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Fuzzy multi-period mathematical programming model for maximal covering location problem

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Abstract

In this paper, a model is presented to locate ambulances, considering backup facility (to increase reliability) and the restriction of ambulance capacity. This model is designed for emergencies. In this model the covered demand for each demand point depends on the number of coverage times and the amount of demand. The demand amount and ambulance coverage radius are considered fuzzy in various periods, with respect to the conditions and application of the model. Ambulances have the ability to be relocated in different periods. In this model we have considered two types of ambulances to locate: ground and air ambulance. Air ambulances are considered as backup facilities. It is assumed that ground ambulances are major facilities, taking into account capacity limitations. To solve this model, making chromosomes (initial solution) is presented in such a way that location chromosome for both ground and air ambulances are appears as a general chromosome. Since this is a complicated model, a population-based simulated annealing algorithm (Multiple Simulated Annealing) with a chromosome combinatorial approach is used to solve it**.** Finally, the results of the algorithm presented to solve the model are compared with the simulated annealing (SA) algorithm. The results showed that the quality of the presented algorithm (MSA) is better than the SA algorithm.

Keywords: Backup covering location, fuzzy dynamic location, Ambulance location, Reliability, Capacity constraints, Multiple simulated annealing

1-Introduction

Along with the development of science and facilities, war and natural disasters are growing, and the demand for ambulance services is increasing to protect people's lives and property. This is one of the main concerns of people in urban and rural areas. The importance of this issue has made optimization scientists pay more attention to this issue recently (ReVelle and Eiselt 2005). Many models have been presented in this field. Most of the models presented in this field are going to minimize uncovered demands and service costs.

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For a more detailed explanation, an appropriate location should be located for stations so that the distance between stations and demand sites decreases and service performance time increases, or the number of stations which have common coverage points is minimized and the third is to choose the more reasonable number of stations maintaining the balance between the unsatisfied demand cost and the costs of establishing new stations. One of the most famous locating models that have been used since the beginning of locating science is covering the location problem (CLP). This model is looking for solutions to cover a set of demand points with respect to how many targets are considered (Berman and Krass 2002).

Coverage problems fall into two general categories: maximal covering location problem (MCLP), and set covering location problem (SCLP). Regardless of the type of problem, the population is considered covered when it is located in one or more facilities in a predetermined period of time or distance. The aim of SCLP is to cover all demand points by the least possible facilities, while MCLP tries to cover the maximum demand points by a predetermined number of facilities. Since then, numerous theoretical applications and developments have been presented on classic MCLP model (Shavandi and Mahlooji 2006).

In the models presented on backup that cover the location there are many crisp and probability model, but there are no models presented for the dynamic backup that covers the location, while in reality amount of demands and the coverage radius are changing during different periods considering road traffic. The first advantage of the suggested model is the demand dependency that covers the amount in the number of covering times per facility and demand amount. The second advantage of the model presented over other presented models in backup coverage subject is that the model is fuzzy dynamic (the parameters of model in periods are fuzzy). The amount of demand and the coverage radius is considered fuzzy dynamic in different periods to be coherent with reality. The third advantage is to consider air ambulance backup that covers the ground ones. Ground ambulances are considered limited, but the capacity of air ambulances (backup) is considered unlimited to cover an area in the case that a ground facility is already involved. Considering changeable demand amount and coverage radius in different periods to maximize the demand that covers the amount during the whole period, ground and air ambulances can exchange in this model.

The rest of this paper is as follows: In section 2 we review the works related to ambulance services. In section 3 we present suggested model and an exhaustive explanation about it. In section 4 there is an explanation about the algorithm presented, and in section 5 we mentioned some suggestions to regulate the algorithm parameters. In section 6 numerical examples have been presented, and in section 7 we have a conclusion**.**

2 - The related works

Facility locating has the main role in supply chain, especially in production and service facilities, so it has been focused by professionals and scholars. We can mention some valuable references such as ReVelle and Eiselt, (2005) as a comprehensive introduction to the location models. Furthermore (Revelle et al., 2008b) a comprehensive source of location problem consisting of p-median, p-center and covering location problem. During the last decade, many articles have been presented on the maximal covering location problem and its solution, which we will mention next. Aytug and Saydam, (2002) have conducted a comparison study on the performance of the genetic algorithm and other heuristic solutions on the maximum covering problems expected in large scales. Espejo et al., (2003a) have studied hierarchy maximal covering location in multi level and have presented a Lagrange release method to solve the problem. In a valuable study conducted by Galvão et al., (2000a)**,** a comparison has been made between initiative method based on Lagrange release and replacement. Galvão et al., (2000a)**,** by applying 331 experimental problems in the literature with 900-55 nodes, they discovered that there is no substantial difference between these two meta-heuristic methods. (Barbas and Marín, 2004) designed a network of combinatorial communication codes in communication systems and solved it through a Lagrange release solution. Canbolat and von Massow, (2009)**,** studied the maximum covering location in order to cover the maximum demand in oval form. This problem was formulated as an integrated non-linear programming problem and simulated annealing algorithm was used to solve the problem. Xia et al., (2009), had presented meta-heuristic methods to solve MCLP have presented and compared that concluded simulated annealing is the most effective method. They concluded that simulated annealing is the most effective method compared to other methods. In addition, there are many articles on the traditional development model of the maximum coverage location. Berman and Krass, (2002) have considered partial customer covering in the general model of the maximum coverage location. Younies and Wesolowsky (2004,) have introduced an integrated zero-one planning complex model for MCLP in locations where demand points are covered by skew trapezoids on the plate. Shavandi and Mahlooji (2006), presented a location-allocation problem in fuzzy method for density of systems, and they called it maximum coverage location-allocation with fuzzy queue. They solved the model with genetic algorithm. The maximal covering location was presented by Araz et al., (2007) had Multiple objectives that considered the uncertainty in multi-objective goal programming to cover model for emergency facility location. The first objective was to maximize demand; another objective was to minimize the total distance between larger distances with a predetermined standard distance. Another interesting article in the literature was Erdemir et al., (2008) that had considered the maximum coverage location in a situation that the demand could develop in nodes and routes. They showed a real example for station cellular location in New York City. Berman and Huang, (2008) have studied the location of facilities in the net to minimize the total demand for coverage. This problem is used in undesirable facility location. They compared the Tabu search method with the Lagrange release method to solve this problem and concluded that Tabu search is a better alternative for large-scale cases. Another conclusion that was reached about the problem of maximal covering location was considering negative weight of the demand nodes by Berman et al., (2009). They presented problems two types of models consist of linear and non-linear models that have been solved by CPLEX and other initiative methods. The results of another study showed that the simple model of maximum covering location was developed by applying the ability of geographic information system (GIS) and the idea of maximal covering to illustrate the demand coverage. O'Hanley and Church, (2011) improved a prevention locating model to maximize initial covering form through facilities developed by, and minimum covering level that occurs while losing essential facilities. They applied analysis algorithm to solve this problem. In addition, there are case studies about the maximal covering location in the literature. Moore and ReVelle, (1982) implemented a model for the case study of the therapeutic facility system in Pakistan using hierarchy location method. Curtin et al., (2010) interrupt Police patrols in Dallas and Texas maximal covering location problem was considered. To solve the maximal covering location problem, Murawski and Church, (2009) presented a model that assumed facility locations were stable and access to demand point should be improved which was named maximal covering improvement. Their model was called maximal covering improvement in the network and was carried out in Gina. The problem of uncertain maximal covering location has attracted a lot of attention in the literature. De Assis Corrêa et al., (2009) evaluated a case study of probable maximum covering with one service provider for each center and for solving used column producing and the covering graph solutions. Batanović et al., (2009) had presented maximal covering location problem on the network with a state of uncertainty. They studied model with the demand and the same importance, and with semi-definite weights of demand nods. Furthermore, they presented an adequate algorithm to solve these models. Berman and Wang, (2011) issued another paper that considers the uncertainty in maximum covering location. They studied a case in which weight of the demands related to the nodes in the network of random variables is in an uncertain probable distribution. The aim of the study is to locate a place where the unrelated covering is minimized.

ReVelle et al., (2008a) have presented a model solution that emphasizes on solving maximum covering location with 900 nodes. This article is a development of the last one with a change to become a multi-period dynamic model. In such models, the solvers are willing to locate p facilities in m time periods. Obviously, the scope of the solution may consist of a large number of feasible solutions and, at the same time with volume increase, this problem becomes strongly complex. This model is not unusual in practice and has different applications in many cases. For instance, it can be used in Police patrol location and fire stations emplacement traffic, vacations, disasters and etc, ambulance locating, or the location of the first air facility to help victims of natural disasters . Curtin et al., (2010) take locating an emergency service center in a crowded area to illustrate the problem. In order to offer an adequate service to people when traffic accidents occur, the number of facilities that must be established fluctuates during different periods of time related to traffic, vacations, weather conditions, and many other factors. Considering that the number of established facilities required is related to a limited budget, the problem of the maximum covering location problem is developed in form of multiple periods and dynamics. Dynamic location models can help managers to make daily or hourly decisions to respond to predictable fluctuations demands in emergency occasions over time and space (Rajagopalan et al., 2008). In these models a facility that is established first in the period, can be closed in the next period. The concept of dynamic covering location is not a new subject in the literature. Schilling,(1980) presented a multi-objective dynamic model to emergency service facilities such as ambulances. Gunawardane, (1982) presented various problems of location of dynamic public facilities.

Repede and Bernardo, (1994) have made another effort to model the dynamic maximal covering location that was modeled and improved the decision support system, and was used in Lousevill model. (Rajagopalan et al., 2008) has studied the periodic total covering model and to solve the simulation was used, Tabu search, and queue theory models. Their work was the development of Marianov and Revelle, (1994) study combined with the probabilities involved. Another article on the dynamic location model was submitted by Gendreau et al., (2001) in order to maximize the backup coverage and minimize costs of relocating. One of the published papers on dynamic maximal covering location is Başar et al., (2011) article that has developed multi-period location problem for the emergency centers so that they can make strategic plan for different periods. A useful Tabu search algorithm was developed to solve the problem and was implemented in Istanbul. This comprehensive literature review indicates that there are many considerations regarding the problem of maximal covering location in various situations, but dynamic fuzzy cases with fuzzy and dynamic parameters like radius, the number of facilities and demand have not been considered until now. In addition, a small number of articles have been submitted about maximal covering location in large scales. This study tries to decrease the gap by presenting a dynamic fuzzy model in emergency situations, and particle swarm optimization algorithm to solve models in large scale. Table1 analyzes the most significant models in the field. While most of them do not take account of the fuzzy and dynamic in their model, they are connected to the model of this paper from at least one perspective. In Table1 the last row shows contribution of this study compared to the literature.

Table1. Related papers in the literature (classified based on model and solution method). (Farahani et al., 2014)

Reference	Problem structure	Dynamic	Deterministic	Probabilistic	Fuzzy	Solving methodology	Exact	Heuristic	Meta-heuristic
(Başar et al., 2011)	Multi period backup double covering	✓				Tabu search algorithm			✓
(Gunawardane, 1982)	Dynamic public facility	✓				preliminary computational experiences	✓		
(Gendreau et al. 2001)	Dynamic Maximizing backup covering	✓				Parallel tabu search		✓	
(Fazel zarandi and et al.2013)	Large scale dynamic maximal	✓				Simulated annealing			✓
(Rajagopalan et al., 2008)	Dynamic relocation/replacemer	✓				Reactive tabu search		✓	
(Galvão, Gonzalo Acosta Espejo et al. 2000)	MCLP		✓			Lagrangean and surrogate relaxation	✓	✓	✓
(Marianov and Serra 2001)	Hierarchical queuing maximal covering location problem (HQMCLP)		✓			GRASP improvement by vertex substitution and Tabu search	✓	✓	✓
(Aytug and Saydam 2002)	Covering location problem(MECLP)			✓		Genetic algorithm			
(Berman, Krass et al. 2007)	Generalized Maximal covering location problem (G-MCLP)		✓			Greedy Heuristic	✓		
(Galvão, Acosta Espejo et al. 2002)	Hierarchical model for perinata facility		✓			Lagrangean Heuristics		✓	
(Espejo, Galvão et al. 2003)	HCLP		✓			Lagrangean and surrogate relaxation Sub-gradient Heuristic	✓	✓	
(Karasakal and Karasakal 2004)	MCLP with partial covering		✓			Lagrangean and relaxation	✓	✓	✓
(Snyder and Daskin 2005)	Reliability in facility location			✓		present an optimal lagrangian relaxation algorithm	✓		✓
(Shavandi and Mahlooji 2006)	Fuzzy queuing maximal coveri location problem				✓	Genetic algorithm	✓		
(ReVelle, Scholssberg et al. 2008)	MCLP		✓			Heuristic concentration	✓		
(Şahin and Süral 2007)	Hierarchical facility location		✓			Review of hierarchical facility location models	✓		
(Shavandi and Mahlooji 2007)	Fuzzy Hierarchical location–allocation maximal covering problem				✓	Probabilistic and fuzzy	✓		✓
(Batanović, Petrović et al. 2009)	MCLP				✓	Heuristics algorithm		✓	
(Canbolat and von Massow 2009)	Maximal covering with ellipse on the plane		✓			Simulated annealing			✓
(de Assis Corrêa, Lorena et al. 2009)	Probabilistic maximal covering location-allocation problem			✓		Heuristics algorithm (decomposition approach)	✓	✓	
(Qu and Weng 2009)	Multiple allocation hub maxim covering		✓			Path relinking procedure			
(Ratick, Osleeb et al. 2009)	Hierarchical maximal covering problem		✓			Optimal solution	✓		
(Lee and Lee 2010)	Generalized Hierarchical cover location (G-(HCLP)		✓			Tabu based Heuristic	✓	✓	
(Shen, Zhan et al. 2011)	Reliable facility location			✓		Approximation algorithm: SSA-greedy adding	✓	✓	✓
(Li, Zeng et al. 2013)	Reliable P-Median problem anc Reliable uncapacitated fixed-charge location problem			✓		Lagrangean relaxation based algorithm		✓	✓
(Farahani, Hassani et al. 2014)	Reliable Hierarchical MCLP		✓			Hybrid artificial bee colony	✓		✓
(Colombo, Cordone et al. 2016)	Multimode covering location problem		✓			Combining two greedy algorithms			
This research	Dynamic covering location-allocation problem	✓			✓	Multiple Simulated Annalinng	✓		

3- The proposed Model

In the proposed model, two types of facilities were considered to cover demand points, including ground ambulances and air ambulances. Ground ambulances are considered as the main facilities and are limited in number which means they can't cover demand points which their demands are more than the capacity and such demand points need to be covered by more than one facility. If it is covered by a single facility and the demand amount is more than the capacity of facility, they would cover just a part (fraction) of the demand of that point. Air ambulances are considered as backup facilities. Backup facility is used to cover the area which is in the domain of a specific ground facility but it is involved. In this model, the number of demands and covering radius in different periods regarding situations during the time has been considered dynamic. The objective of this model is to maximize covered demand amount by ground and air ambulances. The amount of ground and air facilities (backup) is constant and predetermined in different periods. Potential points to create ground and air facilities are in discrete form and different from each other. Ground and air ambulances are able to carry victims in different periods. Proposed model is an integrated and zero-one model which all of its variables are in zero-one form. Proposed model have applications in emergency occasions. The schematic figure of the presented model is as figure1.

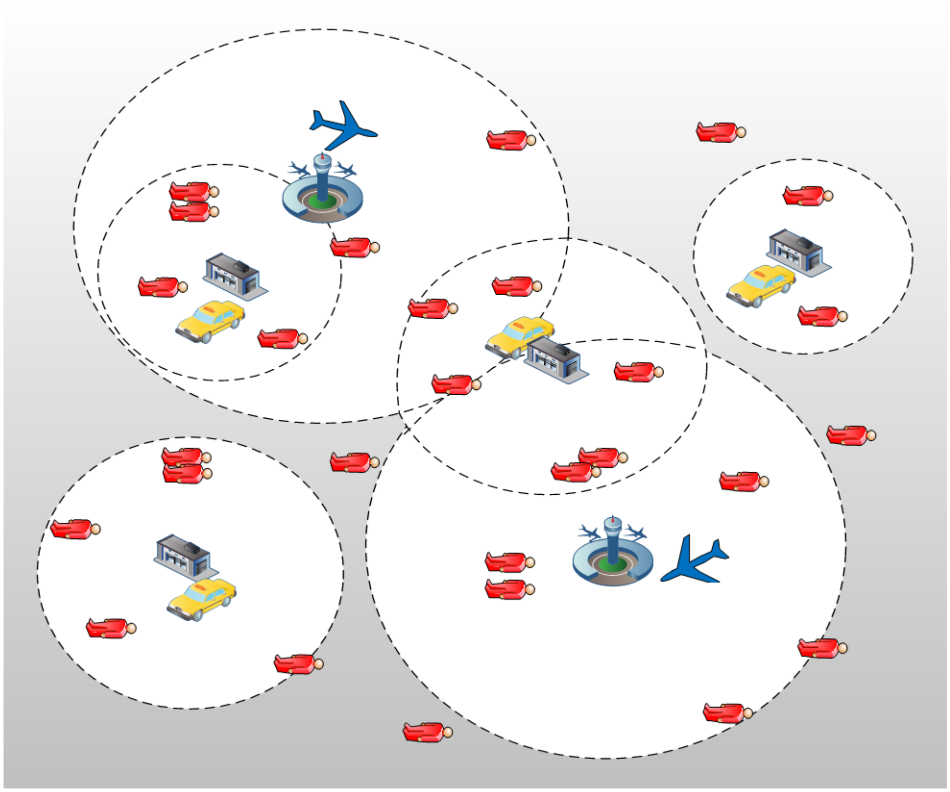

Fig 1. Schematic figure of presented model

3-1- Components of the model

This section presents the components of the mathematical model divided into indices, parameters and decision variables.

Indexes:

$\boldsymbol{i}$: The index and set of demand point (node)

$\boldsymbol{j}$: The index and set of potential points to establish ground ambulances

$\boldsymbol{f}$: The index and set of potential point to establish air ambulances (backup)

$\boldsymbol{t}$: The number of considered periods

$\tilde{\boldsymbol{k}}$: The fuzzy number of facilities which covers each point of demand $\boldsymbol{i}$

$\boldsymbol{a_{ijt}}$: A binary parameter if in period $\boldsymbol{t}$ located ground ambulance in point $\boldsymbol{j}$ covers demand point $\boldsymbol{i}$ is one, otherwise zero

$\boldsymbol{r_{ift}}$: A binary parameter if air ambulance (backup) located in point $\boldsymbol{f}$ in period $\boldsymbol{t}$ covers $\boldsymbol{i}$ demand point, is one, otherwise zero.

$\tilde{\boldsymbol{p}}_{\boldsymbol{t}}^{\boldsymbol{A}}$: Fuzzy number of ground ambulances in each period to be sited.

$\tilde{\boldsymbol{p}}_{\boldsymbol{t}}^{\boldsymbol{B}}$: Fuzzy number of air ambulances (backup) which need to be located in each period.

$\widetilde{\boldsymbol{D}}_{\boldsymbol{it}}$: Fuzzy amount of demand point of $\boldsymbol{i}$ in period $\boldsymbol{t}$.

$\widetilde{\boldsymbol{C}}_{\boldsymbol{itk}}$: Fuzzy fraction of demand point $\boldsymbol{i}$ which is covered by $\boldsymbol{k}$ ground facility in period $\boldsymbol{t}$, and depend on demand amount $\widetilde{\boldsymbol{D}}_{\boldsymbol{it}}$

3.2. Decision variables

$\boldsymbol{x_{jt}^{A}}$: is an integer variable which represents number of ground ambulances in period $\boldsymbol{t}$ which is located in potential point$\boldsymbol{j}$.

$\boldsymbol{x_{jt}^{B}}$: is an integer represents air ambulances (backup) which is located in period $\boldsymbol{t}$ in potential point f.

$\boldsymbol{y_{itk}^{A}}$: A binary variable which states if in period t the demand point $\boldsymbol{i}$ is covered with $\boldsymbol{k}$ ground ambulance, is one, otherwise zero.

$\boldsymbol{y_{it}^{B}}$: A binary variable which shows if demand point $\boldsymbol{i}$ in period $\boldsymbol{t}$ is covered by at least one air ambulance (backup), is one, otherwise zero.

$\boldsymbol{x_{Ai}}$: A binary variable that means a demand point is just covered by one ground ambulance.

$\boldsymbol{x_{Bi}}$: A binary variable which represents a demand point is covered by both ground and air ambulances.

3-3- The mathematical model

The proposed model is as follows:

$$MaxZ = \sum_{\forall i} x_{A_i} \sum_{\forall t} \sum_{\forall k} \widetilde{D}_{it} \tilde{C}_{itk}\, y_{itk}^{A} + \sum_{\forall i} x_{Bi} \sum_{\forall t} \widetilde{D}_{it} y_{it}^{B} \tag{1}$$

ST:

$$\tilde{k} y_{itk}^{A} \lesseqgtr \sum_{j} a_{ijt} x_{jt}^{A} \qquad \forall\, i, t, k \tag{2}$$

$$\sum_{\forall k} y_{itk}^{A} \leq 1 \qquad \forall\, i,t \tag{3}$$

$$y_{it}^{B} \leq \sum_{\forall\, k} y_{itk}^{A} \qquad \forall\, i,t \tag{4}$$

$$y_{it}^{B} \ \leq \sum_{\forall\, f} r_{ift} x_{ft}^{B} \qquad \forall\, i,t \tag{5}$$

$$\sum_{\forall\, j} x_{jt}^{A} \approx \tilde{p}_{t}^{A} \qquad \forall\, t \tag{6}$$

$$\sum_{\forall\, f} x_{ft}^{B} \approx \tilde{p}_{t}^{B} \qquad \forall\, t \tag{7}$$

$$x_{Ai} + x_{Bi} \leq 1 \qquad \forall\, t \tag{8}$$

$$x_{ft}^{B}\,, x_{jt}^{A} \ \geq 0 \quad Integeer \qquad \forall\, i\,, t \tag{9}$$

$$\mathrm{y}_{\mathrm{it}}^{\mathrm{B}} \ \,, y_{itk}^{A}\,, x_{Ai}\ ,\ x_{Bi} \in \{0 \ \ 1\} \qquad \forall\, i\,, t\ k \tag{10}$$

The objective function, equation (1) maximizes the sum of covered demands by ground and air ambulances. Equation (2) states that one demand point is covered $\boldsymbol{k}$ times when located in coverage radius $\boldsymbol{k}$ ambulances. Equation (3) represents that one demand point is covered or not by exactly one definite facility. Equation (4) shows that a demand point is covered by an air ambulance only in the case that the point is located in minimum coverage radius of a ground ambulance. Equation (5) guarantees that demand point is covered when it is in coverage radius of an air ambulance. Equations (6) and (7) represent the number of ground and air ambulances which are located in each period.

3-4- Linearization of the model

We replace $\mathbf{x}_{\mathbf{A,i}} * \mathbf{y}_{\mathbf{i,t,k}} = \mathbf{Z}_{\mathbf{i,t,k}}^{\mathbf{A}}$ by $\mathbf{x}_{\mathbf{B,i}} * \mathbf{y}_{\mathbf{i,t}} = \mathbf{Z}_{\mathbf{i,t}}^{\mathbf{B}}$ in objective function. Objective function changes to linear statement in relation (11).

$$MaxZ = \sum_{\forall\, i} \sum_{\forall\, t} \sum_{\forall\, k} \widetilde{D}_{it} \times \tilde{C}_{itk}\, Z_{itk}^{A} + \sum_{\forall\, i} \sum_{\forall\, t} \widetilde{D}_{it} \times Z_{it}^{B} \tag{11}$$

Instead of writing non-linear constraint $\mathbf{x}_{\mathbf{A,i}} * \mathbf{y}_{\mathbf{i,t,k}} = \mathbf{Z}_{\mathbf{i,t,k}}^{\mathbf{A}}$ and $\mathbf{x}_{\mathbf{B,i}} * \mathbf{y}_{\mathbf{i,t}} = \mathbf{Z}_{\mathbf{i,t}}^{\mathbf{B}}$ in the model, we insert following linear constraint in the model which equals constraint (12)

$$\sum_{\forall\, k} Z_{itk}^{A} + Z_{it}^{B} \leq 1 \tag{12}$$

Constraints which are added to the model include equation (13), (14), (15) and (16)

$$Z_{itk}^{A} \leq y_{itk}^{A} \tag{13}$$

$$\boldsymbol{Z_{it}^{B} \leq y_{it}^{B}} \tag{14}$$

$$\textstyle\sum_{\forall\, k} Z_{itk}^{A} + Z_{it}^{B} \leq 1 \tag{15}$$

$$\boldsymbol{Z_{itk}^{A}} \ \,, \boldsymbol{Z_{it}^{B}} \in \{\mathbf{0} \ \ \mathbf{1}\} \tag{16}$$

3-5- The fuzzy tactical multi-period maximal covering location-allocation problem

In this part, we explain a method to change the FDMCLAP model in the form of an equivalent supplementary crisp DMCLAP model for strategic FLP under demand and number of ambiguities of the facilities. It is crucial to take the methods of fuzzy mathematical programming into account that

necessarily consider the fuzzy coefficients of the objective function and fuzzy constraint: technological and right–hand side coefficients. In this context, several studies are available in the literature. In addition, the authors present a review of fuzzy mathematical programming models in accordance with fuzzy parameters: the fuzzy model with fuzzy aim(s), the fuzzy model with fuzzy objective coefficients and fuzzy right-hand side rates of the restrictions, etc. The readers propose a categorization of fifteen types of the fuzzy mathematical models taking into account all possible mixtures of the fuzzy parameters mentioned above. At the end, they review the exiting solution procedures suggested in the literature to resolve fuzzy mathematical programs (Peidro et al., 2009).
Since the FDMCLAP model considers inadequate or inaccurate information in data (connected to: demand, number of facility, covering radius) and fuzziness connected to adjustable constraints (1),(2),(6,7), a fuzzy escalation approach which simultaneously considers the possible deficiency of knowledge in data and current fuzziness is needed. Therefore, in this study we choose to follow the approach by Cadenas and Verdegay. A regular model for fuzzy linear programming that looks at fuzzy technological, fuzzy cost coefficients and fuzzy right-hand side terms in restrictions is suggested by the authors. Furthermore, in the inequalities that explain the restrictions, fuzziness is considered. This common fuzzy linear programming model is like (17).

$$Maximum\ Z = \sum_{k=1}^{n} \tilde{c}_k x_k$$

$$Subject\ to\ \sum_{k=1}^{n} \tilde{a}_{rk} x_k \lessapprox \tilde{b}_r \tag{17}$$
$$x_k \geq 0\,, \quad r \in D, \quad K \in Z$$

Where the fuzzy parameters are given by:

- For each cost $\exists\ \mu_j \in F(S)\ so\ that\ \mu_j : S \to [0\,,1],\ K \in Z$, which explains the fuzzy costs.
- For each row $\exists\ \mu_r \in F(S)\ so\ that\ \mu_r : S \to [0\,,1],\ r \in D$, which shows the fuzzy number in the right side of constraints.
- For each $r \in D\ and\ K \in Z\ \exists\ \mu_{rk} \in F(S)\ so\ that\ \mu_{rk} : S \to [0\,,1]$, which shows the fuzzy number in the technological matrix.
- For each row $\exists\ \mu_r \in F[F(S)]\ so\ that\ \mu_r : F(S) \to [0\,,1], r \in D$, which supplies the accomplishment degree of the fuzzy number of each $x \in R^n$
  $\tilde{a}_{r1}x_1 + \tilde{a}_{r2}x_2 + \cdots + \tilde{a}_{rn}\,x_n\,, r \in D$

Cadenas and Verdegay (2000), described a resolution method that consists of replacing the problem (17) by a convex fuzzy set by means of classification operation as a contrasting method of fuzzy numbers.

Let $H, V \in F(S)$; a simple approach to classify fuzzy number formed by the determination of a classifying function that places each fuzzy number into the real line, $w\!: F(S) \to S$. If this role of w (0) is obvious, then

$$w(H) < w(V) \Leftrightarrow H\ is\ less\ than\ V$$

$$w(H) > w(V) \Leftrightarrow H\ is\ greater\ than\ V$$

$$w(A) = w(B) \Leftrightarrow H\ is\ equal\ to\ V$$

Usually, w is called a linear ranking function if

$$\forall\ H\,, V \in F(S), \quad w(H+V) = w(H) + w(V)$$

$\forall\, d \in S, \quad d > 0, \quad w(rH) = dw(H) \ \ \forall\, H \in F(S)$

To resolve the problem (12), define: let w be a fuzzy number linear ranking function and given the function, $\Psi: F(S) \times F(S) \rightarrow F(S)$ *in order to*

$$\Psi\left(\tilde{a}_r x, \tilde{b}_r\right) = \begin{cases} \tilde{t}_r & , \quad \tilde{a}_r x \leq_w \tilde{b}_r \\ t_r(-)\tilde{a}_r x(+)\tilde{b}_r & , \quad \tilde{b}_r \leq_w \tilde{a}_r x \leq_w \tilde{b}_r(+)\tilde{t}_r \\ 0 & , \quad \tilde{a}_r x \leq_w \tilde{b}_r(+)\tilde{t}_r \end{cases}$$

Where $\tilde{t}_r \in F(S)$ is a fuzzy number so that its support consists of R^+, and $\leq_w$ is a relationship that evaluates that $H \leq_w V, \forall H, V \in F(S)$ and (-) and (+) are the common operation among fuzzy numbers.

As stated by Cadenas and Verdegay, the membership function connected to the fuzzy constraint $\tilde{a}_r x \tilde{\lesseqgtr} \tilde{b}_r$, with $\tilde{t}_r$ a fuzzy number giving the maximum defilement of the rth constraint is relation (18)

$$\mu^r: F(S) \rightarrow [0,1] / \mu^r\left(\tilde{a}_r x, \tilde{b}_k\right) = \frac{\Psi\left(\tilde{a}_r x, \tilde{b}_k\right)}{g(\tilde{t}_r)} \tag{18}$$

Where w is a linear classifying function.

Taking the problem (17) into account, $\tilde{\lesseqgtr}$ with the membership function (18) and using the decomposition theorem for fuzzy arrangements, the following is acquired:

$$\mu^r\left(\tilde{a}_r x, \tilde{b}_k\right) \geq \alpha \iff \frac{w\left(\Psi\left(\tilde{a}_r x, \tilde{b}_k\right)\right)}{g(\tilde{t}_r)} \geq \alpha \iff \frac{g\left(\tilde{t}_r(-)\tilde{a}_r x(+)\tilde{b}\right)}{g(\tilde{t}_r)} \geq \alpha$$

$$\iff w(\tilde{t}_r) - w(\tilde{a}_r x) + w(\tilde{b}_r) \geq w(\tilde{t}_r)\alpha \iff w(\tilde{a}_r x) \leq w(\tilde{b}_r(+)\tilde{t}_r(1-\alpha))$$

$$\iff \tilde{a}_r x \leq_w \tilde{b}_r + \tilde{t}_r(1-\alpha)$$

Where $\leq_w$ is the relationship corresponding to w

Therefore, an equivalent model to resolve (17) is as below:

$$Maximum \quad Z = \sum_{k=1}^{n} \tilde{c}_k x_k$$

$$subject\ to \sum_{k=1}^{n} a_{rk} x_k \leq_w \tilde{b}_k + \tilde{t}_r(1-\alpha) \tag{19}$$

$$x_k\, , r \in D, k \in Z, \alpha \in [0,1]$$

Resolving the problem (19), we can use the divergent fuzzy numbers ranking ways in both the constraints and the objective function, or to use ranking methods in the constraints and $\alpha - cuts$ in the objective, which will guide us to acquire different conventional models, that allow us to obtain a fuzzy solution. Specifically in this paper and for illustration effects of the method, we apply a linear ranking function, the first index of yager. Although the approach could be easily adapted to the use of any other index. Thus applying the first index of yager and by considering triangular fuzzy numbers, the DMCLAP problem defined in equation (19) is transformed into the crisp equivalent linear programming problem defined in the equation (20).

$$Maximum \quad z = \sum_{k=1}^{n} \left(c_k + \frac{d_{c_k} - d'_{c_k}}{3} \right) x_k$$

$$Subject\ To \sum_{k=1}^{n} \left(a_{rk} + \frac{d_{a_{rk}} - d'_{a_{rk}}}{3} \right) x_k \leq \left(b_k + \frac{d_{b_r} - d'_{b_r}}{3} \right) + \left(t_r + \frac{d_{t_r} - d'_{t_r}}{3} \right) (1 - \alpha) \tag{20}$$

$x_k \geq 0 \, , r \in D \, , k \in Z, \alpha \in [0,1]$

Where, for instance, $\mathrm{d}_{\mathrm{c_k}}$ and $\mathrm{d}'_{\mathrm{c_k}}$ are the lateral margins (right and left, respectively) of the triangular fuzzy number central point $\mathrm{c_k}$(see Fig. 2)

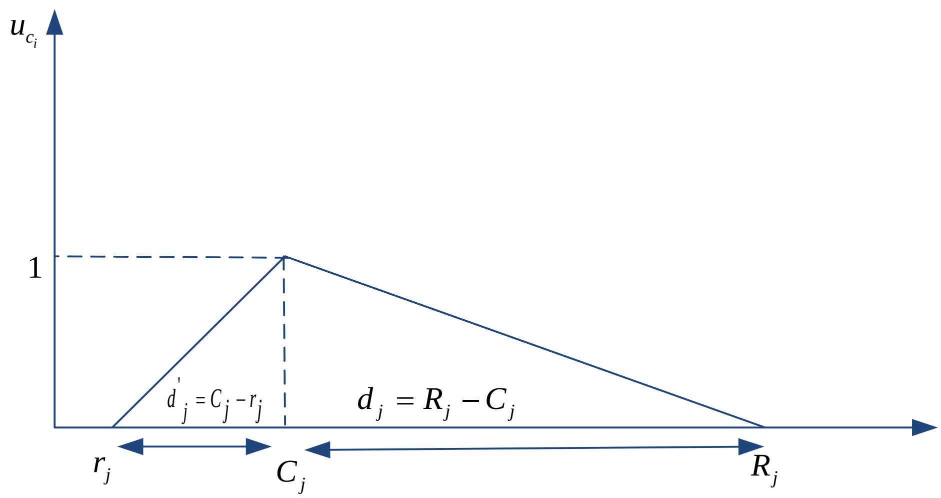

Fig 2. Triangular fuzzy number

Consequently, when applying this approach to the previously defined FDMCLAP model, In this section are presented component of mathematic model and we would obtain an auxiliary crisp DMCLAP model in the following way:

$\boldsymbol{D_{it}}$**:** The central point of the triangular fuzzy number $\boldsymbol{\widetilde{D}_{it}}$

$\boldsymbol{d_{D_{it}}}$**:** The lateral right margins of the triangular fuzzy number of central point$\boldsymbol{\widetilde{D}_{it}}$

$\boldsymbol{d'_{D_{it}}}$**:** The lateral left margins of the triangular fuzzy number of central point $\boldsymbol{\widetilde{D}_{it}}$

$\boldsymbol{C_{itk}}$**:** The central point of the triangular fuzzy number $\boldsymbol{\tilde{\mathrm{C}}_{\mathrm{itk}}}$

$\boldsymbol{d_{C_{itk}}}$**:** The lateral right margins of the triangular fuzzy number of central point $\boldsymbol{\tilde{\mathrm{C}}_{\mathrm{itk}}}$

$\boldsymbol{d'_{C_{itk}}}$**:** The lateral left margins of the triangular fuzzy number of central point $\boldsymbol{\tilde{\mathrm{C}}_{\mathrm{itk}}}$

$\boldsymbol{k}$**:** The central point of the triangular fuzzy number $\boldsymbol{\widetilde{k}}$

$\boldsymbol{d_k}$**:** The lateral right margins of the triangular fuzzy number of central point $\boldsymbol{\widetilde{k}}$

$\boldsymbol{d'_k}$**:** The lateral left margins of the triangular fuzzy number of central point $\boldsymbol{\widetilde{k}}$

$\boldsymbol{p_t^A}$**:** The central point of the triangular fuzzy number $\boldsymbol{\tilde{p}_t^A}$

$\boldsymbol{d_{p_t^A}}$**:** The lateral right margins of the triangular fuzzy number of central point $\boldsymbol{\tilde{p}_t^A}$

$\boldsymbol{d'_{p_t^A}}$: The lateral left margins of the triangular fuzzy number of central point $\boldsymbol{\tilde{p}_t^A}$

$\boldsymbol{p_t^B}$: The central point of the triangular fuzzy number $\boldsymbol{\tilde{p}_t^B}$

$\boldsymbol{d_{p_t^B}}$**:** The lateral right margins of the triangular fuzzy number of central point $\boldsymbol{\tilde{p}_t^B}$

$\boldsymbol{d'_{p_t^B}}$**:** The lateral right margins of the triangular fuzzy number of central point $\boldsymbol{\tilde{p}_t^B}$

$\boldsymbol{t_r}$ **:** The fuzzy number giving the maximum defilement of the **rth** constraint in relations

$$MaxZ = \sum_{\forall i}\sum_{\forall t}\sum_{\forall k}\left[D_{it} + \frac{\left(d_{D_{it}} - d'_{D_{it}}\right)}{3}\right] \times \left[C_{itk} + \frac{\left(d_{C_{itk}} - d'_{C_{itk}}\right)}{3}\right] Z_{itk}^A + \sum_{\forall i}\sum_{\forall t}\left[D_{it} + \frac{\left(d_{D_{it}} - d'_{D_{it}}\right)}{3}\right] \times Z_{it}^B \tag{21}$$

$$Subject\ To$$

$$a_{ijt}x_{jt}^A \geq \sum_{\forall j}\left(k + \frac{(d_k - d'_k)}{3}\right) \times y_{itk}^A \qquad \forall\, i, t, k \tag{22}$$

$$\sum_{\forall k} y_{itk}^A \leq 1 \qquad \forall\, i, t \tag{23}$$

$$y_{it}^B \leq \sum_{\forall k} y_{itk}^A \qquad \forall\, i, t \tag{24}$$

$$y_{it}^B \leq \sum_{\forall f} r_{ift}x_{ft}^B \qquad \forall\, i, t \tag{25}$$

$$\sum_{\forall j} x_{jt}^A \geq \left(p_t^A + \frac{\left(d_{p_t^A} - d'_{p_t^A}\right)}{3}\right) - \left(t_4 + \frac{\left(d_{t_4} - d'_{t_4}\right)}{3}\right) \times (1-\alpha) \qquad \forall\, t \tag{26}$$

$$\sum_{\forall j} x_{jt}^A \leq \left(p_t^A + \frac{\left(d_{p_t^A} - d'_{p_t^A}\right)}{3}\right) + \left(t_5 + \frac{\left(d_{t_5} - d'_{t_5}\right)}{3}\right) \times (1-\alpha) \qquad \forall\, t \tag{27}$$

$$\sum_{\forall j} x_{ft}^B \geq \left(p_t^B + \frac{\left(d_{p_t^B} - d'_{p_t^B}\right)}{3}\right) - \left(t_6 + \frac{\left(d_{t_4} - d'_{t_4}\right)}{3}\right) \times (1-\alpha) \qquad \forall\, t \tag{28}$$

$$\sum_{\forall j} x_{ft}^B \leq \left(p_t^A + \frac{\left(d_{p_t^B} - d'_{p_t^B}\right)}{3}\right) + \left(t_7 + \frac{\left(d_{t_5} - d'_{t_5}\right)}{3}\right) \times (1-\alpha) \qquad \forall\, t \tag{29}$$

$$x_{Ai} + x_{Bi} \leq 1 \qquad \forall\, t \tag{30}$$

$$\sum_{\forall k} Z_{itk}^A + Z_{it}^B \leq 1 \tag{31}$$

$$Z_{itk}^A \leq y_{itk}^A \tag{32}$$

$$Z_{it}^B \leq y_{it}^B \tag{33}$$

$$\mathrm{y}_{\mathrm{it}}^{\mathrm{B}}\ , y_{itk}^A\, , x_{Ai}\ ,\ x_{Bi}\ , Z_{itk}^A\ ,\ Z_{it}^B \in \{0\ \ 1\} \qquad \forall\, i\, , t\ k \tag{34}$$

x_{jt}^{A} , $x_{ft}^{B} \geq 0$ *Integeer* (35)

The non fuzzy constraints (3)-(5), (8) are also included in the model in a similar way.
To solve the problem and according to Eq. (20), $\boldsymbol{\alpha}$ is settled parametrically ($\boldsymbol{\alpha} \in [\mathbf{0}\ ,\mathbf{1}]$) to obtain the value of the objective function for the different levels ($\boldsymbol{\alpha} - \mathbf{cuts}$) of the fuzzy parameters considered in the model. The result is a fuzzy set and the FLP planner has to decide which pair ($\boldsymbol{\alpha}, \boldsymbol{z}$) is most suitable to obtain a crisp solution.

4- The general-optimizer algorithms

4-1- Simulated annealing (SA) algorithm

The SA algorithm was presented by **Metropolice** in 1953 (Metropolis et al., 1953). This algorithm is a random meta-heuristic algorithm to solve combinatorial optimization problems. The main idea of such an algorithm is to select one solution from current solution neighborhood in each step. If the selected neighbor solution improves the objective function, it will be replaced with current solution. Otherwise, a solution between zero and one is replaced by chance. Simulated annealing algorithm is probability of selecting a worse solution for objective function with conformity to the amount of difference between solutions of two previous objective functions. Solutions which have less difference in objective function with current amount of objective function are selected with higher probability, and solutions with larger difference are seldom selected. Therefore, by increasing the number of iterations of this algorithm, the possibility of abandoning local optimum solution increases, on the other hand, along with decreasing probability of solutions with worse objective functions during the time, the algorithm converges in a good local optimum solution due to decreasing the degree of temperature T.

4-1-1- Parameters of SA algorithm

Annealing program has a great effect on the converging of the gradual annealing algorithm. Annealing program identifies the way of controlling algorithm temperature. In an annealing program, some factors need to be identified such as initial temperature, length of Markov chain, rule of temperature decrease, rule of stop. These factors must be adjusted according to investigated problem conditions, and it is due to lack of theoretical results about designing mentioned parameters. In this adjustment, quality of results and algorithm calculation time is considered. Suitable design of mentioned parameters has a substantial impact on the algorithm. In the other word, this is a deficiency in this algorithm comparing other meta-heuristic algorithms. To investigate it, there is an analysis about mentioned factors.

The use of a high initial temperature makes the algorithm have a manner like random search and using lower temperature causes algorithm to be changed to a local search algorithm. In gradual annealing algorithm, while identifying initial temperature, it must be identified in a way that a balance between two mentioned cases occurs. Length of Markov chain to achieve sustained position in any temperature, and enough number of transports (movements) need to be considered. Theoretical studies suggest that the number of iterations need to be based on an exponential function of the problem. Applying this strategy is hard in practice. Generally, the number of iterations must be identified according to the size of the problem and neighborhood. The rule of temperature decrease: Generally, there is a relationship between result quality and annealing speed. The degree temperature has always been positive and when the number of iterations goes up to zero, temperature degree goes toward zero, too. By observing such principals, there are several rules for temperature decrease such as linear rule, Geometric rule, logarithmic rule, slow linear decrease rule, non-uniform rule, dynamic rule, temperature-related geometric rule.

4-1-2 -Steps of simulated annealing algorithm

1. Creating a random initial response and evaluating it.

2. Considering later response as the best answer.

3. Adjusting initial temperature T=T0. Conducting steps 5-8 in determined times.

4. Producing random response in the neighborhood of current response and evaluating it.

5. Accepting new response if it is better (If $\Delta \mathbf{f}(\mathbf{x}) \geq \mathbf{0}$ then $\mathbf{X}^{\mathbf{new}}$ is replaced by X with probability $\mathbf{p} = \mathbf{e}^{\frac{\Delta f}{T}}$

6. Updating the best found solution.

7. Temperature decrease and return to step 4 if needed, otherwise end of procedure.

4-2- Multiple simulated annealing (MSA) algorithm

Steps of simulated annealing algorithm based on population are as following and its figure is as figure3.

1. Producing initial population and its evaluation.

2. Identifying the best found solution.

3. Adjusting initial temperature T=T0.

4. Conducting steps 5-8 in determined times.

5. For each of population members, a specific number of neighbors is created and evaluated.

6. Arranging the members of neighbors population, and selecting the best one to add to the quantity of the main winners.

7. Each of the current members (main members) is compared with one of the neighbor members according to SA.

8. Updating the best found solution.

9. If the end conditions aren't met, we decrease temperature and begin from 4th step.

10. End.

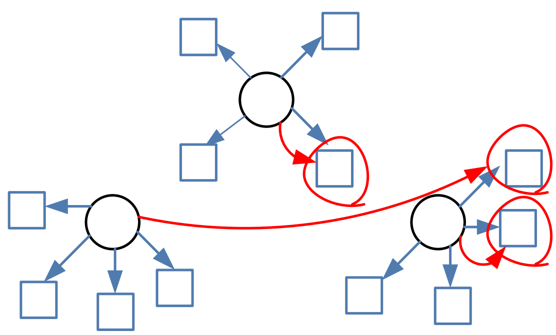

Fig 3. Method of arranging based on SA

4-2-1- Initialize solution for multiple simulated annealing

$$\left[\overbrace{\overbrace{0.1\ \ 0.9}^{p_A}\ \ \overbrace{0.011}^{p_B}}^{t_1} :\ \overbrace{\overbrace{0.42\ \ 0.82}^{p_A}\ \ \overbrace{0.95}^{p_B}}^{t_2} :\ \overbrace{\overbrace{0.09\ \ 0.23}^{p_A}\ \ \overbrace{0.81}^{p_B}}^{t_3}\right]$$

The number of potential points to establish facilities of type A is equal to j in each period and also the number of potential points to establish facilities of type B is equal to f. The number of type A facilities to locate is PA in each period and the number of type B facilities in each period is equal to PB. To locate facilities of **A** type, intervals between zero and one are divided into j equal parts. The numbers that are located in the intervals are allocated to potential corresponding points. For location of type B, we will do the same.

$$\left[\begin{matrix} \overbrace{\overbrace{\mathbf{1\ 0\ 1}}^{\mathbf{j}}\ \overbrace{\mathbf{1\ 0}}^{\mathbf{f}}}^{\mathbf{t_1}} : \overbrace{\overbrace{\mathbf{0\ 1\ 1}}^{\mathbf{j}}\ \overbrace{\mathbf{0\ 1}}^{\mathbf{f}}}^{\mathbf{t_2}} : \overbrace{\overbrace{\mathbf{2\ 0\ 0}}^{\mathbf{j}}\ \overbrace{\mathbf{0\ 1}}^{\mathbf{f}}}^{\mathbf{t_3}} \\ \\ \end{matrix}\right]$$

4-2-2- The method of neighborhood development

In this method, each particle selects two random numbers from chromosome and reverses all the numbers between these two numbers and creates the neighborhood solution. The reason for creating neighborhood is that all neighborhoods (possible solutions) are easily accessible. The way to create neighborhood is as follow:

$$1 \ldots\ldots, i_1 - 1, i_1, i_1 + 1, \ldots\ldots\ldots i_2 - 1,\ i_2\ , i_2 + 1\ , \ldots\ldots.\ n$$

$$\underbrace{1 \ldots\ldots, i_1 - 1}_{1:\, i_1 - 1}\ , \underbrace{i_2, i_2 - 1, \ldots\ldots\ldots i_1 + 1,\ i_1}_{i_2:\, i_1}, \underbrace{i_2 + 1\ , \ldots\ldots.\ n}_{i_2 + 1:n}$$

4-2-3- Parameter setting for multiple simulated annealing

In this section MSA algorithm parameters and operators are identified. Regarding large number of existing parameters in simulated annealing algorithm based on population, the process of finding suitable combination for parameters which improves has great utility. Due to large number of parameters using complete factorial, it is inefficient. To remove this fault, the Taguchi method is usually used. In Taguchi method, we use orthogonal arrays to study a lot of decision variables using small number of experiments. In this section to adjust parameters, a problem in large scale is selected and by applying the main effects diagram, six parameters of MSA algorithm are adjusted for it.

Regarding the diagram in figure3, the parameters of iteration number in third level, subset iteration in the first level, alpha in the first level, initial temperature in the second level, the number of population members in second level, the number of neighborhood in first level have the best optimum value.

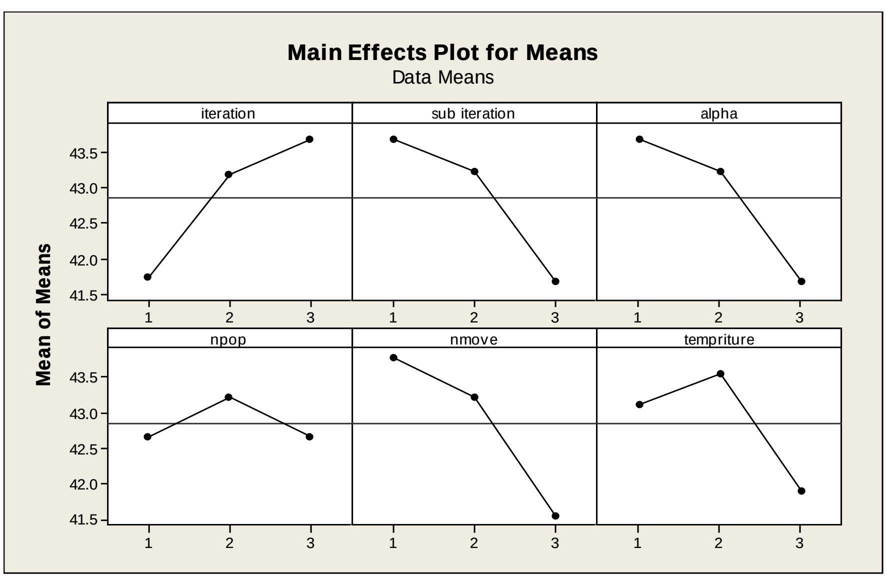

Fig 3. Diagram of great effects for large scale problems

Table 2. Optimum levels for parameters of MSA algorithm

Parameters	Level	Optimum level
Subset Iteration Number	{10,20,30}	10
Iteration Number	{50,60,70}	70
Alpha	{0.85,0.90,0.95}	0.85
Initial Temperature	{30,40,50}	40
Number of Population Members	{4,5,6}	5
Number of Neighborhood	{5,6,7}	5

6-Numerical example

In this section we study the numerical example in different scales. At first, to correct fuzzy dynamic modeling procedure, the presented dynamic procedure is solved in crisp situations by means of GAMS software and is compared. Different examples are solved based on assumed data for more investigations. In this section there will be some comparisons between solution of presented models in fuzzy dynamic and decisive by means of both GAMS and meta-heuristic algorithms to confirm the accuracy of coding by using such algorithms. It should be noted that due to model complexity and large number of constraints and variables, it can’t be solved by using GAMS definite method, so it is necessary to apply meta-heuristic methods to solve the problem. The model Presented with the proposed algorithm is being coded by MATLAB10a software. They are also solved by GAMS in a corei3 CPU computer. 3.1 GHz. and 4GB RAM.

In table number 3, the location model for ambulance services is solved in both states, fuzzy dynamic state by GAMS software and by simulated annealing algorithm SA. Results represent in fuzzy dynamic model case quality of answers increases in a high rate along with increase in period numbers, and also duration of solving the model increases along with increase of time periods. In table 4, ambulance location model has been solved in both definite case and fuzzy dynamic case by means of GAMS and popular based simulated annealing algorithm (MSA). Results show in fuzzy dynamic state quality of answers increases along with number of periods of solving. In table5 the fuzzy dynamic model is solved by two algorithms, simulated annealing SA and popular based simulated algorithm (MSA). Results show MSA is better than SA regarding answer qualities. But regarding time, the SA has preference compare to MSA.

Table 3. Results of the model in deterministic, dynamic and fuzzy multi period state

α-cut and Models		*Number of period*	*Number of (A) facility*	*Number of (B) facility*	*Objective Function (GAMS)*	*CPU Time(GAMS)*	*Objective Function (SA)*	*CPU Time(SA)*	*Objective Function (MSA)*	*CPU Time(MSA)*
Deterministic Model		-	20	5	57	210.78	54	25.11	56	28.13
Dynamic Model		1	20	5	57	210.78	54	25.11	56	28.13
Dynamic Fuzzy Model	**α=0**	1	20	5	76	309.76	71	40.98	76	46.13
	α=0.1				74	300.37	71	39.22	75	45.21
	α=0.2				71	291.23	70	35.23	70	42.98
	α=0.3				69	288.02	66	33.65	67	40.69
	α=0.4				69	287.33	65	32.31	67	39.34
	α=0.5				67	279.37	62	30.45	65	37.23
	α=0.6				65	276.32	64	28.92	65	35.87
	α=0.7				64	240.21	60	27.36	64	33.98
	α=0.8				63	235.11	60	26.21	61	30.34
	α=0.9				56	211.96	52	25.36	55	32.25
	α=1				55	210.78	51	25.11	55	30.45
Dynamic Model		2	20	5	72	365.13	69	35.09	71	45.21
Dynamic Fuzzy Model	**α=0**	2	20	5	96	445.32	94	62.32	94	74.45
	α=0.1				91	435.21	90	62.01	90	75.21
	α=0.2				89	421.34	85	59.11	88	65.09
	α=0.3				88	418.56	84	55.04	87	64.12
	α=0.4				86	410.65	84	50.11	86	61.65
	α=0.5				82	408.32	80	50.58	80	58.23
	α=0.6				78	397.54	74	45.22	76	56.98
	α=0.7				75	390.03	71	40.98	74	56.01
	α=0.8				74	379.98	70	38.36	74	48.23
	α=0.9				73	370.00	69	36.96	72	45.65
	α=1				73	365.13	68	35.09	72	40.75
Dynamic Model		3	20	5	85	443.76	82	58.32	83	67.23
Dynamic Fuzzy Model	**α=0**	3	20	5	112	565.11	110	85.13	110	93.22
	α=0.1				107	542.08	101	79.09	106	90.23
	α=0.2				101	521.10	98	79.23	100	84.32
	α=0.3				99	512.32	94	71.56	97	80.45
	α=0.4				99	501.03	93	70.10	96	78.98
	α=0.5				96	485.36	90	66.78	94	74.32
	α=0.6				94	496.30	89	65.98	93	71.45
	α=0.7				92	476.85	85	64.98	90	66.21
	α=0.8				89	465.32	85	61.23	88	67.76
	α=0.9				87	450.58	82	60.12	86	65.12
	α=1				85	443.76	81	58.32	84	61.09

7-Conclusions and future studies

In the model presented, ground and air (backup) ambulances for maximum coverage in different periods can be replaced and relocated (facility replacement). Air ambulances are considered as backup ambulances for ground ambulances. In presented model, extent of coverage of one demand point is related to demand amount and the number of facilities which this point has under its coverage. It is also considered that the radius coverage and the demand amount are as fuzzy dynamic states to conform with reality and the allocation of service machines to the stations in different periods. The number of service machines in each period in each station could be variable. The proposed model is preferably applied to locate urgency and fire fighting service stations in emergency occasions (crisis management). Proposed algorithm MSA presented a better solution comparing SA in terms of quality.

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