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An integrated heuristic method based on piecewise regression and cluster analysis for fluctuation data (A case study on health-care: Psoriasis patients)

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Abstract

Trend forecasting and proper understanding of the future changes is necessary for planning in health-care area. One of the problems of analytic methods is determination of the number and location of the breakpoints, especially for fluctuation data. In this area, few researches are published when number and location of the nodes are not specified. In this paper, a clustering-based method is developed to obtain the number and the location of breakpoints. We propose an appropriate piecewise regression model to analyze the fluctuation data and predict trends of them. The efficiency of proposed integrated approach is evaluated by using simulated and real examples, and results are compared with results of Mars algorithm. Comparison shows that proposed approach has less sum of square error (SSE) criterion than Mars algorithm with equal number of nods.

Keywords: Piecewise regression, node, clustering, Mars algorithm, health-care systems.

1- Introduction

The fluctuations in observations have significant effect on the fitness of forecasting methods. In other words, the greater fluctuate of the observations lead to lower accuracy in results of simple or multiple regression. Hence, it is necessary to develop models with more capability to regress these data. In much health-care area, there are observations with high fluctuations, because diseases are seasonal. Hence methods by capability of analysis these data should be considered. Piecewise regression is one of the efficient regression methods for fluctuate data. In piecewise regression, data divided in some intervals with breakpoints and each interval evaluated with a different equation (Marsh, 2001).

Three important issues including degree of piecewise regression model, number of nodes and the location of nodes should be considered which have directly effect on the modeling quality. So, according to these issues, we introduce three types of the piecewise regression as follows:

1- Number and location of nodes is specified

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The simplest type of piecewise regression is the cases with specified number and location of nodes.

2- Number of nodes is specified and location of nodes is unknown

When location of nodes in not defined, piecewise regression model becomes more complex. In addition to estimate the regression coefficients, it is necessary to determine the location of nodes.

3- Number and location of nodes are unknown

This case is the most complex types of the piecewise regression model. when the number of nodes is not certain, the mathematical modeling is impossible (Objective function and constraints could not be defined (Jalili, 2016)).

In this paper, a clustering based method is developed to obtain number and location of nodes and an integrated clustering-heuristic method with high quality is modeled to predict the volatile data. The proposed piecewise approach meets the defect in clustering such that the fitness of piecewise regression model is maximized. Note that, this paper considers third case of piecewise regression with unspecified number and location of breakpoints. At the following, some researches in piecewise regression area and Mars algorithm applications are discussed.

2- A literature review on piecewise regression area and Mars algorithm applications

Rung et al. (2001) discussed about the importance of the triangular fuzzy piecewise regression to obtain the change point when the distributions of input data are different and then proposed mixed integer programming model. They supposed that the number of break points is known. Their proposed method is more power than ordinary fuzzy regression whereas, compared with ordinary fuzzy regression is more sensitive to outliers. Note that, results of their method are the global optimum solution. Toms et al. (2003) used univariate piecewise regression model as a tool to identify and obtain breakpoints of environmental data. They compared the sharp-transition model with three models including hyperbolic-tangent, bent-hyperbola and bent-cable and also computed three confidence interval containing standard error of model fitting, confidence interval of empirical self-starter and confidence interval of *f*-test to estimate break point. In addition, they stated that proposed method is applicable for other data like brightness level, the wind speed and population frequency in one region. Meggeo et al. (2003) introduced piecewise regression to model the complex non-linear problems. They studied the heart transplant trend and impact of dust concentration in a region of Germany on the suffering of bronchitis. Results showed that piecewise regression is suitable for analyzing the great fluctuations data. Strikholm et al. (2006) presented a method based on statistical tests to estimate the number of break points for univariate linear piecewise function. An advantage of the proposed method is the use of standard statistical inferences. Finally, they used simulated study to show the efficiency of their methods to obtain the number of break points. Debora et al. (2007) proposed a consistent piecewise regression model to determine the strategic turning point for organizational change. This model considers each first and second type of piecewise regression. They stated that, if number and location of break point are known (first case), then the least square method is used to estimate regression coefficients and for second case, the non-linear least square method is suitable for this aim. Meggeo et al. (2008) proposed an analysis based on segmentation concept. They argue that, the segmentation formulates problems using non-linear models and needs to obtain the segmentation input variables, number of break points and also initial guesses about location of break points (second type of piecewise regression). Malash et al. (2010) used piecewise regression and compared their results with other existed graphical methods. They designed piecewise regression function by solving a non-linear programming model which determined suitable location of break points and consequently proposed heuristic algorithm to obtain the number of break points. Finally, an experienced data are used to evaluate the validation and efficiency of the proposed method. Alessandro (2011) used piecewise regression to recognition of environmental break points. In this research, the Ruling regression is introduced to determine the break points. Arsang et al. (2011) analyzed the TB incidence trend in Iran during the year's 1964 to 2008 using piecewise regression. Hong et al. (2012) proposed a method to design the efficient piecewise regression model through approach development of efficient sample of regression tree for analyzing the semi-conductor performance. This method try by using degrees of freedom of the data set, maximize the estimation of impartially model. Yueju et al. (2013) presented a new mixed method through integration of the fuzzy logic and piecewise regression. This method is implemented on the Yukan River to recognize changes in patterns of spatial and temporal vegetables. Comparing the results with other existed regression model show the better performance of the proposed method for analyzing mentioned real data. Chamroukhi (2014) proposed mixed model with number of multinomial regression so that each piecewise regression is associated with one cluster and in each cluster, each particle is related with one food diet. Note that in this research, the K-means clustering is used. Mattews et al. (2014) applied piecewise regression to show the relationship between the level and distribution of wealth in the region with known number of break points. Briefly, they comparative study on piecewise regression models and then evaluated the performance of these models to obtain the convenient location of changes. Greene et al. (2015) examined the possibility of modeling the relationship between the HRQOL scales before and after treatment by using piecewise regression model. The medical information of 36625 patients before and after illness are collected. They used four regression models and two regression lines with best fitness are designed. Indeed, one break point is obtained. Results showed that piecewise regression had valid estimation from the base line and HRQOL scale size. Werner et al. (2015) applied *chow* test to obtain the break points in piecewise regression model. They examined the efficiency of the proposed method by using a real data set from the northern and southern hemispheres and global climate. Results showed that this method had less error than the other existed regression model. Jafari et al. (2015) analyzed the incidence of kidney failure in patients with type 2 diabetes using piecewise regression based on Lerman and Hadson algorithm. Yang et al. (2016) proposed an efficient model based on integer linear programming and a heuristic algorithm to estimate the number and location of break points in piecewise regression. Presented model is implemented for seven real data sets including Hydrodynamics, Energy Heating, Energy Cooling, Concrete, Airfoil, RedWine and WhiteWine and results showed that presented model had less absolute error for four mentioned area than the existing methods such as linear regression, MARS, Kiriging, MLP, SVR, KNN, Random forest, ALAMO and Pace Regression.

Mars algorithm has many applications including radio channel prediction (Kubin, 1999), environmental modeling (Leathwick, 2005) and prioritize buying, selling credit (Lee, 2006 and York, 2006) and etc. in addition, some researches presented the combined methods with Mars algorithm. For example, a combined Mars and neural network approach is presented by Lee (2005). In addition, De Andres (2011) proposed an integrated Mars algorithm with C-means clustering to predict the economic bankruptcy. Note that, another application of Mars algorithm is in data mining researches.

In this paper, a clustering based method is developed obtain number and location of nodes and a piecewise regression with high quality is modeled to predict the volatile data. In addition, an integrated clustering-heuristic approach is proposed to meet the deficit in clustering such that the fitness of piecewise regression model is maximized.

The rest of this paper is organized as follows: in the next section, we overview on the piecewise regression model, clustering technique and Mars algorithm. In Section 4, mentioned approaches are developed and the steps of heuristic algorithm are defined. The performance of proposed approaches is evaluated through a simulation study in Section 5 and a real data set in health-care systems is applied in section 6 to show the efficiency of proposed approaches in real application. Finally, concluding remarks and future researches are discussed in section 7.

3- An overview on volatile data analysis methods

In this section, we have a discussion on piecewise regression, clustering and Mars algorithm. At the following the piecewise regression model is discussed:

3-1- Piecewise regression model

One of regression models that is less considered, is piecewise regression. In piecewise regression, independent variables divided at intervals and for each interval a separate piece regression line is fitted and bound between parts called as breakpoint or node (Marsh, 2001). This model is very useful when there are many fluctuations in the data (Marsh, 2002). The general form of one degree of piecewise regression is as follows:

$$y = a + bT + cD(T - T_1) + e \tag{1}$$

Where, *a,b,c* are regression coefficients and y is the response variable. .T_1 is the location of node and T is the time period (*T*= 1,2,…, *N*). *D* is dummy variable and is 1 when, $T > T_1$ otherwise is zero and *e* is the error term of piecewise regression. in addition, the two and three degrees of piecewise regression models are defined as Equations 2 and 3 (Jalili, 2016).

$$y = a + bT^2 + cD(T - T_1)^2 + e \tag{2}$$

$$y = a + bT^3 + cD(T - T_1)^3 + e \tag{3}$$

3-2- Clustering technique

Clustering is a statistical method to group data or observations by considering the similarity of the observations (Bashiri and Kamranrad, 2015). Clustering could be categorized as hierarchical and nonhierarchical (K-means clustering). In hierarchical clustering, the number of clusters is not clear, but in nonhierarchical this item is specified (See, Bashiri and Kamranrad, 2015). Since, this paper focus on third case of piecewise regression and it should be specified the number and location of nodes at first hence, we use hierarchical cluster analysis.

3-3- Mars algorithm

Mars algorithm is one the most widely used method to search the node in regression technique first introduced by Friedman (1991). Mars is a nonparametric regression method for modeling between independents and dependent variables using number of basic function (Hastie, 2001). Mars algorithm is operated based on strategy of solution space dividing and it selects the best part of this space and then defines a related regression function. Kink in the curve is determined by Hinge function. Hinge function is very important in Mars model and defined as follows:

$$\max(0, x - c) \qquad x \geq c \tag{4}$$

$$\max(0, c - x) \qquad x < c \tag{5}$$

Where, *c* is constant value and named as Kink. Note that, equations 4 and 5 are formed the Hinge function and guarantee that hinge function always would be positive.

The general form of Mars algorithm is as following equation.

$$\hat{f}(x) = \sum_{i=1}^{K} C_i B_i(x) \tag{6}$$

Where, $\hat{f}(x)$ is the estimated value from *x*. C_i and $B_i(x)$ are the intercept coefficient for each basic function and the basic function, consequently. For basic functions of $B(x)$ there are three general cases including 1) a constant value which is intercept of the curve 2) A hinge function which has one of Equation 4 or 5 form and 3) combination of two or more hinge functions. This combination has ability to model the interaction between two or more variables (Jalili, 2015). The Mars algorithm is done in two main steps described in the following figure:

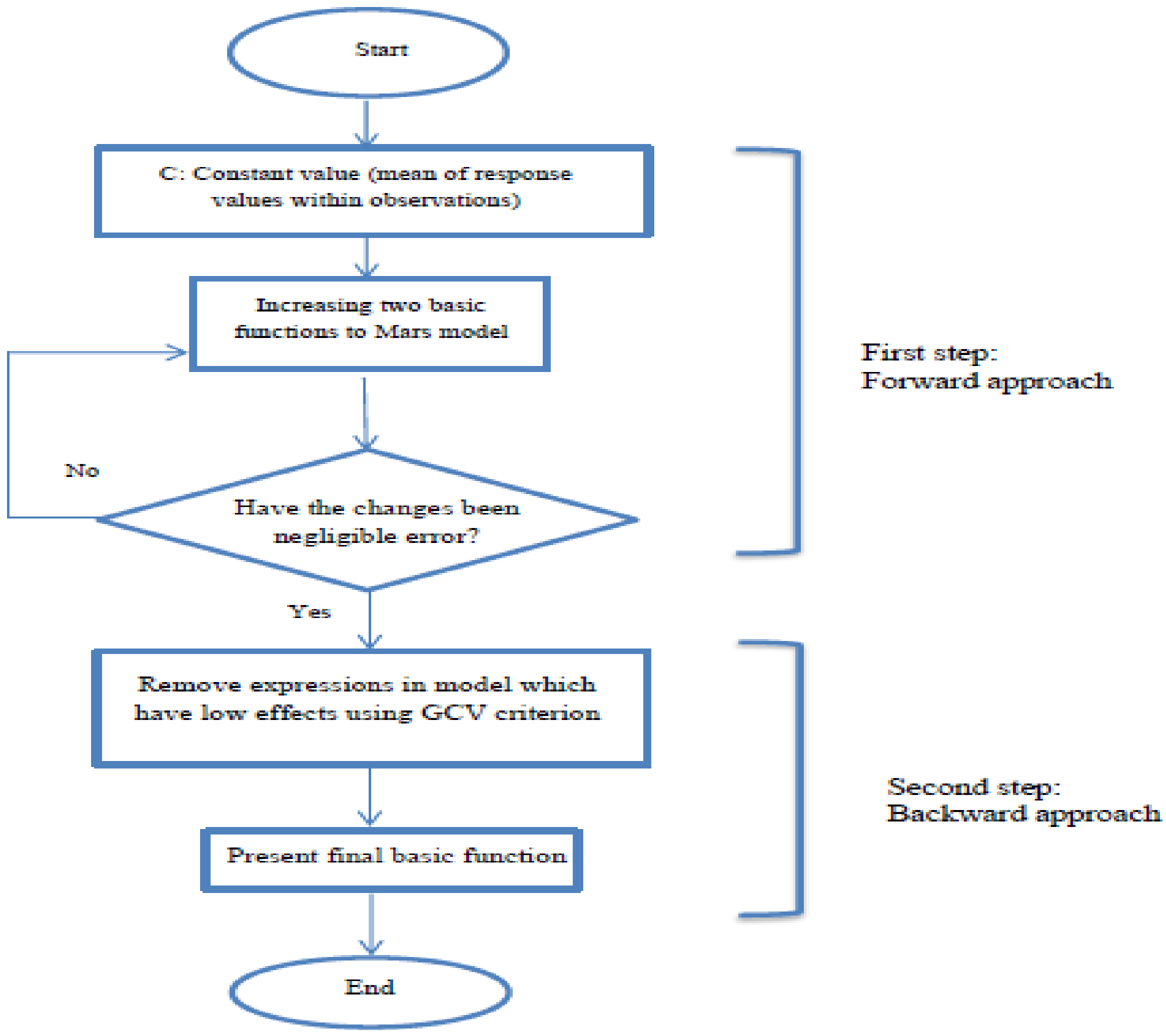

Fig 1. The steps of Mars algorithm

Where, GCV is calculated from the following equation.

$$GCV = \frac{RSS}{(N*(1-\frac{EffectiveNumberOfParameters}{N})^2)} \quad (7)$$

In the above equation, RSS is the root sum of square and N is the number of observations. In addition, the effective number of parameters could be calculated by equation (8).

$$EffectiveNumberOfParameters = NumberOfMARSterms + penalty*(\frac{NumberOfMARSterms-1}{2}) \quad (8)$$

Where, $\frac{NumberOfMARSterms-1}{2}$ is equal to the number of nodes from hinge functions and value of error (penalty) is usually considered as 2 or 3.

As mentioned previous, the aim stage of the start of Mars algorithm is to recognize the nodes location for each variable. This stage is done and calculated by the $Lof(f_M)$ where:

$$Lof(f_M) = GCV(M) = \frac{\frac{1}{N} \sum_{i=1}^{N} [y_i - f_M(x_i)]}{\left[1 - \frac{C(M)}{N}\right]}, \tag{9}$$

Where, M, N and $C(M)$ are the number of identified area (number of basic functions), the total number of observations and the penalty value, respectively. In addition, y_i is the response variable and $f_M(x_i)$ is the predicted value for response variable increases by increasing the number of M and the number of equation variables. Note that, all observations are evaluated to find the suitable location of nodes and node location is where at minimum *Lof* (Jalili, 2015).

4- Problem defenition and proposed method

As mentioned in previous section, clustering is used to assign observations in different groups so that, observations in each group have most similarity and observations from different groups have maximum discrimination from each other. Note that, the bound of each cluster is called breakpoint or node. Indeed in our method, the number of clusters is defined as number of nodes minus one in piecewise regression; for example, with two clusters, we have one node in the regression model. In addition, node location is defined where two clusters are separated. Figure 2 shows the concept of number and location of nodes as well.

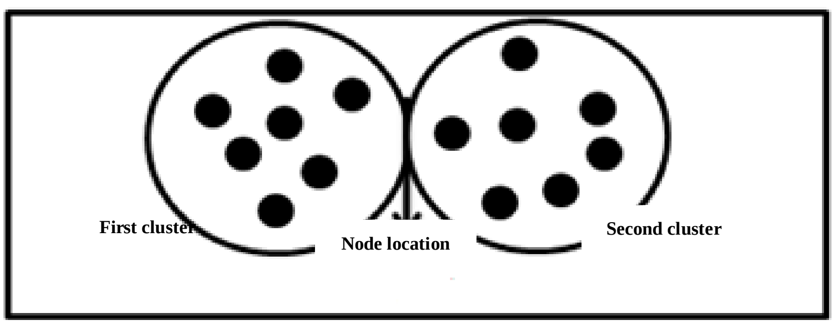

Fig 2. The concept of node and cluster

One of the weaknesses of clustering is the consecutive failure in a cluster. For example suppose that, there are 30 observations and after clustering we expect that observations from 1 to 10 are in cluster 1, observations from 11 to 14 are in cluster 2 and observations from 15 to 30 are in cluster 3. But, it is possible that observations from 1 to 10 are in cluster 1, observations from 15 to 23 are in cluster 2 and observations from 11 to 14 and 24 to 30 are in cluster 3, this means that consecutive observations are not located in a cluster. This event is happened because the clustering method considering only dependent variable (y), and do not attention to the independent (x) variable. So it is possible to some observation with faraway x values have close y values. This defect leads to decrease the adequacy of clustering and consequently decrease of prediction model fitness. To resolve of this defect, we propose new heuristic algorithm. This algorithm could be assigned observations from 11 to 14 to each of cluster 1 or 2 such that prediction model fitness is maximized. Figure 3 shows that existed data in cluster 3 are not consecutive.

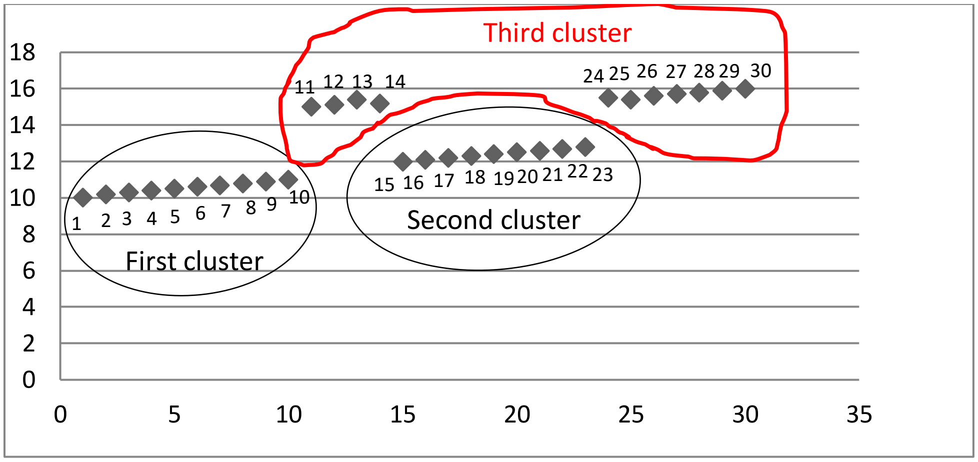

Fig 3. Defect in clustering with volatile data

4-1- Steps of proposed method

As mentioned before, to resolve the clustering defect, a heuristic algorithm is developed. This algorithm is presented by the following figure.

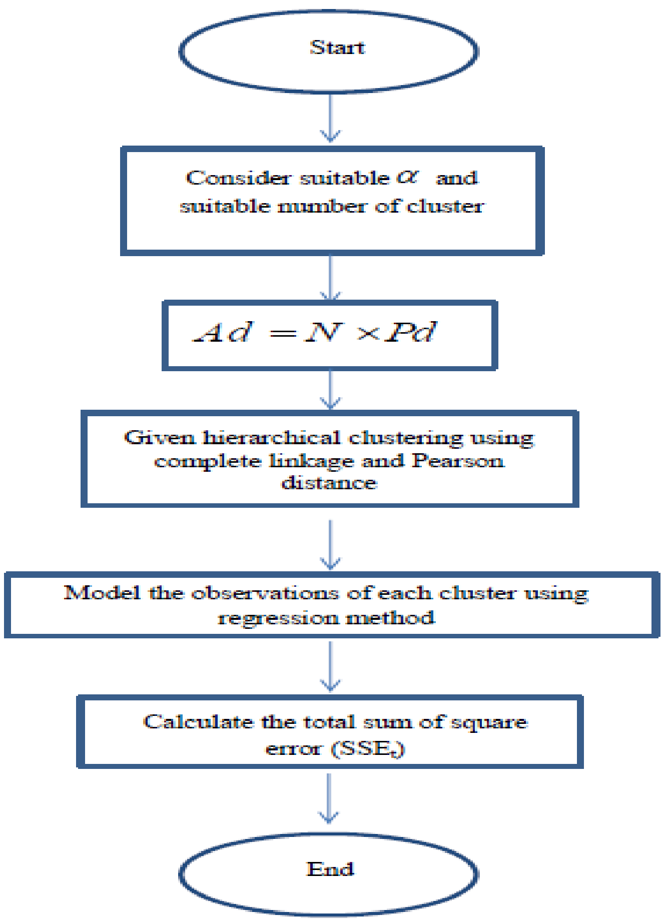

Fig 4. Steps of proposed algorithm

Where, α is the accepted similarity level obtained by expert. In addition, *Ad* is adjusted distance; *N* and *Pd* are the number of observations between each pair and Pearson distance between each pair, respectively. Furthermore, $SSE_T = \sum_{i=1}^{n} SSE_i$ and *n* is the number of cluster.

5- A numerical example

In this section, the performance of proposed approaches is evaluated by a numerical example which is extracted from Jalili (2015). This example involves 250 volatile data and related trend diagram is shown in figure 5.

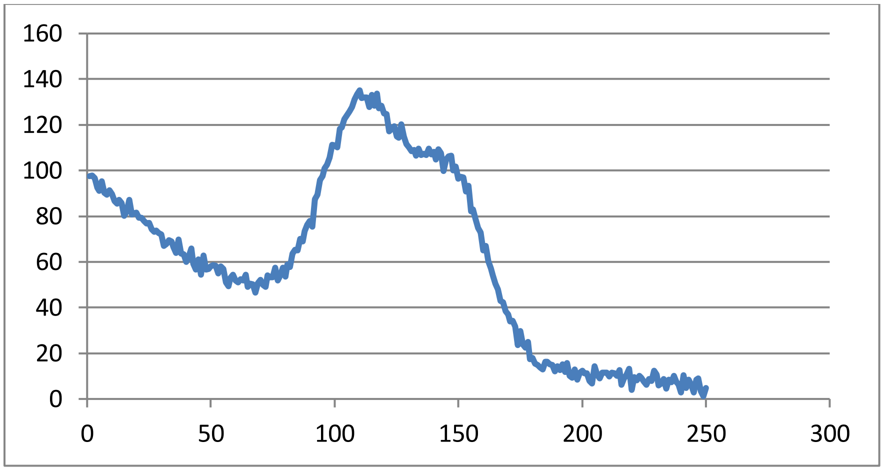

Fig 5. Observations diagram

According to these observations, following results are obtained. Note that, for 75 % similarity level, 8 clusters are selected.

Table 1. Results of proposed approach

Experiment error	Regression equations	Observations	Cluster number
102.33	y = 97.83 - 0.8860 x	1 to 34	Cluster 1
301.15	y = 126.1 - 2.261 x + 0.01722 x^2	35 to 82	Cluster 2
132.74	y = - 191.5 + 3.024 x	83 to 98	Cluster 3
378.44	y = - 4583 + 114.8 x - 0.9228 x^2+ 0.002442 x^3	99 to 146	Cluster 4
123.93	y = 545.2 - 2.976 x	147 to 162	Cluster 5
92.77	y = 426.9 - 2.286 x	163 to 178	Cluster 6
64.802	y = 64.21 - 0.2662 x	179 to 202	Cluster 7
264.33	y = 36.25 - 0.1233 x	203 to 250	Cluster 8

Regression equations are modeled by Minitab 17 software and the total SSE for 250 observations is 1460.49. In order to validate the proposed approach, results are compared with the Mars algorithm. To this aim, previous observations are analyzed by Mars algorithm and result is shown in the figure 6.

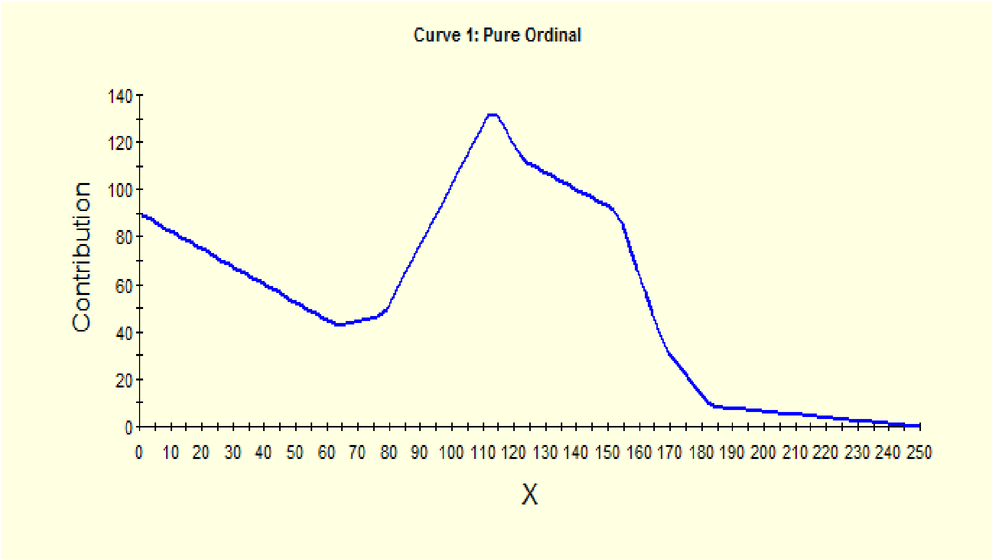

Fig 6. Observations fitting with Mars algorithm

As it is clear from Figure 6, Mars algorithm selects 7 nodes in locations of 63, 78, 113, 123, 153, 168 and 183. Hence, proposed equations by the Mars algorithm are as follows:

$$\begin{aligned} y &= 2.9467 + 0.7521BF_1 + 0.7512BF_2 + 1.0259BF_3 + 1.5472BF_4 - 3.0716BF_5 \\ &- 4.7439BF_6 + 2.2381BF_7 + 2.1248BF_8 \end{aligned} \quad (9)$$

Where,

$$\begin{aligned} BF_1 &= \max(0, x - 123) \\ BF_2 &= \max(0, 123 - x) \\ BF_3 &= \max(0, x - 63) \\ BF_4 &= \max(0, x - 183) \\ BF_5 &= \max(0, x - 153) \\ BF_6 &= \max(0, x - 113) \\ BF_7 &= \max(0, x - 78) \\ BF_8 &= \max(0, x - 168) \end{aligned}$$

The total SSE for regression equation by Mars algorithm is 2083.86. Table 2 compares the results from proposed method and Mars algorithm.

Table 2. Comparing of proposed method and Mars algorithm results

Method	Number of nodes	SSE
Proposed	7	1460.49
Mars	7	2083.86

As it is clear from SSE, proposed method has better performance in prediction modeling. In other words, cluster-heuristic based method could select the location of nodes better by assuming that the number of cluster is equal.

6- A real case study

In this section, performance of the proposed approach is evaluated by a real data set in health-care systems by focusing on Psoriasis patients. Data are collected from a Dermatology center in Tehran for during of 90 months for women and men departments, separately. Collected data for each women and men department are shown in figures 7 and 8.

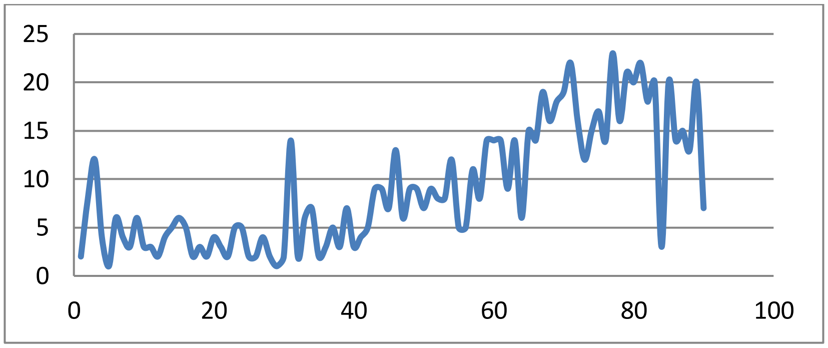

Fig 7. Psoriasis patient's data for women

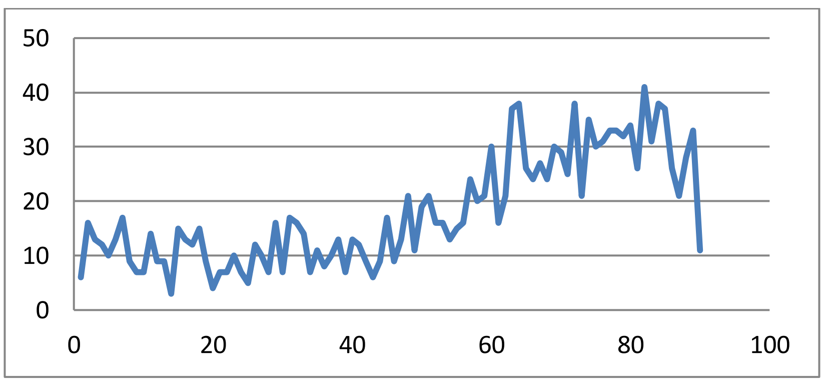

Fig 8. Psoriasis patient's data for men

As it is clear from above charts, data have fluctuation and general linear methods could not estimate regression equation, well. Since, Mars algorithm determines the number of nodes on its own hence, in order to more exact compare between proposed method and Mars algorithm, first observations are analyzed by Mars and then the best fitting and optimum number of nodes are used as selected parameter for proposed approach. For women data, Mars algorithm selects 3 nodes in 27, 55 and 79 locations shown in figure 9.

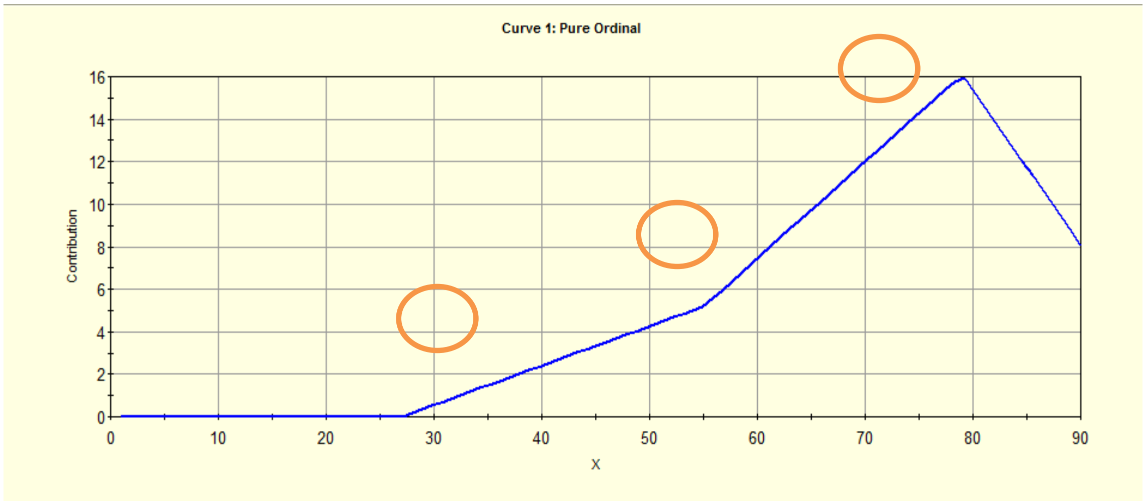

Fig 9. Number and location of nodes for women Psoriasis patients

In addition, regression model by Mars algorithm is proposed as follows:

$$y = 3.824 + 0.1851 BF_1 - 1.1854 BF_2 + 0.2695 BF_3 \tag{10}$$

Where,

$BF_1 = \max(0, x - 27)$

$BF_2 = \max(0, x - 55)$

$BF_3 = \max(0, x - 79)$

Note that, total SSE is 949.270. Now, using number of clusters extracted by Mars algorithm, proposed approach is applied for women data and results are reported in table 3.

Table 3. Results of proposed approach for women patients

Cluster number	Observations	Regression equations	Experiment error
Cluster 1	1 to 16	y = 5.450 - 0.0971 x	104.547
Cluster 2	17 to 42	y = 1.204 + 0.08957 x	163.651
Cluster 3	43 to 66	y = - 1.085 + 0.1996 x	190.158
Cluster 4	67 to 90	y = 31.48 - 0.1887 x	470.386

Note that, SSE for the above table is 928.742. Table 4 compares results of proposed and Mars algorithm in terms of SSE with the same nodes. At the following, observations from men department are analyzed first by Mars algorithm to select the location of nodes. Based on the Mars, two nodes in 35 and 83 locations are considered. Figure 10 shows the regression lines and location of nodes for men patients during 90 months.

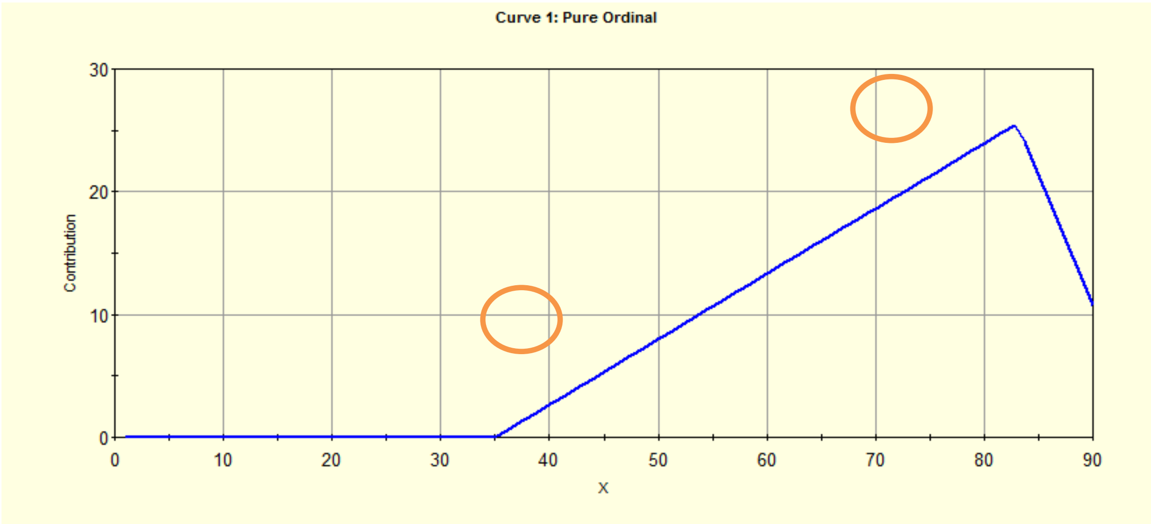

Fig 10. Number and location of nodes for men Psoriasis patients

According to Mars algorithm, the regression equation is proposed as follows:

$$y = 9.6142 + 0.5322BF_1 - 2.6438BF_2 \tag{11}$$

Where,

$BF_1 = \max(0, x - 35)$

$BF_2 = \max(0, x - 83)$

And the SSE for these observations is 2024.32. In addition, results of proposed approach for men patients' data is shown table 4.

Table 4. Results of proposed approach for men patients

Cluster number	Observations	Regression equations	Experiment error
Cluster 1	1 to 28	y = 12.10 - 0.1494 x	333.064
Cluster 2	29 to 52	y = 90.56 - 4.175 x + 0.05397 x^2	295.695
Cluster 3	53 to 90	y = - 173.5 + 5.422 x - 0.03576 x^2	1256.82

Note that, SSE for above table is 1885.575. Table 5, compares the results of proposed and Mars algorithm for both women and men observations.

Table 5. Comparing of proposed and Mars algorithm results

Sex	Method	Number of nodes	SSE
Women	Proposed	3	928.742
	Mars	3	949.270
Men	Proposed	2	1885.575
	Mars	2	2024.32

Table 4 shows that, proposed method has less SSE than Mars algorithm for both women and men observations. Hence, we can conclude that proposed method is more effective for analyzing and fitting volatile data.

7- Conclusion and future research

In this paper a new clustering-based approach based on piecewise regression is developed to obtain the number and location of breakpoints in order to analyze and model the volatile data with high adequacy. In addition, a combined heuristic-clustering approach is proposed to resolve defect of cluster analysis such that fitness of piecewise regression is maximized. To evaluate performance of proposed approach, a numerical example is used and results are compared with an existing algorithm called Mars. Results showed that proposed method has better capability to analyze the volatile data in terms of selecting the location of nodes (by considering the same number of nodes for each method) and fit models for the same observations with less error. In addition, to show the efficiency of proposed method in real application, a real data set in health-care systems based on Psoriasis patients in two women and men groups is used and results are the same with the results of first numerical example. In other words, in both example, proposed method has less SSE then the Mars algorithm for piecewise regression models. As a future research, developing proposed method for multivariate volatile data could be investigated. Furthermore, applying proposed method for other application including transportation industry can be fruitful area for future research.

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