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Optimal control of quality investment in joint venture by Stackelberg game

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Abstract

In projects carried out as joint venture (*JV*), two or more legally independent firms form a strategic alliance to do a project cooperatively and to obtain the necessary credits asked by a contractor. Due to the wide scope of a joint venture project, partners are sometimes from different areas industrial fields or even different countries which might have different quality standards. Such differences in the quality standard result some difficulties in the problem of quality, which entails sufficient planning to avoid or decrease it. In this paper, a cost sharing coordination mechanism based on a two person Stackelberg game is proposed in which, the more qualified partner that acts as the leader invests in the quality promotion of the other partner who acts as the follower and the costs of investment are shared between the partners according to a contract. Based on the dynamic nature of the quality level and the investment programs, the problem is modeled as an optimal control problem for which the necessary and sufficient conditions of the optimal solution are discussed. Also, based on the Hamilton function of the optimal control problem, some alternatives for the path of investment are considered. Then the path which results in the best gain for the partners according to the leader-follower game is chosen as the solution of the problem. The results show that the optimal path of investment is parameter dependent so the sensitivity analysis is done to show how changes in the parameters affect the best path of investment.

Keywords: Joint venture, coordination mechanisms, cost sharing contract, quality investment, Stackelberg game, optimal control theory

1- Introduction

Nowadays, due to the growth in market competition even at global levels, firms have examined different strategies to increase their capabilities and market shares. In this regard, formation of a strategic alliance with other firms is an emerging strategy which facilitates innovation activities and access to new technologies. Joint Venture (*JV*) as a kind of a strategic alliance has remarkable features which make it expedient for temporary alliances formed to carry out a project. Among these features, ease of formation and legal termination could be mentioned. *JV* is formally defined as "an arrangement where there is commitment of funds, facilities, and services by two or more legally separated interests to an enterprise for their mutual benefits for a long period of time" (Hong and WM Chan, 2014).

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JV provides many advantages for partners such as technology transfer (Girmscheid and Brockmann, 2009), extensions of experiences (Norwood and Mansfield, 1999), entrance to a new market, economies of scale by combining resources (Kazaz and Ulubeyli, 2009), overcoming economic or political barriers and also first accessing to a new market (Sahebi et al., 2015).

There are some classifications for types of *JV*. One of the classifications which is more relevant to the present study is the integrated and the non-integrated *JV* (Norwood and Mansfield, 1999; Garb, 1988). In the integrated *JV*, The partners agree to provide the required resources, manage the *JV* and perform the responsibilities jointly and in an integrated manner (Garb, 1988). However the non-integrated *JV* is similar to the managing and performing a business separately and partners usually cooperate with each other to obtain the required credits of contractors. In the latter case, there might also be agreements on sharing the risk of *JV* by the partners (Badger *et al.*, 1993). When a non-integrated *JV* is formed for the implementation of a project or a construction project, each partner has capabilities that enable it to do some tasks of the project without control and interference of the other partners. It is clear that in such cases, no partner has the authority to affect others and to direct them in its favorites even in a state that it cares about the interests of the others. However there are situations in which the cooperation of the partners is undeniable even though the non-integrated agreement of the partners isn’t self-enforcing to support and encourage such cooperation. In this regard, the quality difficulties of the partners are addressed in this paper that entails cooperation of the firms to improve the quality of the less qualified partner and to avoid postponement of the project completion. Here we propose a cost sharing mechanism as a complementary coordination contract between the partners by which they agree to share the cost of investment in order to enhance the quality of the less qualified partners.

In ISO International Standards, the quality of a product is defined as "the degree to which a set of inherent characteristics fulfills requirements’ (Cao et al, 2009). Nowadays in some markets, the firms compete with each other not only in price but also in quality as one of the most important aspects of the customers' satisfaction. So the quality improvement is the focus of many firms to achieve competitive advantages in the market. It is notable that the final product is the result of many firms' efforts that acts as a connected chain until the final product is prepared. Therefore the firms that concern about the quality of their products also concern about the quality of their suppliers' products and that’s why they invest in their quality improvements (Hsieh and Liu, 2010). There are considerable researches in quality enhancement in the context of the supply chain management and these studies mainly focus on the quality investment efforts within the supply chain, the type of contracts that coordinate the parties and the allocation of the resulting profits in quality investment efforts. For example, Reyniers and Tapiero (1995), Singer et al (2003), Balachandran and Radhakrishnan (2005), Zhu et al (2007), Chao et al (2009), El Ouardighi (2014) and Yan (2015) explored the role of each member's quality in a supply chain's revenue and the different ways that the contract of the quality investment can coordinate the supply chain. However it should be noted that there is a significant difference between a supply chain and a *JV* project in the quality improvement problem. Indeed the aim of the quality improvement in a supply chain is to encourage the customers to buy more products, while in the joint venture projects, the quality efforts are implemented to decrease the quality failure (in the contractor's perspective) and to prevent the delay of the project and its subsequent costs.

In this paper, an innovative project is considered in which a local firm from a developing country forms a *JV* with a foreign firm to do a project. Each partner undertakes one work package of the project and when completed, the final product of the project is processed by assembling the work packages of the partners. Also it is assumed that the final product entails high precision of each work package and this is due to the intent of the local partner's host country as the contractor of the project that wants to implement and exploit the radical innovation project or product. The quality difficulty problem that arises here is the inconsistency of the local partner's quality standards with that of the foreign firm which undertakes high tech part of the project. This inconsistency results in the quality failure of the local partner's work package and it's completion in a longer time until the required standard or tolerance is fulfilled.

Although it is common that the cost of the delay related to each partner is determined and this cost is allocated to the relevant partners, but the non-delay partners are also incurred with some costs which is not due to their negligence even though no one is responsible for them. For example in the case of a foreign firm cooperation in *JV*, the cost of foreign workers' residence in a host country or idle resources cost could come into consideration. Therefore the quality improvement of the less qualified partner in a *JV* project is considered as the both partners concern and that is why a coordination contract is necessary for enhancement the performance of the *JV* and the partners.

The aforementioned problem is based on the challenges of a *JV* construction project reported in Scaringella and Burtschell (2015) and indeed this article was our motivation to investigate the quality investment policy in a non-integrated *JV* such as what is used in the context of supply chain studies. The case study of Scaringella and Burtschell (2015) was about the construction of a football stadium in Iran as a developing country by cooperation of a French firm and a local Iranian firm as a joint venture. As reported in Scaringella and Burtschell (2015), the local Iranian firm was in charge of a steel structure production while the French firm was responsible for project management, design of the roof, erection methods, supply and installation of cables, rods, membrane, and elastomeric bearings of the stadium. The actual tolerance of the steel structure was more than the required tolerance of the French firm as the designer of the stadium. This led to the return of the unqualified structure produced by the local firm in the initial phase of the project. After a while, the French firm found that the local partner is unable to produce the steel structure according to the required quality of the first design and therefore the French firm decided to change the first design to neglect some precise requirements so that the completion of the project is facilitated. But this change also didn't work out, so the French firm finally took part in the quality management and control of the local firms due to the pressure of the contractor. The final strategy led to the improvement of the local partner's quality but passing of the time caused the local partner and French firms to incur high costs.

The aforementioned case study indicates that the quality investment in the less qualified partner is necessary in *JV* projects and if this is planned and agreed in the first stage of the *JV* formation, the possible difficulties will not happen in advance. Therefore the proposed cost sharing contract that specifies quality investment plans in time scope of the project could be a proper tool to coordinate the partners and to provide sufficient motivation for partners to pursue quality efforts plan.

Cost sharing contract is a well-known coordination mechanism in the context of a decentralized decision making such as the non-integrated *JV*. The cost sharing mechanism for coordination of the Decentralized Assembly Supply Chains (Leng and Parlar, 2010), coordination and profit division in a three-echelon supply chain (Panda et al, 2014) and coordination of quality improvement efforts in a supply chain (Cao et al, 2009; He et al, 2016) could be referred as the existing examples. It should be pointed out that in the cost sharing contract and other coordination contracts, game theory acts as a worthwhile tool for the determination of the contract. We refer to the three widely used game theory concepts in parameter setting of a coordination contract which are the Stackelberg game, the synchronized move game and the cooperative game theory. The Stackelberg game arises when two players of game act independently and the more powerful player moves first as the leader of the game. Then the other player as the follower chooses its best response actions based on the leader's action. For example, the cooperative advertising in a supply chain with demand disruption proposed by Huang and li (2001) is among the Stackelberg game based contract. They considered the manufacturer as the leader and the retailers as the followers and used the revenue sharing contract for coordination of them.

In the synchronized move game, the players are similar in power and moves simultaneously. He et al (2016) modeled quality improvement contract between a manufacture and its supplier in both the synchronized move game and the Stackelberg game. Finally the cooperative game theory is exercised when two players decide to integrate and form an alliance. In this case, they use the cooperative game theory concept such as Shapley value or the Nash bargaining solution to divide the profit of integration between themselves. One example of this approach can be found in the revenue sharing contract for multi stage supply chain coordination proposed by Moon et al (2014). They suggested a contract in which all members are willing to determine their decision variables such that the coordinated supply chain resembles an integrated and centralized supply chain. It should be noted that in all of the games considered above, the aim is to propose a contract that directs the players towards the integrated decisions which improve their individual and whole performance by providing sufficient motivations. Because of the fact that in the non-integrated *JV*, the partners are independent in nature, the quality investment contract addressed in this paper, is categorized as the Stackelberg-game-based mechanism.

Taking into account the aforementioned literature, the contributions of the paper are presented as follow: First, the quality investment problem is defined in the context of the project management using mathematical formulations. We highlight our meaning about the concept of quality in the project and develop the relationship between cost and quality to express the objective function of the problem as a single objective function. Moreover, due to the non-integrated nature of the construction *JV* considered in this paper, the problem is modeled as a bi-level model such that at the first level, the objective function of the foreign firm as the leader is optimized and at the second level, the objective function of the local firm is optimized based on the decisions at the first level. Finally, the problem is modeled as an optimal control problem. Doing so, we have novelty in both the modelling approach and the dynamic nature of quality investment in the project management literature.

In the following, the model of quality investment in the non-integrated *JV* is presented in section 2. The conclusion remark is presented in section 3 and finally the results of the paper and suggestions for future research direction are provided in section 4.

2- Model

In the present study, we consider a foreign firm (***A***) that is engaged with a local firm (***B***) in a non-integrated *JV* to do a project jointly. Here it is assumed that each of them undertakes one part of the project and the final product of the project is completed by integration of the partners' task or the partners' modules. The network of the project is shown in figure (1).

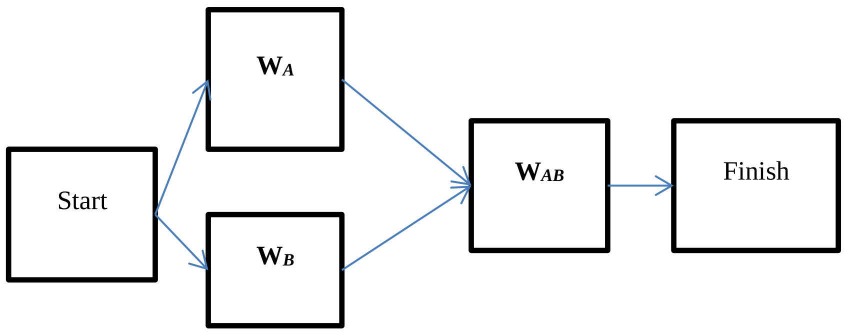

Fig.1. The network of the *JV* project activities

Also the following assumptions are considered in the above-mentioned problem:

1. ***A*** has high technological capabilities and its processes are done by high precision and quality. This high quality is the results of quality management practice, high level of employees training and high level of acceptable quality standard in ***A***'s country.
2. Mainly, implementation and usage of high technology entails high precision and quality i.e. some competitive advantages of ***A*** might be the results of the high quality practice.
3. The quality level of ***B*** as well as the acceptable quality standard in ***B***'s country is significantly much less than that of ***A***.
4. Both work packages W_A and W_B require high quality level and if these requirements aren't satisfied, then the work packages are rejected, the overworking is increased and the project is delayed. This fact is also due to the high quality obligations of the innovative project that brings off by assembling of W_A and W_B to prepare W_{AB} as the final deliverable of the project.
5. The provision that each partner is responsible for the resulting delay of its negligence is identified in the prescript contract between ***A*** and ***B***.
6. The quality improvement efforts resume up to a quality level which is the result of doing tradeoff between investment cost and delay cost of the project. This assumption is somehow in contrast with the quality efforts in competitive market that act as a tool for absorbing the customers.

7. The completion time of the project is equal to sum of the completion time of work package $\mathbf{W}_{B}$ and $\mathbf{W}_{AB}$. The firm ***A*** has sufficient experience to perform its duties but the local firm ***B*** sometimes leads to a delay in the project due to insufficient expertise. Here, it is assumed that if $\mathbf{W}_{B}$ is done by the maximum quality level then there will be no delay in the project but otherwise some delay will be occurred due to the reworking of fault work packages.
8. Although it is agreed that the local firm ***B*** is responsible for the delay cost of the project relevant to its shortcoming but if a delay is happened, the firm ***A*** bears some cost such as opportunity cost or the cost of additional residence in the host county that isn't in charge of the firm ***B***. For such reasons, the firm ***A*** is enthusiastic to participate in the quality improvement effort of the firm ***B*** and to avoid the delay of the project.

Regarding the above assumptions, it is clear that the foreign firm needs to provide incentives for increasing the capacity and quality capabilities of its local partner and by the way decreases some of its incurred cost. In the other hand, when the partners contribute to each other, they strengthen the trust between themselves which is essential for forming and continuing an alliance (Child et al, 2005). However as mentioned in Scaringella and Burtschell (2015), the structure of the non-integrated *JV* doesn't provide the authority for the foreign firm in inducing the local firms to modify and adjust the quality level of its work package. Therefore, a mechanism is needed to encourage the partners to cooperate with each other. In this mechanism, the foreign firms invest in quality improvement programs of the local partner, by providing necessary training, skilled worker, quality control manager, advanced equipment and etc. and share the cost of such investment with the local partner.

To clarify the quality concept and its impact on the completion time of a project, let the failure rate of product failure in time *t* is denoted by $1-\lambda(t)$. Moreover, we assume that in the lack of failure i.e. $\lambda(t)=1$, the completion time of the project is equal to T_c . T_c is identified based on expert knowledge or date base of organization in the context of the project management. Therefore, $\lambda(t)$ could be used as a measure of the quality level in time *t* which is measured by the ratio of the non-defective products to the total products in time *t*. Now if the time stream of the project completion is regarded as Figure (2), then from time *t* to *t*+*dt*, when the local partner do the project for *dt* time, it needs additional reworking time equals to $(1-\lambda(t)).dt$ and the fault work of the reworked work also entails the additional time equals to $(1-\lambda(t))^2.dt$ and so on. Therefore, the real time period of doing the project from *t* to *t*+*dt* in the time stream of the project is $dt+(1-\lambda(t)).dt+(1-\lambda(t))^2.dt+\ldots=\dfrac{dt}{1-(1-\lambda(t))}=\dfrac{dt}{\lambda(t)}dt+\dfrac{(1-\lambda(t))}{\lambda(t)}.dt$. This means that each dt period of the project started at any time *t*, needs additional time equal to $\dfrac{(1-\lambda(t))}{\lambda(t)}.dt$. Summing up such time periods by using an integral term, leads to the real completion time of the project equal to $\int_0^{T_C}\dfrac{dt}{\lambda(t)}$.

Fig. 2. The time stream of the project completion

So, the quality improvement problem in this paper is about the investment in the quality level that leads to decrease in the failure rate and consequently leads to save in the time and cost of the project. After description of the problem, the notation as below is considered to explain the coordination mechanism of the non-integrated JV that improves the quality level and the performance of the *JV*:

π_A the cost function of ***A***

π_B	the cost function of ***B***
$\lambda(t)$	the failure rate of ***B***'s product at time *t* (state variable)
λ_B	the failure rate of ***B***'s product in the beginning of *JV*
$s(t)$	the investment rate of quality improvement in ***B*** (control variable)
s_0	the initial investment in quality level of ***B***
$S_{\max}$	the maximum rate of investment in quality improvement
α	the elasticity of the quality level with respect to investment function
φ	the percentage of total investment in quality improvement of ***B*** afforded by ***B*** itself.
c_A	the cost of delay that firm ***A*** incurs for each time unit
c_B	the cost of delay that firm ***B*** incurs for each time unit
T_c	project delivery time
r	discount rate

It is notable that in the literature, the cost of quality improvement *S* has been modeled as a quadratic function of the quality level *q* i.e. $\alpha S = q^2$ (Kopalle and Winer, 1996; Gavious and Lowengart, 2012). To take into account the dynamic investment in the quality, the quality level at the beginning of the project is denoted by λ_B and equations (1) and (2) as the following is considered:

$$\alpha.(\int_0^t s(u).du + s_0) = \lambda(t)^2 \tag{1}$$

$$\alpha.s_0 = \lambda_B^2 \tag{2}$$

Now by differentiating equation (1) with respect to *t*, the transition function of the state variable $\lambda(t)$, could be derived as equation (3):

$$\frac{d\lambda(t)}{dt} = \frac{\alpha.s(t)}{2\lambda(t)} \tag{3}$$

$$\lambda(0) = \lambda_B \tag{4}$$

The notations above are used to model the mentioned problem as a leader-follower Stackelberg game based on the following sequence. At first, ***A*** as the leader of the game declares its desired percentage of quality investment $(1-\varphi)$ which contribute to the quality promotion of ***B***. Based on the declared value of $(1-\varphi)$, ***B***; the follower of the game; determines the amount of investment in the quality promotion at any point in time i.e. *s* (*t*). The aim of both leader and follower are to minimize their cost function by solving a bi-level optimization problem as follow:

Model* (1)*: The Leader-Follower model of quality investment in Non-integrated JV

$$\min \pi_A(\lambda(t), s(t), \varphi) = c_A.(\int_0^{T_c} \frac{dt}{\lambda(t)} - T_C) + (1-\varphi).\int_0^{T_c} e^{-rt}.s(t).dt \tag{5}$$

$s.t.$

$$0 \le \varphi \le 1 \tag{6}$$

$$\min \pi_B(\lambda(t), s(t)) = c_B.(\int_0^{T_c} \frac{dt}{\lambda(t)} - T_C) + \varphi.\int_0^{T_c} e^{-rt}.s(t).dt \tag{7}$$

$s.t.$

$$\frac{d\lambda(t)}{dt} = \frac{\alpha.s(t)}{2\lambda(t)} \tag{8}$$

$$0 \le \lambda(t) \le 1 \tag{9}$$

$$\lambda(0) = \lambda_B \tag{10}$$

$$0 \le s(t) \le S_{\max} \tag{11}$$

$$s(t) \ge 0$$

The cost function of both partner (equations (5) and (7)) are composed of delay cost of the project and the incurred cost of investment by considering time value of the money using the discount rate *r*. To solve *Model (1),* one could consider the minimization of the objective function in (7) with the constraints (8), (9), (10) and (11) and regard φ as a known parameter to find $\lambda(t)$ and $s(t)$ as functions of φ. Then by replacing the functional value of $\lambda(t)$ and $s(t)$ in the objective function (5) and minimizing it by taking into account constrain (6), the optimal φ could be determined. Doing so, the optimal value of $\lambda(t)$ and $s(t)$ will be specified consequently. Such procedure could be found in He et al (2016).

Also, some notes should be pointed out about the leader and the follower of the game. As mentioned in the problem statement, the quality difficulty problem is mainly the great concern of the local firm that sometimes has low level of quality. This low quality level of the local firm might be incompatible with that of the foreign firms which has high level of quality, high technology capability and high precision of deliverables. Also, we reviewed the study of Scaringella and Burtschell (2015) as our main motivation for developing optimal quality investment model. In that study, it was indicated that the local firm finally went bankrupt due to the low level of quality in its deliverables. On the other hand, we referred to the non-integrated structure of the considered *JV* which are relevant in construction joint ventures and we proposed the quality investment problem for this type of *JV* structure. Based on this discussion, the game between the local firm and the foreign firm will be in a non-integrated manner wherein the foreign firm acts as the leader of the game. In this manner, the role of foreign firm is to contribute in the quality investment program of the local firm in order to provide sufficient motivation for him to do investment more efficiently. Moreover, as the results of the model indicates, where the local firm losses a lot for the sake of low quality, there is no need for the foreign firm to afford investment cost of the local firm i.e. $\varphi = 1$.

Therefore, first, the minimization of the objective function (4) as the second level of the above problem is taken into account. The Hamiltonian function of such optimal control problem will be as equation (12):

$$H(\lambda, s) = \frac{c_B}{\lambda(t)} + (1-\varphi).s(t).e^{-r.t} + \delta(t).\frac{\alpha.s(t)}{2\lambda(t)} \tag{12}$$

The variable $\delta(t)$ is called *adjoint variable* of the Hamilton function which is regarded as a Lagrangian multiplier in the optimal control problem. To derive the optimal solution of an optimal control problem, the sufficient condition is the convexity of the Hamilton function with respect to the control variable (Maurer, 1981). In equation (12), the Hamilton function is convex with respect to control variable $s(t)$. Now the optimal solution is such that the first order conditions of the Hamilton function as follow is hold (Macki & Strauss, 2012):

$$\frac{\partial H(.)}{\partial s} = 0 \tag{13}$$

$$\frac{\partial H}{\partial \lambda} = -\dot{\delta} \tag{14}$$

$$\frac{d\lambda(t)}{dt} = \frac{\alpha.s(t)}{2\lambda(t)} \tag{15}$$

Here it is notable that in the first order condition (13), the variable $s(t)$ isn't existed and this means that we couldn't use equation (13) as a necessary condition for the optimal solution. This is due to the linearity of the Hamilton function with respect to $s(t)$. In such situation, which is called "bang-bang" situation, it is remarkable that the expression (13) is a special case of the more general minimum condition as equation (16) (Anderson &moor, 1971):

$$s^* = \arg\min H(\lambda, s) \tag{16}$$

So, taking into account equation (16), to derive the optimal solution of the problem in the viewpoint of ***B***, the following equations should be satisfied:

$$\frac{\partial H}{\partial \lambda} = -\frac{c_B}{(\lambda(t))^2} - \delta \frac{\alpha.s(t)}{2(\lambda(t))^2} = -\dot{\delta} \tag{17}$$

$$\frac{d\lambda(t)}{dt} = \frac{\alpha.s(t)}{2\lambda(t)} \tag{18}$$

$$H(\lambda, s) = \frac{c_B}{\lambda(t)} + (1-\varphi).s(t).e^{-r.t} + \delta(t).\frac{\alpha.s(t)}{2\lambda(t)} \tag{19}$$

$$0 \le s(t) \le S_{\max} \tag{20}$$

Now, the value of $s(t)$ which maximizes the Hamilton function is determined by conditions which is shown in (21):

$$s(t) = \begin{cases} S_{\max} & \varphi.e^{-r.t} > \dfrac{-\alpha.\delta(t)}{2\lambda(t)} \\ 0 & \varphi.e^{-r.t} \le \dfrac{-\alpha.\delta(t)}{2\lambda(t)} \end{cases} \tag{21}$$

Here, one could use $s(t)$ in (21) to find the quality level $\lambda(t)$ which is denoted by a differential equation in (18). If $\varphi.e^{-r.t} \le \dfrac{-\alpha.\delta(t)}{2\lambda(t)}$, then the equation (18) will be according to equation (22) as follow:

$$\frac{d\lambda(t)}{dt} = 0 \rightarrow \lambda(t) = C_1 \text{ a constant value} \tag{22}$$

Substituting $\lambda(t)$ in equation (22) and $s(t) = 0$ into equation (17), the adjoint variable is identified as equation (23):

$$\delta(t) = \frac{c_B}{C_1^2} t + C_2 \tag{23}$$

Hence, regarding $\lambda(t)$ in equation (22) and $\delta(t)$ in equation (23), it could be said that $s(t)$ is equal to zero whenever inequality (24) is hold:

$$\varphi.e^{-r.t} \leq \frac{-\alpha.(\frac{c_B}{C_1^{\,2}}t + C_2)}{2C_1} = -\alpha.(\frac{c_B}{2C_1^{\,3}}t + \frac{C_2}{2C_1}) \rightarrow G(t, C_1, C_2) = \varphi.e^{-r.t} + \alpha.(\frac{c_B}{2C_1^{\,3}}t + \frac{C_2}{2C_1}) \leq 0 \quad (24)$$

Also if $\varphi.e^{-r.t} > \frac{-\alpha.\delta(t)}{2\lambda(t)}$ then $s(t)$ is equal to the maximum investment rate i.e. S_{max}. So, equation (25) is satisfied in this case:

$$\frac{d\,\lambda(t)}{dt} = \frac{\alpha S_{\max}}{2\lambda(t)} \rightarrow \lambda(t) = (\frac{a S_{\max}}{2}t)^{\frac{1}{2}} + C_2 \quad (25)$$

Where C_1 and C_2 are constant value and they should be determined by considering the boundary conditions of the optimal control problem. It is clear that the boundary condition on the adjoint variable depends on the final condition of the related state variables. Therefore the condition in equation (21) couldn’t be judged preliminary. So, with loss of the generality and for the sake of the simplicity, we turn our attention to the cases where the patterns of the state variables are assumed to be specific and actually we aim to find the optimal parameters of the considered patterns. In this article, four patterns for the state variable $\lambda(t)$ are considered which are depicted in Figures (3) to (6). In all considered patterns, there are only one investment period. Hence, it is enough to specify the start time and the final time of investment for each pattern in order to characterize it. Regarding these patterns, the optimal control problem is converted to a time switching problem for which the switching times should be determined. A switching time is a time in which at least one control variable is changed from its lower bound to its upper bound or vice versa. Finally each pattern that results in the minimum value for the objective function is regarded as the best solution of the aforementioned problem. It should also be noted that all considered patterns are developed by taking into account the necessary conditions of the optimality. Hence, these patterns are good candidates for the optimal solution though might not have the sufficient conditions of the optimality.

2-1- *Policy* (1)

In ***Policy*** **(1)** which is depicted in figure (3), the investment in quality was postponed until time t_1. After t_1, based on equation (22), the investment is started at the maximum rate S_{max} and continues until time t_2. In t_2, the investment is stopped and we don’t have any investment up to the final time *T*.

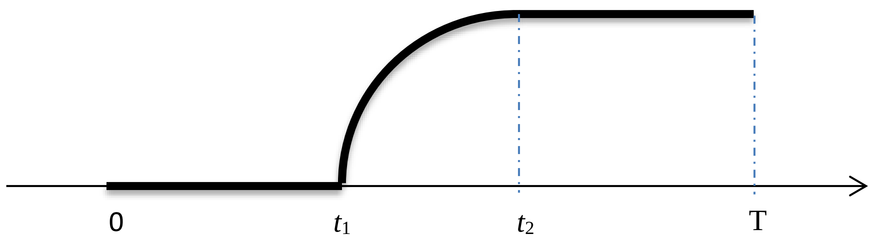

Fig. 3. The first regarded path of the state variable $\lambda(t)$

It should be pointed out that in the final time *T*, two conditions could be occurred. These conditions are relevant not only for policy (1) but also are applicable for all other policies if they are the optimal path of investment. These conditions could be explained based on the *K.K.T* conditions. In *K.K.T*, each inequality constraint is added to the objective function by a multiplier which sometimes are named as "Lagrangian Multiplier". Doing so, if in the optimal solution, the constraint is not hold as equality, then the multiplier should be equal to zero. Such necessary conditions modifies the optimal value of variables such that the constraint is satisfied. Moreover, for finding the optimal solution by regarding this fact, first the multiplier sets to zero assuming that the constraint isn't violated at the optimal solution. However, if the constraint is violated in the optimal solution, then the optimal solution is explored by regarding the constraint as an equality constraint and solving the resulting problem. In the considered problem, we have the inequality $0 \le \lambda(t) \le 1$ as the constraint of the optimal control problem which must be true in all time *t* of the planning horizon. For the Non-descending nature of $\lambda(t)$ based on equation (8), if the constraint is satisfied at the final time *T*, then it is also valid in all other time *t*. Furthermore, it is known that if a state variable is regarded as a free variable at the final time *T*, then its corresponding adjoin variable should be equal to zero at final time *T*. Otherwise, if there are some restrictions or limitations on the state variable at the final time, such limitations are considered as the boundary conditions of the problem and its adjoin variable should not be equal to zero any more.

Therefore, the final state $\lambda(T) = 1$ is regarded for the case of the constraint violation. In the other so, if the constraint (11) isn't violated then we have free final state variable and the boundary condition on the adjoint variable should be considered as $\delta(T) = 0$. Thus, first of all, the problem is solved by considering free final state variable. Then, if the state variable constraint is violated, this solution is regarded as an invalid solution and the problem is solved by considering the final condition $\lambda(T) = 1$. So, in this case, the time switching problem are reached based on the below formulation:

- $0 \le t \le t_1$

$$\lambda(t) = \lambda_B \tag{26}$$

$$\frac{\partial H}{\partial \lambda} = -\frac{c_B}{\lambda_B{}^2} = -\dot{\delta} \rightarrow \delta(t) = \frac{c_B}{\lambda_B{}^2} t + C_1 \tag{27}$$

$$\varphi.\exp(-r.t_1) = -\frac{\alpha.\delta(t_1)}{2\lambda_B} = -\frac{\alpha.(\frac{c_B}{\lambda_B{}^2} t_1 + C_1)}{2\lambda_B} \tag{28}$$

- $t_1 \le t \le t_2$

$$\frac{d\lambda(t)}{dt} = \frac{\alpha.s(t)}{2\lambda(t)} \rightarrow \lambda^2(t) = \frac{\alpha S_{\max}}{2} t + C_2 \xrightarrow{\lambda(t_1)=\lambda_B} C_2 = \lambda_B^2 - \frac{\alpha S_{\max}}{2} t_1 \rightarrow$$
$$\lambda^2(t) = \frac{\alpha S_{\max}}{2}(t - t_1) + \lambda_B^2 \tag{29}$$

$$\frac{\partial H}{\partial \lambda} = -\frac{c_B}{\left(\frac{\alpha S_{\max}}{2}(t-t_1) + \lambda_B^2\right)} - \delta \frac{\alpha S_{\max}}{2\left(\frac{\alpha S_{\max}}{2}(t-t_1) + \lambda_B^2\right)} = -\dot{\delta} \rightarrow \tag{30}$$
$$\delta(t) = \frac{-(2c_B - C_3(2.\lambda_B{}^2 + \alpha S_{\max}(t - t_1)))}{\alpha S_{\max}}$$

$$\varphi.\exp(-r.t_2) = -\frac{\alpha.\delta(t_2)}{2\lambda(t_2)} = -\frac{\alpha.\frac{-(2c_B - C_3(2.\lambda_B{}^2 + \alpha S_{\max}(t_2 - t_1)))}{\alpha S_{\max}}}{2\sqrt{\frac{\alpha S_{\max}}{2}(t - t_1) + \lambda_B^2}} \tag{31}$$

$$\varphi.\exp(-r.t_1) = -\frac{\alpha.\delta(t_1)}{2\lambda(t_1)} = -\frac{\alpha.\frac{-(2c_B - C_3(2.\lambda_B{}^2))}{\alpha S_{\max}}}{2\lambda_B} \tag{32}$$

- $t_2 \le t \le T$

$$\lambda(t)=\lambda(t_2)=\sqrt{\frac{\alpha S_{\max}}{2}(t_2-t_1)+\lambda_B^2} \tag{33}$$

$$\frac{\partial H}{\partial \lambda}=-\frac{c_B}{\lambda^2}=-\dot{\delta}\rightarrow\delta(t)=\frac{c_B}{\frac{\alpha S_{\max}}{2}(t_2-t_1)+\lambda_B^2}t+C_4 \tag{34}$$

$$\varphi.\exp(-r.t_2)=-\frac{\alpha.\delta(t_2)}{2\lambda(t_2)}=-\frac{\alpha.\frac{c_B}{\frac{\alpha S_{\max}}{2}(t_2-t_1)+\lambda_B^2}t_2+C_4}{2\sqrt{\frac{\alpha S_{\max}}{2}(t_2-t_1)+\lambda_B^2}} \tag{35}$$

$$\delta(T)=0\rightarrow\frac{c_B}{\frac{\alpha S_{\max}}{2}(t_2-t_1)+\lambda_B^2}T+C_4=0 \tag{36}$$

Here, there are unknown variables t_1, t_2, C_1, C_3 and C_4 which is determined by solving System equation (1). So, to find the optimal parameters of ***Policy* (1)**, ***Lemma* (1)** is stated as bellow:

Lemma* (*1*)*: The ***System equation* (1)** should be solved in order to find the optimal parameters of the state variable, control variable and adjoint variable of ***Policy* (1)**.

System equation* (1)*:

$$\varphi.\exp(-r.t_1)=-\frac{\alpha.(\frac{c_B}{\lambda_B{}^2}t_1+C_1)}{2\lambda_B} \tag{37}$$

$$\varphi.\exp(-r.t_2)=-\frac{\alpha.\frac{-(2c_B-C_3(2.\lambda_B{}^2+\alpha S_{\max}(t_2-t_1)))}{\alpha S_{\max}}}{2\sqrt{\frac{\alpha S_{\max}}{2}(t_2-t_1)+\lambda_B^2}} \tag{38}$$

$$\varphi.\exp(-r.t_1)=-\frac{\alpha.\delta(t_1)}{2\lambda(t_1)}=-\frac{\alpha.\frac{-(2c_B-C_3(2.\lambda_B{}^2))}{\alpha S_{\max}}}{2\lambda_B} \tag{39}$$

$$\varphi.\exp(-r.t_2)=-\frac{\alpha.\frac{c_B}{\frac{\alpha S_{\max}}{2}(t_2-t_1)+\lambda_B^2}t_2+C_4}{2\sqrt{\frac{\alpha S_{\max}}{2}(t_2-t_1)+\lambda_B^2}} \tag{40}$$

$$\frac{c_B}{\frac{\alpha S_{\max}}{2}(t_2-t_1)+\lambda_B^2}T+C_4=0 \qquad if\ \ \lambda^*(T)\le 1 \tag{41}$$

$$\lambda(T)=1 \qquad otherwise \tag{42}$$

2-2- *Policy* (2)

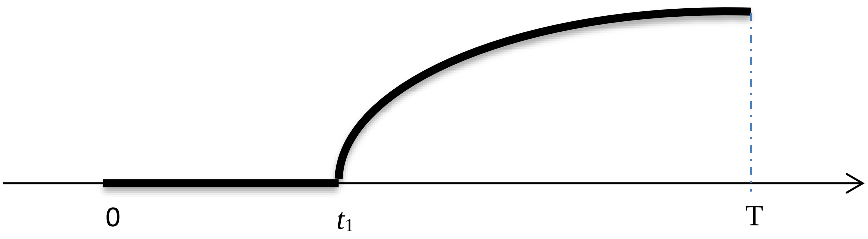

Fig. 4. The second regarded path of the state variable $\lambda(t)$

In ***Policy* (2)** which has been depicted in figure (4), it is assumed that the investment is postponed until time t_1 and after that it starts and doesn't stop until the final time *T*. Again two final conditions might be occurred. If the problem is regarded as free final state variable problem, the constraint $\delta(T)=0$ is imposed. Also if the final state variable $\lambda(T)>1$, then this solution is invalid and we impose the boundary condition $\lambda(T)=1$ instead of $\delta(T)=0$ and solve the problem again. As before if the solution of this policy is invalid, this solution doesn't take into account. In this policy, the time switching problem is reached based on the below formulation:

- $0 \le t \le t_1$

$$\lambda(t)=\lambda_B \tag{43}$$

$$\frac{\partial H}{\partial \lambda}=-\frac{c_B}{\lambda_B{}^2}=-\dot{\delta}\rightarrow \delta(t)=\frac{c_B}{\lambda_B{}^2}t+C_1 \tag{44}$$

$$\varphi.\exp(-r.t_1)=-\frac{\alpha.\delta(t_1)}{2\lambda_B}=-\frac{\alpha.(\frac{c_B}{\lambda_B{}^2}t_1+C_1)}{2\lambda_B} \tag{45}$$

- $t_1 \le t \le T$

$$\frac{d\lambda(t)}{dt}=\frac{\alpha.s(t)}{2\lambda(t)}\rightarrow \lambda^2(t)=\frac{\alpha S_{\max}}{2}t+C_2\xrightarrow{\lambda(t_1)=\lambda_B}C_2=\lambda_B^2-\frac{\alpha S_{\max}}{2}t_1\rightarrow \tag{46}$$

$$\lambda^2(t)=\frac{\alpha S_{\max}}{2}(t-t_1)+\lambda_B^2 \tag{47}$$

$$\frac{\partial H}{\partial \lambda}=-\frac{c_B}{\left(\frac{\alpha S_{\max}}{2}(t-t_1)+\lambda_B^2\right)}-\delta\frac{\alpha S_{\max}}{2\left(\frac{\alpha S_{\max}}{2}(t-t_1)+\lambda_B^2\right)}=-\dot{\delta}\rightarrow \tag{48}$$

$$\delta(t)=\frac{-(2c_B-C_3(2.\lambda_B{}^2+\alpha S_{\max}(t-t_1)))}{\alpha S_{\max}} \tag{49}$$

$$\varphi.\exp(-r.t_1)=-\frac{\alpha.\delta(t_1)}{2\lambda(t_1)}=-\frac{\alpha.\frac{-(2c_B-C_3(2.\lambda_B{}^2))}{\alpha S_{\max}}}{2\lambda_B} \tag{50}$$

$$\delta(T)=0 \rightarrow \frac{-(2c_B - C_3(2.\lambda_B{}^2+\alpha S_{\max}(T-t_1)))}{\alpha S_{\max}}=0 \tag{51}$$

Here, the unknown variables are t_1, C_1 and C_3 that are determined by ***System equation* (2)** as below. Again, if the solution is invalid, the policy won't be considered. Also, ***Lemma* (2)** is stated as the following:

***System equation* (2)**

$$\begin{cases} \varphi.\exp(-r.t_1)=-\dfrac{\alpha.\delta(t_1)}{2\lambda_B}=-\dfrac{\alpha.(\frac{c_B}{\lambda_B{}^2}t_1+C_1)}{2\lambda_B} & (52) \\ \varphi.\exp(-r.t_1)=-\dfrac{\alpha.\delta(t_1)}{2\lambda(t_1)}=-\dfrac{\alpha.\frac{-(2c_B-C_3(2.\lambda_B{}^2))}{\alpha S_{\max}}}{2\lambda_B} & (53) \\ \begin{cases} \dfrac{-(2c_B-C_3(2.\lambda_B{}^2+\alpha S_{\max}(T-t_1)))}{\alpha S_{\max}}=0 & if\ \lambda^*(T)\le 1 \quad (54) \\ \lambda(T)=1 & otherwise \quad (55) \end{cases} \end{cases}$$

Lemma* (2):* The optimal parameters for the state, control and adjoint variable in ***Policy* (2)** is determined by solving ***System equation* (2)**.

2-3- *Policy* (3)

In ***Policy* (3)**, the investment is started from the beginning of the project and continues until the final time T (figure (4)). The following equations are used to determine the optimal state and adjoint variables over time:

- $0 \le t \le T$

$$\frac{d\lambda(t)}{dt}=\frac{\alpha.s(t)}{2\lambda(t)} \rightarrow \lambda^2(t)=\frac{\alpha S_{\max}}{2}t+C_2 \xrightarrow{\lambda(0)=\lambda_B} C_2=\lambda_B^2 \rightarrow \lambda^2(t)=\frac{\alpha S_{\max}}{2}t+\lambda_B^2 \tag{56}$$

$$\frac{\partial H}{\partial \lambda}=-\frac{c_B}{\left(\frac{\alpha S_{\max}}{2}t+\lambda_B^2\right)}-\delta\frac{\alpha S_{\max}}{2\left(\frac{\alpha S_{\max}}{2}t+\lambda_B^2\right)}=-\dot{\delta}\rightarrow \tag{57}$$

$$\delta(t)=\frac{-(2c_B-C_3(2.\lambda_B{}^2+\alpha S_{\max}.t))}{\alpha S_{\max}} \tag{58}$$

$$\delta(T)=0 \rightarrow \frac{-(2c_B-C_3(2.\lambda_B{}^2+\alpha S_{\max}.T))}{\alpha S_{\max}}=0 \tag{59}$$

It is notable that this policy is possible if and only if $\lambda^2(T)=\frac{\alpha S_{\max}}{2}T+\lambda_B^2 \le 1$. If the solution is possible, then ***System equation* (3)** as below should be solved to find the optimal path of the state variable in this policy.

System equation **(3)*:***

$$\begin{cases} \dfrac{-(2c_B - C_3(2.\lambda_B^{\ 2} + \alpha S_{\max} T))}{\alpha S_{\max}} = 0 & if \quad \lambda^*(T) \le 1 & (60) \\ The\ solution\ is\ invalid & otherwise & (61) \end{cases}$$

Hence, ***Lemma*** **(3)** is stated as the following for ***Policy*** **(3)**:

Lemma **(3)***:* The investment in quality as ***Policy*** **(2)** is invalid if $\lambda^2(T) = \dfrac{\alpha S_{\max}}{2} T + \lambda_B^2 \le 1$. When this condition is hold, the parameter of the optimal control problem is determined by solving ***System equation*** **(3)**.

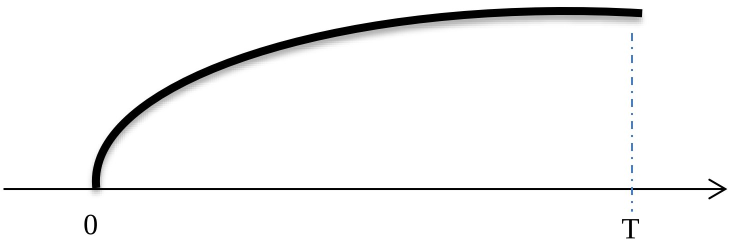

Fig .4. The third regarded path of the state variable $\lambda(t)$

2-4- *Policy* (4)

In figure (5), the path of the state variable in this policy has been depicted. The reason behind considering ***Policy*** **(4)** is that sometimes it is optimal to start investment from the beginning of the project and when the quality gets its highest desired level at time t_1, the firms benefit from this advantage in time saving, cost saving and other positive aspects in advance.

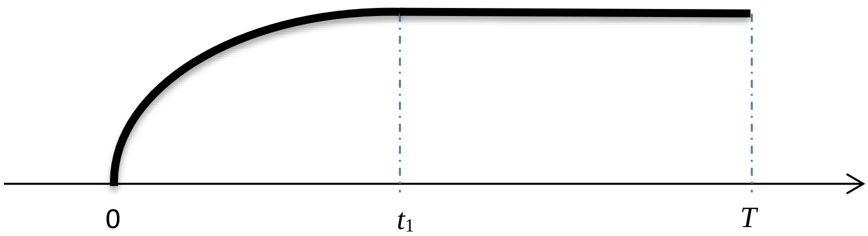

Fig .5 . The forth regarded path of the state variable $\lambda(t)$

In this policy, the following equations should be hold in order to finding the optimal solution.

- $0 \le t \le t_1$

$$\frac{d\lambda(t)}{dt} = \frac{\alpha.s(t)}{2\lambda(t)} \rightarrow \lambda^2(t) = \frac{\alpha S_{\max}}{2} t + C_2 \xrightarrow{\lambda(0)=\lambda_B} C_2 = \lambda_B^2 \rightarrow \lambda^2(t) = \frac{\alpha S_{\max}}{2} t + \lambda_B^2 \qquad (62)$$

$$\frac{\partial H}{\partial \lambda} = -\frac{c_B}{\left(\frac{\alpha S_{\max}}{2}(t) + \lambda_B^2\right)} - \delta \frac{\alpha S_{\max}}{2\left(\frac{\alpha S_{\max}}{2}(t) + \lambda_B^2\right)} = -\dot{\delta} \rightarrow \tag{63}$$

$$\delta(t) = \frac{-(2c_B - C_3(2.\lambda_B{}^2 + \alpha S_{\max}.\,t))}{\alpha S_{\max}} \tag{64}$$

$$\varphi.\exp(-r\,t_1) = -\frac{\alpha.\delta(t_1)}{2\lambda(t_1)} = -\frac{\alpha.\dfrac{-(2c_B - C_3(2.\lambda_B{}^2 + \alpha S_{\max}.\,t_1))}{\alpha S_{\max}}}{2\sqrt{\dfrac{\alpha S_{\max}}{2}t_1 + \lambda_B^2}} \tag{65}$$

- $t_1 \le t \le T$

$$\lambda(t) = \sqrt{\frac{\alpha S_{\max}}{2}t_1 + \lambda_B^2} \tag{66}$$

$$\frac{\partial H}{\partial \lambda} = -\frac{c_B}{\frac{\alpha S_{\max}}{2}t_1 + \lambda_B^2} = -\dot{\delta} \rightarrow \delta(t) = \frac{c_B}{\frac{\alpha S_{\max}}{2}t_1 + \lambda_B^2}t + C_1 \tag{67}$$

$$\delta(T) = 0 \rightarrow \frac{c_B}{\frac{\alpha S_{\max}}{2}t_1 + \lambda_B^2}T + C_1 = 0 \tag{68}$$

In ***Policy*** **(4)**, the unknowns' t_1, C_1 and C_2 should be determined to identify the configuration of the depicted path in figure (5). To do so, the ***System equation*** **(4)** as follow could be solved. Again, if ***System equation*** **(4)** has invalid solution, this policy is aborted. The final Lemma is stated for ***Policy*** **(4)** as below:

System equation **(4)***:*

$$\varphi.\exp(-r\,t_1) = -\frac{\alpha.\delta(t_1)}{2\lambda(t_1)} = -\frac{\alpha.\dfrac{-(2c_B - C_3(2.\lambda_B{}^2 + \alpha S_{\max}.\,t_1))}{\alpha S_{\max}}}{2\sqrt{\dfrac{\alpha S_{\max}}{2}t_1 + \lambda_B^2}} \tag{69}$$

$$\varphi.\exp(-r\,t_1) = -\frac{\alpha.\delta(t_1)}{2\lambda(t_1)} = -\frac{\dfrac{c_B}{\frac{\alpha S_{\max}}{2}t_1 + \lambda_B^2}t_1 + C_1}{2\sqrt{\dfrac{\alpha S_{\max}}{2}t_1 + \lambda_B^2}} \tag{70}$$

$$\delta(T) = \frac{c_B}{\frac{\alpha S_{\max}}{2}t_1 + \lambda_B^2}T + C_1 = 0 \qquad if\ \ \lambda^*(T) \le 1 \tag{71}$$

$$\lambda(T) = 1 \qquad\qquad otherwise \tag{72}$$

Lemma **(4)***:* The optimal parameters for the path of the state, control and adjoint variable in ***Policy*** **(2)** is determined by solving ***System equation*** **(2)**.

2-5-The proposed algorithm

The above results as ***System equation*** **(1)** to ***System equation*** **(4)** are related to decisions of the local partner that has low level of quality. In these systems equations, it is assumed that the decision variable φ of ***A*** is known. Actually, this is a first level variable in *Model* (1) that should be determined by ***A***. However, it is not straightforward to determine the optimal investment path of ***B*** with respect to the parametric value of φ. So in this paper, a heuristic procedure is used to find a nearly optimal solution for the problem. This procedure is described as follow:

Quality Investment determination algorithm

Initialize $\varphi = \varphi^* = 1, \pi_A^* = \infty$, ε (increment in φ).

Step **(1)**: Solve System equation (1)-(4). If the solution of the system equation is valid, consider them as $S_1 = (\lambda_1^*, s_1^*)$, $S_2 = (\lambda_2^*, s_2^*)$, $S_3 = (\lambda_3^*, s_3^*)$, $S_4 = (\lambda_4^*, s_4^*)$. It is notable that S_1, S_2 , S_3 and S_4 are corresponding to the control variable and state variable of Policy (1) to Policy (4) respectively which was explained above. Moreover, if Policy (i) |i=1, 2, 3, 4 is impossible then we set $\pi_B(S_i) = \infty$.

Step **(2)**: Set $(S^*, \lambda^*) = \underset{S_i, \lambda_i; i=1,2,3,4}{\arg\min} \ \pi_B(S_i, \lambda_i)$ and $\pi_B^* = \pi_B(S^*)$.

Step **(3)**: If $\pi_A(S^*, \varphi) \le \pi_A^*$ and $\varphi \le 1$ then

$\pi_A^* = \pi_A(S^*, \varphi)$, $\varphi^* = \varphi$, $\varphi = \varphi - \varepsilon$

If $\varphi < 0$ then

Terminate algorithm

Else

Return to ***Step*** **(1)**

End

End

In the proposed procedure, the foreign firm changes its share of investment i.e. φ iteratively, until no reduction in its cost function is obtained. In fact, the path of investment, $\lambda(t)$ and $s(t)$ are determined in ***Step*** **(2)** and these variables are used to calculate the objective function of ***A*** using equation (5).

3-Numerical example

To illustrate the proposed model, a numerical example is considered and the sensitivity analysis on the model parameters is illustrated. We assume that the model parameters are as table (1):

Table 1 . Value of the parameters for numerical example

Parameters	$S_{\max}$	α	c_A	c_B	T_c	r	λ_B
Value	100 $/time unit	0.0005 ((qualified product/product)^2)/$	0.04 $	0.05 $	100 time unit	0.1 /time unit	0.1 (qualified product/product)

The solution of the problem using the proposed algorithm and the solution without quality investment is shown in table (2):

Table 2 . The comparison of the solution in the numerical example problem with investment in quality and without investment in quality

	φ	Optimal Policy	Constant term of the optimal policy	Start time of investment in the quality	Finish time of investment in the quality	π_A	π_B
Solution with investment in quality	1	Policy 2	$C_1 = 397.2$ $C_2 = 2.61$	60.4	100	27.04	36.13
solution without investment in quality	-	-	-	-	-	50	40

The results in table (2) indicate that the investment in quality could result in significant benefit for both local partner and foreign firms. Also the results show that the foreign firm is reluctant to share investment cost of the local firm. These results are according to the assumption that the foreign firm has more negotiation power than the local firm as suggested in the previous researches.

Also, the proposed procedure finds the best policy among the considered policies. Hence, in Table (2), only the policy (2); the best policy with the best objective function for ***A***; are considered as the leader of game. However, to compare the optimality of ***Policy* (2)** than other Policy, the results of other policies when $\varphi = 1$ are shown in table (3) and as these results indicate, the objective function of ***A*** as the leader of game in ***Policy* (2)** is the best.

Table 3. the cost functions of *A* and *B* for different policies

Policy	Constant term of the optimal policy	Start time of investment in the quality	Finish time of investment in the quality	π_A	π_B
Policy (1)	$C_1 = -200$ $C_3 = -459.3$ $C_4 = -200$	7.73	7.73 (the policy resemble to no-investment in quality)	50	40
Policy (2)	$C_1 = 397.2$ $C_2 = 2.61$	60.4	100	27.04	36.13
Policy (3)	infeasible	-	-	inf	inf
Policy (4)	$C_1 = -0.83$ $C_3 = -1.88$	0	39.6	397.2	983.5

Nonetheless, it could be notable to argue whether changes in the partners' parameters would change the decision variables of the proposed model or not. So, we investigate these conditions using sensitivity analysis on the model parameters. In all cases of the sensitivity analysis shown below, only the considered parameters will be changed and all others parameters are assumed in their initial value shown in table (1).

3-1- Change in the delivery time of the project

The results of the sensitivity analysis on the delivery time are shown in table (4). Moreover, the optimal policy and the share of investment for ***B*** have been depicted in figure (6).

Table 4 . The results of the sensitivity analysis on the delivery time of the project

Project delivery time	Optimal policy	Start time of investment in the quality	Finish time of investment in the quality	φ	$\lambda(T)$
10000	***Policy* (4)**	0	39.6	1	1
5000	***Policy* (4)**	0	39.6	0.99	1
2000	***Policy* (4)**	0	39.6	0.89	1
1500	***Policy* (4)**	0	39.6	0.66	1
1350	***Policy* (4)**	0	39.6	0.62	1
1300	***Policy* (2)**	1260.4	1300	1	1
100	***Policy* (2)**	60.4	100	1	1
80	***Policy* (2)**	53.21	80	1	0.82
70	***Policy* (2)**	53.21	70	1	0.65
60	***Policy* (2)**	52.25	60	0.92	0.43
50	***Policy* (2)**	42.78	50	0.5	0.26
40	***Policy* (2)**	37.14	40	0.18	0.285
35	***Policy* (2)**	32.28	35	0.11	0.27
30	***Policy* (2)**	25.7	30	0.06	0.34
25	***Policy* (2)**	22.07	25	0.04	0.28
20	***Policy* (2)**	14.57	20	0.02	0.38
15	***Policy* (2)**	11.33	15	0.014	0.2722
10	***Policy* (2)**	7.22	10	0.009	0.281
5	***Policy* (2)**	0.93	5	0.004	0.339
1	***Policy* (2)**	0.46	1	0.002	0.15

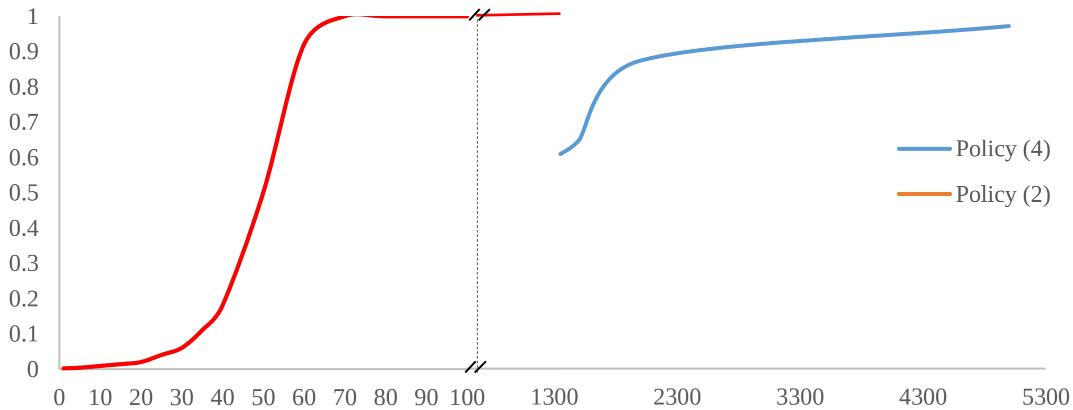

Fig.6 . The optimal share of investment vs the delivery time of the project

The results of table (4) and figure (7) indicate that by increasing in the delivery time of the project from 100 to 1300, neither the optimal policy nor the share of partners in investment will change but the switching time of investment (time t_1 in policy (2)) will change. Also, it is seen when the completion time exceeds 1300, the optimal policy will be changed to policy (4). Indeed when time of the project delivery increases there are motivations by the partner to start the quality investment as soon as possible. For example when delivery time is equal to 1350, ***A*** affords about 40 percent of investment cost in order to motivate the local partner ***B*** for changing its policy to policy (2). This leads to save in time and cost of the project delay. In the aforementioned problem, despite the fact that ***A*** acts as the leader and consequently has more negotiation power than ***B***, it's delay cost is more than that of ***B*** and this is why starting investment at the beginning of the project has more benefit for it than ***B***. Therefore when delivery time is equal to 1350, ***A*** affords about 40 percent of the investment cost. However, as shown in table (4), when the delivery time increases more than 1350, the resulting benefit of starting the investment at the beginning of the project is enough for ***B*** to accept less investment share of ***A*** and be willing to start the investment as soon as possible. When the delivery time decreases from 100, it is seen that there is a time at which ***A*** find the cooperation in the quality investment beneficial. So, ***A*** increases its share of investment to motivate ***B*** in starting the quality investment sooner. For example, when delivery time is equal to 60, ***A*** increase its investment share from 0 to 0.13 in order to encourage ***B*** to change the start time of the investment from 53.13 to 52.25. Also, it is observed that by reduction in the delivery time, the start time of the investment is decreased; the share of ***A*** is increased and this tradeoff between cost and investment for ***A*** continues until it finds the optimal share of its investment.

3-2- Change in the maximum rate of investment

The results of the proposed algorithm for changes in the maximum rate of the investment are shown in table (5). These results indicate that when the maximum rate of investment increases, there is no change in the policy of the investment (***Policy*** **(2)**). However, the initial time for investment is increased which is due to the selection of the ***Policy*** **(2)** as the best investment policy and lack of change in the maximum value of the state variable i.e. $\lambda(T)$. However when the maximum rate of investment decreases from the initial value, this changes first affect the value of the state variable at the final time. This reduction decrease until the time at which the ***Policy*** **(2)** is not optimal anymore and the policy is changed to ***Policy*** **(4)**. Such pattern could be explained by the fact that the investment in the quality always provides benefits for the local partner but he/she always seeks the best way for investment, does tradeoff between cost saving due to reduction in the completion time of the project and the investment cost by regarding time value of the financial resources. So by doing such tradeoff, he first decides to start the investment at a time that results in less value for the final state variable than 1. Then further reduction in maximum rate of investment leads to the reduction in cost saving benefit of quality investment. So with low rate of investment for example when $S_{max} = 0.1$, it is affordable to start investment as soon as possible in order to benefiting the advantages of the quality enhancement.

3-3- Change in the elasticity of the quality level (α)

The results of the changes in the elasticity of the quality level are shown in Table (6). Also, the changes in the share of ***B*** versus the elasticity level have been depicted in Figure (8). The results denote that when the elasticity is low, a reduction in the elasticity has no effect on the optimal investment policy although it could change the start time and the final value of the state variable. However when the elasticity coefficient increases up to 0.05, the benefit of quality promotion in a sooner time is enough to start the investment at the beginning of the project and this leads to converting the policy to ***Policy*** **(4)**.

Table 5. The results of the sensitivity analysis on the maximum rate of the investment

Maximum rate of investment	Optimal policy	Start time of investment in the quality	Finish time of investment in the quality	φ	λ(T)
1000	***Policy* (2)**	96.04	100	1	1
500	***Policy* (2)**	92.08	100	1	1
200	***Policy* (2)**	80.2	100	1	1
100	***Policy* (2)**	60.4	100	1	1
80	***Policy* (2)**	50.85	100	1	0.9964
70	***Policy* (2)**	49.52	100	1	0.9451
60	***Policy* (2)**	48	100	1	0.88
50	***Policy* (2)**	46.19	100	1	0.8261
40	***Policy* (2)**	43.99	100	1	0.75
35	***Policy* (2)**	42.68	100	1	0.7152
30	***Policy* (2)**	41.16	100	1	0.6717
25	***Policy* (2)**	39.38	100	1	0.62
20	***Policy* (2)**	37.2	100	1	0.56
15	***Policy* (2)**	34.41	100	1	0.5
10	***Policy* (2)**	30.5	100	1	0.42
5	***Policy* (2)**	24.02	100	1	0.32
1	***Policy* (2)**	10.63	100	1	0.17
0.1	***Policy* (4)**	0	99.99	1	0.11
0.05	***Policy* (4)**	0	99.999	1	0.106
0.01	***Policy* (4)**	0	99.9999	1	0.102
0.000001	***Policy* (3)**	0	100	1	0.1

Table 6. The results of the sensitivity analysis on the elasticity of the quality level

α	Optimal policy	Start time of investment in the quality	Finish time of investment in the quality	φ	λ(T)
1	Policy (4)	0	0.0198	1	1
0.5	Policy (4)	0	0.0396	1	1
0.05	Policy (4)	0	0.396	1	1
0.005	Policy (2)	96.04	100	1	1
0.0005	Policy (2)	60.4	100	1	1
0.00005	Policy (2)	53.81	100	1	0.354
0.000005	Policy (2)	59.9	100	1	0.14

3-4- Change in the quality level in the beginning of the project

The variations of the quality level at the beginning of the project don't change policy investment and investment share of the partners as shown in table (7). But the tradeoff between investment cost and saving cost of the quality promotion sometimes lead to the reduction in the final value of the state variable for example when $\lambda_B = 0.3$. Also the results in table (7) show that when the final state variable is equal to 1, as much as the quality level in the beginning of the project is higher, the starting time of the quality investment is higher and this is due to the dominance of the ***Policy* (2)** over another policies in the aforementioned problem.

Table 7. The results of the sensitivity analysis on the quality level in the beginning of the project

Quality level in the beginning of the JV	Optimal policy	Start time of investment in the quality	Finish time of investment in the quality	φ	$\lambda(T)$
0.05	***Policy* (2)**	60.1	100	1	1
0.1	***Policy* (2)**	60.4	100	1	1
0.2	***Policy* (2)**	61.6	100	1	1
0.3	***Policy* (2)**	64.9	100	1	0.98
0.4	***Policy* (2)**	68.7	100	1	0.97
0.5	***Policy* (2)**	72.14	100	1	0.97
0.6	***Policy* (2)**	75.5	100	1	0.98.5
0.7	***Policy* (2)**	79.6	100	1	1
0.8	***Policy* (2)**	85.6	100	1	1
0.9	***Policy* (2)**	92.4	100	1	1

3-5- Change in the cost of *B* per unit delay

There are different components of the delay cost for ***B***. For example, ***B*** affords the delay cost of the project, the reworking cost, the residence cost of its employees and resources non-utilization cost. Such cost parameters are related to the time of completion in this paper. For example, when the failed deliverables are useless then the reworking cost per unit time is determined by the cost of production occurred in a unit time. In table (8), the results of the model with respect to the change in c_B are shown. By changing c_B , the share of investment is altered. So, in figure (7), the value of φ versus c_B has been depicted.

The results show that by reduction in c_B , the negotiation power of ***B*** will be increased. This in turn reduces the share of ***B*** in quality investment. Also when the cost of delay increases up to 2 for ***B***, the investment in quality at the beginning of the project leads to more benefits than the postponement of the investment.

Table 8. Share and pattern of partner investment by changing c_B

c_B	Optimal policy	Start time of investment in the quality	Finish time of investment in the quality	φ	$\lambda(T)$
2	***Policy* (4)**	0	39.6	1	1
1.5	***Policy* (4)**	0	39.6	1	1
1	***Policy* (2)**	60.4	100	1	1
0.05	***Policy* (2)**	60.4	100	1	1
0.04	***Policy* (2)**	60.4	100	1	1
0.02	***Policy* (2)**	60.4	100	0.8	1
0.01	***Policy* (2)**	60.4	100	0.4	1
0.005	***Policy* (2)**	60.4	100	0.2	1
0.001	***Policy* (2)**	60.4	100	0.04	1
0.0005	***Policy* (2)**	60.4	100	0.02	1
0.0001	***Policy* (2)**	60.4	100	0.004	1
0.00005	***Policy* (2)**	60.4	100	0.002	1
0.000005	***Policy* (2)**	60.4	100	0.001	1

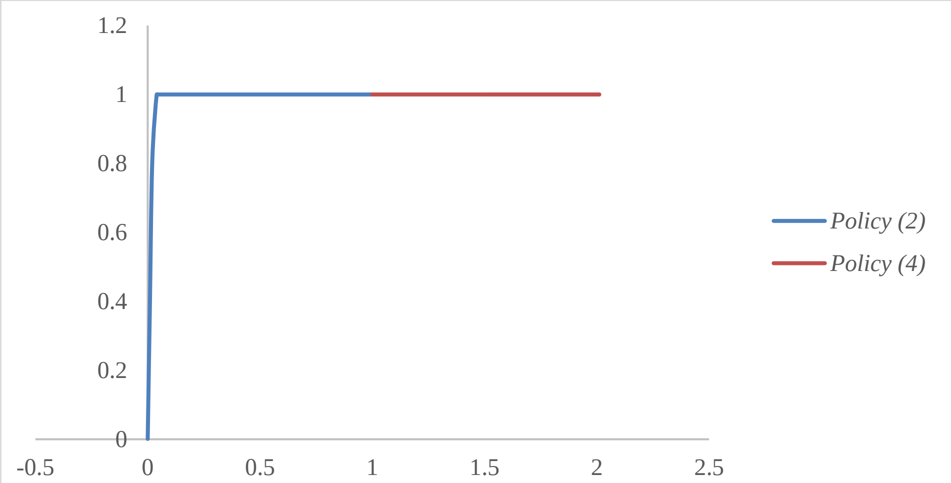

Fig. 7. The optimal share of investment vs the cost of *B*

3-6- Change in the cost of *A* per unit delay

We assumed in this paper that the delay cost of the project are mainly related to the unqualified work packages of ***B***. So, the delay cost of ***A*** are negligible and the components of c_A include the residence cost and the resource non-utilization cost of employees. The results for different value this parameter are presented in table (9).

Table 9. Share and pattern of partner investment by changing c_A

c_A	Optimal policy	Start time of investment in the quality	Finish time of investment in the quality	φ	$\lambda(T)$
3	***Policy* (4)**	0	39.6	0	1
2	***Policy* (4)**	0	39.6	0	1
1.75	***Policy* (2)**	60.4	100	0	1
1.5	***Policy* (2)**	60.4	100	1	1
1	***Policy* (2)**	60.4	100	1	1
0.05	***Policy* (2)**	60.4	100	1	1
0.04	***Policy* (2)**	60.4	100	1	1
0.02	***Policy* (2)**	60.4	100	1	1
0.01	***Policy* (2)**	60.4	100	1	1
0.005	***Policy* (2)**	60.4	100	1	1
0.001	***Policy* (2)**	60.4	100	1	1
0.0005	***Policy* (2)**	60.4	100	1	1
0.0001	***Policy* (2)**	60.4	100	1	1
0.00005	***Policy* (2)**	60.4	100	1	1
0.000005	***Policy* (2)**	60.4	100	1	1

The results of table (9) indicate that two conditions for the optimal policy exist based on the values of c_A . The first one is when c_A is not so much to make the quality investment at the beginning of *JV* beneficial. In this condition, ***A*** also doesn't need to contribute in the quality investment of ***B*** i.e. $\varphi = 1$. However, when this cost exceeds from a threshold value (In this numerical study $c_A = 2$), then ***Policy* (4)** is optimal and ***A*** affords all the quality investment cost because otherwise ***B*** decides to postpone investment and agree on ***Policy* (2)**.

4- Results and discussion

In this paper, the joint investment in the quality promotion of a less qualified partner in a non-integrated joint venture was investigated. This problem was modeled as a Stackelberg leader-follower game wherein the qualified partner acts as the leader and the less qualified partner as the follower of the non-cooperative game. In this article, the investment in quality level was considered as a dynamic problem in which the cost of the partners is composed of the investment cost by regarding the time value of money and the delay cost of the project related to each partner. Here based on the literature review, the dynamic nature of the quality level was generated and the near optimal solution of the problem is searched by utilizing the sufficient and necessary condition of the Hamilton function. However base on the adjoint and state dependent nature of the optimal path investment, the exact solution of the problem was identified to be complex effort. So we explored the solution among some possible paths although the equations of Hamilton function were regarded. We generated a solution procedure to find the good solution and using sensitivity analysis, it was shown that how the partners parameters affects the decisions about the investment in the quality promotion.

The results show that the different setting of the parameters value could yield different result and paths for investment. Such variations in the best chosen path of the investment might be a reason for the applicability of the proposed algorithm in the generation of the best path of the investment. The results of the sensitivity analysis showed the compliance of the proposed approach with the expected nature of the quality investment in the real world problems. For example, when the qualified partner is the leader of game, usually the less qualified partner should be afford all investment cost. Nevertheless when the cost of delay for the less qualified partner is decreased, then the share of the qualified partner in quality investment is increased. This is due to the fact that more investment by the qualified partner will encourage the local partner to increase the quality level and by the way, the foreign firm cost is deceased. But in reality, the need of quality improvement for local partners is more than that of the foreign firms and therefore the quality partner as the leader of the game incurred less cost of investment than local partners.

The joint venture project between an Iranian and a French company that is mentioned in the introduction of the paper, are a good example of the requirement of less qualified partner to engage in quality improvement effort. The aforementioned example was shown that neglecting the quality difficulty between the partners might result in bankruptcy of the less qualified partner and incur considerable cost for qualified partner too. Therefore, the qualified partner could be save the quality related costs by participation in the quality improvement efforts.

For the future researches, the shared investment in the quality improvement problem by considering the intellectual asset rights and the necessary protections against knowledge leakage is suggested. This research concerns is remarkable because sometime shared quality investment might lead to knowledge leakage of the partners and therefore this problem should be taken into account not only by considering the cost of quality but also by regarding the consequences of the knowledge leakages.

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