

# **A hybrid metaheuristic algorithm for the robust pollution-routing problem**

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## **Abstract**

Emissions resulted from transportation activities may lead to dangerous effects on the whole environment and human health. According to sustainability principles, in recent years researchers attempt to consider the environmental burden of logistics activities in traditional logistics problems such as vehicle routing problems (VRPs). The pollution-routing problem (PRP) is an extension of the VRP which consists of routing a number of vehicles to serve a set of customers and determining their speed on each route segment so as to minimize a function of comprising fuel, emissions and driver costs. This paper proposes an adaptive large neighborhood search for the robust PRP (RPRP) under demand uncertainty. The achieved results indicate a premium performance of the solutions obtained by the proposed robust models.

**Keywords:** Green vehicle routing, pollution-routing problem, robust optimization, metaheuristic algorithm

## **1- Introduction**

Green vehicle routing is related to dispatching goods not only based on economic goals, but also by considering the relevant harmful impacts on the environment (Sbihi and Eglese 2007). Transportation has dangerous effects on the environment such as resource depletion, land use, acidification, toxic effects on ecosystems and humans, noise and the impacts induced by Greenhouse Gas (GHG) emissions (Knörr, 2008; Kwon et al., 2013). GHGs and in particular, CO<sub>2</sub>-equivalent emissions are the most disturbing matters about the environment in the last decade (Bektaş and Laporte, 2011; Li et al., 2015).

As vehicle planning has a significant effect on the environment and GHG emissions; a number of authors (e.g., McKinnon, 2007; Sbihi and Eglese, 2007; Lee et al., 2014) mention that there are several opportunities for reducing GHG emissions by developing the classical VRP objectives to account for environmental and social burdens rather than just considering the economic profit (Bektaş and Laporte, 2011).

The Vehicle Routing Problem (VRP) is a well-known NP-hard problem which was firstly introduced by Dantzig and Ramser (1959). Since that time, VRP has been a topic of numerous studies

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in the literature of operations research. The traditional VRP includes a set of customers with known demands, a depot and a fleet of vehicles by objective of determining the optimized routes to minimize the total travel cost. The literature on VRP and its variants are very widespread and include many different models and extensions (see for example the latest surveys by Golden et al., 2008; Eksioglu et al., 2009; De Jaegere et al., 2014). Also various exact (e.g. Drexl 2014; Almoustafa et al., 2013; Baldacci et al., 2012; Dabia et al.; 2017) and heuristics (e.g. Demir et al., 2012; Demir et al., 2014; Kopfer et al., 2014; Franceschetti et al., 2017) solution methods are suggested to solve these problems.

In the real-world applications the input data of VRP is highly tainted with uncertainty. Therefore, several studies accomplished in this field and proposed mathematical approaches in order to handle the imprecision of input parameters in VRP. For example, Gendreau et al. (1996) illustrated different types and solution approaches for stochastic VRP (SVRP). The most common variant of SVRP is the VRP with stochastic customer demand. There are two main approaches in the literature for dealing with uncertain parameters in VRP: (1) stochastic programming which has been extensively applied in the condition that the probability distribution of the uncertain parameter is available according to sufficient and reliable historical data (e.g. Li et al., 2010; Juan et al., 2011), and (2) robust optimization which firstly used by Bertsimas and Simchi-Levi (1996) for SVRP and mostly applied when the probability distribution of uncertain parameter is unknown (i.e., deep uncertainty).

Sungur et al. (2008) introduced a robust optimization approach to solve the capacitated VRP (CVRP) with uncertain demand and compared the performance of robust solutions to deterministic ones. Adulyasak and Jaillet (2015) described a number models and algorithms for stochastic and robust vehicle routing problem (RVRP).

Green vehicle routing problem (GVRP) which was principally probed since 2006, is characterized by the objective of accounting of environmental burdens in addition to the economic costs (Lin et al., 2014). Lin et al. (2014) illustrated the three major directs of GVRP in their survey which includes GVRP, Pollution Routing Problem (PRP), and VRP in reverse logistics. The objective of PRP is to determine the vehicles path with less pollution. Demir et al. (2014) described main effective factors in fuel consumption and variant fuel consumption models in their review paper.

Despite the fact that the number of researches on PRPs is increasing in recent years, the studies are still restricted and have not covered many needed aspects. The uncertainty of input data, such as uncertain demand or travel time, is one of the important properties of real world problems. Many reasons like inventory fluctuations and customer interests may lead to demand uncertainty. Also any daily usual events can cause travel time uncertainty, such as road traffic, road accident and weather conditions. Eshtehadi et al. (2017) proposed several robust optimization models for the robust pollution routing problem (RPRP) regarding the PRP with uncertain demands (PRPUD) involve Hard worst case model, soft worst case model and chance constraint model. They show that the classical hard worst case approach commonly used for VRP, cannot achieve the right worst case solution of PRP. In this paper, we propose a solution method based on an enhanced adaptive large neighborhood search (ALNS) and a specialized speed optimization algorithm described in Demir et al. (2012) for the worst case robust PRP model is introduced by Eshtehadi et al. (2017). The results of adapted algorithm can show the effect of demand uncertainty in large scale instance and help decision makers to choose more suitable models in different uncertainty level.

The rest of this paper is organized as follows. In section 2, the Robust Pollution-Routing Problem is described and the related mathematical model is presented. Section 3 includes a brief description of the metaheuristic algorithm. The results of extensive numerical experiments presented in section 4. Finally, Section 5 concludes this paper and introduces some future research directions.

## **2- The robust pollution-routing problem**

In this section the RPRP is described and the corresponding mathematical model is presented based on Eshtehadi et al. (2017) as the foundation of the proposed robust PRP models.

### **2-1- Problem description**

The PRPUD is defined on a complete directed graph  $G = (V, A)$  with  $V = \{0, 1, 2, \dots, n\}$  as the set of nodes that node 0 considered as depot.  $A = \{(i, j): i, j \in V, i \neq j\}$  is the set of arcs and the distance from node  $i$  to node  $j$  is shown by  $d_{ij}$ . The number of homogeneous vehicles is a deterministic

exogenous parameter and set of vehicles is represented by  $k = \{1, 2, \dots, m\}$  and the capacity of each vehicle is equal to  $Q$ .

The tilde ( $\sim$ ), bar ( $\bar{\cdot}$ ) and hat ( $\hat{\cdot}$ ) accents are distinguish the uncertain parameters, nominal value (which is used in deterministic modeling) and uncertain part of each uncertain parameter respectively; therefore,  $\tilde{q}_i$  is a uncertain parameter which is donated to customers demand with nominal value of  $\bar{q}_i$  and maximum perturbation of  $\hat{q}_i$  over and under nominal value ( $\tilde{q}_{ij}$  uncertainty interval is  $[\bar{q}_{ij} - \hat{q}_{ij}, \bar{q}_{ij} + \hat{q}_{ij}]$ ). Uncertain demand results from a mixture of reasons such as some variation in customer interests, modes, styles and requirements as well as the number of them and their strategies. These factors may lead in undulation in real customer demand to expectations.

### 2-1-1- Diesel fuel consumption and CO<sub>2</sub> emissions

The PRP model is introduced by Bektaş and Laporte (2011) based on the comprehensive emission models presented by Barth and Boriboonsomsin (2008), Barth et al. (2005) and Scora and Barth (2006). Later, Demir et al. (2012) have proposed a more useful formulation for fuel consumption rate that can be applied appropriately in PRP models. In this formulation the fuel consumption rate is calculated as the following:

$$F(v) = \lambda(KNv + w\gamma av + \gamma\alpha f v + \beta\gamma v^3)d/v \quad (1)$$

Where  $v, \tau, \theta, d$  and  $w$  denote the vehicle speed, amount of acceleration, road angle, distance, and curb weight of an empty vehicle, respectively. Moreover,  $f$  denotes the vehicle load and  $\alpha, \beta, \lambda$  and  $\gamma$  are constants which their formulations are as follows:

$$\alpha = \tau + g\sin\theta + gC_r\cos, \quad (2)$$

$$\beta = 0.5C_d\rho A, \quad (3)$$

$$\lambda = \xi/k\psi, \quad (4)$$

$$\gamma = 1/1000n\eta\eta_{tf}. \quad (5)$$

The definition of all parameters and their typical values are given in Table 1 based on Eshtehadi et al. (2017). The cost of fuel and CO<sub>2</sub> emissions can be estimated as  $C=F(v)f_c$ , where  $f_c$  is the unit cost of fuel and CO<sub>2</sub> emissions.

**Table 1.** Parameters used in the PRP model

Notation	Description	Typical values
$w$	Curb-weight (kilogram)	6350
$\xi$	Fuel-to-air mass ratio	1
$K$	Engine friction factor (kilojoule/rev/liter)	0.23
$N$	Engine speed (rev/second)	37
$V$	Engine displacement (liters)	5
$g$	Gravitational constant (meter/second <sup>2</sup> )	9.81
$C_d$	Coefficient of aerodynamic drag	0.7
$\rho$	Air density (kilogram/meter <sup>3</sup> )	1.2041
$A$	Frontal surface area (meter <sup>2</sup> )	3.912
$C_r$	Coefficient of rolling resistance	0.01
$n_{tf}$	Vehicle drive train efficiency	0.4
$\eta$	Efficiency parameter for diesel engines	0.9
$f_c$	Fuel and CO <sub>2</sub> e emissions cost per liter (£)	1.4
$f_d$	Driver wage per second (€)	0.0022
$k$	Heating value of a typical diesel fuel	44

Table 1. Continued

Notation	Description	Typical values
	(kilojoule/gram)	
$\psi$	Conversion factor (gram/second to liter/second)	737
$v^l$	Lower speed limit (meter/second)	5.55 (or 20 kilo meters/hour)
$v^u$	Upper speed limit (meter/second)	25 (or 90 kilo meters/hour)

## 2-2- The PRP Model formulation

In this section we present the model formulation for the PRP. Speed function should be discretized, that defined by R non-reducing speed levels  $\bar{v}^r$  ( $r = 1, 2, \dots, R$ ). Binary variables  $x_{ij}$  are equal to 1 if arc  $(i, j)$  appears in solution.  $z_{ij}^r$  is a binary variable equal to 1 if arc  $(i, j) \in A$  is crossed by a speed level  $r$ , and 0 otherwise. Continuous variables  $f_{ij}$  defined the total amount of flow on each arc.  $y_j$  is a non-negative continuous variable representing the time at that service starts at node  $j \in N_0$ . Accordingly, the integer linear programming formulation of the PRP is shown in below:

$$\begin{aligned}
 \text{Minimize } & \sum_{i=0}^n \sum_{j=0}^n f_c kNV\lambda d_{ij} \sum_{r=1}^R z_{ij}^r / \bar{v}^r + \sum_{i=0}^n \sum_{j=0}^n f_c w\gamma\lambda\alpha_{ij} d_{ij} x_{ij} \\
 & + \sum_{i=0}^n \sum_{j=0}^n f_c \gamma\lambda\alpha_{ij} d_{ij} f_{ij} + \sum_{i=0}^n \sum_{j=0}^n f_c \beta\gamma\lambda d_{ij} \sum_{r=1}^R z_{ij}^r (\bar{v}^{r^2}) \\
 & + \sum_{i=0}^n s_i f_d
 \end{aligned} \tag{6}$$

Subject to:

$$\sum_{i=1}^n x_{0i} \leq m \tag{7}$$

$$\sum_{i=0}^n x_{ij} = 1 \quad \forall i \in N_0 \tag{8}$$

$$\sum_{j=0}^n x_{ij} = 1 \quad \forall j \in N_0 \tag{9}$$

$$\sum_{j=0, j \neq i}^n f_{ij} - \sum_{j=0, j \neq i}^n f_{ji} = q_i \quad \forall i \in N_0 \tag{10}$$

$$q_j x_{ij} \leq f_{ij} \leq (Q - q_i) x_{ij} \quad (i, j) \in A \tag{11}$$

$$y_i - y_j + t_i + \sum_{r=1}^R d_{ij} z_{ij}^r / \bar{v}^r \leq K_{ij} (1 - x_{ij}) \quad \forall i \in N, \forall j \in N_0, i \neq j \tag{12}$$

$$y_j - s_j + t_j + \sum_{r=1}^R d_{j0} z_{j0}^r / \bar{v}^r \leq L (1 - x_{j0}) \quad \forall j \in N_0 \tag{13}$$

$$a_i \leq y_i \leq b_i \quad \forall i \in N_0 \tag{14}$$

$$\sum_{r=1}^R z_{ij}^r = x_{ij} \quad (i, j) \in A \tag{15}$$

$$x_{ij} \in \{0, 1\} \quad (i, j) \in A \tag{16}$$

$$z_{ij}^r \in \{0, 1\} \quad (i, j) \in A, r = 1, 2, \dots, R \tag{17}$$

$$f_{ij} \geq 0 \quad (i, j) \in A \quad (18)$$

$$y_i \geq 0 \quad \forall i \in N_0. \quad (19)$$

The objective function (6) minimizes the quantity of total fuel consumption is derived from (1) and drivers wage..Constraint (7) guaranteed that maximum used vehicles must less than available vehicles. Constraints (8) and (9) assure that each customer should be visited exactly one time through the tour. Constraint (10) defines the arc flows as well as eliminating the sub-tours. Constraint (11) enforces the capacity limitations on the load of each vehicle. Constraints (12) - (14), which  $K_{ij} = \max\{0, b_i + t_i + \frac{d_{ij}}{l_{ij}} - a_j\}$  and  $L$  is a large number, enforce the time window restrictions ( $l_{ij}$ ) Is the minimum speed level. Constraints (15) ensure that only one speed level is selected for each arc. Finally, constraints (16) - (19) enforce the binary and non-negativity restrictions on decision variables.

### 2-3- The hard worst case (HWC) robust optimization approach

Stochastic programming is the most important classical approach to handle uncertainty in optimization problems; however, sufficient and reliable historical data is a necessary requirement while applying this approach in order to find probability distribution of uncertain parameters. In spite of this requirement, in real world problems it is very hard and in some cases impossible to achieve accurate and reliable data. As an alternative in robust optimization (RO), the distribution of uncertain parameters is not needed to be known and only the range for each of them is required.

As the pioneer in RO area, Soyster(1973)presented a conservative linear optimization model which considers the worst case condition of uncertain parameters. Two decades later, Ben-Tal and Nemirovski (1998, 2000) and Bertsimas and Sim (2003, 2004)came up with some less conservatism robust optimization approaches which are based on box+ellipsoidal and box+polyheadral uncertainty sets, respectively. In this section, two robust optimization models are developed in order to cope with PRPUD. First, a new hard worst case model is proposed and thereafter a soft worst-case robust optimization approaches presented for the PRPUD.

A solution of an optimization problem is called “robust” if it stays feasible for roughly all possible values of uncertain parameters (i.e., feasibility robustness) and also results in objective function values which have the least deviation from the planned optimal value for almost all possible values of uncertain parameters (i.e., optimality robustness) (Pishvae et al. 2012). Therefore, robust optimization (RO) approach tries to enhance the performance of the obtained solutions in both feasibility and optimality robustness aspects. As Pishvae et al. (2012) mentioned, various RO approaches can be classified in three main categories including (1) hard worst case, (2) soft worst case and (3) realistic approaches.

Among the aforementioned categories, the hard worst case (HWC) approach provides the maximum conservatism against uncertainty for the decision maker. In other words, the obtained solution from this approach is always feasible for all possible values of uncertain parameters. Regarding the value of objective function, this approach assures that the value of objective function never violates the optimal planned value. HWC models are applied in the fields namely logistics management and different problems in this context such as VRP.

Eshthead et al.(2017) proposed a hard worst case robust optimization modelfor PRPUD that used in this paper. They extended deterministic model to hard worst case robust by making the following changes:

$$\begin{aligned}
\text{Minimize } & \sum_{i=0}^n \sum_{j=0}^n f_c kNV \lambda d_{ij} \sum_{r=1}^R z_{ij}^r / \bar{v}^r + \sum_{i=0}^n \sum_{j=0}^n f_c w \gamma \lambda \alpha_{ij} d_{ij} x_{ij} \\
& + \sum_{i=0}^n \sum_{j=0}^n f_c \gamma \lambda \alpha_{ij} d_{ij} f'_{ij} + \sum_{i=0}^n \sum_{j=0}^n f_c \beta \gamma \lambda d_{ij} \sum_{r=1}^R z_{ij}^r (\bar{v}^{r^2}) \\
& + \sum_{i=0}^n s_i f_d
\end{aligned} \tag{20}$$

*Subject to:*

constraints (7) to (9), (12)to (19) and:

$$\sum_{j=0, j \neq i}^n f'_{ji} - \sum_{j=0, j \neq i}^n f'_{ij} = \bar{q}_i + \hat{q}_i, \quad i = 1, 2, \dots, n \tag{21}$$

$$f'_{ij} \leq (Q - \bar{q}_i - \hat{q}_i) x_{ij}, \quad \forall (i, j) \in A \tag{22}$$

$$f'_{ij} \geq (\bar{q}_j + \hat{q}_j) x_{ij}, \quad \forall (i, j) \in A \tag{23}$$

$$f_{0i} \leq f'_{0i}, \quad i = 1, 2, \dots, n \tag{24}$$

$$\sum_{j=0, j \neq i}^n f_{ji} - \sum_{j=0, j \neq i}^n f_{ij} - \sum_{j=0, j \neq i}^n f'_{ji} + \sum_{j=0, j \neq i}^n f'_{ij} = 2\hat{q}_i, \quad i = 1, 2, \dots, n \tag{25}$$

$$f'_{ij} \geq 0, \quad \forall (i, j) \in A \tag{26}$$

Which continuous variables  $f'_{ij}$  defined the total amount of flow on arc  $(i, j)$ . If each customer takes the lower bound value of demand parameter while the vehicle's load takes the upper bound value of demand, variable  $f_{ij}$  shows the total amount of flow on the arc  $(i, j)$ . Moreover, if both customer and vehicle's load takes the upper bound value of demand parameters, then the load of vehicles in each arc should be computed in hard worst case condition. Constraint (21) defines the hard worst case arc flows as well as eliminating the sub-tours like Constraint (10). Constraints (22) to (24) enforce the capacity limitations on each vehicle. Constraint (25) ensures the flow balance according to relations between variables  $f_{ij}$  and  $f'_{ij}$ .

### 3- An adaptive large neighborhood heuristic algorithm for the RPRP

As the main propose of this paper is analyzing the effect of uncertainty in RPRP, we use an enhanced version of the ALNS algorithm introduced by Demir et al. (2012) with the same parameters that results of computational experimentation confirmed the efficiency of them and we just adapted a part of algorithm for the RPRP. We describe the ALNS briefly here and refer the interested reader to Demir et al. (2012) for comprehensive details of this algorithm and parameters' values.

The ALNS metaheuristic first proposed by Ropke and Pisinger (2006) as an extension of the large neighborhood search (LNS) heuristic introduced by Shaw (1998). Ropke and Pisinger (2006) used ALNS to solve variants of the VRP and shown the perfect efficiency of ALNS for these problems.

ALNS algorithm based on the idea of improving an initial solution by means of some repair and destroy operators. In each iteration one removal operator, as destroy function, and one insertion operator, as repair operator, are chosen by roulette wheel mechanism according to their past performance. Repair operator inserting the nodes in the removal list, which removed by destroy operator from current solution, back into the routes. As like as Demir et al. (2012) we use the classical Clarke and Wright (1964) heuristic to construct an initial solution and the same 12 removal and five insertion operators in the ALNS algorithm. The new solution is accepted if it satisfies a criterion defined by the simulated annealing (Kirkpatrick, Gelatt, & Vecchi, 1983) used as local search framework used at the last loop. The score of used operators in each iteration updated regarding to

new solution objective function. At the end, the speed optimization procedure (SOP) is used for computing the optimal speed on each arc of the route in order to minimize objective function comprising fuel consumption and driving wage. This is the same algorithm as illustrated in Demir et al. (2012) that interested readers can refer.

#### **4- Computational results**

In this section the performance of the proposed robust models are investigated in 10, 50,100 and 200 nodes PRP data sets. Twenty instances of each groups of PRP library that are used to analyze the effect of the data uncertainty in RPRP based on hard worse case model. All the used instances are achievable from <http://www.apollo.management.soton.ac.uk/prplib.htm>. The computational experiments are conducted using Core i5 CPU 1.68 GHZ personal computer with 8 GB RAM.

##### **4-1- Performance on RPRP instances**

In tables 2 to 5, the results of different uncertainty levels are shown for each group of instances. Three levels of uncertainty (10, 25 and 50 percent) considered behind deterministic value to analyze the effect of uncertainty. For example, when using 10 percent uncertainty level, real demand for each customer changes between  $[0.9 \times \text{nominal value}, 1.1 \times \text{nominal value}]$ .

We use three abbreviations in the tables. S.C. shows the Solution cost (£), D.K. used instead of total routes distances in kilometers, and finally N.V. shows the Number of used vehicles.

**Table 2.** Computational results for 10-node instances

Instances	Data uncertainty											
	Deterministic			10%			25%			50%		
	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.
UK10_01	178.82	408.98	2	182.31	418.67	2	193.23	430.56	2	217.42	500.64	3
UK10_02	216.02	529.71	2	218.37	529.71	2	235.45	578.28	2	239.22	578.28	3
UK10_03	210.43	507.27	2	215.44	507.27	2	214.5	502.01	2	218.57	502.01	3
UK10_04	199.52	480.09	2	201.35	480.09	2	204.08	480.09	2	221.21	515.1	3
UK10_05	184.47	446.96	2	186.1	446.96	2	191.44	454.16	2	195.53	454.16	2
UK10_06	199.99	483.16	2	227.44	546.4	2	238.51	578.57	2	247.87	584.54	2
UK10_07	199.7	494.69	2	208.86	541.65	2	211.63	541.65	2	217.55	573.57	2
UK10_08	233.13	567.8	2	235.85	567.8	2	249.08	587.54	2	267.82	639.67	3
UK10_09	183.37	457.04	2	184.88	457.04	2	187.14	457.04	2	195.12	466.97	2
UK10_10	200.38	546.4	2	201.99	483.16	2	204.99	483.16	2	235.19	558.18	3
UK10_11	275.54	697.16	2	278.33	697.16	2	282.52	697.16	2	299.48	719.21	2
UK10_12	192.13	460.26	2	193.91	460.26	2	196.58	460.26	2	215.09	489.73	2
UK10_13	205.83	510.49	2	208.56	510.49	2	209.93	540.34	2	214.05	510.49	2
UK10_14	171.15	397.75	2	177.63	417.93	2	200.48	478.73	2	205.09	478.73	2
UK10_15	132.89	291.37	2	134.6	291.37	2	153.44	353.39	2	163.57	377.35	2
UK10_16	196.64	451.06	2	199.13	451.06	2	241.12	558.75	3	265.32	613.06	3
UK10_17	166.84	387.52	2	180.52	421.41	2	198.85	468.79	2	215.06	499.98	2
UK10_18	169.91	401.48	2	175.37	405.02	2	177.9	405.02	2	184.75	413.57	2
UK10_19	177.65	414.46	2	184.27	438.6	2	187.51	434.04	2	220.65	520.17	2
UK10_20	176.78	412.78	2	178.66	412.78	2	181.46	412.78	2	192.79	442.98	3
Average	193.56	467.326	2	198.68	474.24	2	207.99	495.11	2.05	221.56	521.92	2.4



**Table 3.** Computational results for 50-node instances

Instances	Data uncertainty											
	Deterministic			10%			25%			50%		
	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.
UK50_01	624.39	1362.24	7	668.09	1484.28	8	719.78	1600.27	9	798.42	1758.86	10
UK50_02	627.74	1396.23	7	665.72	1485.49	7	717.35	1594.55	8	830.57	1872.64	10
UK50_03	659.59	1460.34	7	692.98	1545.64	8	740.36	1645.98	9	836	1846.72	11
UK50_04	786.97	1792.26	8	820.26	1869.28	8	898.8	2045.16	9	1020.97	2337.61	11
UK50_05	672.81	1525.28	6	701.5	1589.09	7	752.85	1692.38	8	816.79	1836.64	9
UK50_06	615.29	1300.47	8	633.52	1340.98	9	688.2	1477.36	10	790.5	1705.13	12
UK50_07	569.02	1249.42	7	588.58	1261.71	7	616.26	1345.21	8	703.6	1523.57	10
UK50_08	600.23	1330.55	7	606.12	1330.09	7	666.46	1455.8	8	737.42	1611.45	10
UK50_09	721.97	1611.5	7	768.16	1731.55	8	824	1849.52	9	931.8	2105.26	11
UK50_10	705.38	1585.49	7	738.92	1670.09	8	799.17	1797.64	9	917.33	2090.13	11
UK50_11	663.09	1484.66	7	691.72	1539.38	7	744.17	1662.11	8	812.65	1826.67	10
UK50_12	598.83	1334.46	6	628.21	1401.18	7	680.23	1504.26	8	759.77	1700.48	10
UK50_13	617.82	1349.99	7	666.85	1455	8	700.49	1515.11	9	783.39	1697.22	11
UK50_14	686.05	1543.69	7	722.79	1641.76	7	789.85	1784.51	8	903.44	2071.86	10
UK50_15	611.86	1378.4	6	655.03	1464.97	7	692.01	1553.63	8	781.23	1748.2	10
UK50_16	613.43	1353.84	7	646.71	1433.05	7	703.03	1575.23	8	798.72	1794.13	10
UK50_17	484.14	951.71	7	512.75	1014.37	8	536.65	1080.31	9	587.47	1191.89	11
UK50_18	720.15	1587.38	8	776.86	1727.16	9	819.59	1808.56	10	935.19	2072.49	12
UK50_19	637.6	1404.13	7	668.11	1460.78	8	711.87	1566.45	9	789.87	1761.24	10
UK50_20	713.03	1606.75	7	759.63	1693.44	8	811.24	1831.79	9	918.66	2089.79	11
Average	646.47	1430.44	7.00	680.63	1506.96	7.65	730.62	1619.29	8.65	822.69	1832.10	10.50

**Table 4.** Computational results for 100-node instances

Instances	Data uncertainty											
	Deterministic			10%			25%			50%		
	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.
UK100_01	1305.87	2871.56	14	1358.25	3028.2	15	1469.28	3269.34	17	1652.44	3763.16	21
UK100_02	1256.96	2786.41	13	1296.76	2868.87	14	1415.35	3146.95	16	1560.92	3507.91	19
UK100_03	1168.91	2560.93	13	1233.67	2701.29	14	1335.91	2933.43	16	1497.97	3385	20
UK100_04	1167.54	2468.76	14	1243.13	2624.41	16	1334.57	2803.67	18	1486.89	3214.23	21
UK100_05	1110.27	2323.95	14	1176.67	2474.09	16	1257.22	2646.16	18	1445.98	3061.46	22
UK100_06	1277.47	2792.6	14	1355.59	2965.01	16	1495.21	3287.95	18	1685.78	3728.4	22
UK100_07	1123.12	2473.58	12	1205.84	2635.94	13	1293.36	2846.65	15	1414.28	3121.08	18
UK100_08	1186.88	2624.85	12	1241.49	2747.92	14	1355.9	2975.13	16	1515.5	3371.35	19
UK100_09	1057.36	2232.07	13	1120.73	2370.97	14	1184.53	2508	16	1331.93	2846.11	19
UK100_10	1142.64	2513.49	12	1204.01	2649.11	13	1266.37	2785	15	1443.76	3180.08	18
UK100_11	1289.31	2793.99	15	1372.48	2964.14	16	1493.09	3231.5	18	1666.75	3641.27	22
UK100_12	1133.13	2453.79	12	1194.73	2622.72	14	1293.85	2811.62	15	1458.13	3195.73	19
UK100_13	1221.48	2677.64	13	1311.76	2889.26	15	1402.08	3070.24	17	1567.77	3482.1	20
UK100_14	1334.77	2987.47	14	1427.09	3176.34	15	1556.51	3465.29	18	1752.21	3941.11	21
UK100_15	1399.68	3068.16	15	1470.53	3233.86	16	1617.36	3578	19	1830.29	4056.68	23
UK100_16	1067.37	2263.26	12	1106.98	2378.26	13	1206.29	2573.51	15	1336.51	2861.2	19
UK100_17	1351.6	2965	15	1426.78	3115.01	16	1549.96	3387.59	19	1777.07	3917.69	23
UK100_18	1168.72	2529.78	13	1229.88	2692.39	14	1314.22	2869.81	16	1529.97	3379.04	19
UK100_19	1101.4	2334.31	13	1145.38	2417.31	14	1259.38	2684.7	17	1390.04	2969.33	20
UK100_20	1339.53	2970.4	14	1425.48	3173.15	15	1543.63	3411.96	17	1763.32	3948.72	21
Average	1210.20	2634.60	13.35	1277.36	2786.41	14.65	1382.20	3014.33	16.80	1555.38	3428.58	20.30

**Table 5.** Computational results for 200-node instances

Instances	Data uncertainty											
	Deterministic			10%			25%			50%		
	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.
UK200_01	2199.64	4622.78	28	2348.37	4927.98	31	2508.04	5360.51	35	2901.65	6235.24	42
UK200_02	2095.47	4461.41	24	2251.07	4717.88	27	2417.82	5142.17	31	2726.35	5915.66	37
UK200_03	2119.95	4422.59	27	2288.1	4770.39	30	2447	5107.17	35	2808.42	6009.61	42
UK200_04	2011.08	4154.9	26	2167.03	4458.73	30	2334.85	4847.07	33	2599.11	5570.53	40
UK200_05	2276.32	4876.52	27	2452.36	5275.14	29	2653.49	5725.44	33	3029.85	6643.64	41
UK200_06	1971.79	3961.48	27	2068.38	4214.12	29	2283.34	4682.5	33	2564.97	5274.85	41
UK200_07	2115.11	4377.76	27	2239.39	4656.27	30	2461.31	5126.27	34	2754.59	5758.9	40
UK200_08	2225.69	4659.72	27	2358.47	4983.69	29	2539.59	5405.53	34	2902.51	6154.17	41
UK200_09	1974.06	4028.42	25	2096.24	4317.46	28	2263.76	4675.03	32	2510.14	5283.1	38
UK200_10	2313.49	4909.19	28	2458.49	5309.76	31	2696.32	5801.58	35	3070.34	6705.42	42
UK200_11	2031.52	4142.29	27	2168.66	4399.75	30	2322.62	4748.05	34	2613.73	5463.09	41
UK200_12	2230.13	4837.74	25	2389.2	5180.81	28	2612.88	5709.04	32	2997.23	6594.85	38
UK200_13	2242.82	4788.94	26	2375.49	5070.53	28	2613.29	5686.38	32	3002.49	6542.87	38
UK200_14	2108.54	4366.28	27	2270.69	4715.2	30	2442.17	5143.69	33	2789.45	5911.34	41
UK200_15	2210.02	4742.3	26	2354.24	5038.68	28	2563.9	5513.61	32	2952.69	6358.59	39
UK200_16	2166.42	4536.67	27	2319.65	4924.33	30	2534.13	5315.97	34	2840.28	6132.65	40
UK200_17	2317.72	4992.11	26	2460.85	5336.04	29	2647.6	5788.8	32	3071.65	6763.62	39
UK200_18	2131.26	4378.23	27	2280.91	4799.19	29	2451.36	5161.64	34	2819.24	6053.86	40
UK200_19	1944.76	3970.12	25	2043.73	4240.3	27	2221.67	4590.42	31	2518.55	5280.18	38
UK200_20	2256.26	4760.51	27	2393.84	5111.48	29	2590.69	5532.53	33	2975.24	6483.21	40
Average	2147.10	4499.50	26.45	2289.26	4822.39	29.10	2480.29	5253.17	33.10	2822.42	6056.77	39.90

Table 6 shows the summary of results for all instances. Each cell shows the average percent of increase in each factor in comparison with deterministic solution without uncertainty. For all test problems groups, the increase in uncertainty level caused in higher value of objective function, total distances and number of vehicles. Moreover, the effect of uncertainty is more observable in larger instances.

For 10-node instances, results show 14.47 percent increase in solution cost of maximum uncertainty level (50%) in comparison with deterministic solution cost, so decision makers should use robust solution with acceptable increase in their costs. But this amount increase to 31.45percent in average for 200-node instances and decision makers maybe want to choose other approaches with lower cost.

The results show that data uncertainty in PRP, lead to significant increase in fuel consumption, driver wage and required fleet size, especially for large scale problems. In this situation, decision makers will have three choices to confront with this cost:

- Accept the risk of unmet demand cost and use deterministic models or soft worse case PRP models with less robustness level.
- Try to decrease data uncertainty level.
- Use any intelligent transportation system (ITS) as like as RFID (Radio-frequency identification) equipment for their fleet, and use Dynamic models or backup fleet for neutralize the effect of uncertainty.

If decision maker confronts with high uncertainty information and large scale problem, they should estimate the cost of each customer unmet demands and required ITS equipment for decrease uncertainty level, and then they can compare the surplus cost of robustness with two mentioned costs and choose the best solution.

**Table 6.** Summary of changes Percent relative to deterministic model

Instances	Data uncertainty								
	10%			25%			50%		
	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.	S.C.(£)	D.K.	N.V.
10-node instances	2.64	1.48	0.00	7.46	5.95	2.50	14.47	11.68	20.00
50-node instances	5.28	5.35	9.29	13.02	13.20	23.57	27.26	28.08	50.00
100-node instances	5.55	5.76	9.74	14.21	14.41	25.84	28.52	30.14	52.06
200-node instances	6.62	7.18	10.02	15.52	16.75	25.14	31.45	34.61	50.85
Average	5.02	4.94	7.26	12.55	12.58	19.26	25.43	26.13	43.23

## 5- Conclusions

In this paper an adaptive large neighborhood search algorithm used for solving the Robust Pollution-Routing Problem. The ALNS algorithm adapted for hard worst case robust optimization PRP model and results of large scale instances data sets confirm the results of small size instances, that shown the direct relationship between objective function (involve fuel cost and driver wage) and data uncertainty level.

Results show necessity of using ITS systems for decrease the level of uncertainty and its effect, if the cost of unmet demands is high, or unacceptable. Otherwise decision makers should compare the cost of unmet demand and preparing intelligent fleet, or accept the cost of the RPRP solution.

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