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Equitable multi objective model for public facility location using RLTP technique

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Abstract

In the present research, a multi-objective model is proposed, which considers equity among the citizens in addition to the cost criterion. Then, the model will be solved using Reservation Level Tchebycheff Procedure (RLTP), which is one of the interactive multi-objective decision-making techniques. Subsequently, the obtained results will be compared with those of the single-objective models to determine the effect of considering and not considering the equity criterion on public facilities location. Results of the present study show that the basic models of public facilities location do not consider the equity criterion; thus, in order to protect citizens' rights, it is necessary for decision-makers of the urban management and planning to consider the objective of equity, along with other objectives of the project, as a multi-objective model in public facilities location problems. The proposed multi-objective model has also desirable and acceptable performance, which can be used in the public facilities location problems.

Keywords: Citizenship equity, urban management and planning, public facilities location, reservation level Tchebycheff procedure (RLTP)

1- Introduction

The problem of location or selecting a location for constructing facilities from among several candidate locations is one of the major challenges of decision-makers at the onset of any project. That is because selecting the appropriate location for the construction of facilities can guarantee many of the objectives of decision makers such as increased sale, reduced costs, etc., which would ultimately lead to the increased material or immaterial profitability of the project. Immaterial profitability refers to the profit in nonprofit projects, in which the objective is not to obtain revenue, but regarding the type of the project, the objective can be ease of access, reduced traffic, increased customer satisfaction, public safety, established equity, etc.

One of the major concerns in facility location is to guarantee provision of fair services for customers; in particular, in public facility location models, equity of access to the facilities is considered as one of the main requirements in an executable solution. Just imagine what irreparable losses may result from the lack of establishment of equity in facilities location such as a hospital, EMS center, or a fire station. Nevertheless, despite such great importance, there are only a very limited number of articles which have been focused on the issue of equity in location and presented a

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mathematical model. Therefore, it is very essential to investigate and develop a model, which can execute equity in the problem of public facilities location among citizens, so the present study is aimed to achieve this goal.
Equity has two dimensions, each of which has different types. Figure 1 shows dimensions of equity as well as its various types.

According to Karsu and Morton (2015), equity has two dimensions, including horizontal equity and vertical equity. By means of an example, these two dimensions are defined below:
Suppose that a road hospital is going to be constructed in a province. For considering equity in the construction of this hospital, there are two different thoughts. First, regardless of the other conditions and considering the identical importance of the roads, the hospital is built in a place with higher centrality. However the second thought states that, initially, other conditions such as road crash statistics of this province over the past years should be observed and, then, the hospital should be rather close to a place with higher crash statistics.

These two thoughts indicate horizontal equity and vertical equity, respectively. Horizontal equity considers all the individuals or regions the same, while vertical equity classifies the individuals or regions.

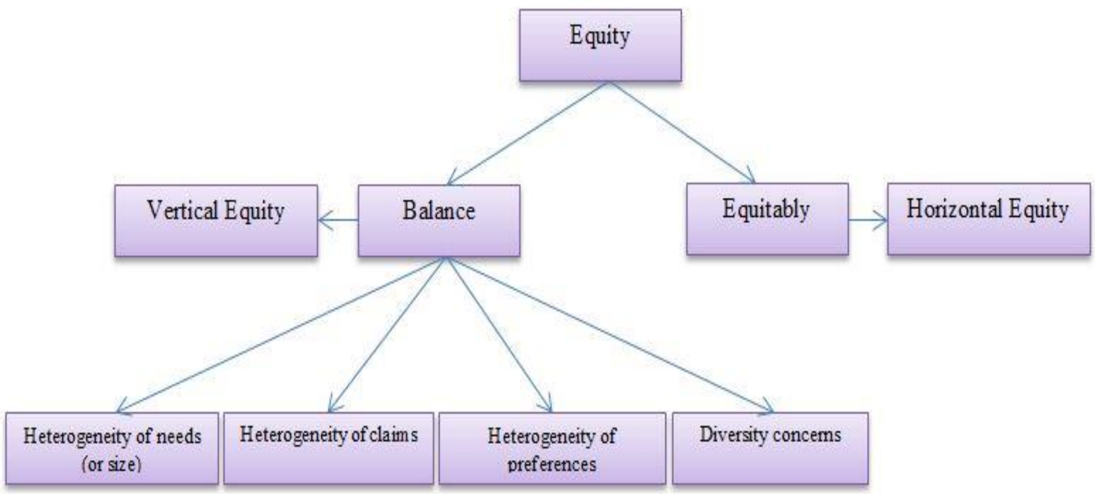

Fig 1. Dimensions of equity

On the other hand, Karsu and Morton (2015) introduced and defined the two terms “equitably” and “balance” as follows: in equitably, all the individuals, regions, and, in general, all the demand points are considered equal and without any priority and are also viewed as the same. But in balance, the individuals, regions, and, in general, all the demand points are classified with regard to the needs, preferences, etc., and also are not considered the same. Accordingly, equitably and balance can be considered as concordant to horizontal and vertical equity, respectively.

In some articles, instead of equity, the terms “equality” and “justice and fairness” have been used, which imply equitably and balance, respectively (Karsu and Morton, 2015).
In this research, first, the most widely used facilities location models as well as some of the most important models, which have been presented for considering equity in facilities location, will be investigated and compared. Then, regarding these comparisons, a multi-objective model will be presented, which considers the equity criterion, in addition to cost (median location), in public facilities location. Subsequently, the proposed model will be solved using RLTP technique and, then, the obtained results will be compared with those of the single-objective models in order to assess performance of the proposed model.

2- Literature review

This section provides a review of the literature in the field of equity in facility location models. Then, these models will be classified based on the equity consideration method. As previously mentioned, one of the main issues in facility location is to guarantee provision of fair services for clients (population). Equity has two dimensions, including equitably and balance, which are used by researchers in accordance with their needs regarding the type of the investigated problem. Classification of the studies on the issue of equity in public facility location in terms of these two dimensions is provided in table (1).

As can be seen, most of the articles, in which the concept of equity and inequality aversion has been discussed under the topic of location, have emphasized on the fairness dimension of equity, implying that the individuals (demand points) have been indistinguishable and have had no superiority over each other.

As stated earlier, the subject of equity has been discussed in many problems, but there has not been an identical approach for dealing with and considering equity.

One of the most common and simplest ways for considering equity is to minimize (maximize) the individuals' access level, which is called Rawlsian principle. This approach, by focusing on the worst individual (region), defines a minimum access level for him/her. In this regard, many of the previous studies have attempted to maximize the worst output so that the other outputs remain higher than the predetermined values (Rawls, 1971). The Rawlsian principle is justified using a *veil of ignorance* concept, which assumes that the entities do not know what their positions (the worst-off, the second worst-off etc.) will be in the distribution. To illustrate, suppose that you are given two distributions over two people generically named A and B, such as (5, 50) and (30, 25). You have to choose one of the allocations and then will learn whether you are A or B. You would seriously consider choosing (30, 25) as you might be the worse-off person in a distribution and would get only 5 units if you choose (5, 50). This ignorance is a reason to consider the worst-off entities in the distribution as any entity should find the distribution acceptable after learning its position. This approach, however, fails to capture the difference between distributions that give the same amount to the worst-off entity: two distributions such as (1,1,9) and (1,5,5) are indistinguishable in terms of inequity from a Rawlsian point of view although the latter is significantly more equitable from a common sense point of view (Karsu and Morton, 2015).

Table 1. Dimensions of equity in previous studies

Year	Author	Equitably	Balance	Year	Author	Equitably	Balance
2003	Melachrinoudis & Xanthopulos	*		2012	Mestre et al.	*	
2003	Ohsawa & Tamura	*		2012	Maliszewski et al.	*	
2003	Mladenović et al.	*		2013	López-de-los-Mozos et al.	*	
2003	Johnson		*	2013	Lejeune & Prasad	*	
2006	Galvão et al.	*		2014	Batta et al.	*	
2007	Jia et al.	*		2014	Chanta et al.		*
2007	Baron et al.	*		2014	Davari et al.	*	
2007	Caballero et al.		*	2015	Khodaparasti et al.	*	
2008	Boffey et al.		*	2015	Barbati et al.	*	
2008	Ohsawa et al.		*	2015	Barbati & Piccolo	*	
2009	Berman et al.	*		2015	Kalcsics et al.	*	
2009	Suzuki & Drezner	*		2015	Caramia & Mari	*	
2011	Bell et al.	*		2016	Khodaparasti et al.	*	
2011	Chanta et al.	*		2016	Romero et al.	*	

Another approach, which has been used in many studies to consider the equity, is the inequality indicators; accordingly, Barbati and Piccolo (2015) presented a list of the most widely used equity measures used in the articles. The symbols used in these scales and the list of these measures are presented in tables (2) and (3), respectively.

Table 2. Symbols in equity measures

Notation	Description
$I = \{1,\ldots,n\}$	the set of the n demand points
d_i	the distance between the demand point i and its assigned facility
$\bar{d} = \sum_I \frac{d_i}{n}$	the average distance between the demand points and their assigned facilities

Table 3. Equity Measures

Code	Measure	Formulation
CEN	Center (Hakimi 1965)	$Max_{i \in I} d_i$
RG	Range (Gopalan 1990)	$Max_{i \in I} d_i - Min_{i \in I} d_i$
MAD	Mean Absolute Deviation (Mulligan 1991)	$\frac{1}{n}\sum_{i \in I} \lvert d_i - \bar{d} \rvert$
VAR	Variance (Morrill 1977)	$\frac{1}{n}\sum_{i \in I} \left(d_i - \bar{d}\right)^2$
MD	Maximum Deviation (Erkut 1992)	$Max_{i \in I} d_i - \bar{d}$
AD	Absolute Difference (Erkut 1992)	$\frac{1}{n^2}\sum_{c \in I, d \in I} \lvert d_c - d_i \rvert$
SMDA	SumMaxDiffAbs (Erkut 1992)	$\frac{1}{n}\sum_{c \in I} \max_{d \in I} \lvert d_c - d_d \rvert$
SI	Schutz's Index (Erkut 1992)	$\frac{1}{n\bar{d}}\sum_{i \in I} \lvert d_i - \bar{d} \rvert$
CV	Coefficient of Variation (Coulter 1980)	$\sqrt{\frac{\frac{1}{n}\sum_{i \in I}\left(d_i - \bar{d}\right)^2}{\bar{d}}}$
GC	Gini Coefficient (Gini 1912)	$\frac{\sum_{c \in I, d \in I} \lvert d_c - d_d \rvert}{2n^2\bar{d}}$

Table (4) shows approach of the studies conducted on considering the equity criterion in facility location problems.

Table 4. Approach of equity in previous studies

Year	Author	Rawlsian	inequality indicator	Year	Author	Rawlsian	inequality indicator
2003	Melachrinoudis & Xanthopulos	*		2012	Mestre et al.	*	
2003	Ohsawa & Tamura	*		2012	Maliszewski et al.	*	
2003	Mladenović et al.	*		2013	López-de-los-Mozos et al.	*	
2003	Johnson	*		2013	Lejeune & Prasad		*
2006	Galvão et al.		*	2014	Batta et al.	*	
2007	Jia et al.	*		2014	Chanta et al.	*	
2007	Baron et al.	*		2014	Davari et al.	*	
2007	Caballero et al.	*		2015	Khodaparasti et al.	*	*
2008	Boffey et al.	*		2015	Barbati et al.		*
2008	Ohsawa et al.		*	2015	Barbati & Piccolo		*
2009	Berman et al.	*		2015	Kalcsics et al.		*
2009	Suzuki & Drezner		*	2015	Caramia & Mari	*	
2011	Bell et al.		*	2016	Khodaparasti et al.		*
2011	Chanta et al.	*		2016	Romero et al.		*

The onset of the attention and focus on facility location models should be assumed from the papers of Mumphery et al. (1971), McAllister (1976), and Savas (1978). In general, there are two research policies in these articles, the first of which includes general aspects such as proper definitions of equity and its properties as well as a comparison between the solutions provided by equity measurement scales, while the second research policy focuses on optimizing the problems and algorithms in order to solve the location problems with a regard to equity.

Marsh and Schilling (1994), in their review paper, presented 20 scales for measuring equity in facility location, some of which include center location, variance, mean absolute deviation, total absolute deviation, maximum absolute deviation, amplitude, Gini coefficient, etc. Berman and Kaplan (1990) were the first ones who used the total weighted absolute deviation scale to consider the equity criterion. They obtained a time algorithm for public networks. Discussing the meaning of equity, Ogryczak (2009) regarded it as concordant to the minimization of the distance and considering it as equal to the distance from the customers.

Another problem is the problem of equitable load among the facilities, which considers equity for the facilities to ensure that the load has been distributed equitably among the facilities and no excessive load has been exerted on a facility so that it can result in the increased efficiency of the facilities. Furthermore, this problem leads to the increased equity among the citizens, prevents long waiting queues, and also facilitates providing the services for them in a shorter time. Berman et al. (2009), Baron et al. (2007), Suzuki and Drenzer (2009), and Galvão et al. (2006) have discussed equity from this point of view in their works.

Table (5) presents the studies, which have particularly included the issue of equity in the mathematical model of public facility location problem, along with the features considered by them.

3- Mathematical model

In this section, by investigating the studies conducted on the field of equity in facility location, a multi-objective model will be presented for public facility location with regard to the equity criterion, which includes p-median, minimum p-envy, and equitable loads. Due to the great importance of costs in all the problems including location problems, the p-median was considered in this model. Besides, to consider the equity criterion, the minimum p-envy was used. The reason for considering this objective, in comparison with other above-mentioned objectives of equity, was its relative superiority in the comparisons (Chanta et al. 2014). The objective of equitable loads was considered since, in addition to its great importance, it considers equity both for demanders and the facilities, so that efficiency of the facilities is increased.

3-1- Assumptions

Assumptions are as the following.

1. Population of each demand center is considered as the potential demand of each center.
2. Capacity of the facility is considered unlimited.
3. The problem space is discrete.
4. Demanders can refer only to one of the constructed facilities.
5. It is impossible to close the facilities that have been already constructed.
6. The number of facilities that are scheduled to be constructed is predetermined.
7. The distance between demand points and public facilities should be fair.
8. The same distance between different demand points and public facility is desirable.

3-2- Notations

The notation used is stated as:

i = the index and set of exist and candidate sites

k, s = the index and set of demand points

d_{is}= the distance between candidate sate I and demand point s

f_s= population of demand points s

h_s= weight of demand points s

P = number of facilities can be located

e_{sk}= the envy of demand point s relative to demand point k

$$Y_i = \begin{cases} 1 & \text{if a facility is located at conditate site i} \\ 0 & \text{otherwise} \end{cases}$$

$$x_{is} = \begin{cases} 1 & \text{if a facility at conditate site i assigned to demand point s} \\ 0 & \text{otherwise} \end{cases}$$

G = maximal demand rate to a facility

Table 5. Features of mathematical models in previous studies

Year	Author	Type of facility	Number of objectives	P-dispersion	Distance Equity	Time Equity	Maximum coverage	Equitable load	P-median	Considering population
2003	Johnson	Favorable facility	2				*	*		*
2006	Galvão et al.	Favorable facility	1		*					
2007	Jia et al.	Favorable facility	1		*		*		*	*
2007	Baron et al.	Favorable facility	1	*	*			*		
2007	Caballero et al.	Obnoxious facility	2						*	
2008	Boffey et al.	Obnoxious facility	4					*	*	
2008	Ohsawa et al.	Favorable facility	2		*				*	
2009	Berman et al.	Favorable facility	1					*		
2009	Suzuki & Drezner	Favorable facility	3		*			*		
2011	Bell et al.	Favorable facility	1		*		*		*	
2012	Mestre et al.	Favorable facility	1			*			*	*
2012	Maliszewski et al.	Favorable facility	2	*	*		*		*	*
2012	Smith et al.	Favorable facility	2		*		*		*	*
2013	Lejeune & Prasad	Favorable facility	2		*				*	
2014	Batta et al.	Obnoxious facility	1	*	*					*
2014	Chanta et al.	Favorable facility	2		*		*			
2014	Davari et al.	Favorable facility	2			*				
2015	Khodaparasti et al.	Favorable facility	2			*				
2015	Barbati et al.	Semi-obnoxious facility	1		*					
2015	Barbati & Piccolo	Favorable facility	1		*					
2015	Kalcsics et al.	Favorable facility	1		*					
2016	Khodaparasti et al.	Favorable facility	3	*	*				*	*
2016	Romero et al.	Obnoxious facility	2						*	*

3-3- The proposed model

The proposed multi-objective model for equitable public facility location is as follows:

$$Z_1 : Min \sum_{i=1}^{n} \sum_{s=1}^{m} f_s d_{is} x_{is} \tag{1}$$

$$Z_2 : Min \sum_{s} \sum_{k} h_s e_{sk} \tag{2}$$

$$Z_3 : Min\, G \tag{3}$$

s.t.

$$e_{sk} \geq \sum_{i} d_{is} x_{is} - \sum_{i} d_{ik} x_{ik} \qquad \forall s,k ; s \neq k \tag{4}$$

$$e_{sk} \geq 0 \qquad \forall s,k ; s \neq k \tag{5}$$

$$G \geq \sum_{s} f_s x_{is} \qquad \forall i \tag{6}$$

$$\sum_{i} X_{is} = 1 \qquad \forall s \tag{7}$$

$$\sum_{i} Y_i = P \tag{8}$$

$$x_{is} \leq y_i \qquad \forall i,s \tag{9}$$

$$x_{is} \in \{0.1\} \qquad \forall i,s \tag{10}$$

$$Y_i \in \{0.1\} \qquad \forall i \tag{11}$$

The objective function (1) seeks to minimize the travel costs from demand centers to constructed facilities. The objective function (2) seeks to minimize the total envy of the demand centers regarding the weight of the centers, meaning that it attempts to assign lower envy to the center with higher population weight. The objective function (1) minimizes the total cost while Objective function (2) aims to achieve equity, even if it increases costs. This model somehow tries to balance these goals. The objective function (3) reduces the load exerted on the facility with the highest demand value as much as possible. The constraints (4) and (5) have been used for proper calculation of envy in each region. The constraint (6) is related to proper calculation of the maximum load on the facilities. The constraint (7) states that the demand of each demander should be met only by a single facility. The constraint (8) represents that a certain number of facilities (P) should be located. The constraint (9) indicates that the candidate location *i* can cover the demand of the demand center *s* (x_{is}=1) only if a facility has been constructed at location *i* (Y_i = 1). The constraints (10) and (11) indicate that the two variables x_{is} and Y_i are binary variables and only take the values of 0 or 1.

Furthermore, in order to use the minimum envy criterion, first, a function is considered as the envy function, which can be represented in terms of time, distance, or even, according to Chanta et al.'s study (2014) on EMS stations location, as the demander rescue function. In this research, the rescue function is based on the demander's distance from the facilities. Envy of the demander at demand center *s* relative to the demander at demand center *k* (e_{sk}) is calculated as follows:

$$e_{sk} = Max \{0, d_{is} x_{is} - d_{ik} x_{ik}\} \tag{12}$$

The constraint (12) states that if the distance of the demander at location *s* from the facility at location *i* is more than the distance of the demander at location *k* from facility at location *i*, the demander *s* will envy the demander *k* as much as the difference of these distances; while, if the former distance is less than the latter one, there will be no envy ($e_{sk} = 0$).

4- Solution approach

In this research, the proposed multi-objective model was solved using RLTP technique. This method is one of the multi-objective decision-making methods interacting with the decision-maker. Conventionally, there are several non-dominated solutions in a multi-objective problem; therefore, many interactive methods have been developed to help the decision makers narrow their selection space in order to achieve the most preferable solution. The Tchebycheff-based methods are among the most popular multi-objective interactive methods, which systematically reduce the non-dominated solutions set with regard to the decision maker's viewpoint to achieve the most preferable solution for the decision maker.

RLTP is one of these methods proposed by Reeves and Macleod (1999), which has higher flexibility than other methods Tchebycheff-based methods including Interactive Weighted Tchebycheff Procedure (IWTP) proposed by Steuer and Choo (1983). Flexibility refers to the application of a more number of decision maker's opinions such as a series of solutions with more or less importance, instead of selection of a preferable solution by the decision maker.

This method reduces the space of objectives using the Reservation Level (RL) to limit the set of non-dominated solutions from one repetition to the next repetition, which is repeated to the extent that the most preferable non-dominated solution is achieved (Reeves and Macleod, 1999).

Including the decision maker's viewpoints in the problem to reduce the solution space requires an interactive multi-objective decision making method. Besides, as mentioned earlier, the RLTP method has higher optimality and superiority over the other Tchebycheff-based methods; thus, it is used to solve the proposed multi-objective problem. The algorithm of this technique is as follows:

Step-1: Initialization

1.1. Determining the number of solutions provided for the decision maker (N) in each repetition, in which $N \geq P$, where P indicates the number of objectives.

1.2. Calculating the objective reference vector (y^u) using equation (13). This vector is used in the third step to solve the Tchebycheff program.

$$y^u = (y_1^u, y_2^u, ..., y_p^u): \qquad \forall k = 1,...,P \tag{13}$$
$$y_k^u = Min\{f_k(x); x \in X\} - \varepsilon_k$$

Where ε_k is small positive values that are applied in the process of solving the Tchebycheff program.

In fact, y_k^u are optimal values of the objective functions when used alone in the model.

1.3. Putting $RL_k = +\infty$ for $k=1,...,P$.

1.4. Determining the allowed number of repetitions.

Step-2: Sampling

Obtaining 2N scattered weight vectors using equation (14):

$$\Lambda = \left\{ \lambda \in R^p \middle| \lambda_k \in (0,1), \sum_{k=1}^{P} \lambda_k = 1 \right\} \tag{14}$$

Step-3: Problem solving

In this step, for each weight vector created in the second step, the following model is implemented and, then, the results are calculated:

$$Min \left\{ \alpha - \rho \sum_{k=1}^{P} f_k(x) \right\} \tag{15}$$

s.t.

$$x \in X \tag{16}$$
$$\alpha \geq \lambda_k \left(f_k(x) - y_k^u \right) \qquad \forall k = 1,...,P \tag{17}$$
$$f_k(x) \leq RL_k \qquad \forall k = 1,...,P \tag{18}$$

According to Steuer and Choo (1983), ρ is a small positive numerical value that is suggested to be selected between 0.0001 and 0.01; on this basis, N solutions with maximum dispersion will be obtained to provide for the decision maker. If the decision maker is looking for a better solution, we go to step (4); otherwise, we stop and the decision maker selects the most preferable solution.

Step-4: Adjusting

In this step, regarding the decision-maker's viewpoints, the current solutions are classified into preferential subsets; then, the Reservation Levels (RL) are determined and we return to step (2). In order to calculate the RL, two points should be taken into account:

First, the reservation level for each objective should be equalized as worse than or equal to the worst value of that objective among the current solutions with more preference. Second, at least one RL should be better than the current value of that objective among the solutions with less preference.

If the decision maker agrees, the reservation levels values can be calculated automatically using equation (19):

$$RL_k = MPWV_k - r\left(MPWV_k - CSWV_k\right) \quad (19)$$

Where $CSWV_k$ is equal to the worst value of the objective function k among all the currently existing solutions and $MPWV_k$ is equal to the worst value of the objective function k among all the currently existing solutions with higher preferences.

Also, r is a reduction rate that can take values between 0 and 1 so that the smaller the value of r, the faster the reduction rate of the objective space would be.

To apply the decision maker's preferences from a function with regard to the normal values of the objectives, Equation (20) is used:

$$U(z) = \sum_{k=1}^{p} \alpha_k \times \frac{z_k - z_k^*}{z_k^- - z_k^*} \quad (20)$$

where z_k is equal to the value obtained for the function k, z_k^* is the optimal value of the objective function k, and z_k^- is the worst value of the objective function k. As can be seen, the lower the value of this functions, the better the efficiency of the model. The weights are selected (α_k) with regard to the importance of the objectives and viewpoints of the decision makers, and also the sum of the weights should be equal to 1.

5- Research findings

In this section, the proposed multi-objective model for public facility location is solved, with regard to the equity criterion, using the reservation level chebycheff procedure (RLTP) technique for the construction of three facilities in order to compare the obtained results with those of the single-objective models.

Four solutions are provided for the decision-maker in each repetition (N = 4). To calculate the target reference vector (y_u), three single-objective problems are solved for each objective, the results of which are presented in table (6).

Table 6. Single objective and optimal results

Objective \ Results	Z_1	Z_2	Z_3
Only Z_1	752553.100	27.872	99293
Only Z_2	1966007.600	10.745	114145
Only Z_3	1395286.500	37.753	78467
Optimal results (Z^*)	752553.100	10.745	78467

Values of ε are also considered equal to 0.001 since the normalized values of the objectives are used in the model. Then, eight random weight vectors ($2N = 8$) are generated, by the use of which eight solutions are obtained for being provided for the decision maker in accordance with table (7). Besides, the value of ρ is considered equal to 0.01.

Table 7. Results of the first repetition of the model with RLTP

Objective	1	2	3	4	5	6	7	8
Z_1	6.1113767	1.1194278	2.1098013	1.1194278	3.1344039	4.936560	6.1465420	6.1164259
Z_2	269.19	483.16	130.19	483.16	878.13	784.21	512.12	038.18
Z_3	79702	79793	103587	79793	80096	105155	89665	90895

Then, from among the obtained solutions, four solutions with higher preference are selected to be presented to the decision maker. In this problem, the preferential solutions are specified regarding the greater importance of equity objective (minimum envy) in accordance to table (8).

Table 8. Higher preference results of the first repetition

Objective	2	5	7	8
Z_1	100.1194278	300.1344039	600.1465420	600.1164259
Z_2	483.16	878.13	512.12	038.18
Z_3	79793	80096	89665	90895
U(z)	238.0	250.0	324.0	313.0

According to the above table, the decision maker selects Solution (2) as the most preferable solution with regard to the minimum value of U(z); then, the RL values are calculated in accordance to equation (22) and adjustment of r equal to 0.2 as follows:

$$RL_1 = 1465420.600 - 0.2(1465420.600 - 1465420.600) = 1465420.600$$
$$RL_2 = 18.038 - 0.2(18.038 - 21.784) = 18.787$$
$$RL_3 = 90895 - 0.2(90895 - 105155) = 93747$$

After adjusting the RL values, we return to step (2). This problem reach the optimal solution accepted by the decision maker after three repetitions, in which the values of p-median, p-envy, equitable load, and U(z) function are equal to 1194278.100, 16.483, 79793, and 0.238, respectively. As can be inferred from values of the objectives, the proposed multi-objective model has yielded much better results than the single-objective models and it had only a slight difference from the main objective of the single-objective models. This is a natural case, because consideration of several objectives at the same time causes the values of the objective to become worse than the optimal values in a single model. However, such a slight difference indicates good performance and high efficiency of the multi-objective model. This comparison can be seen more clearly in figure (2), in which normal values of the objectives are used.

As can be seen, the proposed multi-objective model has less efficiency than the main objective of the single-objective models; but in the meantime, it has the best performance among all the three objectives, so that it has accomplished establishing a balance among the objectives, which indicates the appropriate performance and efficiency of the proposed model.

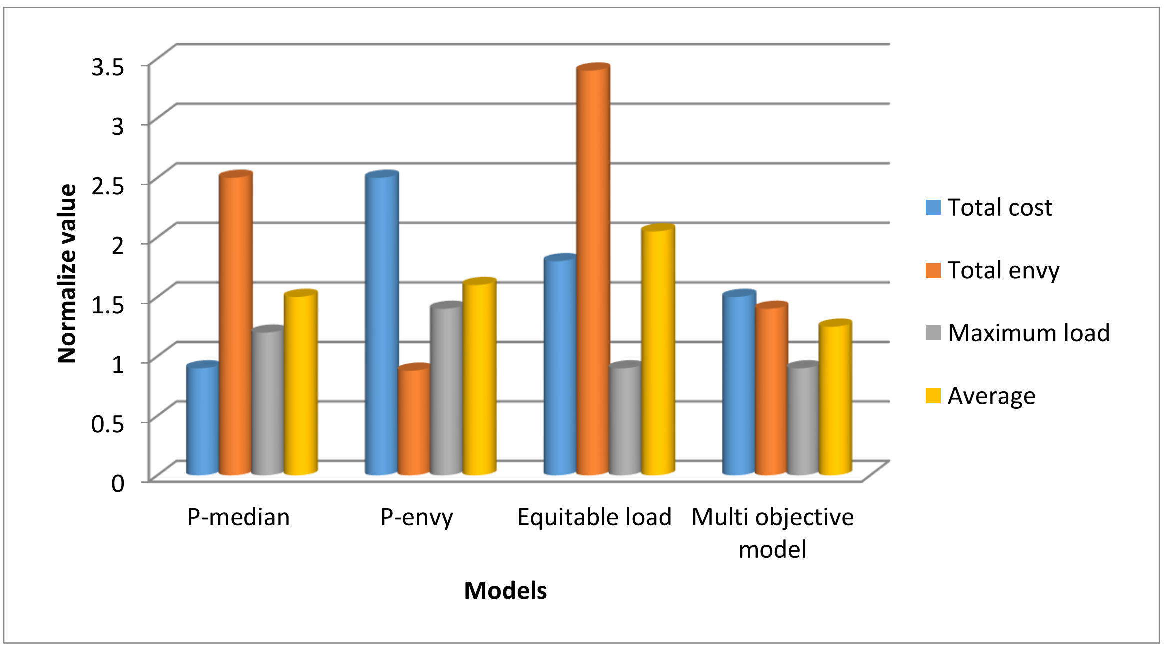

Fig 2. Comparison of proposed multi-objective model with single-objective models

6- Conclusion

In this research, a multi-objective model, including p-median, minimum p-envy, and equitable load, was proposed for considering equity objective along with other objectives considered in construction of the facilities as well as establishing balance among these objectives. Subsequently, the proposed multi-objective model was solved using RLTP exact solution method from the set of multi-objective decision making techniques. Then, the results were compared with those of the single-objective models. Accordingly, the good performance of the proposed model was confirmed.

According to the investigations, it was found that the basic models of facility location problem (median location model, etc.) do not consider to the issue of equity in location and, thus, selection of the places with regard to the results obtained from these models might be completely inequitable, leading to the protest of the citizens living in deprived regions and irreparable losses to the society. Not paying attention to the equitable load objective would result in the allocation of very large amount of demand to some of the facilities and, in the meantime, a very small amount of demand to some other facilities. This is an unpleasant case since it might reduce the efficiency and service-providing level in facilities with high demand and some of the individuals who provide services in the facilities might think that equity is not observed. Therefore, it is recommended to use this objective as one of the objectives in the form of a multi-objective model in public facility location.

Another notable point is that the models considering equity in facility location do not consider other objectives of the problems, such as cost that is one of the most important objectives in most of the projects; thus, the results extracted from these models might impose costs several times more than the cost imposed by the median location model. This issue, regarding the constraints of any project, especially the financial constraints, would cause its specific problems as well as numerous concerns. Regarding the above-mentioned points, it is essential to consider multiple objectives in the public facility location problems and take the problems from a multi-dimensional point of view. Therefore, it seems necessary to use a multi-objective model for public facility location in order to create interaction and balance among various objectives of the decision-makers in problems. Such a model should consider not only the basic objectives of the location problem (e.g. cost), but also other important objectives (e.g. equity) in order to achieve a result with higher acceptability.

For further studies in future, it is suggested to expand the proposed model for location of specific public facilities such as fire stations, hospitals, etc. with regard to the constraints of each. Furthermore, the proposed model can be solved using other multi-objective techniques and, then, the obtained results can be compared in order to determine efficiency of the techniques.

References

Barbati, M., Bruno, G., & Marín, A. (2015). Balancing the arrival times of users in a two-stage location problem. *Annals of Operations Research*, 1-16.

Barbati, M., & Piccolo, C. (2015). Equality measures properties for location problems. *Optimization Letters*, 1-18.

Baron, O., Berman, O., Krass, D., & Wang, Q. (2007). The equitable location problem on the plane. *European Journal of Operational Research*, *183*(2), 578-590.

Batta, R., Lejeune, M., & Prasad, S. (2014). Public facility location using dispersion, population, and equity criteria. *European Journal of Operational Research*, *234*(3), 819-829.

Bell, J. E., Griffis, S. E., Cunningham, W. A., & Eberlan, J. A. (2011). Location optimization of strategic alert sites for homeland defense. *Omega*,*39*(2), 151-158.

Berman, O., Drezner, Z., Tamir, A., & Wesolowsky, G. O. (2009). Optimal location with equitable loads. *Annals of operations research*, *167*(1), 307-325.

Berman, O., & Kaplan, E. H. (1990). Equity maximizing facility location schemes. *Transportation Science*, *24*(2), 137-144.

Boffey, T. B., Mesa, J. A., Ortega, F. A., & Rodrigues, J. I. (2008). Locating a low-level waste disposal site. *Computers & Operations Rearches*, *35*(3), 701-716.

Caballero, R., González, M., Guerrero, F. M., Molina, J., & Paralera, C. (2007). Solving a multiobjective location routing problem with a metaheuristic based on tabu search. Application to a real case in Andalusia. *European Journal of Operational Research*, *177*(3), 1751-1763.

Caramia, M., & Mari, R. (2015). A decomposition approach to solve a bilevel capacitated facility location problem with equity constraints. *Optimization Letters*, 1-23.

Chanta, S., Mayorga, M. E., & McLay, L. A. (2011). Improving emergency service in rural areas: a bi-objective covering location model for EMS systems. *Annals of Operations Research*, *221*(1), 133-159.

Chanta, S., Mayorga, M. E., & McLay, L. A. (2014). The minimum p-envy location problem with requirement on minimum survival rate. *Computers & Industrial Engineering*, 74, 228-239.

Coulter, P.B. (1980), "Measuring the inequity of urban publicservices: A methodological discussion with applications", *Policy Studies Journal 8*, 683-698.

Erkut, E. (1992), "Inequality measures for location problems", *University of Alberta Research Report No. 91-2*, September 1992.

Galvão, R. D., Espejo, L. G. A., Boffey, B., & Yates, D. (2006). Load balancing and capacity constraints in a hierarchical location model.*European Journal of Operational Research*, *172*(2), 631-646.

Gini, C. (1912), Variabilita e Mutabilita, Bologna. Gopalan, R., Batta, R., and Karwan, M.H. (1990), "The equity constrained shortest path problem", *Computers & Operations Research*, *17/3*, 297-307.

Gopalan, R., Kolluri, K.S., Batta, R., and Karwan, M.H. (1990), "Modeling equity in the transportation of hazardous materials*", Operations Research*, *38*, 961-973.

Hakimi, S.L. (1965), "Optimal distribution of switching centers in a communications network and some related graph theoretic problem", *Operations Research 13*, 462-475.

Jia, H., Ordóñez, F., & Dessouky, M. (2007). A modeling framework for facility location of medical services for large-scale emergencies. *IIE transactions*, *39*(1), 41-55.

Johnson, M. P. (2003). Single-period location models for subsidized housing: Tenant-based subsidies. *Annals of Operations Research*, *123*(1-4), 105-124.

Kalcsics, J., Nickel, S., Puerto, J., & Rodríguez-Chía, A. M. (2015). Several 2-facility location problems on networks with equity objectives. *Networks*, *65*(1), 1-9.

Karsu, Ö., & Morton, A. (2015). Inequity averse optimization in operational research. *European Journal of Operational Research*, *245*(2), 343-359.

Khodaparasti, S., Maleki, H. R., Bruni, M. E., Jahedi, S., Beraldi, P., & Conforti, D. (2015). Balancing efficiency and equity in location-allocation models with an application to strategic EMS design. *Optimization Letters*, 1-18.

Khodaparasti, S., Maleki, H. R., Jahedi, S., Bruni, M. E., & Beraldi, P. (2016). Enhancing community based health programs in Iran: a multi-objective location-allocation model. *Health care management science*, 1-15.

Lejeune, M. A., & Prasad, S. Y. (2013). Effectiveness–equity models for facility location problems on tree networks. *Networks*, *62*(4), 243-254.

López-de-los-Mozos, M. C., Puerto, J., & Rodríguez-Chía, A. M. (2013). Robust mean absolute deviation problems on networks with linear vertex weights. *Networks*, *61*(1), 76-85.

Maliszewski, P. J., Kuby, M. J., & Horner, M. W. (2012). A comparison of multi-objective spatial dispersion models for managing critical assets in urban areas. *Computers, Environment and Urban Systems*, *36*(4), 331-341.

Marsh, M. T., & Schilling, D. A. (1994). Equity measurement in facility location analysis: A review and framework. *European Journal of Operational Research*,*74*(1), 1-17.

McAllister, D. M. (1976). Equity and efficiency in public facility location. *Geographical Analysis*, *8*(1), 47-63.

Melachrinoudis, E., & Xanthopulos, Z. (2003). Semi-obnoxious single facility location in Euclidean space. *Computers & Operations Research*, *30*(14), 2191-2209.

Mestre, A. M., Oliveira, M. D., & Barbosa-Póvoa, A. (2012). Organizing hospitals into networks: a hierarchical and multiservice model to define location, supply and referrals in planned hospital systems. *OR spectrum*,*34*(2), 319-348.

Mladenović, N., Labbé, M., & Hansen, P. (2003). Solving the p-Center problem with Tabu Search and Variable Neighborhood Search. *Networks*,*42*(1), 48-64.

Morrill, R.L., and Symons, J. (1977), "Efficiency and equity aspects of optimum location", *Geographical Analysis 9*, 216-225.

Mulligan, G.F. (1991), "Equality measures and facility location", *Regional Science 7/4*, 548-56l.

Mumphrey, A. J., Seley, J. E., & Wolpert, J. (1971). A decision model for locating controversial facilities. *Journal of the American Institute of Planners*,*37*(6), 397-402.

Ogryczak, W. (2009). Inequality measures and equitable locations. *Annals of Operations Research*, *167*(1), 61-86.

Ohsawa, Y., & Tamura, K. (2003). Efficient location for a semi-obnoxious facility. *Annals of Operations Research*, *123*(1-4), 173-188.

Ohsawa, Y., Ozaki, N., & Plastria, F. (2008). Equity-efficiency bicriteria location with squared Euclidean distances. *Operations research*, *56*(1), 79-87.

Rawls, J. (1971). A Theory of Justice (Cambridge. *Mass.: Harvard University*.

Romero, N., Nozick, L. K., & Xu, N. (2016). Hazmat facility location and routing analysis with explicit consideration of equity using the Gini coefficient. *Transportation Research Part E: Logistics and Transportation Review*, *89*, 165-181.

Savas, E. S. (1978). On equity in providing public services. *Management Science*, *24*(8), 800-808.

Steuer, R. E., & Choo, E. U. (1983). An interactive weighted Tchebycheff procedure for multiple objective programming. *Mathematical programming*,*26*(3), 326-344.

Suzuki, A., & Drezner, Z. (2009). The minimum equitable radius location problem with continuous demand. *European Journal of Operational Research*, *195*(1), 17-30.