac{-d - \tau s + (\mu - \gamma)(p - w)}{2\mu - 2\gamma} \tag{B-7}$$

$${x_2}^{S-CN} = \frac{-d - \tau s + (\mu - \gamma)(p - w)}{2\mu - 2\gamma} \tag{B-8}$$

By substituting the optimal variables of the retailers in the manufacturer's profit function, the manufacturer profit function is:

$$\Pi_m^{S-CN}(s) = (w - e)\left(2d + 2\tau s + 2(\mu - \gamma)(\frac{-d - \tau s + (\mu - \gamma)(p - w)}{2\mu - 2\gamma})\right) - \frac{1}{2}\eta s^2 \tag{B-9}$$

To prove concavity of the manufacturer's profit function, the first and second derivatives with respect to s is computed.

$$\frac{\partial \Pi_m}{\partial s} = (w - e)(\tau) - \eta s \tag{B-10}$$

$$\frac{\partial^2 \Pi_m}{\partial s^2} = -\eta < 0 \tag{B-11}$$

Since the second order derivative is always negative, therefore the manufacturer's profit function is concave and the optimal value is calculated as follows:

$$s^{S-CN} = \frac{\tau(w - e)}{\eta} \tag{B-12}$$

By substituting Eq. (B-12) in Eqs. (B-7) and (B-8), the optimal value of x_1, x_2 can be calculated as:

$${x_1}^{S-CN} = \frac{\eta\big(-d + (\mu - \gamma)(p - w)\big) - \tau^2(w - e)}{\eta(2\mu - 2\gamma)} \tag{B-13}$$

$${x_2}^{S-CN} = \frac{\eta\big(-d + (\mu - \gamma)(p - w)\big) - \tau^2(w - e)}{\eta(2\mu - 2\gamma)} \tag{B-14}$$

Appendix C.

To prove the concavity of the supply chain profit function with respect to x_1, x_2, s, the Hessian matrix is calculated.

$$H(\Pi_{sc}) = \begin{bmatrix} \frac{\partial^2 \Pi_{SC}}{\partial {x_1}^2} & \frac{\partial^2 \Pi_{SC}}{\partial x_1 \partial x_2} & \frac{\partial^2 \Pi_{SC}}{\partial x_1 \partial s} \\ \frac{\partial^2 \Pi_{SC}}{\partial x_2 \partial x_1} & \frac{\partial^2 \Pi_{SC}}{\partial {x_2}^2} & \frac{\partial^2 \Pi_{SC}}{\partial x_2 \partial s} \\ \frac{\partial^2 \Pi_{SC}}{\partial s \partial x_1} & \frac{\partial^2 \Pi_{SC}}{\partial s \partial x_2} & \frac{\partial^2 \Pi_{SC}}{\partial s^2} \end{bmatrix}$$

$$\frac{\partial \Pi_{sc}}{\partial x_1} = -d - \tau s - 2\mu x_1 + 2\gamma x_2 + (\mu - \gamma)(p - e) \tag{C-1}$$

$$H_{11} = \frac{\partial^2 \Pi_{sc}}{\partial {x_1}^2} = -2\mu < 0 \tag{C-2}$$

The first principal minor of Hessian matrix (H_{11}) has a negative value.

$$\frac{\partial \Pi_{sc}}{\partial x_2} = -d - \tau s - 2\mu x_2 + 2\gamma x_1 + (\mu - \gamma)(p - e) \tag{C-3}$$

$$\frac{\partial^2 \Pi_{sc}}{\partial {x_2}^2} = -2\mu < 0 \tag{C-4}$$

$$\frac{\partial^2 \Pi_{sc}}{\partial x_1 \partial x_2} = \frac{\partial^2 \Pi_{sc}}{\partial x_2 \partial x_1} = 2\gamma \tag{C-5}$$

$$H_{22} = 4(\mu^2 - \gamma^2) > 0 \tag{C-6}$$

The second principal minor of Hessian matrix (H_{22}) has a positive value because the effect of CSR investment of retailer i is more than the effect of CSR investment of rival retailer on the demand ($\mu > \eta$).

$$\frac{\partial \Pi_{sc}}{\partial s} = 2\tau(p - e) - \tau(x_1 + x_2) - \eta s \tag{C-7}$$

$$\frac{\partial^2 \Pi_{sc}}{\partial s^2} = -\eta \tag{C-8}$$

$$\frac{\partial^2 \Pi_{sc}}{\partial x_1 \partial s} = \frac{\partial^2 \Pi_{sc}}{\partial s \partial x_1} = \frac{\partial^2 \Pi_{sc}}{\partial x_2 \partial s} = \frac{\partial^2 \Pi_{sc}}{\partial s \partial x_2} = -\tau \tag{C-9}$$

$$H_{33} = -4\eta(\mu^2 - \gamma^2) + \tau^2(4\mu + 4\gamma) < 0 \tag{C-10}$$

The third principal minor of Hessian matrix (H_{33}) is negative under the following condition.

$$\mu - \gamma > \frac{\tau^2}{\eta} \tag{C-11}$$

The above condition is tested by a wide range of parameters and the data of three test problems as denoted in table C1.

Table C1. Results of third principal minor (H_{33}) under three test problems

Test problems	1	2	3
Third principal minor (H_{33})	-44620	-96976	-135660

Therefore, the supply chain profit function is concave and the optimal values are as follows:

$$s^{CEN} = \frac{2\tau[(p - e)(\mu - \gamma) + d]}{(2\mu - 2\gamma)\eta - 2\tau^2} \tag{C-12}$$

$${x_1}^{CEN} = \frac{[\eta(2\mu - 2\gamma) - 2\tau^2][-d + (\mu - \gamma)(p - e)] - 2\tau^2[(p - e)(\mu - \gamma) + d]}{\eta(2\mu - 2\gamma)^2 - 2\tau^2(2\mu - 2\gamma)} \tag{C-13}$$

$${x_2}^{CEN} = \frac{[\eta(2\mu - 2\gamma) - 2\tau^2][-d + (\mu - \gamma)(p - e)] - 2\tau^2[(p - e)(\mu - \gamma) + d]}{\eta(2\mu - 2\gamma)^2 - 2\tau^2(2\mu - 2\gamma)} \tag{C-14}$$