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Multi-item inventory model with probabilistic demand function under permissible delay in payment and fuzzy-stochastic budget constraint: A signomial geometric programming method

Masoud Rabbani[1*], Leila Aliabadi[1]

[1]*School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*

mrabani@ut.ac.ir , Leyla.aliabadi@ut.ac.ir

Abstract

This study proposes a new multi-item inventory model with hybrid cost parameters under a fuzzy-stochastic constraint and permissible delay in payment. The price and marketing expenditure dependent stochastic demand and the demand dependent the unit production cost are considered. Shortages are allowed and partially backordered. The main objective of this paper is to determine selling price, marketing expenditure, credit period, and variables of inventory control simultaneously for maximizing the total profit. To solve the problem, first some transformations are applied to convert the original problem into a multi-objective nonlinear programming problem, of which each objective has signomial terms. Then, the multi-objective nonlinear programming problem is solved by first converting it into a single objective problem and then by using global optimization of signomial geometric programming problems. At the end, several numerical examples and sensitivity analysis are done to test model and solution procedure and also obtain managerial insights.

Keywords: Signomial geometric programming, delay in payment, fuzzy-stochastic recourse, price and marketing dependent stochastic demand, EOQ.

1- Introduction

By changing market trends and increasing competition in business world, the trade credit is gaining popularity among many retail establishments. Under this policy, sellers offer a specified period to buyers to pay its payments without penalty in order to stimulate sales and decrease the cost of holding inventory. In practice, a permissible delayed payment reduces the holding cost because under this policy the amount of capital invested in inventory during the credit period decreases. Moreover, during the credit period, buyers can accumulate revenue on sales and earn interest on that revenue by banking business or share marketing investment. In today's competition market, most companies use the trade credit strategy to increase the sales and attract more customers. Therefore, the trade credit strategy plays a main role in modern business operations. In recent years, a substantial amount of research has been dedicated to model

*Corresponding author
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inventory policies involving trade credit policy. For the first time, Goyal (1985) developed an EOQ model under permissible delay in payment. Then, Aggarwal and Jaggi (1995) extended this model for deteriorating items. Jamal et al. (1997) first formulated an EOQ model with allowable shortages and permissible delayed payments. Chung and Huang (2003) generalized the model of Goyal (1985) from the EOQ model to the EPQ model. Huang (2007) supposed the supplier would suggest partially permissible delayed payment if the order quantity is smaller than a pre specified quantity. Liang and Zhou (2011) proposed a two-warehouse inventory model for deteriorating item with allowable delay in payments. Taleizadeh et al. (2013) considered an EOQ problem with partial delay in payments and partial backordering. Sarkar et al. (2015) developed an inventory model for deteriorating items under two level trade credit and time - dependent determination rate.

In all above cited articles, it is assumed that demand rate and production cost is constant while these considerations are not true in real world markets. Some researchers considered unit production cost as a function of demand (Islam and Roy 2006; Panda et al. 2008) or order quantity (Samadi et al. 2013; Tabatabaei et al. 2017), or quality (Cheng 1991). Moreover, in real situation, demand rate depends on different parameters such as selling price and marketing expenditure. Pricing is an important strategy for companies to enhance their profit. In fact, there is a negative correlation among selling price and demand rate. That is, demand rate decreases as selling price increases. Ho et al. (2008) analyzed an integrated inventory model with price dependent demand under permissible delay in payment. They determined the optimal ordering, pricing, payment period, and shipping to maximize the total profit. Soni (2013) formulated an inventory model with assumption that demand rate is a multivariate function of selling price and inventory and delay in payment is permitted. Other works that considered price dependent demand and trade credit simultaneously are as follows: Soni and Patel (2012), Maihami and Abadi (2012), Chung et al. (2015), Maihami et al. (2017) and etc.

Apart from the selling price, in most conditions, marketing expenditure is also important in influencing demand. A company can stimulate demand by increasing advertising, hiring more sales people, providing attractive space, and etc. All of those activities are costly. There are a lot of works that have been considered demand rate as a function of marketing expenditure; for example He et al. (2009), Pang et al. (2014), Samadi et al. (2013), De and Sana (2015), Tabatabaei et al. (2017), and etc.

Recently, to better demonstrate the real situation, some researches formulated their models with stochastic demand. He et al. (2009) investigated the issue of supply chain coordination by considering price and marketing dependent stochastic demand. Maihami and Karimi (2014) proposed an EOQ model with price dependent stochastic demand and partial backordering for non-instantaneous deteriorating items. Maihami et al. (2017) developed an pricing inventory model for non-instantaneous deteriorating items with considering partial backordering, price dependent stochastic demand under two- level trade credit policy.

One of the extensions of the inventory models that has received more academic attention in the recent years, is imprecision in defining input parameters. In general, the existing information can be deterministic, fuzzy or probabilistic. Pramanik et al. (2017) developed an inventory model with fuzzy cost parameters under three level trade credit policy and price dependent demand. Das et al. (2004) formulated multi-item stochastic and fuzzy-stochastic inventory models under space and budgetary constraints. In the both models, demand and budgetary resource are considered random. They considered space resource as fuzzy number in fuzzy-stochastic model. But in many real situations, an organization may face situation that several cost parameters may change in such way that a part is random and another part is fuzzy. These cost parameters are called hybrid cost parameters. Panda et al. (2008) proposed two inventory models with hybrid cost parameters. In model 1: They considered resource parameters as fuzzy number; in model 2: some resource parameters were considered as fuzzy stochastic and some as fuzzy. They provided a framework for an EOQ model in fuzzy- stochastic environment and solved their problem by using Geometric Programming (GP) method.

GP problem is a class of non-linear optimization problems that has particular objective functions and constrains. This method has very useful computational and theoretical properties to solve complex optimization problems in different fields such as engineering, management, science, etc. This technique was extended rapidly by researchers, especially engineering designers. Signomial Geometric Programming (SGP) problem was the first extension of GP problems. SGP problems are categorized in class of non- convex optimization problems and NP- hard problems. SGP technique is well used for solving inventory models in literature (Mandal et al. 2006; Samadi et al. 2013; Sadjadi et al. 2015). In this technique degree of difficulty (DD[2]) has an important role. When DD ≤ 2, many researchers have applied dual geometric programing for solving inventory models. But if DD ≥ 3,, solving inventory models will be difficult. Since, the important section SGP is the method used.

A comparison of mentioned papers is illustrated in Table 1. From the Table 1, some of the major shortcomings of previous papers in the formulation of inventory models can be summarized as follows:

- Most inventory models with delayed payments have failed to consider uncertain demand.
- Most previous studies have assumed the unit cost is constant.
- No inventory model with delayed payments is developed in a fuzzy-stochastic environment.
- No inventory model with delayed payments has considered the price and marketing cost dependent demand.

Incorporating all phenomena mentioned above, this paper develops a multi-item EOQ model under budgetary constraint with considering the probabilistic demand and permissible delay in payment in a fuzzy-stochastic environment. Shortages are allowed and partially backordered. We consider the price and marketing expenditure dependent stochastic demand function. We also adopt the demand depended unit production cost. The cost parameters are represented by hybrid numbers and the total budget to purchase inventory is considered as fuzzy-stochastic quantity. The main objective of this paper is to determine selling price, marketing expenditure, credit period, and variables of inventory control simultaneously for maximizing the total profit. For solving our problem, we first convert out model into a multi-objective nonlinear programming (MONP) problem, of which each objective has signomial terms, with using the methods to turn the fuzzy- random parameters to crisp ones. Then, we solve the MONP problem by first converting it into a single objective problem and then by using global optimization method discussed by Xu (2014) for solving SGP problems.

The rest of this paper is been organized as follows: assumptions and notations that are required to model the proposed problem are given in section 2. The mathematical formulation of the problem is presented in Section 3. Section 4 provides the solution method. Numerical examples and sensitivity analysis are done to test model and solution method and also obtain managerial insights in sections 5 and 6. Finally, conclusions with future research are given in section 7.

Table 1. Brief review of mentioned studies

Studies	Unit cost	Demand						DP	FSC	Shortage	
		C	P-M	O	D	S	F			Full	Partial
Huang (2007)	Constant	*			*						
Panda et al. 2008	Demand dependent	*			*				*		
Liang and Zhou (2011)	Constant	*			*			*			
Taleizadeh et al. (2013)	Constant	*			*			*			*
Samadi et al. 2013	Order quantity dependent		*				*			*	
Maihami and Karimi (2014)	Constant		*			*					*
De and Sana (2015)	Constant		*		*		*			*	
Tabatabaei et al. 2017	Order quantity dependent		*		*						
Maihami et al. (2017)	Constant			*		*		*			
Pramanik et al. (2017)	Constant			*	*			*			
This study	Demand dependent		*			*		*	*		*

Note: Constant (C), Price-Marketing dependent (P-M), Other (O), Deterministic (D), Stochastic (S), Fuzzy(F), Delay in Payment (DP), Fuzzy-Stochastic Constraint (FSC).

[1] DD = the number of decision variables + the numbers of terms in objective functions and constraints -1

2- Notation and assumption

We formulate our problem by following notations and assumptions:

2-1- Notations

indices:

i Sets of product types $i = 1.2.3 \dots . n$

Crisp parameters:

I_e Interest earned rate ($/year)

I_p Interest charged rate ($/year)

β_i The percentage of shortages that will be backordered for each item i

C_i Unit purchasing cost of an item ($/unit)

α_i Price elasticity to demand

χ_i Marketing expenditure elasticity to demand

γ_i Demand elasticity to purchasing cost

M_0 Upper limit of credit period

Hybrid parameters:

$\underline{\tilde{A}}_i$ Ordering cost ($/order)

$\underline{\tilde{\pi}}_i$ Backordering cost ($/unit/year)

$\underline{\tilde{g}}_i$ Goodwill loss for unit lost sales

$\underline{\tilde{h}}_i$ Holding cost ($/unit/year)

Fuzzy-stochastic parameter:

$\hat{\tilde{y}}$ Total available production cost

Decision variables:

P_i The portion of demand that will be satisfied from warehouse
T_i The length of an inventory cycle time
S_i The unit selling price of item i
G_i Marketing expenditure per unit of item i
M_i The period of permissible delay in payment of item i (credit period)

Independent decision variable:

λ_i Demand rate of item i
Q_i The order quantity of item i
B_i Partial backordered amount at time T_i

Note: $\sim$ and $\wedge$ denote randomization and fuzzification of the parameters, $\hat{\tilde{y}}$ and $\underline{\tilde{b}}$ denote that y and b are fuzzy-stochastic parameter and hybrid parameter, respectively.

2-2-Assumptions

- The demand rate of item i , $\lambda_i = \lambda_i(S_i.G_i) + \xi_i$, contains two parts:
  - $\lambda_i(S_i.G_i)$: a power function of selling price and marketing expenditure as follows:
    $$\lambda_i(S_i.G_i) = V_i S_i^{-\alpha_i} G_i^{\chi_i} \quad (1)$$
    where V_i is scaling factor and $\alpha_i > 1$ and $\chi_i > 0$ are selling price elasticity and marketing elasticity, respectively.
  - ξ_i: a continuous random variable by specified and time – independent distribution function $E(\xi_i) = \mu_i$.
- Unit cost is a decreasing function of demand rate which is calculated as follows:
  $$C_i = U_i \lambda_i^{-\gamma_i} \quad (2)$$
- Shortages are allowed and are as combination of lost sales and backorders.
- There is no deterioration.
- Replenishment rate is instantaneous and lead time is zero.
- The time horizon is infinite.
- There is a limitation on the total production cost with fuzzy- stochastic quantity.
- For each item, ordering cost, holding cost, and shortage costs $(\underline{\tilde{A}}_i.\underline{\tilde{h}}_i.\underline{\tilde{\pi}}_i.\underline{\tilde{g}}_i)$ are considered as hybrid numbers.
- In the presented supply chain, the retailer purchases the items in each cycle under the trade credit strategy provided by the supplier. It means the supplier gives a full credit period of M_i years for each item to the retailer. During the credit period M_i, the retailer sells the products and collects the sale revenue and obtains interest at a rate I_e ; the retailer must settle the account at time M_i for each item and pays for interest charges on goods in stock with rate I_p.

3- Model formulation

The behavior of the considered inventory system with price and marketing expenditure dependent stochastic demand and demand dependent unit cost under permissible delayed payment is shown in Fig 1. According to Fig 1, the order quantity of item $i, i = 1.2.3 \dots .n$, is obtained as:

$$Q_i = P_i T_i \lambda_i + \beta_i \lambda_i (1 - P_i) T_i = \left(V_i S_i^{-\alpha_i} G_i^{\chi_i} + \xi_i\right)\left(\beta_i + P_i(1 - \beta_i)\right) T_i \quad (3)$$

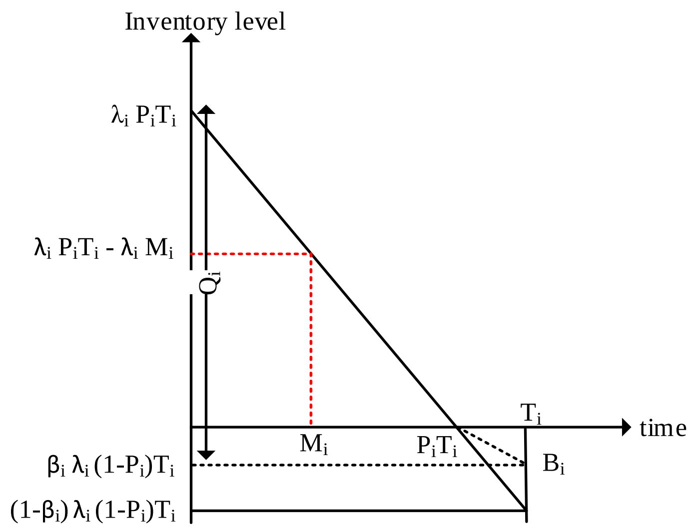


Fig 1. Inventory diagram

The main goal of the problem is to determine the selling price (S_i), marketing expenditure (G_i), credit period (M_i), cycle time (T_i), and the portion of demand that will be satisfied from stock (P_i) so that the total average profit of the inventory system is maximized. So, the following are components of the total annual profit:

The expected sales revenue (SR_i) for the $i$the item per cycle is:

$$SR_i = E(S_iQ_i) = \left(V_iS_i^{-\alpha_i}G_i^{\chi_i} + \mu_i\right)\left(\beta_i + P_i(1-\beta_i)\right)S_iT_i \tag{4}$$

The expected marketing expenditure (CM_i) for the $i$the item per cycle is :

$$CM_i = E(G_iQ_i) = \left(V_iS_i^{-\alpha_i}G_i^{\chi_i} + \mu_i\right)\left(\beta_i + P_i(1-\beta_i)\right)G_iT_i \tag{5}$$

The expected holding cost (CH_i) for the $i$the item per cycle is :

$$CH_i = E\left(\underset{\sim}{\tilde{h}}_i \frac{\lambda_iP_i \times P_iT_i}{2}\right) = 0.5\underset{\sim}{\tilde{h}}\left(V_iS_i^{-\alpha_i}G_i^{\chi_i} + \mu_i\right)P_i^2T_i^2 \tag{6}$$

Where $\underset{\sim}{\tilde{h}}_i = (h_{i1}.h_{i2}.h_{i3})(+)'\left(\mu_{h_i} + \sigma_{h_i}^2\right)$

The expected production cost (CP_i) for the $i$the item per cycle is :

$$CP_i = E(C_iQ_i) = U_i\left(V_iS_i^{-\alpha_i}G_i^{\chi_i} + \mu_i\right)^{1-\gamma_i}\left(\beta_i + P_i(1-\beta_i)\right)T_i \tag{7}$$

The ordering cost (CO_i) for the $i$the item per cycle is :

$$CO_i = \underset{\sim}{\tilde{A}}_i \tag{8}$$

Where $\underset{\sim}{\tilde{A}}_i = (A_{i1}.A_{i2}.A_{i3})(+)'\left(\mu_{A_i} + \sigma_{A_i}^2\right)$

The expected backorder cost (CB_i) for the $i$the item per cycle is :

$$CB_i = E\left(\underset{\sim}{\tilde{\pi}}_i \frac{\beta_i\lambda_i(1-P_i)T_i \times (1-P_i)T_i}{2}\right) = 0.5\underset{\sim}{\tilde{\pi}}_i\beta_i\left(V_iS_i^{-\alpha_i}G_i^{\chi_i} + \mu_i\right)(1-P_i)^2T_i^2 \tag{9}$$

Where $\underset{\sim}{\tilde{\pi}}_i = (\pi_{i1}.\pi_{i2}.\pi_{i3})(+)'\left(\mu_{\pi_i} + \sigma_{\pi_i}^2\right)$

The expected lost sale cost (CL_i) for the $i$the item per cycle is:

$$CL_i = E\left(\underset{\sim}{g}_i(1-\beta_i)\lambda_i(1-P_i)T_i\right) = \underset{\sim}{g}_i(1-\beta_i)\left(V_iS_i^{-\alpha_i}G_i^{\chi_i}+\mu_i\right)(1-P_i)T_i \tag{10}$$

Where $\underset{\sim}{g}_i = (g_{i1}.g_{i2}.g_{i3})(+)'\left(\mu_{g_i}+\sigma_{g_i}^2\right)$

The interest payable per cycle and the interest earned per cycle are calculated by the relationship of credit period (M_i) and the length of time in which no inventory shortage happens(P_iT_i) , hence we consider the following two cases:

Case 1- $M_i \leq P_iT_i$

In this case, the expected interest payable (IP_{1i}) per cycle for the items not sold after the time M_i is as follows (see Fig 2):

$$IP_{1i} = E\left(C_iI_p\frac{\lambda_i(P_iT_i-M_i)\times(P_iT_i-M_i)}{2}\right) = 0.5CU_iI_p\left(V_iS_i^{-\alpha_i}G_i^{\chi_i}+\mu_i\right)^{1-\gamma_i}(P_iT_i-M_i)^{\,2} \tag{11}$$

The expected interest earned (IE_{1i}) per cycle during the positive inventory is as follows (see figure 2):

$$\begin{aligned} IE_{1i} &= E\left(I_eS_i\left(\beta_i\lambda_i(1-P_i)T_iM_i+\frac{\lambda_iM_i^2}{2}\right)\right) \\ &= I_eS_i\left(\beta_i(1-P_i)T_iM_i+0.5M_i^2\right)\left(V_iS_i^{-\alpha_i}G_i^{\chi_i}+\mu_i\right) \end{aligned} \tag{12}$$

Case 2- $P_iT_i \leq M_i \leq M_0$

In this case, the expected interest earned (IE_{2i}) per cycle during $[0.M_i]$ is (see Fig 2):

$$\begin{aligned} IE_{2i} &= E\left(I_eS_i\left(\beta_i\lambda_i(1-P_i)T_iM_i+\frac{\lambda_iP_i^2T_i^2}{2}+\lambda_iP_iT_i(M_i-P_iT_i)\right)\right) \\ &= I_eS_i\left(\beta_iT_iM_i-0.5P_i^2T_i^2+(1-\beta_i)P_iT_iM_i\right)\left(V_iS_i^{-\alpha_i}G_i^{\chi_i}+\mu_i\right) \end{aligned} \tag{13}$$

In this case, the retailer does not need to pay any interest, that is $IP_{2i} = 0$.

Therefore, the average total profit per year for n items for case 1 (ATP_1) and case 2 (ATP_2) is :

$$ATP_j = \sum_{i=1}^{n}\left[\frac{1}{T_i}\left(SR_i-CM_i-CH_i-CP_i-CO_i-CB_i-CL_i-IP_{ji}+IE_{ji}\right)\right] \qquad j = 1.2 \tag{14}$$

After simplification, the following results are obtained:

$$\begin{aligned} ATP_1(x) = \sum_{i=1}^{n}&\Big(N_iX_iS_i-N_iX_iG_i-0.5\left(\underset{\sim}{h}_i+\theta_{1i}\underset{\approx}{\pi}_i\right)X_iP_i^2T_i+\theta_{1i}\underset{\approx}{\pi}_iX_iP_iT_i-0.5\theta_{1i}\underset{\approx}{\pi}_iX_iT_i \\ &-\theta_{2i}\underset{\sim}{g}_iX_i+\theta_{2i}\underset{\sim}{g}_iX_iP_i-\theta_{3i}N_iX_i^{1-\gamma_i}-\theta_{4i}X_i^{1-\gamma_i}P_i^2T_i-\theta_{4i}X_i^{1-\gamma_i}M_i^2T_i^{-1}+2\theta_{4i}X_i^{1-\gamma_i}P_iM_i \\ &+\theta_{5i}X_iS_iM_i-\theta_{5i}X_iS_iM_iP_i+\theta_{6i}X_iS_iM_i^2T_i^{-1}-\underset{\sim}{A}_iT_i^{-1}\Big) \end{aligned} \tag{15}$$

$$\begin{aligned} ATP_2(x) = \sum_{i=1}^{n}&\Big(N_iX_iS_i-N_iX_iG_i-0.5\left(\underset{\sim}{h}_i+\theta_{1i}\underset{\approx}{\pi}_i\right)X_iP_i^2T_i+\theta_{1i}\underset{\approx}{\pi}_iX_iP_iT_i-0.5\theta_{1i}\underset{\approx}{\pi}_iX_iT_i \\ &-\theta_{2i}\underset{\sim}{g}_iX_i+\theta_{2i}\underset{\sim}{g}_iX_iP_i-\theta_{3i}N_iX_i^{1-\gamma_i}+\theta_{5i}X_iS_iM_i-\theta_{6i}X_iS_iM_iP_i^2T_i+\theta_{7i}X_iS_iM_iP_i \\ &-\underset{\sim}{A}_iT_i^{-1}\Big) \end{aligned} \tag{16}$$

Where

$X_i = V_i S_i^{-\alpha_i} G_i^{\chi_i} + \mu_i$ (17- 1)

$N_i = \beta_i + P_i(1 - \beta_i)$ (17- 2)

$\theta_{1i} = \beta_i > 0$ (17- 3)

$\theta_{2i} = 1 - \beta_i > 0$ (17- 4)

$\theta_{3i} = U_i > 0$ (17- 5)

$\theta_{4i} = 0.5 U_i I_p > 0$ (17- 6)

$\theta_{5i} = \beta_i I_e > 0$ (17- 7)

$\theta_{6i} = 0.5 I_e > 0$ (17- 8)

$\theta_{6i} = (1 - \beta_i) I_e > 0$ (17- 9)

$x = (S_i . T_i . G_i . M_i . P_i . X_i . N_i) > 0$ (17- 10)

Whit,

$\underline{\tilde{h}}_i = (h_{i1} . h_{i2} . h_{i3})(+)'(\mu_{h_i} + \sigma_{h_i}^2)$, $\underline{\tilde{\pi}}_i = (\pi_{i1} . \pi_{i2} . \pi_{i3})(+)'(\mu_{\pi_i} + \sigma_{\pi_i}^2)$, $\underline{\tilde{g}}_i = (g_{i1} . g_{i2} . g_{i3})(+)'(\mu_{g_i} + \sigma_{g_i}^2)$, $\underline{\tilde{A}}_i = (A_{i1} . A_{i2} . A_{i3})(+)'(\mu_{A_i} + \sigma_{A_i}^2)$, and $i = 1.2.3 \dots . n$.

As explained above, we consider a limitation on the total budget for purchasing inventory with fuzzy stochastic quantity as follows:

$$\sum_{i=1}^{n} CP_i \leq \hat{\tilde{y}} \;\Rightarrow\; \sum_{i=1}^{n} \theta_{3i} N_i X_i^{1-\gamma_i} T_i \leq \hat{\tilde{y}} \tag{18}$$

Where $\hat{\tilde{y}} = \left(\left((y_1^1 . y_1) . q_1 \right) ; \left((y_2^1 . y_2) . q_2 \right) ; \left((y_3^1 . y_3) . q_3 \right) \right)$.

Therefore, the mathematical model of the problem is:

$$Max \;\; ATP_j \qquad j = 1.2 \tag{19}$$

s.t. $$\sum_{i=1}^{n} \theta_{3i} N_i X_i^{1-\gamma_i} T_i \leq \hat{\tilde{y}} \tag{20}$$

$$X_i = V_i S_i^{-\alpha_i} G_i^{\chi_i} + \mu_i \tag{21}$$

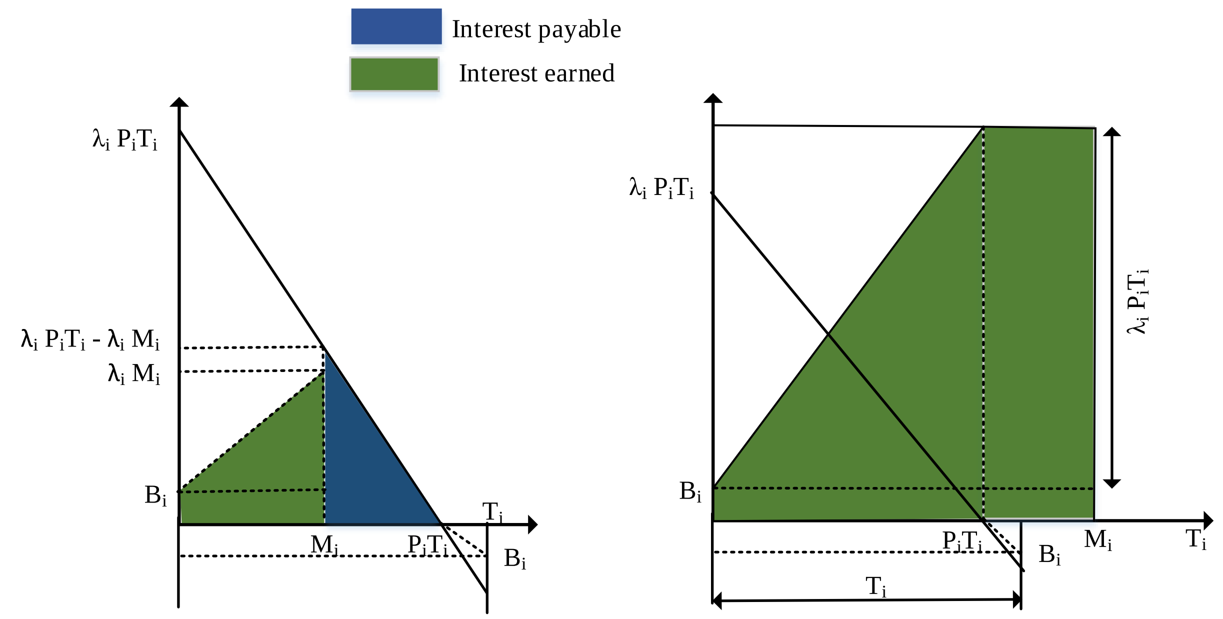
$$N_i = \beta_i + P_i(1 - \beta_i) \tag{22}$$

$$x = (S_i . T_i . G_i . M_i . P_i . X_i . N_i) > 0 \tag{23}$$

$$M_i \leq P_i T_i \quad \text{for } j = 1 \tag{24}$$

$$P_i T_i \leq M_i \leq M_0 \quad \text{for } j = 2 \tag{25}$$

Where, $\hat{\tilde{y}} = \left(\left((y_1^1 . y_1) . q_1 \right) ; \left((y_2^1 . y_2) . q_2 \right) ; \left((y_3^1 . y_3) . q_3 \right) \right)$ and $i = 1.2.3 \dots . n$.



Case 1: $M_i \leq P_iT_i$ Case 2: $P_iT_i \leq M_i$

Fig 2. Inventory diagram for cases 1 and 2

4- Solution method

In this section, we first convert out model into a multi-objective nonlinear programming (MONP) problem, of which each objective has signomial terms, with using the methods of converting the fuzzy-random parameters to crisp one. Then, we solve the MONP problem by first converting it into a single objective problem and then by using global optimization method discussed by Xu (2014) for solving SGP problems.

Case 1- $M_i \leq P_iT_i$

Following example-1 in Luhandjula (1983) , we first convert the fuzzy-stochastic constraint (20) into the following deterministic form:

$$q_1 \frac{\left(\sum_{i=1}^{n} \theta_{3i} N_i X_i^{1-\gamma_i} T_i\right) - y_1^1}{y_1 - y_1^1} + q_2 \frac{\left(\sum_{i=1}^{n} \theta_{3i} N_i X_i^{1-\gamma_i} T_i\right) - y_2^1}{y_2 - y_2^1} + q_3 \frac{\left(\sum_{i=1}^{n} \theta_{3i} N_i X_i^{1-\gamma_i} T_i\right) - y_3^1}{y_3 - y_3^1} \geq \alpha \tag{26}$$

After simplification, we have:

$$-\frac{\left(\frac{q_1}{y_1-y_1^1} + \frac{q_2}{y_2-y_2^1} + \frac{q_3}{y_3-y_3^1}\right)}{\left(\frac{q_1 y_1^1}{y_1-y_1^1} + \frac{q_2 y_2^1}{y_2-y_2^1} + \frac{q_3 y_3^1}{y_3-y_3^1} + \alpha\right)} \left(\sum_{i=1}^{n} \theta_{3i} N_i X_i^{1-\gamma_i} T_i\right) + 1 \leq 0 \tag{27}$$

Then, we rewrite the constraint (21) as follows:

$$X_i = V_i S_i^{-\alpha_i} G_i^{\chi_i} + \mu_i \Rightarrow \begin{cases} X_i \leq V_i S_i^{-\alpha_i} G_i^{\chi_i} + \mu_i & \boxed{1} \\ X_i \geq V_i S_i^{-\alpha_i} G_i^{\chi_i} + \mu_i & \boxed{2} \end{cases} \tag{28}$$

So, we have:

$$\overset{\boxed{1}}{\Rightarrow} \; X_i \leq V_i S_i^{-\alpha_i} G_i^{\chi_i} + \mu_i \;\Rightarrow\; X_i - V_i S_i^{-\alpha_i} G_i^{-\chi_i} \leq \mu_i \;\Rightarrow\; \mu_i^{-1} X_i - \mu_i^{-1} V_i S_i^{-\alpha_i} G_i^{\chi_i} \leq 1 \tag{29}$$

$$\Rightarrow X_i \geq V_i S_i^{-\alpha_i} G_i^{\chi_i} + \mu_i \;\Rightarrow\; V_i S_i^{-\alpha_i} G_i^{\chi_i} X_i^{-1} + \mu_i X_i^{-1} \leq 1 \tag{30}$$

Following the same manner as described for constraint (21), we convert constraints (22) and (24) into the following form:

$$N_i = \beta_i + P_i(1-\beta_i) \Rightarrow \begin{cases} \beta_i^{-1} N_i - \beta_i^{-1}(1-\beta_i)P_i \leq 1 \\ \beta_i N_i^{-1} + (1-\beta_i)P_i N_i^{-1} \leq 1 \end{cases} \tag{31}$$

$$M_i P_i^{-1} T_i^{-1} \leq 1 \tag{32}$$

The objective function of the problem is maximizing the total profit and is written as: $Max\ ATP_1(x)$. Since, $Max\ ATP_1(x)$ is equivalent $-Min\left(\underbrace{-ATP_1(x)}_{Z_1(x)}\right)$, thus, the problem (19)-(24) can be rewritten as follows:

$$Min\ Z_1(x) \tag{33}$$

s.t.

$$\mu_i^{-1} X_i - \mu_i^{-1} V_i S_i^{-\alpha_i} G_i^{\chi_i} \leq 1 \tag{34}$$

$$V_i S_i^{-\alpha_i} G_i^{\chi_i} X_i^{-1} + \mu_i X_i^{-1} \leq 1 \tag{35}$$

$$\beta_i^{-1} N_i - \beta_i^{-1}(1-\beta_i)P_i \leq 1 \tag{36}$$

$$\beta_i N_i^{-1} + (1-\beta_i)P_i N_i^{-1} \leq 1 \tag{37}$$

$$-\frac{\left(\frac{q_1}{y_1-y_1^1} + \frac{q_2}{y_2-y_2^1} + \frac{q_3}{y_3-y_3^1}\right)}{\left(\frac{q_1 y_1^1}{y_1-y_1^1} + \frac{q_2 y_2^1}{y_2-y_2^1} + \frac{q_3 y_3^1}{y_3-y_3^1} + \alpha\right)}\left(\sum_{i=1}^{n} \theta_{3i} N_i X_i^{1-\gamma_i} T_i\right) + 1 \leq 0 \tag{38}$$

$$x = (S_i . T_i . G_i . M_i . P_i . X_i . N_i) > 0 \tag{39}$$

$$M_i P_i^{-1} T_i^{-1} \leq 1 \tag{40}$$

According to the hybrid numbers theory as explained by Panda *et al*. (2008) the problem (33)-(40) reduces to:

$$Min\ EVZ_1(x) = E\hat{Z}_{01}(x)(+)'\big(0.V_1(x)\big) \tag{41}$$

s.t. Constraints (34)-(40)

Where $E\hat{Z}_{01}(x) = \big(EZ_{11}(x).EZ_{21}(x).EZ_{31}(x)\big)$ with

$$\begin{aligned} EZ_{k1}(x) = \sum_{i=1}^{n} \Big(& -N_i X_i S_i + N_i X_i G_i + 0.5\left(h_{ik} + \mu_{h_i} + \theta_{1i}(\pi_{ik} + \mu_{\pi_i})\right) X_i P_i^2 T_i \\ & -\theta_{1i}(\pi_{ik} + \mu_{\pi_i}) X_i P_i T_i + 0.5\theta_{1i}(\pi_{ik} + \mu_{\pi_i}) X_i T_i + \theta_{2i}(g_{ik} + \mu_{g_i}) X_i - \theta_{2i}(g_{ik} + \mu_{g_i}) X_i P_i \\ & +\theta_{3i} N_i X_i^{1-\gamma_i} + \theta_{4i} X_i^{1-\gamma_i} P_i^2 T_i + \theta_{4i} X_i^{1-\gamma_i} M_i^2 T_i^{-1} - 2\theta_{4i} X_i^{1-\gamma_i} P_i M_i - \theta_{5i} X_i S_i M_i \\ & +\theta_{5i} X_i S_i M_i P_i - \theta_{6i} X_i S_i M_i^2 T_i^{-1} + \underline{\tilde{A}}_i T_i^{-1} \Big) \qquad k = 1.2.3. \end{aligned} \tag{42}$$

$$\begin{aligned} V_1(x) = \sum_{i=1}^{n} \Big(& 0.25(\sigma_{h_i}^2 + \theta_{1i}^2 \sigma_{\pi_i}^2) X_i^2 P_i^4 T_i^2 + \theta_{1i}^2 \sigma_{\pi_i}^2 X_i^2 P_i^2 T_i^2 + 0.25\theta_{1i}^2 \sigma_{\pi_i}^2 X_i^2 T_i^2 + \theta_{2i}^2 \sigma_{g_i}^2 X_i^2 \\ & +\theta_{2i}^2 \sigma_{g_i}^2 X_i^2 P_i^2 + \sigma_{A_i}^2 T_i^{-2} \Big) \end{aligned} \tag{43}$$

and $= 1.2.3 \ldots . n$, $\underline{\widetilde{h}}_i = (h_{i1}.h_{i2}.h_{i3})(+)'\left(\mu_{h_i} + \sigma_{h_i}^2\right)$, $\underline{\tilde{\pi}}_i = (\pi_{i1}.\pi_{i2}.\pi_{i3})(+)'\left(\mu_{\pi_i} + \sigma_{\pi_i}^2\right)$, $\underline{\tilde{g}}_i = (g_{i1}.g_{i2}.g_{i3})(+)'\left(\mu_{g_i} + \sigma_{g_i}^2\right)$, and $\underline{\tilde{A}}_i = (A_{i1}.A_{i2}.A_{i3})(+)'\left(\mu_{A_i} + \sigma_{A_i}^2\right)$.

Referring to Kauffman and Gupta (1991), the approximated value of triangular fuzzy number $\tilde{b} = (b_1.b_2.b_3)$ is calculated as $\hat{b} = \frac{b_1+2b_1+b_3}{4}$. Therefore, an approximated value of $E\hat{Z}_0(x)$ is as follows:

$$AEZ_{01}(x) = \frac{EZ_{11}(x) + 2EZ_{21}(x) + EZ_{31}(x)}{4} \tag{44}$$

$$= \sum_{i=1}^{n} \Big(-N_iX_iS_i + N_iX_iG_i + 0.5\left(\hat{h}_i + \mu_{h_i} + \theta_{1i}\left(\hat{\pi}_i + \mu_{\pi_i}\right)\right)X_iP_i^2T_i - \theta_{1i}\left(\hat{\pi}_i + \mu_{\pi_i}\right)X_iP_iT_i$$

$$+0.5\theta_{1i}\left(\hat{\pi}_i + \mu_{\pi_i}\right)X_iT_i + \theta_{2i}\left(\hat{g}_{ik} + \mu_{g_i}\right)X_i - \theta_{2i}\left(\hat{g}_{ik} + \mu_{g_i}\right)X_iP_i + \theta_{3i}N_iX_i^{1-\gamma_i}$$

$$+\theta_{4i}X_i^{1-\gamma_i}P_i^2T_i + \theta_{4i}X_i^{1-\gamma_i}M_i^2T_i^{-1} - 2\theta_{4i}X_i^{1-\gamma_i}P_iM_i - \theta_{5i}X_iS_iM_i + \theta_{5i}X_iS_iM_iP_i$$

$$-\theta_{6i}X_iS_iM_i^2T_i^{-1} + \underline{\tilde{A}}_iT_i^{-1}\Big)$$

So, problem (33) -(40) is reduced to the following multi-objective nonlinear programming problem, of which each objective has signomial terms:

$$Min \;\; EVZ(x) = [AEZ_{01}(x).V_1(x)] \tag{45}$$

s.t. Constraints (34)-(40)

In what following, we solve the multi-objective nonlinear programming problem (34) -(40) and (45) by first converting it into a single objective problem by the following steps and then using global optimization approach discovered by Xu (2014) for solving SGP problems.

***Step 1*:** Solve the problem (34) -(40) and (45) with considering only objective function $AEZ_{01}(x)$ and solve it using the SGP algorithm of Xu (2014). Let $x^{(1)} = \left(S_i^{(1)}.T_i^{(1)}.G_i^{(1)}.M_i^{(1)}.P_i^{(1)}.X_i^{(1)}.N_i^{(1)}\right)$be the optimal solutions for decision variables and so the optimal amount of objective function is $AEZ_{01}\left(x^{(1)}\right)$. Next calculate the amount of the second objective function $V_1(x)$ in $x^{(1)}$, say $V_1\left(x^{(1)}\right)$.

Step 2: Consider just the second objective function $V_1(x)$ and solve it using SGP approach said in Step 1 and obtain the optimal solutions for decision variables and objective function as $x^{(2)} = \left(S_i^{(2)}.T_i^{(2)}.G_i^{(2)}.M_i^{(2)}.P_i^{(2)}.X_i^{(2)}.N_i^{(2)}\right)$ and $V_1\left(x^{(2)}\right)$, respectively. Next compute the amount of the first objective function $AEZ_{01}(x)$ in $x^{(2)}$, say $AEZ_{01}\left(x^{(2)}\right)$.

Step 3: There are the following relation among objective functions: $AEZ_{01}\left(x^{(1)}\right) < AEZ_{01}(x) < AEZ_{01}\left(x^{(2)}\right)$ and $V_1\left(x^{(2)}\right) < V_1(x) < V_1\left(x^{(1)}\right)$.

Step 4: Formulate the membership functions for the objective functions of (45) as follows:

$$\mu_{AEZ_0}(x) = \begin{cases} 1 & AEZ_{01}(x)\ (x) \leq AEZ_{01}\left(x^{(1)}\right) \\ \dfrac{AEZ_{01}\left(x^{(2)}\right) - AEZ_{01}(x)}{AEZ_{01}(x^{(2)}) - AEZ_{01}(x^{(1)})} & AEZ_{01}\left(x^{(1)}\right) \leq AEZ_{01}(x) \leq AEZ_{01}\left(x^{(2)}\right) \\ 0 & AEZ_{01}\left(x^{(2)}\right) \leq AEZ_{01}(x) \end{cases} \tag{46}$$

$$\mu_{V_1}(x) = \begin{cases} 1 & V_1(x) \le V_1\left(x^{(2)}\right) \\ \dfrac{V_1\left(x^{(1)}\right) - V_1(x)}{V_1(x^{(1)}) - V_1(x^{(2)})} & V_1\left(x^{(2)}\right) \le V_1(x) \le V_1\left(x^{(1)}\right) \\ 0 & V_1\left(x^{(1)}\right) \le V_1(x) \end{cases} \quad (47)$$

Step 5: According to Tiwari *et al.* (1987), the membership functions are maximizing by max-convex combination operator through following equations :

$$Max\ \ MZ_1(x) = f_1 \mu_{AEZ_{01}}(x) + f_2 \mu_{V_1}(x) \quad (48)$$

s.t. Constraints (34)-(40)

Where the weights f_1 and f_2 are corresponding to the member functions $\mu_{AEZ_{01}}(x)$ and $\mu_{V_1}(x)$, respectively. So, the problem (34) -(40) and (48) can be rewritten as the following constrained SGP problem:

$$Min\ \ Z'_1(x) = \frac{f_1}{AEZ_{01}(x^{(2)}) - AEZ_{01}(x^{(1)})} AEZ_{01}(x) + \frac{f_2}{V_1(x^{(1)}) - V_1(x^{(2)})} V_1(x) \quad (49)$$

s.t. Constraints (34) -(40)

Now problem (34) -(40) and (49) can be solved using global optimization of SGP problem discussed in Appendix.

Case 2- $\boldsymbol{P_i T_i \le M_i \le M_0}$

The mathematical model for case 2 is:

$$Max\ \ ATP_2 \quad (50)$$

s.t. Constraints (20)-(23) and (25)

All procedure to solve the above problem is similar to the procedure used to solve case 1. Following the same procedure used for case 1, the constrained SGP problem for case 2 is:

$$Min\ \ Z'_2{}^{(x)} = \frac{f_1}{AEZ_{02}(x^{(2)}) - AEZ_{02}(x^{(1)})} AEZ_{02}(x) + \frac{f_2}{V_2(x^{(1)}) - V_2(x^{(2)})} V_2(x) \quad (51)$$

s.t. $P_i T_i M_i^{-1} \le 1$ (52)

$M_0^{-1} M_i \le 1$ (53)

And constraints (34) -(39)

5- Numerical example

In this Section, an example is designed to demonstrate the application of the model and solution procedure proposed above for a particular retailer that orders three types of products from the supplier ($n = 3$). The retailer has a limitation on the total budget for purchasing units which is fuzzy stochastic. The budget amount here lies within \$(232, 280) with probability 0.5; within \$(245, 320) with probability 0.35; within \$(255, 310) with probability 0.4. According to the past reorders, the annual demand rate of three items are calculated as $10^6 S_1^{-3.5} G_1^{0.007} + \xi_1$, $1.5 \times 10^6 S_2^{-3.8} G_2^{0.005} + \xi_2$, and $1.8 \times 10^6 S_3^{-3.1} G_3^{0.01} + \xi_3$. The crisp parameters for all items are $I_e = 0.05, I_p = 0.1$, $\beta_1 = 0.6$, $\beta_2 = 0.65$, $\beta_3 = 0.7$, $\alpha = 0.85$, $\gamma_1 = 1.6$, $\gamma_2 = 1.5$, $\gamma_3 = 1.7$, $\xi_1 \sim N\ (2.1)$, $\xi_2 \sim N\ (3.1)$, $\xi_3 \sim N\ (1.1)$, and the hybrid parameters are listed in table 2.

Table 2. Hybrid parameters for each item

i	$\tilde{h}_i$	$\tilde{\pi}_i$	$\tilde{A}_i$	$\tilde{g}_i$
1	(0.8, 0.9,0.95) (+)' (0.85,0.06)	(2, 2.5, 3) (+)' (2.5, 1)	(100, 112, 115) (+)' (100, 25)	(1, 1.5, 2) (+)' (2.5, 1)
2	(0.85, 0.93, 1) (+)' (0.9, 0.065)	(2.5, 3, 3.5) (+)' (3, 1)	(105, 112, 117) (+)' (100, 25)	(1.5, 2, 2.5) (+)' (3, 1.5)
3	(1, 1.2,1.5) (+)' (1,0.07)	(3, 3.2, 3.5) (+)' (3,1)	(109, 115, 120) (+)' (100, 25)	(2, 2.2, 2.5) (+)' (3,1)

The payoff matrix of problem (19) -(24), which is needed to transform problem (19) -(24), into problem (34) -(40) and (49), is as following:

$$\begin{bmatrix} AEZ_{01}(x^{(1)}) & V_1(x^{(1)}) \\ AEZ_{01}(x^{(2)}) & V_1(x^{(2)}) \end{bmatrix} = \begin{bmatrix} -18.5899 & 8.099 \\ 221.1500 & 5 \end{bmatrix}$$

Similarly, the payoff matrix of case 2 is:

$$\begin{bmatrix} AEZ_{02}(x^{(1)}) & V_2(x^{(1)}) \\ AEZ_{02}(x^{(2)}) & V_2(x^{(2)}) \end{bmatrix} = \begin{bmatrix} -16.5562 & 8.1201 \\ 235.2 & 5.1 \end{bmatrix}$$

Calculating these pay off matrixes and considering the weights 0.9 and 0.1 plus the provided data, it is possible to solve the problem (34) -(40) and (49) for case 1 and the problem (34) -(39) and (51) -(53) using global optimization method. The proposed algorithm is coded in MATLAB R2014b software and implemented on an Intel Core i5 PC with CPU of 1.4 GHz and 4.00 GB RAM using GGPLAB solver (Mutapcic *et al.* 2006). The optimal values of decision variables along with the optimal values of mean profit function($EATP$) and the optimal values of variance profit function ($VATP$) for the both cases and all items are reported in tables 3-5.

Table 3. Optimal solutions of item 1 for the both cases

Case	S_1^*	G_1^*	M_1^*	T_1^*	P_1^*	Q_1^*	B_1^*	$EATP$	$VATP$
1	6.0912	0.0061	0.1489	1.2345	0.6085	147.2328	68.3499	500.3933	9.1737
2	5.7640	0.0069	0.5785	0.5868	0.3688	147.9103	124.8986	500.2987	9.2155

Table 4. Optimal solutions of item 2 for the both cases

Case	S_2^*	G_2^*	M_2^*	T_2^*	P_2^*	Q_2^*	B_2^*	$EATP$	$VATP$
1	5.6891	0.0062	0.1529	1.1641	0.6082	148.3110	68.8989	500.3933	9.1737
2	5.6137	0.0074	0.5799	0.5871	0.3677	145.1583	122.8687	500.2987	9.2155

Table 5. Optimal solutions of item 3 for the both cases

Case	S_2^*	G_2^*	M_2^*	T_2^*	P_2^*	Q_2^*	B_2^*	$EATP$	$VATP$
1	6.7985	0.0062	0.1513	1.1500	0.6085	149.0210	68.3419	500.3933	9.1737
2	6.6237	0.0081	0.5787	0.5761	0.3667	148.8599	123.6844	500.2987	9.2155

6- Sensitivity analysis

Sensitivity analyses for the proposed problem are done to analyze the impacts of changes in the key parameter values on the optimal solutions. For simplicity, we assume there is an item (item 1) with $P_1T_1 \leq M_1$. We first consider the effect of changes in values of α_1 and $\chi_1$on the selling price, marketing expenditure, order quantity, and mean profit function. The calculated results are shown in Figs 3 -6. We observe from figures 3 and 4 that when the amount of α_1 increase, selling price, marketing expenditure, order quantity, and mean profit function decrease. Moreover, when the amount of χ_1 increases, other parameters like the selling price, marketing expenditure, order quantity and mean profit function also increase (see figures 5 and 6). This is because when the price elasticy to demand increase, demand rate and order quantity decrease; thus, the mean profit function decreases. In contrast, when the amount of χ_1 increase, demand rate and order quantity increase; thus, the mean profit function increases, which agrees with reality.

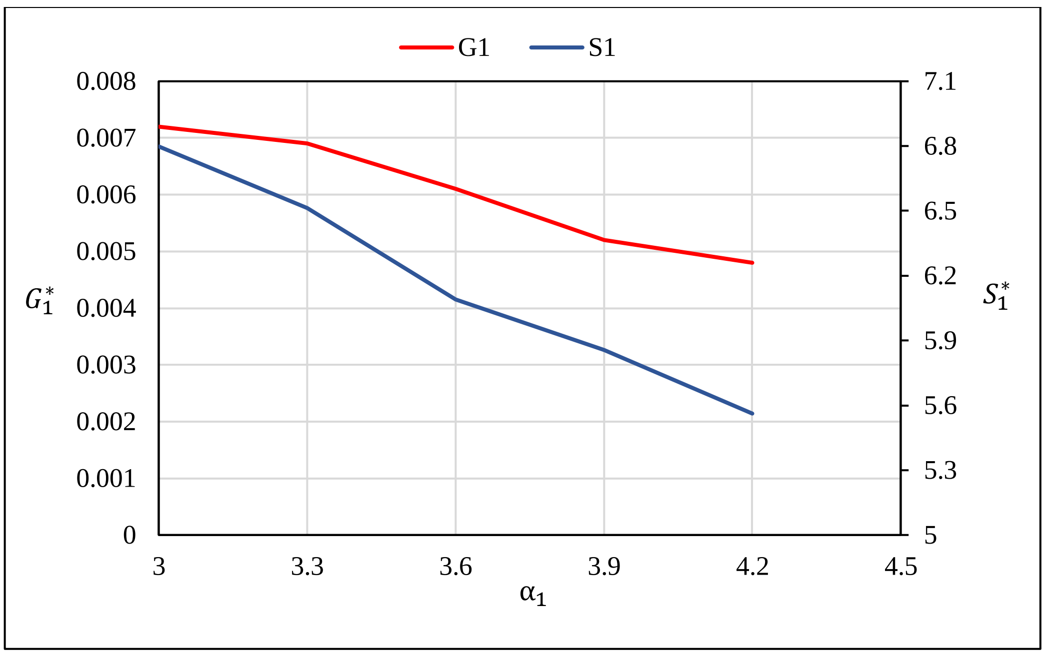


Fig 3. The effect of change of α_1 on the selling price and marketing expenditure

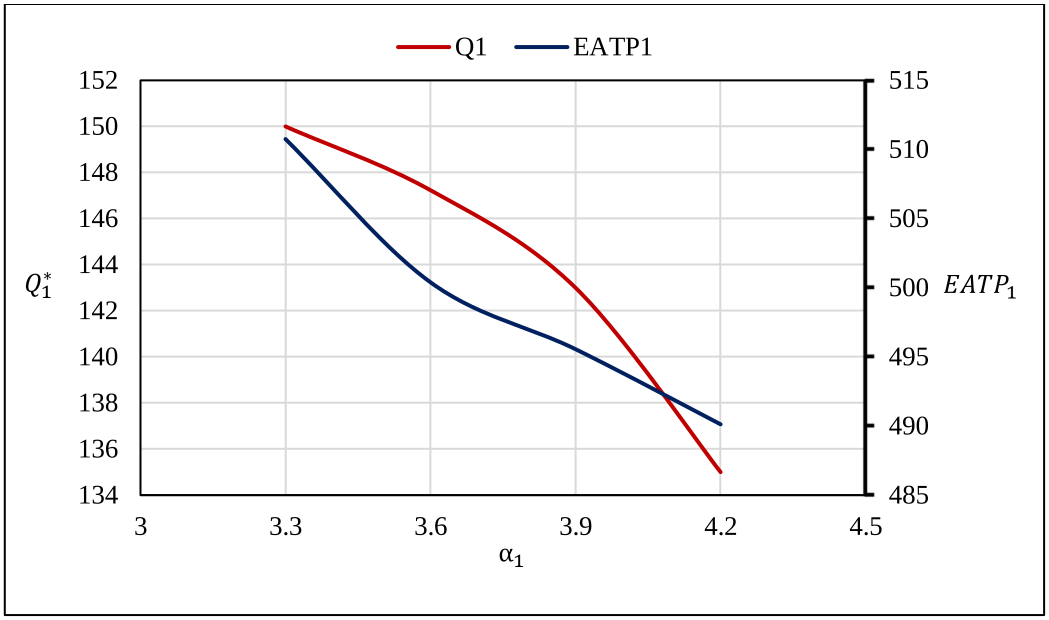


Fig 4. The effect of change of α_1 on the order quantity and mean profit function

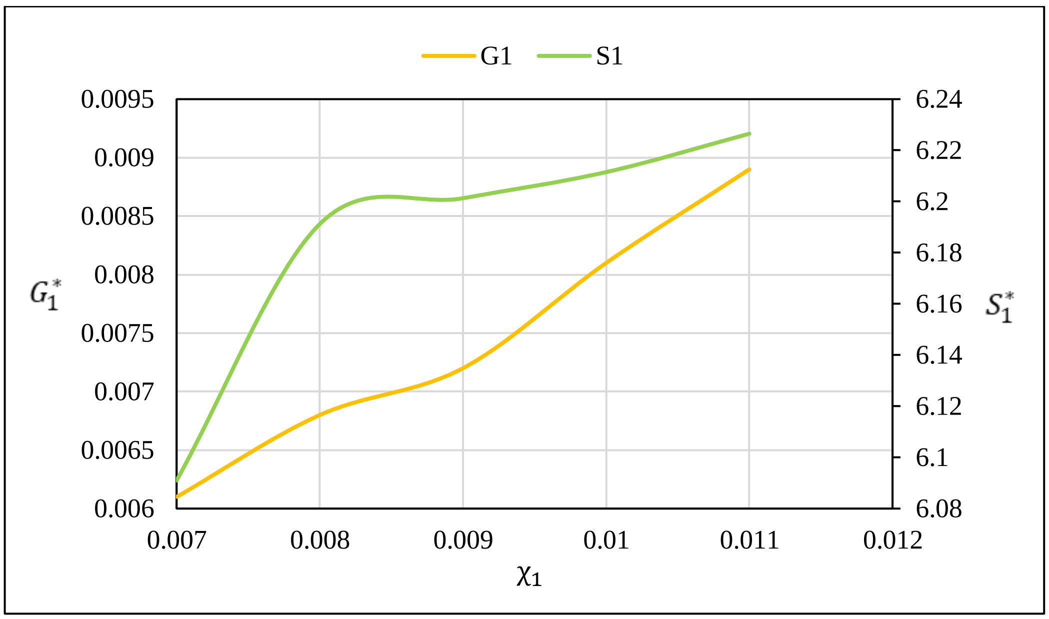


Fig 5. The effect of change of χ_1 on the selling price and marketing expenditure

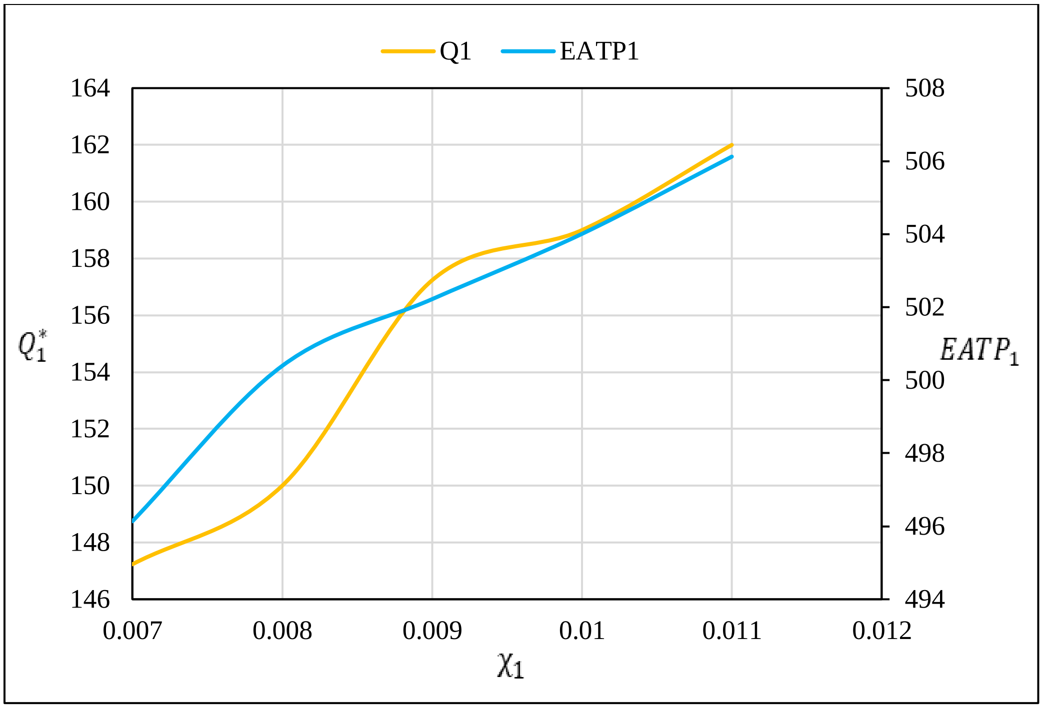


Fig 6. The effect of change of χ_1 on the order quantity and mean profit function

We also investigate the sensitivity analyses on the optimal solutions due to the parameters I_p , I_e , and β_1. The impact of the changes is reported in Table 6 and the following results can be viewed:

- When the parameter $\mathrm{I_p}$ increases, the amount of S_1^* and G_1^* will increase, whereas the amounts of M_1^* , T_1^*, P_1^*, Q_1^* , and $EATP_1$will decrease.
- When the parameter $\mathrm{I_e}$ increases, the amount of G_1^* and $EATP_1$ will increase, whereas the amounts of M_1^* , T_1^*, P_1^*, Q_1^* , and S_1^* will decrease.
- When the parameter β_1. increases, the amount of M_1^* , P_1^*, Q_1^* , and $EATP_1$ will increase, whereas the amounts of T_1^*, G_1^* , and S_1^* will

Table 6. Sensitivity analysis on the parameters I_p , I_e , and β_1

Parameters	S_1^*	G_1^*	M_1^*	T_1^*	P_1^*	Q_1^*	$EATP_1$
$I_p = 0.1$	6.0912	0.0061	0.1489	1.2345	0.6085	147.2328	500.3933
$I_p = 0.15$	6.1012	0.007	0.1471	1.2320	0.6062	147.2216	495.8620
$I_p = 0.2$	6.1152	0.0081	0.1452	1.2215	0.6047	147.2056	498.8752
$I_p = 0.25$	6.1301	0.009	0.1419	1.2117	0.6010	147.1388	491.4250
$I_p = 0.3$	6.1430	0.0095	0.1383	1.2101	0.6000	147.1015	485.8457
$I_e = 0.05$	6.0912	0.0061	0.1489	1.2345	0.6085	147.2328	500.3933
$I_e = 0.09$	5.8321	0.0068	0.1462	1.2118	0.6055	145.3523	505.7652
$I_e = 0.12$	5.0100	0.0072	0.1441	1.2069	0.6032	143. 1668	512.4562
$I_e = 0.16$	4.1458	0.0081	0.1417	1.19975	0.6011	140.1700	515.3441
$I_e = 0.2$	3.4452	0.0089	0.1383	1.1942	0.6005	138.3556	518.2546
$\beta_1 = 0.5$	6.0910	0.0061	0.1387	1.2371	0.6826	149.6412	496.1354
$\beta_1 = 0.6$	6.0902	0.0061	0.1489	1.2345	0.7085	150.2328	500.3933
$\beta_1 = 0.7$	6.0902	0.0060	0.1502	1.2310	0.7675	153.3245	502.2198
$\beta_1 = 0.8$	6.0896	0.0055	0.1563	1.2294	0.8132	158.9431	504.0085
$\beta_1 = 0.9$	6.0865	0.0053	0.1589	1.2256	0.8875	160.6825	506.1244

Finally, the changes in mean and variance profit function with respect to weight parameter $f_1(= 1 - f_2)$ are illustrated in figure (7). From this figure, when f_1 increases, the mean profit function will decrease, while, the variance profit function will increase. This is because if f_1 increases, f_2 decreases, therefore, the variance profit function and the mean profit function contradicts each other. That is, if one decreases, next the other increases.

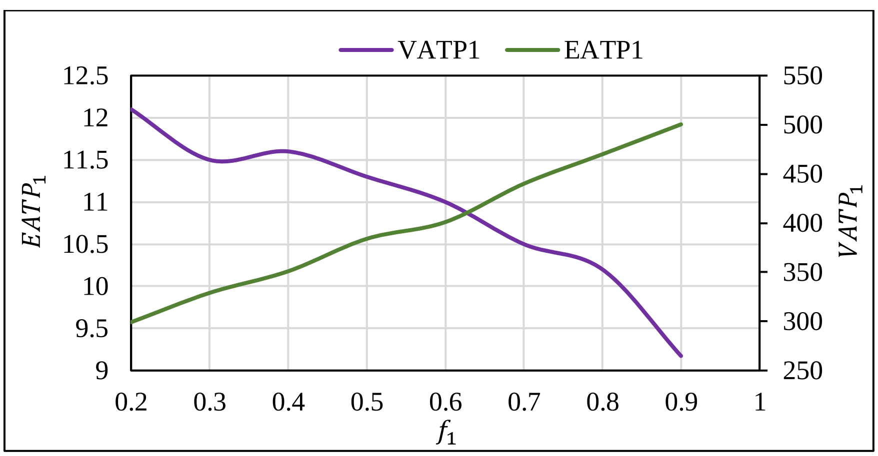


Fig 7. The effect of weight parameter f_1 on the mean and variance profit function

7- Conclusion

In this study, for the first time a multi-item EOQ model has been developed with price and marketing cost dependent stochastic demand under permissible delay in payment. We considered some cost parameters as hybrid number. Moreover, a limitation on the total budget to purchase inventory was considered with fuzzy-stochastic quantity. Shortages are permitted and partially backordered. We solved our problem with using the methods of converting fuzzy- random parameters to crisp one and obtaining the global optimum of SGP problems. Finally, several numerical examples and a sensitivity analysis of the main parameters were provided to demonstrate the formulated model. Our study can be extended for deteriorating items. Moreover, a multi- item EOQ model with variable lead time and considering the issues of sustainability can be developed.

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Appendix. Transforming SGP problems into a series of standard GP problems

As mention earlier, a global optimization method is applied for solving SGP problem proposed in Steps 1, 2, and 5. So in this section, we first present a SGP problem, and then explain this approach in detail for transforming the SGP problem to a series of standard GP problem according to type of our problem.

1. SGP program

A SGP problem is equal to an optimization problem as follows:

$$Min\ \psi_0(y) = \sum_{k=1}^{n_0} \theta_{0k} c_{0k} \prod_{i=1}^{m} y_i^{a_{0ik}} \qquad c_{0k} > 0,\ \theta_{0k} = \pm 1 \tag{1}$$

$$\text{s.t} \quad \psi_j(y) = \sum_{k=1}^{n_j} \theta_{jk} c_{jk} \prod_{i=1}^{m} y_i^{a_{jik}} \leq 1 \qquad c_{jk} > 0,\ \theta_{jk} = \pm 1,\ a_{jik} \in R,\ j = 1.2. \dots .t \tag{2}$$

$$y_i > 0, ,\ i = 1.2. \dots .m \tag{3}$$

$n_j (j = 0.1.2. \dots .t)$ show the number of elements of the objective function and constraints. $\psi_j (j = 0.1.2. \dots .t)$ is a signomial function.

2. Global optimization approach

This method defines all functions $\psi_j (j = 0.1.2. \dots .t)$as:

$$\psi_j(y) = \psi_j^+(y) - \psi_j^-(y) \qquad j = 0.1.2. \dots .t \tag{4}$$

Where $\psi_j^+(y)$ and $\psi_j^-(y)$ are formulated as:

$$\psi_j^+(y) = \sum_{k=1}^{n_j} \theta_{jk} c_{jk} \prod_{i=1}^{m} y_i^{a_{jik}} \qquad \theta_{jk} = +1,\ j = 0.1.2. \dots .t \tag{5}$$

$$\psi_j^-(y) = \sum_{k=1}^{n_j} \theta_{jk} c_{jk} \prod_{i=1}^{m} y_i^{a_{jik}} \qquad \theta_{jk} = -1,\ j = 0.1.2. \dots .t \tag{6}$$

Next it defines a large number, > 0 , so that $\psi_j^+(y) - \psi_j^-(y) + L > 0$ and rewrites the model (1)-(3) as the following problem:

$$Min\ \psi_0(y) = \psi_0^+(y) - \psi_0^-(y) + L \tag{7}$$

$$\text{s.t} \quad \psi_j^+(y) - \psi_j^-(y) + L \leq 1 \qquad j = 1.2. \dots .t \tag{8}$$

$$y_i > 0,\ i = 1.2. \dots .m \tag{9}$$

The model (7)-(9) converts to the following optimization problem, by introducing an extra variable y_0 in order to express constraints and objective function as quotient and linear form, respectively.

$$Min\ \ y_0 \tag{10}$$

$$\text{s.t} \quad \frac{\psi_0^+(y) + L}{\psi_0^-(y) - y_0} \leq 1 \tag{11}$$

$$\frac{\psi_j^+(y)}{\psi_j^-(y) + 1} \leq 1 \qquad j \in j_1, j = 1.2. \dots .t \tag{12}$$

$$\frac{\psi_j^+(y)}{\psi_j^-(y) + 1} \leq 1 \qquad j \in j_2, j = 1.2. \dots .t \tag{13}$$

$$y_i > 0,\ i = 1.2. \dots .m \tag{14}$$

Where, $j_1 = \{j \mid \psi_j^-(y) + 1 \text{ are monomials}\}$ and $j_2 = \{j \mid j \notin j_1\}$. In the above model, the objective function (10) is a posynomial function, constraint (12) is a posynomial inequality, and constraint (14) is a monomial inequality that all three equations are allowable in standard GP problem, but constraints (11) and (13) are not permitted in a standard GP problem. So this method used from arithmetic–geometric mean approximation to approximate every denominator of constraints (11) and (13) with monomial functions as follows:

$$f(y) \geq \hat{f}(y) = \prod_u \left(\frac{v_u(y)}{w_u(x)}\right)^{w_u(x)} \tag{15}$$

Where the parameters $w_u(x)$ can be computed as:

$$w_u(x) = \frac{v_u(x)}{f(x)} \qquad \forall\, u \tag{16}$$

And $f(y) = \sum_u v_u(y)$ is a posynomial function, $v_u(y)$ are monomial terms, and $x > 0$ is a fixed point. Using the proposed monomial approximation approach to every denominator of constraints (11) and (13), finally we have:

$$Min \quad y_0 \tag{17}$$

$$\text{s.t} \quad \frac{\psi_0^+(y) + L}{\psi_0^-(y.y_0)} \leq 1 \tag{18}$$

$$\frac{\psi_j^+(y)}{\psi_j^-(y) + 1} \leq 1 \qquad j \in j_1, j = 1.2. \dots . t \tag{19}$$

$$\frac{\psi_j^+(y)}{\psi_{2j}^-(y)} \leq 1 \qquad j \in j_2, j = 1.2. \dots . t \tag{20}$$

$$y_i > 0, i = 1.2. \dots . m \tag{21}$$

Where $\psi_0^-(y.y_0)$ and $\psi_{2j}^-(y)$are the corresponding monomial functions approximated using Equation (15). Now, the problem (17)-(21) is a standard geometric programming that can be optimized efficiently using GGPLAB solver in MATLAB (Mutapcic *et al*. 2006). So, the proposed algorithm can be summarized as an iterative algorithm as follows:

Algorithm

Step 0: Select an initial solution for decision variables y_0 and y, $y_0^{(0)}$and $y^{(0)}$ respectively. Consider a solution accuracy $\varepsilon > 0$ and put iteration counter $r = 0$.

Step1: In iteration r, calculate the monomial components in the denominator posynomials of Equations (11) and (13) by the determined $y_0^{(r-1)}$and $y^{(r-1)}$. Calculate their corresponding parameters $w_u\left(y_0^{(r-1)}.y^{(r-1)}\right)$ using equation (16).

Step2: Do the condensation on the denominator posynomials of equations(11) and (13) using Equation (15) by parameters $w_u\left(y_0^{(r-1)}.y^{(r-1)}\right)$.

Step3: Solve the standard GP (17)-(21) to obtain $\left(y_0^{(r)}.y^{(r)}\right)$.

Step4: If $\left\|y^{(r)} - y^{(r-1)}\right\| \leq \varepsilon$, so stop. Else $r = r + 1$ and return to Step1.
