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A credit period contract towards coordination of pharmaceutical supply chain: The case of inventory-level-dependent demand

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Abstract

This paper is considered a two stage pharmaceutical supply chain (PSC) consisting of a pharmaceutical manufacturer (pharma-manufacturer) that suppling one type of pharmaceutical product to a pharma-retailer. The customer demand rate for the pharmaceutical product is dependent on the pharma-retailer's current-inventory-level. The pharma-retailer determines the order quantity (Q) value as decision variable and the pharma-manufacturer uses EPQ system that usually the economic order quantity value of retailer is less than the optimal production quantity value of manufacturer. First, the problem is investigated in decentralized decision-making and accordingly, a coordination incentive based on credit payment period policy to coordinate the mentioned PSC in two structures is proposed: independent optimization and centralized model with credit policy. Moreover, numerical examples and sensitivity analysis are considered to illustrate the results of the presented coordination structures toward decentralized model.

Keywords: Pharmaceutical supply chain, Inventory-dependent demand, Production, credit payment period, Coordination

1- Introduction and literature review

In developed countries, two sources of the subject of Healthcare Supply Chain Management (HSCM) and inventory management have not been given much attention. The inventory investments are estimated between 10% and 18% of total revenues in healthcare range by several researchers (Holmgren and Wentz 1982); (Gary Jarrett 1998). In many real life cases, the demand rate may be directly related to the inventory level, especially some seasonal/perishable products, for example pharmaceutical, vegetables, meat and some oil derivatives such as gas, gasoline often deteriorate over the time. The pharmaceutical industry is also defined as a system of processes, operations, and so on organizations involved in the discovery, development, and production of narcotic and drugs. The pharmaceutical supply chain (PSC) means the path that comes from Through it, quality pharmaceutical products in Properly place and time distributed among final consumers (Bishara 2006). The statistics show that pharmaceuticals are an important part of healthcare expenditures.

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According to statistics published by National Center for Health Statistics (2016), 9.8 % of all national healthcare expenditures in the United States is related to expenditures for prescription drugs in 2014. Narayana, Pati et al. (2014) considered the pharmaceutical industry as a key sector of the health systems and introduced effective management of pharmaceutical supply chain (PSC) as one of the most important factors in managing healthcare expenditures. Since the pharmaceuticals are perishable vital goods, it's very important to be careful about production quantity and sales time. Hence, the manufacturer should to create the balance between the production quantity under condition of inventory level and market demand rate duo to the inventory-dependent demand. In inventory models, many researchers considered quantity discount contract to induce retailers to increase its order quantity more than economic order quantity (EOQ). Of course using a policy of delay in payment (credit period contract) had more effective for seasonal/perishable products, especially pharmaceutical products, because, retailers may save interests and increase capital in this policy. We considered a credit period contract to coordinate the supply chain with inventory-dependent demand rate for pharmaceutical products in our research.

Chung (2012) considered trade credit as a flexible source for short-term financing for firms mainly because it is followed by the firm's purchases. In the literature, credit period (delay in payment) is an incentive mechanism of coordination that has received much attention, first, Goyal (1985) proposed the economic order quantity model under conditions of permissible credit periods and assumed that the supplier would offer the retailer a credit period. Another one of the researches that consider offering of credit period from a manufacturer's point of view in a non-cooperative supply chain is presented by Kim, Hwang et al. (1995). Later, Jaber and Osman (2006) and Chan, Lee et al. (2010) extended Goyal (1985) to offer a fixed trade credit period to the retailer by the manufacturer. A two-level supply chain with credit scheme as an incentive to induce the retailer to increase its order quantity is considered by Jaber and Osman (2006). Chan, Lee et al. (2010) considered a single-manufacturer-multi-retailer supply chain and used a credit incentive to coordinate production and ordering quantities. Recently, credit period contracts have been developed such as (Ho, Ouyang et al. 2008, Heydari, Rastegar et al. 2017) to coordinate supply chain decisions. Moreover, other mechanisms of coordination have been proposed, for example, quantity discount contracts (Taleizadeh and Pentico 2014, Heydari and Norouzinasab 2016), buy back contracts (Wu 2013), revenue sharing contracts (Palsule-Desai 2013), sales rebate contracts (Saha 2013) and so on.

Another one of this research's basic category is coordination of order quantity decisions. Li and Liu (2006) made decision on order quantity to coordinate supply chain with quantity discount policy. Jung, Jeong et al. (2008) proposed a partial information sharing for coordinating supply quantities of products in a two-echelon supply chain; they proved that profitability of full information sharing solutions compare to partial information sharing is negligibly higher. a win–win situation for coordinating a single-buyer and multiple-competing suppliers considering a price-restricted quantity discount policy in an e-marketplace has been proposed (Chen 2012); It was illustrated that superior suppliers tend to choose catalogue policy, while inferior suppliers prefer a wholesale price discount in coordinating mechanism. Du, Banerjee et al. (2013) investigated a coordinated model based on credit incentive and/or wholesale price discount for order quantity, production batch size, and retail price in a two-stage supply chain. Of course there are another important decision which need to be coordinated for supply chains such as pricing (Chung, Talluri et al. 2015, Taleizadeh, Noori-daryan et al. 2015), lead time (Arkan and Hejazi 2012), reorder point and safety stock (Nematollahi, Hosseini-Motlagh et al. 2017), replenishment (Heydari 2015, Heydari and Norouzinasab 2016) and corporate social responsibility (CSR) (Nematollahi, Hosseini-Motlagh et al. 2017) decisions. In this proposed model, the optimal order and the reorder point are illustrating the performance of the inventory system.

In this type of product in this paper, it is important that the impact of demand is determined by what factors. Most of study, however, assumed that the market demand rate was either price sensitive or constant such as (Yao, Leung et al. 2008, Sundar, Narayanan et al. 2012, Saha and Goyal 2015). Of course the market demand rate may be influenced by other factors such as cycle time (Hou and Lin 2006), lead time (Braglia, Castellano et al. 2016), credit period (Chung 2012, Yang, Hong et al. 2014, Heydari, Rastegar et al. 2017), service level (Ha and Tong 2008), advertising (Wang, Zhou et al. 2010) and so on.

In this manner, the market demand rate in some research may be dependent on more than one factors simultaneously, for example, selling price and cycle time (Maihami and Abadi 2012), selling price and credit period (Giri and Maiti 2013), selling price and lead time (Zhu 2015, Heydari and Norouzinasab 2016). Moreover, the market demand rate can also be dependent on the inventory level in real life. There are two kinds of inventory-dependent demand in inventory models (1) the linear-form of $\alpha + \beta I(t)$ and (2) the power-form of $\alpha I(t)^{\beta}$, where $I(t)$ is the inventory level at time 't', α and β are constants. The power-form of inventory-dependent demand rate, which is proposed in this paper, is presented first in inventory models by (Baker and Urban 1988). They considered a deterministic inventory system with an inventory-level-dependent demand rate, which would decline along with the inventory-level throughout the cycle. Datta and Pal (1990) and Goh (1994) modified the model of Baker and Urban (1988) by relaxing the assumption that the inventory-dependent demand was down to a given level of inventory. Later, more practical issues of the inventory model with inventory-dependent demand are considered by researchers, such as considering deterioration (Giri, Pal et al. 1996, Jolai, Tavakkoli-Moghaddam et al. 2006), shortages (Urban 1995), lost sales (Wu, Ouyang et al. 2006) and fixed life time (Zhou and Yang 2003), etc. Recently, the inventory models with inventory-dependent demand rate are developed (Zhou, Min et al. 2008, Sajadieh, Thorstenson et al. 2010, Yang, Teng et al. 2010).

In this paper, a two stage pharmaceutical supply chain (PSC) of one pharma-retailer and one pharma-manufacturer is investigated. The demand rate at the pharma-retailer's end is dependent on the instantaneous inventory level and replenishment policies consist of (a) make decision on order quantity (lot size) or production rate and (b) make decision on reorder point. Retailer decides on order quantity as decision variable. The pharma-manufacturer uses EPQ system and follows the lot-for-lot policy. The mathematical models are developed under three various structures: (1) decentralized decision-making, (2) coordinated decision-making. Firstly, in the decentralized decision making, each PSC member maximizes its own average profit and achieves optimal decision variables without considering one decision maker as an integrated firm. In this case, under the decentralized structure, the pharma-retailer determines economic order quantity, which is a locally optimal solution from the entire PSC viewpoint. In this manner, usually the Pharma-manufacturer produces higher than pharma-retailer's order quantity value. In another decision-making, the coordination of decision regarding order quantity in a two member PSC is considered. in this research, two model based on a credit period contract to coordinate the both members in two frameworks are proposed; independent optimization and centralized model with credit policy. In the independent optimization decision-making with credit policy, the Pharma-manufacturer to encourage the pharma-retailer to increase its order quantity, proposes a mechanism of coordination as delay in payment (credit period). In the centralized model with credit policy, it is assumed that there is one main PSC as an integrated firm which determines optimal decision variable to maximize the whole PSC profit. In reality, profit function of this centralized model with credit policy is equal to the sum of PSC members profit functions in independent optimization mode. The contract model determines optimal order quantity to maximize the whole PSC average profits and also incentivizing members to participate, when the demand rate is dependent on the pharma-retailer's instantaneous inventory level. It means that in this decision process, the optimum profitability of PSC is guaranteed as well as more profitability for all members than decentralized mode. The results of numerical experiments confirm that the proposed model can achieve more profit.

The rest of this paper is organized as follows. In the following section, the problem description, assumptions and notations are presented, in addition, mathematical models are proposed in two structures decentralized decision-making and coordinated decision-making scenarios. Section 3 contains numerical examples and sensitivity analyses with discussions and finally, section 4 provides conclusions and potential future studies.

2- Problem description and mathematical models

In this paper, a two stage pharmaceutical supply chain consisting of a pharma-manufacturer and pharma-retailer with one type of pharmaceutical product is considered. The demand rate at the pharma-retailer's end is dependent on the current-inventory-level. The following assumptions and notations are used through the whole paper.

2-1-Assumptions

- This two stage PSC consists of a pharma-manufacturer suppling one type of pharmaceutical product to a pharma-retailer.
- The market demand rate $D(t)$ of the drug that the pharma-retailer faces, which is dependent on the instantaneous inventory level $I(t)$, and is assumed to be in the following polynomial power function: $D(t) = \alpha I(t)^{\beta}$, $0 \leq t \leq T$ where $\alpha > 0$ and $0 < \beta < 1$ are market scale and shape parameters. The shape parameter b is the elasticity of the demand with respect to the current-inventory-level. There are advantages of this type of demand pattern can be seen in (Baker and Urban 1988) paper.
- The pharma-manufacturer uses EPQ system to replenish its inventory with production rate R at production length per cycle $T_m = \frac{Q}{R}$, $(T_m \leq T)$, and he/she produces the same amount as the pharma-retailer orders.
- Since pharmaceuticals are vital products, the pharma-retailer has to stowage products to the amount of mQ for critical times, where $0 < m < 1$. We also assume that pharma-retailer has specified sufficient shelf space for displaying pharmaceuticals received from pharma-manufacturer. Therefore, the pharma-retailer replenishes when its inventory level becomes to reorder point mQ. It means that The pharma-retailer's inventory is depleting at a decreasing rate due to the stock-dependent demand until the inventory becomes to reorder point (see Figure 1).
- The order costs consist of two parts: fixed costs and variable costs. When the demand is fixed and there are no lost sales, the order cost can be assumed constant because the variable part of the order cost does not affect decision on lot-size. However, the order cost could be related to the order quantity when the demand rate is variable, pointed by researchers like (Zhou and Lau 2000, Zhou, Min et al. 2008). Today, many retailers can order their requirements directly through the Internet, which made the portion of that the fixed part of the order cost insignificant that can be neglected. Therefore, it is assumed that the order cost is dependent to the replenishment lot-size in this paper.
- The pharma-manufacturer follows the "lot-for-lot" policy.
- Shortages are not allowed to occur.
- The lead time is zero.

2-2- Notations

Q: Pharma-retailer's order quantity (decision variable)

c_0: Pharma-manufacturer's production cost of the drug per unit

w: Wholesale price charged of the drug by the pharma-manufacturer to the pharma-retailer

p: Retail price of the drug per unit ($c_0 < w < p$)

R: pharma-manufacturer's production rate

$I(t)$: Pharma-retailer's inventory level at time t

f: the fixed cost per shipment

μ: the unit transportation cost

k_{h2}: Pharma-retailer's opportunity cost of capital

s_{h2}: Pharma-retailer's physical inventory holding cost

c_{h2}: Unit inventory holding cost per unit time for the pharma-retailer, where

$c_{h2} = k_{h2} + s_{h2}$

k_{h1}: Pharma-manufacturer's opportunity cost of capital

s_{h1}: Pharma-manufacturer's physical inventory holding cost

c_{h1}: Unit inventory holding cost per unit time for the pharma-manufacturer, where

$c_{h1} = k_{h1} + s_{h1}$

T: Replenishment cycle length

T_1: Pharma-manufacturer's production length per cycle

τ: Offered credit payment period from pharma-manufacturer (pharma-manufacturer's decision variable)

π_{p2}: Pharma-retailer's average profit

π_{p1}: Pharma-manufacturer's average profit

π_{psc}: Pharma-supply chain's average profit

Since the demand rate is equal to the decrease in the inventory level, the pharma-retailer's inventory level $I(t)$ can be described by the following differential equation:

$$\begin{cases} \frac{dI(t)}{dt} = -\alpha I(t)^{\beta}. & 0 \leq t \leq T \\ I(0) = Q. & I(T) = mQ \end{cases} \tag{1}$$

Where mQ can get as reorder point for pharma-retailer and m is a exogenous parameter for convenient and perfect purpose.

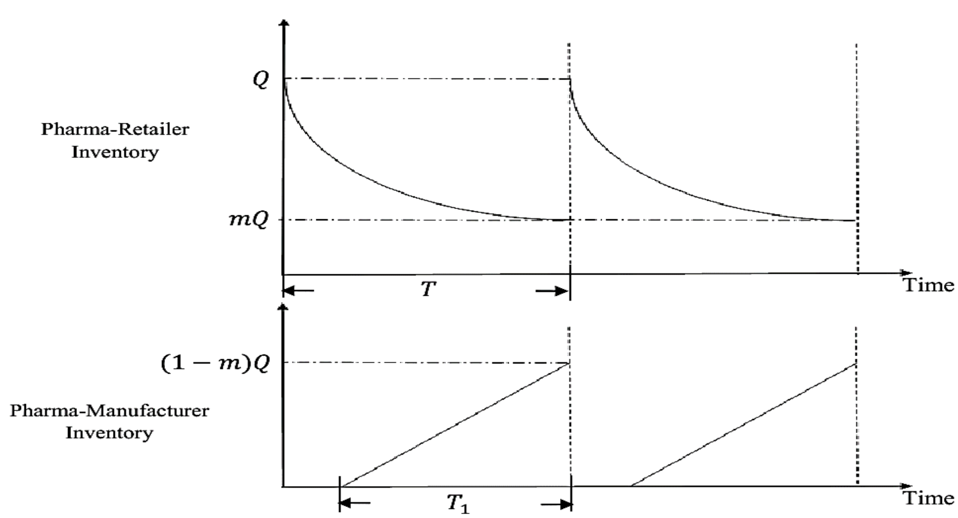


Fig 1. Inventory levels of Pharma-Retailer and Pharma-manufacturer

By integrating equation (1) and its solution we have:

$I(t) = \left[Q^{1-\beta} - \alpha(1-\beta)t\right]^{1/1-\beta}$, and

$$T = \frac{(1-m^{1-\beta})Q^{1-\beta}}{\alpha(1-\beta)}. \quad (2)$$

In this paper, our variables are pharma-retailer's inventory at time t and order quantity Q, which, by integrating and solving according to the above equations, our variable is the same order quantity Q.

2-3- Decentralized decision-making

Under decentralized decision-making mode, members decide base on their own profits and determine the production and the order quantities as the basic decision variable. The pharma-retailer determines order quantity Q to maximize the following average profit:

$$\pi_{p2}^{dc} = \frac{1}{T}\left[(p-w)(Q-mQ) - \varphi(Q-mQ) - c_{h2}\int_0^T I(t)\,d(t)\right] = \frac{1}{T}\left\{\begin{matrix} (1-m)(p-w-\varphi)Q - \frac{c_{h2}}{\alpha(2-\beta)} \\ [Q^{2-b} - [Q^{1-b} - a(1-b)T]^{2-b/1-b}] \end{matrix}\right\} \quad (3)$$

The elements of equation (3) are sales revenue, purchasing cost, order cost and holding cost. Substituting equation (2) into equation (3), we get

$$\pi_{p2}^{dc} = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})Q^{1-\beta}}\left[(1-m)(p-w-\varphi)Q - \frac{(1-m^{2-\beta})c_{h2}}{\alpha(2-\beta)}Q^{2-\beta}\right]. \quad (4)$$

Proposition 1: The pharma-retailer's profit function in this model is concave with respect to Q and the optimal order quantity Q^* can be calculated at $d\pi_{p2}^{dc}/dQ = 0$:

$$Q^* = \left[\frac{\alpha\beta(2-\beta)(1-m)(p-w-\varphi)}{(1-m^{2-\beta})c_{h2}}\right]^{1/1-\beta}. \quad (5)$$

Proof: taking the second derivatives of π_r^{dc} with respect to Q gives:

$$\frac{d^2\pi_{p2}^{dc}}{dQ^2} = -\frac{\alpha\beta(1-\beta)^2(1-m)(p-w-\varphi)Q^{\beta-2}}{(1-m^{1-\beta})} < 0. \quad (6)$$

Since always$p > w + \varphi$, equation (6) is negative and therefore the profit function is concave over$Q$.

Thereafter, substituting Q^* into equation (4), we can get the pharma-retailer' optimal average profit, π_{p2}^{dc*}.

Under decentralized mode, the pharma-manufacturer has to follow the pharma-retailer's decision and determines the production quantity. The elements of the pharma-manufacturer's profit function are as follows: sales revenue, production cost and holding cost. The pharma-manufacturer's objective function is

$$\pi_{p1}^{dc} = \frac{1}{T}\left[(w-c_0)(1-m)Q - \frac{1}{2}c_{h1}(1-m)QT_1\right]. \quad (7)$$

In equation (8), $T_1 = \frac{(1-m)Q}{R}$ and substituting equation (2) into equation (8) gives:

$$\pi_{p1}^{dc} = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})Q^{1-\beta}}\left[(w-c_0)(1-m)Q - \frac{c_{h1}((1-m)Q)^2}{2R}\right]. \quad (8)$$

Substituting Q^* into equation (8), we can get the pharma-manufacturer's average profit, π_{p1}^{dc*}. Since, the retailer's optimal economic quantity is usually different from the pharma-manufacturer's economic production quantity, in most cases, pharma-manufacturer's production quantity is larger than the retailer's order quantity.

Proposition 2: The pharma-manufacturer profit function is concave with respect to Q.

Proof: taking the second derivative of π_{p1}^{dc} with respect to Q in Eq. (9), we find out that π_{p1}^{dc} is concave in$Q$.

$$\frac{d^2\pi_{p1}^{dc}}{dQ^2} = -\frac{\alpha(1-\beta)}{(1-m^{1-\beta})}\left[\beta(1-\beta)(w-c_0)(1-m)Q^{\beta-2} + \frac{\beta(\beta+1)(1-m)^2c_{h1}}{2R}Q^{\beta-1}\right] < 0. \quad (9)$$

Therefore, we can get the pharma-manufacturer's optimal production quantity at $d\pi_1^{dc}/dQ = 0$ in equation (10).

$$\frac{d\pi_{p1}^{dc}}{dQ} = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})}\left[\beta(w-c_0)(1-m)Q^{\beta-1} - \frac{(\beta+1)(1-m)^2c_{h1}}{2R}Q^{\beta}\right] = 0. \quad (10)$$

If the pharma-retailer's order quantity Q^* is less than the economic production quantity obtained by equation (10), the pharma-manufacturer can get more profit when the pharma-retailer orders more.

2-4- Coordinated decision-making

In this section, the coordination of decision regarding order quantity in a two members of PSC is considered. in this research, two model based on a credit payment period to coordinate the members in two single frameworks is proposed:

2-4-1- Credit period contract in independent optimization model

In the first framework, the pharma-manufacturer proposes a credit payment scheme to induce the pharma-retailer and to increase the order quantity that the pharma-manufacturer can get more profit. In this mode, the pharma-manufacturer offers the pharma-retailer a credit payment period τ that dependent on order quantity, in which the pharma-retailer can save opportunity to capitalization. The Independent model with offering credit payment period is

$Maximize$:

$$\pi_{p1}^{co}(Q.\tau) = \pi_{p1}^{dc} - (1-m)Qk_{h1}\tau. \quad (11)$$

$Subject\ to$:

$$\Delta\pi_{p2} = \pi_{p2}^{co}(Q.\tau) - \pi_{p2}^{dc*} = [\pi_{p2}^{dc} + (1-m)Qk_{h1}\tau] - \pi_{p2}^{dc*} \geq 0. \quad (12)$$

Using Eq. (4) and Eq. (8) into above model, we have

$Maximize$:

$$\pi_{p1}^{co}(Q.\tau) = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})Q^{1-\beta}}\left[(w-c_0)(1-m)Q - \frac{c_{h1}\left((1-m)Q\right)^2}{2R} - (1-m)Qk_{h1}\tau\right]. \quad (13)$$

$Subject\ to$:

$$\Delta\pi_{p2} = \pi_{p2}^{co} - \pi_{p2}^{dc*} = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})Q^{1-\beta}}[(p-w)(Q-mQ) - \varphi(Q-mQ) - \frac{(1-m^{2-\beta})c_{h2}}{\alpha(2-\beta)}Q^{2-\beta} + (Q-mQ)k_{h2}\tau] - \pi_{p2}^{dc*} \geq 0. \tag{14}$$

Where the constraint ensures the pharma-retailer gets no less than decentralized mode. There are two case for optimal solution Q^* in above model:

Case I: The minimum $\Delta\pi_{p2}$ ($\overleftarrow{\Delta\pi_{p2}}$) can be resulted when the pharma-manufacturer give to pharma-retailer the same profit as decentralized mode, $\overleftarrow{\Delta\pi_{p2}} = \pi_{p2}^{co} - \pi_{p2}^{dc*} = 0$. Thus, the pharma-manufacturer can get higher profit.

When $\pi_{p2}^{co} - \pi_{p2}^{dc*} = 0$, we can obtain

$$\tau(Q) = \frac{(1-m^{1-\beta})*\pi_{p2}^{dc*}}{\alpha(1-\beta)(1-m)k_{h2}}Q^{-\beta} + \frac{(1-m^{2-\beta})c_{h2}}{\alpha(2-\beta)(1-m)k_{h2}}Q^{1-\beta} - \frac{(p-w-\varphi)}{k_{h2}}. \tag{15}$$

Substituting equation (15) into π_{p1}^{co}, we get

$$\pi_{p1}^{co} = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})}[\left((w-c_0)(1-m) + \frac{(p-w-\varphi)(1-m)k_{h1}}{k_{h2}}\right)Q^{\beta} - \frac{c_{h1}(1-m)^2}{2R}Q^{\beta+1} - \frac{c_{h2}(1-m^{2-\beta})k_{h1}}{\alpha(2-\beta)k_{h2}}Q - \frac{\pi_{p2}^{dc*}(1-m^{1-\beta})k_{h1}}{\alpha(1-\beta)k_{h2}}]. \tag{16}$$

Proposition 3: The profit function π_{p1}^{co} is concave with respect to Q.

Proof: Taking the second derivative of π_{p1}^{co} with respect to Q in Eq. (23), we find out that π_{p1}^{co} is concave in Q.

$$\frac{d^2\pi_{p1}^{co}}{dQ^2} = -\frac{\alpha(1-\beta)}{(1-m^{1-\beta})}\left[\left(\beta(1-\beta)(w-c_0)(1-m) + \frac{(p-w-\varphi)(1-m)k_{h1}}{k_{h2}}\right)Q^{\beta-2} + \frac{\beta(\beta+1)(1-m)^2c_{h1}}{2R}Q^{\beta-1}\right] < 0. \tag{17}$$

Thereafter, we can get the Q^* in following.

$$\frac{d\pi_{p1}^{co}}{dQ} = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})}\left[\left(\beta(w-c_0)(1-m) + \frac{(p-w-\varphi)(1-m)k_{h1}}{k_{h2}}\right)Q^{\beta-1} - \frac{(\beta+1)(1-m)^2c_{h1}}{2R}Q^{\beta} - \frac{c_{h2}(1-m^{2-\beta})k_{h1}}{\alpha(2-\beta)k_{h2}}\right] = 0. \tag{18}$$

Substituting Q^*into equation (16), we can get the pharma-manufacturer‘s optimal profit value π_{p1}^{co*}.

According case I, the pharma-retailer may be acquiesced or may not be satisfied. Therefore, the pharma-retailer requires higher profit from the pharma-manufacturer. We assume the pharma-manufacturer accepts that the pharma-retailer to get $\Delta\pi_{p2}$ more than its profit in the decentralized mode. Thus,

$$\pi_{p2}^{co} - \pi_{p2}^{dc*} = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})Q^{1-\beta}}\left[(p-w)(Q-mQ) - \varphi(Q-mQ) - \frac{(1-m^{2-\beta})c_{h2}}{\alpha(2-\beta)}Q^{2-\beta} + (Q-mQ)k_{h2}\tau\right] - \pi_{p2}^{dc*} = \Delta\pi_{p2}. \tag{19}$$

When $\pi_{p2}^{co} - \pi_{p2}^{dc*} = \Delta\pi_{p2}$, we can obtain

$$\tau(Q)=\frac{(1-m^{1-\beta})(\pi_{p2}^{dc*}+\Delta\pi_{p2})}{\alpha(1-\beta)(1-m)k_{h2}}Q^{-\beta}+\frac{(1-m^{2-\beta})c_{h2}}{\alpha(2-\beta)(1-m)k_{h2}}Q^{1-\beta}-\frac{(p-w-\varphi)}{k_{h2}}. \quad (20)$$

Substituting equation (20) into π_{p1}^{co}, we get

$$\pi_{p1}^{co}=\frac{\alpha(1-\beta)}{(1-m^{1-\beta})}[\left((w-c_0)(1-m)+\frac{(p-w-\varphi)(1-m)k_{h1}}{k_{h2}}\right)Q^{\beta}-\frac{c_{h1}(1-m)^2}{2R}Q^{\beta+1}-\frac{c_{h2}(1-m^{2-\beta})k_{h1}}{\alpha(2-\beta)k_{h2}}Q-\frac{(\pi_{p2}^{dc*}+\Delta\pi_{p2})(1-m^{1-\beta})k_{h1}}{\alpha(1-\beta)k_{h2}}]. \quad (21)$$

Proposition 4: According to proposition 3, the profit function π_{p1}^{co} is concave with respect to Q.

Proof: taking the second derivative of π_{p1}^{co} with respect to Q in equation (22), we find out that π_{p1}^{co} is concave in Q.

$$\frac{d^2\pi_{p1}^{co}}{dQ^2}=-\frac{\alpha(1-\beta)}{(1-m^{1-\beta})}\left[\left(\beta(1-\beta)(w-c_0)(1-m)+\frac{(p-w-\varphi)(1-m)k_{h1}}{k_{h2}}\right)Q^{\beta-2}+\frac{\beta(\beta+1)(1-m)^2c_{h1}}{2R}Q^{\beta-1}\right]<0. \quad (22)$$

Thus, we can get the Q^* in $\frac{d\pi_{p1}^{co}}{dQ}=0$.

$$\frac{d\pi_{p1}^{co}}{dQ}=\frac{\alpha(1-\beta)}{(1-m^{1-\beta})}\left[\beta\left((w-c_0)(1-m)+\frac{(p-w-\varphi)(1-m)k_{h1}}{k_{h2}}\right)Q^{\beta-1}-\frac{(\beta+1)(1-m)^2c_{h1}}{2R}Q^{\beta}-\frac{c_{h2}(1-m^{2-\beta})k_{h1}}{\alpha(2-\beta)k_{h2}}\right]=0.$$
(23)

In the above equation, it is clear that there is no $\Delta\pi_{p2}$, hence $\Delta\pi_{p2}$ does not change the solution. Therefore, the optimal production quantity of the pharma-manufacturer is the same Q^* obtained in equation (18).

Case II: The maximum $\Delta\pi_{p2}$ ($\overrightarrow{\Delta\pi_{p2}}$) can be resulted when the pharma-manufacturer gets the same profit as decentralized mode, $\overrightarrow{\Delta\pi_{p2}}=\pi_{p1}^{co}-\pi_{p1}^{dc*}=0$. Thus, the pharma-retailer can get higher profit.

$$\overrightarrow{\Delta\pi_{p2}}=\frac{\alpha(1-\beta)k_{h2}}{(1-m^{1-\beta})k_{h1}}\left[\left((w-c_0)(1-m)+\frac{(p-w-\varphi)(1-m)k_{h1}}{k_{h2}}\right)Q^{*\beta}-\frac{c_{h1}(1-m)^2}{2R}Q^{*\beta+1}-\frac{c_{h2}(1-m^{2-\beta})k_{h1}}{\alpha(2-\beta)k_{h2}}Q^*-\frac{(1-m^{1-\beta})\pi_{p1}^{dc*}}{\alpha(1-\beta)}\right].$$
(24)

Therefore, the minimum profit that the pharma-retailer can get is $\overleftarrow{\Delta\pi_{p2}}=\pi_{p2}^{co}-\pi_{p2}^{dc*}=0$, and the maximum profit the pharma-retailer can get is $\overrightarrow{\Delta\pi_{p2}}=\pi_{p1}^{co}-\pi_{p1}^{dc*}=0$.

2-4-2- Centralized model with credit period contract

In second framework under centralized mode, one decision maker as an integrated firm determines the order quantity to maximize the entire PSC profit. In this case, the decision variable is expected to be globally optimized. Profit function of the joint PSC with credit period contract is equal to the sum of PSC members profit functions in first framework of coordinated decision-making. Joint PSC expected profit function can be formulated as:

$$\pi_{psc}^{c} = \frac{1}{T}[(p - c_0 - \varphi)(Q - mQ) - \frac{(1-m^{2-\beta})c_{h2}}{\alpha(2-\beta)}Q^{2-\beta} - \frac{c_{h1}((1-m)Q)^2}{2R} + (k_{h2} - k_{h1})\tau(Q - mQ)]. \quad (25)$$

Some simplifications give:

$$\pi_{psc}^{c} = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})}[(1-m)(p - c_0 - \varphi)Q^{\beta} - \frac{(1-m^{2-\beta})c_{h2}}{\alpha(2-\beta)}Q - \frac{c_{h1}(1-m)^2}{2R}Q^{\beta+1} + (1-m)(k_{h2} - k_{h1})\tau Q^{\beta}]. \quad (26)$$

The second derivative of π_{psc}^{c} with respect to Q can be obtained in

$$\frac{d^2\pi_{psc}^{c}}{dQ^2} = -\frac{\alpha(1-\beta)}{(1-m^{1-\beta})}\left[\beta(1-\beta)(1-m)(p - c_0 - \varphi)Q^{\beta-1} + \frac{c_{h1}\beta(\beta+1)(1-m)^2}{2R}Q^{\beta-1} + \beta(1-\beta)(1-m)(k_{h2} - k_{h1})\tau Q^{\beta-2}\right]. \quad (27)$$

In equation (26), it is clear that if the pharma-retailer's opportunity cost of capital exceeds that of the pharma-manufacturer, π_{psc}^{c} increases as τ goes up, accordingly, the optimal τ is positive value, $\tau = \tau_{max}$. Also, when $k_{h2} = k_{h1}$, no credit payment period should be offered, τ can be any value. On the other hand, when $k_{h2} < k_{h1}$, if τ can be less than zero, then $\tau = \tau_{min}$, otherwise, $\tau = 0$. Thus, $d^2\pi_{psc}^{c}/dQ^2 < 0$, π_{psc}^{c} is concave in Q.

Thereafter, we can get the PSC's economic order quantity Q^* at $d\pi_{psc}^{c}/dQ = 0$.

$$\frac{d\pi_{psc}^{c}}{dQ} = \frac{\alpha(1-\beta)}{(1-m^{1-\beta})}[\beta(1-m)(p - c_0 - \varphi)Q^{\beta-1} - \frac{(1-m^{2-\beta})c_{h2}}{\alpha(2-\beta)} - \frac{c_{h1}(\beta+1)(1-m)^2}{2R}Q^{\beta} + \beta(1-m)(k_{h2} - k_{h1})\tau Q^{\beta-1}] = 0. \quad (28)$$

Substituting Q^*into equation (26), we can get the pharma-manufacturer's optimal profit value π_{p1}^{co*}.

Therefore, this proposed framework maximizes the whole PSC profit while each member's profit would not get a value less than the decentralized decision-making.

3- Numerical examples and sensitivity analysis

In this section, the proposed model is tested by various shape of parameter β and various retailers' opportunity cost of capital k_{h2} in two test problems, because these two parameters have a major influence on the profit and order quantity between all of the parameters.

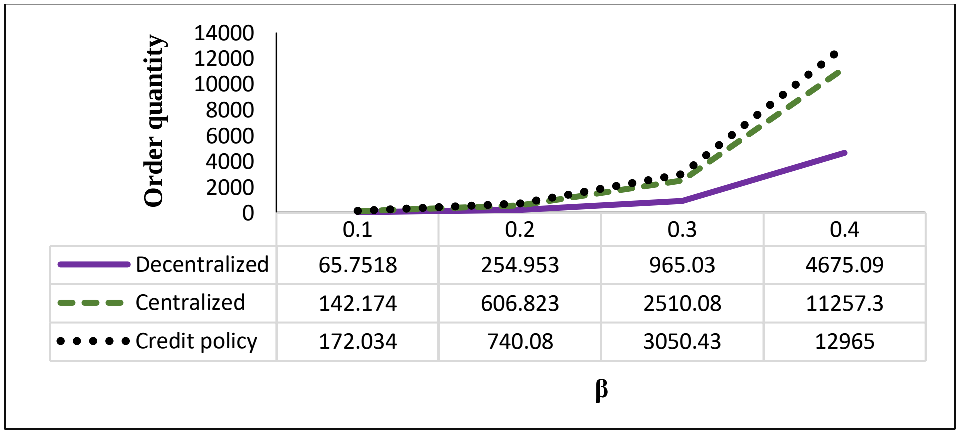
Test problem 1: In this part, we probe the fluctuations of the order quantity and expected profits under three structures responding to various demand elasticity β from 0.1 to 0.4. Also we assume that the pharma-retailer gets the same profit of decentralized mode in the centralized and credit policy modes, and the pharma-manufacturer gets all of the surplus gross profit in centralized and credit policy modes. Because we can find that the order quantity be determined by pharma-retailer and he/she gets the same possible maximum profit in decentralized mode. Therefore, the pharma-retailer's profit in centralized and credit policy modes is the same profit in decentralized mode.

In this part, the parameter values are

α =40, p =22, w =15, c_0 =10, φ =2, m =0.5, s_{h1} =0.1, s_{h2} =0.25,

k_{h1}=0.25, k_{h2}=0.35, c_{h1}=0.35, c_{h2}=0.6, R=2000.

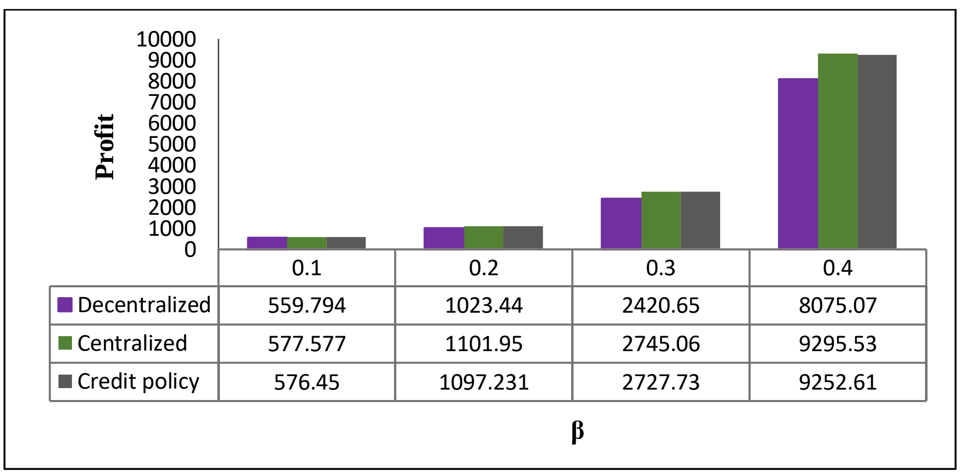
In figure 2, we can find that the order quantity can be grown when the shape parameter β is increased. As expected, the order quantity value under coordinated decision-making with credit policy is greater than decentralized model which implies that the coordination decision-making is applicable from both else structures' perspective.



	0.1	0.2	0.3	0.4
Decentralized	65.7518	254.953	965.03	4675.09
Centralized	142.174	606.823	2510.08	11257.3
Credit policy	172.034	740.08	3050.43	12965

Fig 2. Variation of order quantity under decentralized, centralized and credit policy by increasing **β**.

Figure 3 shows the expected profits of the pharmaceutical supply chain (PSC) under three structures responding to various β. As expressed, the PSC expected profit increases in all three structures when the demand elasticity increases. As can be seen, the supply chain expected profit in the credit policy model is always higher than profits the two other models.



	0.1	0.2	0.3	0.4
Decentralized	559.794	1023.44	2420.65	8075.07
Centralized	577.577	1101.95	2745.06	9295.53
Credit policy	576.45	1097.231	2727.73	9252.61

Fig 3. Variation of PSC profit under decentralized, centralized and credit policy by increasing **β**.

Therefore, figure 2 and 3 show that the order quantity and profit of three structures increases when the demand elasticity β increases, But the increase in the order quantity is larger than profits when the demand elasticity β increases.

Test problem 2: In this part, we probe the fluctuations of the order quantity and expected profits under three structures responding to various pharma-retailer's opportunity cost of capital k_{h2} when $k_{h2} < k_{h1}$, $k_{h2} = k_{h1}$ and $k_{h2} > k_{h1}$. In the centralized and credit policy modes, we assume the pharma-retailer gets the same profit of decentralized mode and the pharma-manufacturer gets all of the surplus gross profit. For τ is an unbounded variable in the centralized PSC mode, we assume:

a) If $k_{h2} < k_{h1}$, let $\tau = -0.1$,

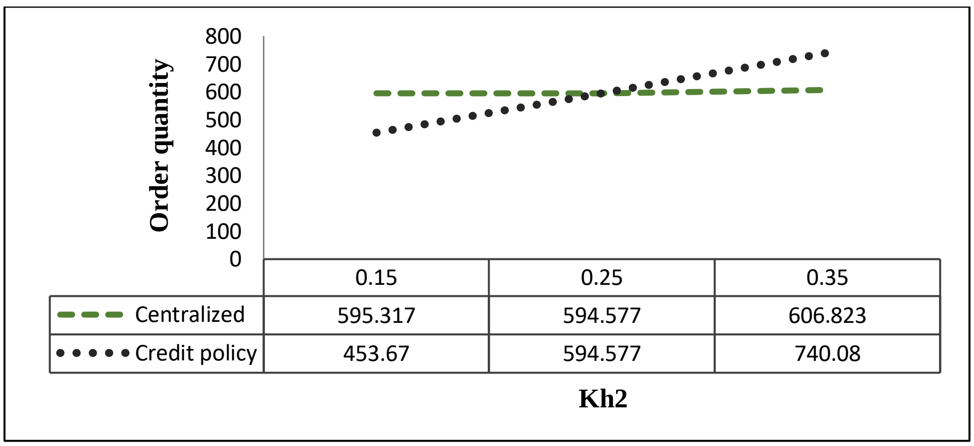
b) There is no τ when $k_{h2} = k_{h1}$,

c) Let τ equals the same value of τ in credit policy mode when $k_{h2} > k_{h1}$ like the values of the parameters in test problem 1.

In this part, the parameter values are

β= 0.2, α =40, p = 22, w =15, c_0 =10, φ =2, m = 0.5, s_{h1} =0.1,
k_{h1}= 0.25, c_{h1}= 0.35, c_{h2}= 0.6, R= 2000

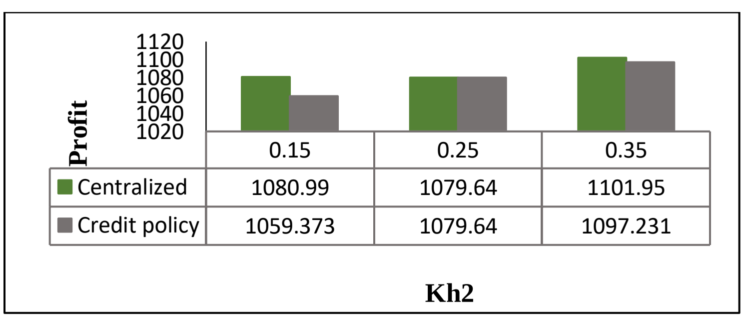
Figure 4 illustrates that the order quantity of the pharmaceutical supply chain (PSC) under credit policy mode increases when retailer's opportunity cost of capital k_{h2} increases. It is therefore clear that the pharma-retailer can get higher credit payment period with more orders. In this Figure, we can also see that the order quantity values of centralized and credit policy modes are similar together in k_{h2}= 0.25, because there is no τ when $k_{h2} = k_{h1}$.



	0.15	0.25	0.35
Centralized	595.317	594.577	606.823
Credit policy	453.67	594.577	740.08

Fig 4. Variation of order quantity under centralized and credit policy by different $\mathbf{k_{h2}}$.

In figure 5, for all value of k_{h2}, the profit is always no less in the centralized mode than in the credit policy mode. It can also be seen that the profit of the (PSC) under credit policy mode increases by increasingk_{h2}. We can also see that the profit values of centralized and credit policy modes are similar together in k_{h2}= 0.25, because there is no τ when $k_{h2} = k_{h1}$. Moreover, when the pharma-retailer's opportunity cost of capital is more than the manufacturer's, the pharma-manufacturer prefers the independent optimization with credit policy.



	0.15	0.25	0.35
Centralized	1080.99	1079.64	1101.95
Credit policy	1059.373	1079.64	1097.231

Fig 5. Variation of PSC profit under centralized and credit policy by different $\mathbf{k_{h2}}$

Finally, the results shown in above figures illustrate that the proposed coordination model mitigates positive effects of increasing β and k_{h2} on order quantity and PSC profitability with respect to decentralized decision-making model.

4- Conclusion

In this research, a mathematical model was developed for coordination decision-making on order quantity in a two stage PSC that consisting of a single pharma-manufacturer and a single pharma-retailer with one type of pharmaceutical product and the demand rate for the pharmaceutical product is dependent on the pharma-retailer's inventory-level on display. The pharma-retailer storage pharmaceutical product up to the reorder point for critical times. Two various decision-making models are considered for the proposed mathematical model. (a) Decentralized decision making; in this mode, each member determines its decision variable to maximize its own profit without considering objectives of other PSC members. Therefore, the pharma-retailer determined the economic order quantity value as its decision variable to optimize its own objective that usually the pharma-manufacturer's optimal production quantity is more than the retailer's economic order quantity. (b) Coordinated decision-making; in this mode, the pharma-manufacturer offers a credit payment period as an incentive contract to induce the pharma-retailer to increase its order quantity that we consider this policy in two frameworks: independent optimization and centralized model with credit policy. The pharma-manufacturer prefers the independent optimization with credit policy when the pharma-retailer's opportunity cost of capital is more than the pharma-manufacturer's; otherwise the pharma-manufacturer prefers the centralized model with credit policy.

Several research gaps can be existed to be extended in the future studies. First, we can be considering elasticity of demand to retailer price or rival price. Second, we can be using other concepts in supply chain, for example, backlogging shortage, disruption and multi stage inventory system with one or multiple members. Moreover, other coordination incentive contract can be extending such as profit-sharing, buy-back, discount quantity and quantity-flexibility.

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