Journal of Industrial and Systems Engineering
Vol. 10, Special Issue on Scheduling, pp. 38- 57
Autumn (November) 2017

Solving a nurse rostering problem considering nurses preferences by graph theory approach

Masoud Rabbani[1*], Mehrdad Niyazi[1]

[1]*School of industrial engineering, University of Tehran, Tehran, Iran*

mrabani@ut.ac.ir, nyazi.mehrdad@ut.ac.ir

Abstract

Nurse Rostering Problem (NRP) or the Nurse Scheduling Problem (NSP) is a complex scheduling problem that affects hospital personnel on a daily basis all over the world and is known to be NP-hard. The problem is to decide which members of a team of nurses should be on duty at any time, during a rostering period of, typically, one month. It is very important to efficiently utilize time and effort, to evenly balance the workload among people and to attempt to satisfy personnel preferences. With demand ever fluctuating, designing a timetable to define a work schedule for each nurse is not an easy task. A NRP deals with a very high number of constraints. A lot of big healthcare organizations around the world still construct nurses' duty roster manually. Many optimization algorithms have been proposed to solve NRPs such as exact algorithms and (Meta) heuristic algorithms. In this paper we propose an approach that uses the graph theory concept to solve the problem. We use the graph coloring and bipartite graph concept. In our approach we first formulize the problem and solve it with exact algorithm and then by using the graph concept, the solution is improved. Finally by results obtained from the graph approaches the final timetable is available. In order to validate the proposed approach some problems with different scales are solved. We solved the problems for 30, 40, 45 and 50 nurses. In all problems the proposed approach is efficient and for instance the relationship between the nurses is presented.
Keywords: Nurse rostering, graph theory, graph coloring, bipartite graph, DSATUR algorithm

1- Introduction

Staff scheduling and rostering problems, with application in several application areas, from transportation systems to hospitals, have been widely addressed by researchers (Rocha et al., 2012). Personnel scheduling, or rostering, is the process of constructing work timetables for its staff so that an organization can satisfy the demand for its goods or services. The scheduling of hospital personnel is particularly challenging because of different staffing needs on different days and shifts. Nurse Rostering Problem (NRP) or the Nurse Scheduling Problem (NSP) is a complex scheduling problem that affects hospital personnel on a daily basis all over the world (Mueller and McCloskey, 1990; Oldenkamp, 1992).

*Corresponding author

The problem is to decide which members of a team of nurses should be on duty at any time, during a rostering period of, typically, one month (Glass and Knight, 2010). A feasible schedule satisfies a set of constraints that reflect the local regulations, human resources' policies, nurses' preferences, and any context specific requirement. Unlike many other organizations, healthcare institutions work around the clock. It is very important to efficiently utilize time and effort, to evenly balance the workload among people and to attempt to satisfy personnel preferences. A high quality roster can lead to a more contented and thus more effective workforce (Mueller and McCloskey, 1990; Oldenkamp, 1992). Irregular shift work has an effect on the nurses' well-being and job satisfaction and can affect the personal life of staff nurses, increase job dissatisfaction, and thus result in high staff turnover (Hadwan et al., 2013).

The nurses' time tabling problem is a decision problem. Generally, it is transformed into an optimization problem whose objective function assesses the quality or attractiveness of the time table. Examples of objective functions are minimizing the number of nurses and minimizing total overtime. Finding feasible solutions to this NP hard combinatorial problem (Osogami and Imai, 2000; Winstanley, 2004)) is usually a challenge (Ernst et al., 2004). Meeting this challenge is further complicated when the objective is satisfying nurses' preferences, balancing their workload, or ensuring equitable schedules (Ernst et al., 2004).This optimization problem is tackled using a multitude of approaches. Modeling approaches vary from mathematical programming to expert systems and artificial intelligence. Solution approaches vary from exact approaches to meta-heuristics. The size and complexity of the problem dictate these choices. However, there is no universal model, solution procedure, or DSS applicable to every health care unit. In fact, the specific labor laws of each country, the different circumstances defining the resources and demand of each hospital, the scarcity/availability of nurses, and the nurses' credentials make nurses' timetabling context dependent. The use of graph theory is an interesting approach in many problems particularly in rostering and scheduling problems. Graph coloring has been studied extensively for the past decades and there are several interesting practical and feasible problems that can be modeled using graph coloring. The surge in recent times has resulted in countless real-life problem applications, which include; time tabling scheduling problems, frequency assignment, register allocation. Also bipartite graph concept is a very good and practical approach for assigning and scheduling problems. In order to validate the proposed approach some problems with different scales are solved. We solved the problems for 30, 40, 45 and 50 nurses. In all problems the proposed approach is efficient and for instance the relationship between the nurses is presented.

This paper includes the following structure: the related literature is reviewed in the section 2. Section 3 presents our proposed approach. In section 4computational results are presented. Last but not least is the conclusion of this study in section 5.

2- Literature review

Cheang et al. (2003), Burke et al. (2004), and Ernst et al. (2004) give a detailed overview of the abundant literature on nurse scheduling. Cheang et al. (2003) classify some of this literature per problem type and area of application, enumerate and model the most frequent constraints and objective functions, and specify general frameworks of solution approaches. Blochlifer (2004) offers a tutorial on modeling the subject. Brucker et al. (2011) classify personnel scheduling problems, present a model (among others) for the nurses' case and discuss its complexity. Mohan (2008) considers nurse scheduling as a special case of scheduling days off, shifts, or tours. Levner et al. (2010) explain that nurses' schedules might be cyclic or periodic (2010). Aickelin and Dowsland (2003) apply a genetic algorithm to establish weekly schedules for up to 30 ward nurses of a hospital in the United Kingdom. The schedules cover the ward demand requirements while maximizing nurses' preferences and ensuring the even distribution of unpopular shifts. Bard and Purnomo (2005) use column generation to maximize the satisfaction of the preferences of twenty to one hundred nurses while guaranteeing demand satisfaction per shift per skill, non violation of nurse contracted working hours, rest days, minimal rest time between consecutive shifts, and fulfillment of union and hospitals rules. They model their nurse scheduling problem as a set covering one with time-related constraints, and obtain an upper bound. Gutjahr and Rauner (2007) apply ant colony optimization (ACO) to generate a minimal cost dynamic schedule of a fixed number of pool nurses that serve the fifteen hospitals of Vienna. The schedule has a lead time of two weeks from the time the demand for nurses becomes known. The generated schedule accounts for the nurses' expressed preferences. Tsai and Li (2009) tackle the scheduling problem of the fifteen nurses of an otolaryngology hospital in Tainan, Taiwan, using a two-stage genetic algorithm (GA).Topaloglu (2009) models the case of emergency medical residents of a local Turkish hospital as a goal program, and solves it using an off-the-shelf optimizer for one-month periods. Carrasco (2010) studies the problem of assigning the twenty to thirty physicians of the pediatrics department of a hospital in Spain to guard shifts over a one-year planning horizon. Physicians like to have their shifts uniformly distributed over the planning horizon. The physician's wishes are subject to coverage constraints and workload restrictions. Millar and Kiragu (1992) used a network model, which is in fact a shortest-path problem with side constraints, for cyclic and non-cyclic nurse scheduling with two work shift types. Jaumard et al. (1998) presented a generalized 0–1 column generation model with a resource constrained shortest path auxiliary problem for nurse rostering. Burns (1978) used a cyclic model to study the case of 10 working days in a 14-day period with variable demands and alternate weekend's off. Berrada et al. (1996) formulated the NRP as a multi-objective MP model. In this model, hard constraints must be satisfied, while soft constraints are treated as goals to be reached. The overall objective is to get as close as possible to these goals. Many heuristic approaches were straightforward automation of manual practices, which have been widely studied and documented in nursing administration literature Howell (1966) and Marchionno (1987) described the necessary steps to develop cyclic schedules. Smith and Wiggins (1977) presented a three-phase scheduling algorithm which first collects a summary of rostering data, then generates tentative shift schedules indicating shortages and averages in each unit, and finally manually adjusts the tentative shift schedules to produce final schedules. Dowsland (1998) used Tabu Search (TS) with strategic oscillation to tackle the NRP in a large hospital. The objective is to ensure enough nurses are on duty at all times while taking account of individual preferences and requests for days off. Jan et al. (2000) used GA for a problem with multiple criteria where the concept of a Pareto optimality scheme is used for the evaluation of the multi-criteria objective function. Aickelin and Dowsl and (2001) developed a GA approach to solve an NRP. There has been some use of simulated annealing techniques for the NRP. For example, Thompson (1996) presented a SA heuristic for shift-scheduling using non-continuously available employees.

Kumara and Perera (2011) provided effective method for solving NRP by satisfying the nurses, patients and hospital requirements, by using graph coloring. They argued that using graph coloring could reduce the complexity of the NSP problem. The second study is done by Amponsah et al. (2011) for a hospital in Ghana. They constructed a conflict graphs for the nurses. The aim of their study was to satisfy the various applications of graph coloring and to come out with a model in solving the schedule problem for nurses using graph coloring techniques. They argued that the proposed model can be used multiple times, incorporating different sets of preferential conditions each time, until finally arriving at a shift table that is ultimately most suitable.

3- The proposed approach

In this section we propose our approach to solve the NRP. Firstly we present the hard and soft constraints of the problem and then by introducing the parameters and variables we formulate the problem as a mathematical model. Then the mathematical model is solved by using the GAMS and after that by using the graph coloring and bipartite concepts we improve the solution. Our approach is illustrated in figure 1.

3-1- Model description

Many public health centers operate around the clock based on three shifts per day. Shift 1 or the morning duty is a seven-hour shift spanning from 7:00 AM to 2:00 PM. Shift 2 or the afternoon duty is an eight-hour shift spanning from 2:00 PM to 10:00 PM. Shift 3 or the night duty is a nine-hour shift spanning from 10:00 PM to 7:00 AM. Therefore, a_t, the number of working hours per shift t, $t = 1, 2, 3$, is defined as: $a_1 = 7$, $a_2 = 8$, and $a_3 = 9$.A center has a set of n permanent nurses, but can outsource nurses from other health care units. The objective of the head nurse, who generates the nurses' weekly timetables manually, is to reduce the number of outsourced nurses.

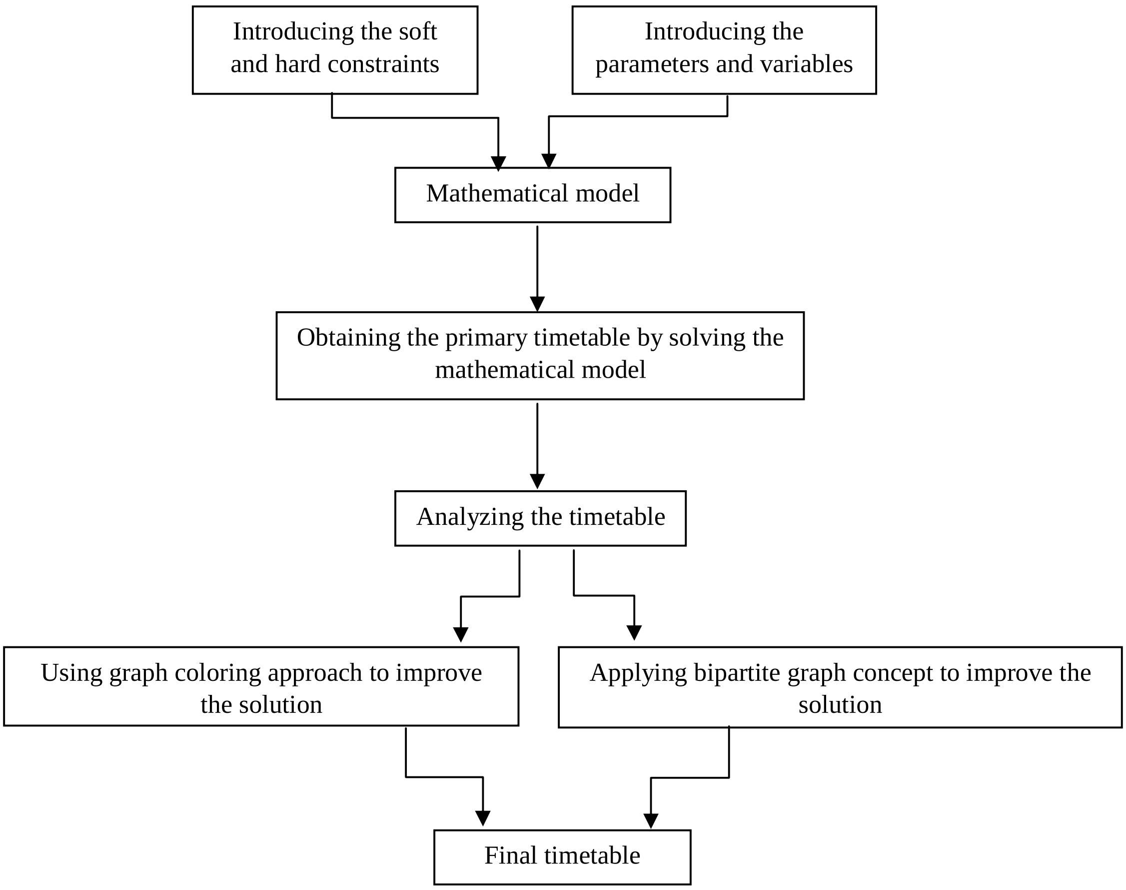

Fig 1. The structure of the proposed approach

The unit has a nurses' request book, where nurses can request to work or rest on specific shifts of the week. Currently, the head nurse ignores these requests but allows nurses to trade their shifts. Let $f_i = 1$ if nurse i, $i = 1, ..., n$, is female and 0 otherwise. A nurse is classified as a head nurse ($s_i = 1$), a staff nurse ($s_i = 2$), or a nurse in training ($s_i = 3$). This classification is important since it determines the timetabling rules in vigor for each nurse. Herein, there is three head nurse, denoted nurse $i = 1, 2, 3$ and four nurse in training denoted nurse $i = 21, 22, 23$ and 24.The goal is to design a timetable for the n nurses of the health unit for a review period of n_w weeks.

The timetable should minimize the number of outsourced nurses during this period while satisfying all requirements of the health unit and accounting for nurses' special requests. These constraints follow.

- There must be n_{jt} nurses for shift t, t = 1, 2, 3, of day j, j = 1, ..., 7.
- The weekly workload of a nurse must be 45 hours.
- Except for the head nurse, a nurse should work at most one shift per day.
- Every staff nurse should work five night shifts during one of the n_w weeks.
- If not assigned five night shifts, a staff nurse should work at least two and at most three afternoon shifts per week.
- No nurse works a morning shift after two consecutive afternoon shifts.
- Each nurse gets one rest day per week.
- A nurse who works five night shifts in a given week gets an extra rest day during that week.
- There must be at least $\underline{nf}_{jt}$ female and $\underline{nm}_{jt}$ male nurses per shift t, t = 1, 2, 3, of day j, j = 1, ..., 7.

3-2- The mathematical programming model

This nurses' time tabling problem can be modeled as a mixed integer program with four decisions variables:

- x_{ijtw} = 1 if nurse i, i = 1, ... , n, works on shift t, t = 1, 2, 3, of day j, j = 1, ... , 7, of week w, w = 1, ... , n_w, and 0 otherwise;
- y_{iw} = 1 if nurse i, i = 1, ... , n, has a night shift during week w, w = 1, ... , n_w, and 0 otherwise;
- d_{ijw} = 1 if nurse i, i = 1, ... , n, has day j, j = 1, ... , 7, off during week w, w = 1, ... , n_w, and 0 otherwise; and
- s_{gjtw} is the number of outsourced nurses of gender g, g = 0 for male and g = 1 for female, during shift t, t = 1, 2, 3, of day j, j = 1, ... , 7, of week w, w = 1, ... , n_w.

The first three types of decision variables are binary whereas the last one is integer. Using these decision variables, we model the problem as follows:

$$\text{Min} \sum_{g=0}^{1} \sum_{j=1}^{7} \sum_{t=1}^{3} \sum_{w=1}^{n_w} s_{gjtw} \tag{1}$$

$$\sum_{i=1}^{n} x_{ijtw} + \sum_{g=0}^{1} s_{gjtw} \geq n_{jt} \qquad j = 1, \dots, 7;\; t = 1, 2, 3;\; w = 1, \dots, n_w \tag{2}$$

$$\sum_{t=1}^{3} \sum_{j=1}^{7} a_t x_{ijtw} = 45 \qquad i = 1, \dots, n;\; w = 1, \dots, n_w \tag{3}$$

$$\sum_{t=1}^{3} x_{ijtw} \leq 1 \qquad i = 1, \dots, n \text{ and } s_i \neq 1,2,3;\; j = 1, \dots, 7;\; w = 1, \dots, n_w \tag{4}$$

$$\sum_{w=1}^{n_w} y_{iw} = 1 \qquad i = 1, \dots, n \; and \; s_i = 2 \qquad (5a)$$

$$\sum_{j=1}^{7} x_{ij3w} \geq 5y_{iw} \qquad \begin{array}{l} i = 1, \dots, n \; and \; s_i = 2 \\ w = 1, \dots, n_w \end{array} \qquad (5b)$$

$$\sum_{j=1}^{7} x_{ij3w} \leq 5 \qquad \begin{array}{l} i = 1, \dots, n \; and \; s_i = 2 \\ w = 1, \dots, n_w \end{array} \qquad (5c)$$

$$\sum_{j=1}^{7} x_{ij2w} \geq 2(1 - y_{iw}) \qquad \begin{array}{l} i = 1, \dots, n \; and \; s_i = 2 \\ w = 1, \dots, n_w \end{array} \qquad (6a)$$

$$\sum_{j=1}^{7} x_{ij2w} \leq 3(1 - y_{iw}) \qquad \begin{array}{l} i = 1, \dots, n \; and \; s_i = 2 \\ w = 1, \dots, n_w \end{array} \qquad (6b)$$

$$x_{ij2w} + x_{i(j+1)2w} + x_{i(j+2)1w} \leq 2 \qquad \begin{array}{l} i = 1, \dots, n \\ j = 1, \dots, 5 \\ w = 1, \dots, n_w \end{array} \qquad (7a)$$

$$x_{i62w} + x_{i72w} + x_{i11(w+1)} \leq 2 \qquad \begin{array}{l} i = 1, \dots, n \\ w = 1, \dots, n_w \end{array} \qquad (7b)$$

$$x_{i72w} + x_{i12(w+1)} + x_{i21(w+1)} \leq 2 \qquad \begin{array}{l} i = 1, \dots, n \\ w = 1, \dots, n_w \end{array} \qquad (7c)$$

$$\sum_{t=1}^{3} x_{ijtw} = 1 - d_{ijw} \qquad \begin{array}{l} i = 1, \dots, n \\ j = 1, \dots, 7 \\ w = 1, \dots, n_w \end{array} \qquad (8)$$

$$\sum_{j=1}^{7} d_{ijw} = 1 + y_{iw} \qquad \begin{array}{l} i = 1, \dots, n \; and \; s_i \neq 1 \\ w = 1, \dots, n_w \end{array} \qquad (9)$$

$$\sum_{i=1}^{n} f_i x_{ijtw} + s_{1jtw} \geq \underline{nf}_{jt} \qquad \begin{array}{l} j = 1, \dots, 7 \\ t = 1, 2, 3 \\ w = 1, \dots, n_w \end{array} \tag{10a}$$

$$\sum_{i=1}^{n} (1 - f_i) x_{ijtw} + s_{0jtw} \geq \underline{nm}_{jt} \qquad \begin{array}{l} j = 1, \dots, 7 \\ t = 1, 2, 3 \\ w = 1, \dots, n_w \end{array} \tag{10b}$$

Equation (1) indicates that the objective of a health care unit is to minimize the number of outsourced nurses. Equation (2) states that the number of nurses available during shift *t* of day *j* of week *w* must be at least n_{jt}, where this total is the sum of the number of nurses assigned to shift t and the number of outsourced nurses. Equation (3) specify that a nurse *i*, *i* = 1, ... , *n*, should work 45 h during a week *w*. Equation (4) stipulates that any nurse except the head nurse works at most one shift per day. Equation (5a) requires every staff nurse to take an all-week night duty for one of the n_w weeks whereas equations (5b) and (5c) restrict the number of night shifts during the all-night shifts week to five. Equations (6a) and (6b) force every staff nurse who is not assigned an all night shifts week to work at least two and at most three afternoon shifts per week. Equations (7a) – (7c) guarantee that a nurse does not work a morning shift after two consecutive afternoon shifts. Equation (8) ensures that a nurse does not work during his/her day off. Equation (9) fixes the total number of rest days per week per nurse to one day, unless s/he has worked an all night-shift week; in which case, she gets an extra day of rest. Equations (10a) and (10b) set the minimal number of female and male nurses to $\underline{nf}_{jt}$ and $\underline{nm}_{jt}$, respectively.

To account for nurses' requests, we append the pertinent constraints to the model as follows. If nurse i'wishes to work on shift $t'$of day $j'$of week w', then $x_{i'j't'w'}$ is set to 1. If on the other hand, a nurse i'does not wish to work on shift $t'$of day j' of weekw', then$x_{i'j't'w'} = 0$.Finally, if nurse i'wishes to have day $j'$of week w'off, then$\sum_{t=1}^{3} x_{i'j'tw'} = 0$. This is equivalent to fixing the values of a subset of variables.

4- Computational results

Now we solve the above model for n=24 nurses and after that by using the graph coloring and bipartite graph approaches we improve the results and take into account the relation between nurses.

The minimal number of nurses per shift of day is shown in Table 1 as follows:

Table 1. Minimal number of female and male nurses per shift per day

t	*j* = 1, …, 5		*j* = 6, 7	
	$\underline{nm}_{jt}$	$\underline{nf}_{jt}$	$\underline{nm}_{jt}$	$\underline{nf}_{jt}$
1	6	2	8	2
2	5	2	8	2
3	2	1	2	1

In table 2 the results of the mathematical model which is obtain by using CPLEX solver in GAMS is shown. Table 2shows the timetable of the nurses for the 4 week period. Each cell indicates the shift *t* of nurse *i*, *i* = 1,... , 24, for day *j* of week *w*, where *t* = 1, 2, 3 denotes the morning, afternoon, and night shifts, respectively, whereas an empty cell indicates that the nurse is off-duty. The resulting timetable satisfies all the constraints but in this problem there are some issues that should be considered. In our problem the relationship between the nurses are so important. The hospital policy is to avoid of assigning the nurses with bad relationship in one shift and hospital management is interested to assign the nurses with good relationship to same shifts. The relationship between nurses is categorized by "good", "neutral" and "bad". The relationship between the nurses is shown in table 3. The nurses 21-24 are not in table 2 because these nurses are the nurses in training and they can't have the bad relationship with other nurses and they have to assign to shifts which the head nurses are assigned. In the next step we show how to assign these nurses by the bipartite graph.

Now we analyze the results obtained by the mathematical model. We check the shifts of the nurses with bad relationship and if they are assigned to different shift there is no problem; but if they are in same shift we use the graph coloring approach to assign them to other shifts. After that we check the required number of nurses for the shifts that nurses with bad relationship are assigned to; if this limitation is satisfied there is no problem but if there is shortage in some shift there are two possible job: 1) outsource nurses from other healthcare units2) assign the off-nurses to these shifts.

Now we check the shifts of the nurses with bad relationship. After that with the DSATUR Algorithm of graph coloring we calculate the coloring number that is equal to the number shifts for the nurses with bad relationship. The algorithm is as follows:

4-1- DSATUR algorithm

Let G be a simple graph and C a partial coloration of G vertices. We define the saturation degree of a vertex as the number of different colors to which it is adjacent (colored vertices).
This algorithm as called DSATUR because it uses saturation degree and the steps is as follows and Pseudo code of the algorithm is shown below.

1. Arrange the vertices by decreasing order of degrees.

2. Start with the node that has the maximum degree. Color the current node with the lowest available color.

3. Select the next node by selecting the node with the maximum degree of saturation. This means that you have to select the node that has the most number of unique neighboring colors. In case of a tie, use the node with the largest degree.

4. Color the chosen vertex with the least possible (lowest numbered) color.

5. If all the vertices are colored, stop. Otherwise, return to 3.

In our problem we first use the nurses with bad relationship as nodes of the graph and if they are in same shifts we draw an edge between the corresponding nodes. For example the corresponding graph to Tuesday of week 2 is shown in figure 2. As it shown in this figure, nurses 5, 9, 16, 17 and 20 are linked together because they are in same shift (shift 1). Now we color the vertices by the DSATUR algorithm. If the colors are less than three we can assign the nurses in different 3 shifts; but if the colors are greater and equal to four they can't be assigned to shifts of a day and they should be assign to other days or they should be off-day in that day. When we can't assign the nurses to shifts in one day we change the nodes to the days (days are the nodes of the graph) and assign the nurses to different days and after that again we use the first approach (nurses are the nodes) to assign the nurses to the shifts of a day. For example nurse 9 and 19 is assigned to shift 2 of day 1 of week 1 and these nurses have bad relationship to each other so they should not be assign to same shift. In this case by graph coloring we can assign the nurse 19 to shift 3 without any problem. Nurses 5, 12 and 19 are assigned to shift 1 of day 3 of week 4 and in this case we can't assign these 3 nurses to different shift in same day, therefore we assign two of these nurses to other days or assign the day-off to them. In this cases we try to assign the nurses to the shift type that they can't assign to; it means that for example nurse 12 can be assigned to shift 3 of day 1 of week 4 (it isn't forbidden because there is no nurse with bad relationship in this shift) and nurse 19 should be assign today-off because there is no proper shift for this nurse.

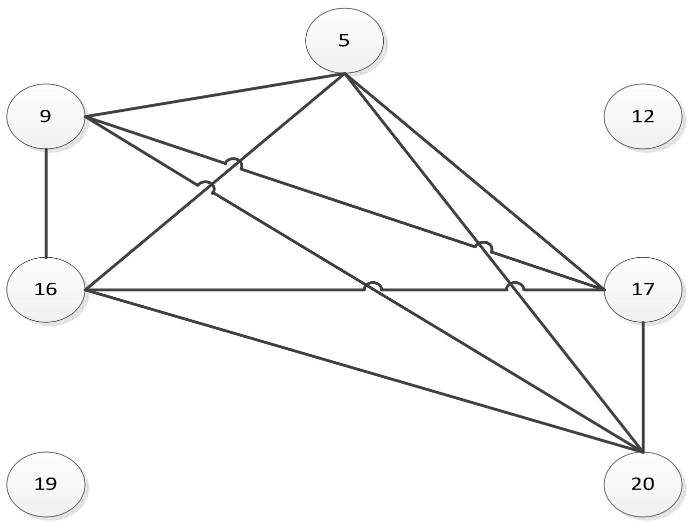

Fig 2. Graph for the nurses with bad relationship

The pseudo code of DSATUR algorithm is as follows:

```
function[coloring] = dsatur(V,E)
n = size(V, 1);
coloring = zeros(n, 1);
available colors = 1;
fori = 1 to n
v=i;
Degrees(i, 1)=size([E(find(E(:, 1) == v), 2); E(find(E(:, 2) == v), 1)], 1);
end
Degrees of saturation = zeros(n, 1);
fori = 1 to n
ifi == 1
[value index] = max(Degrees);
v = index(1);
coloring(v) = 1;
assigned color v = 1;
else
Uncolored = find(coloring == 0);
index temp=find(Degrees of saturation(Uncolored)==max(Degress of saturation(Uncolored)));
```

```
index = Uncolored(index temp);
if size(index, 1) > 1
[value1 index1] = max(Degrees(index));
v = index(index1);
else
v = index;
end
neighbors=[E(find(E(:, 1) == v, 2); E(find(E(:, 2) == v), 1)];
for j=1 to available colors
if size(find(coloring(neighbors) == j), 1) == 0
coloring(v) = j;
assigned color v = j;
break;
end
end
if coloring(v) == 0
available colors = available colors + 1;
coloring(v) = available colors;
assigned color v = available colors;
end
end
neighbors v = [E(find(E(:, 1) == v), 2); E(find(E(:, 2) == v), 1)];
for j = 1 to size(neighbors v, 1)
u = neighbors v(j);
neighbors u = [E(find(:, 1) == u), 2); E(find(E(:, 2) == u), 1)];
if size(find(coloring(neighbors u) == assigned color v), 1)==1
Degress of saturation(u, 1) = Degress of saturation(u, 1) + 1;
end
end
end
```

When we use this algorithm, nurse shortage may occur in some shifts and management policy is that nurses with bad relationship should not assign to same shifts and in this case we outsource the nurses from other healthcare units.

After using this algorithm the nurse timetable is obtained which is shown in table 4. In this table for some nurses the previews shifts are changed and * denotes that in this shift outsourcing should be done or off-day nurses should be assigned to these shifts and it depends on the off-day nurses to accept the task or not. For this purpose we propose the off-day Table for the nurses (variable d_{ijw}) and according to this table and nurses decision we can assign nurses to shifts that need nurses (Table 5).

For example nurse 2 is off in day 2 of week 1 and she/he is interested in work in this day. Then this nurse is assigned to this shift and therefore there is no need to outsource nurse from other healthcare units. For this purpose the head nurse ask the nurses interested in working in day-off or not and according to this the final timetable can be obtained.

4-2- Bipartite graph concept

Finally by bipartite graph concept we assign the "in training nurses" with head nurses to the same shifts. In fact if they are not assigned in same shift in the initial timetable the bipartite graph help us to assign these nurses in same shifts. The steps of the algorithm are as follows:

1) Draw two lines of dots. You need a line for each set, with one dot for each option within the set.
2) Label these dots to prevent confusion later.
3) Join up all of the possible matches.
4) Choose an initial matching by pairing up the sets along these lines. Don't bother trying to find the best way to pair them up unless it's really obvious. This is just to provide something to work within the next steps. If you do manage to match up all the pairs, you're done.
5) Create a path. Make the path around your graph, going from an unmatched dot on the left side, to an unmatched dot on the right side, considering all "paired" lines to "travel" from right to left while all "unpaired" lines travel from left to right, and switching the "direction" or color of each line you go down. This will give a different matching with one more pair than the original.
6) Repeat. Repeat step 5 until a complete matching has been achieved or there is no suitable path, and then remove all unused (red) lines. What is left is your maximal matching.

We use this algorithm to match the head nurses with nurses in training. We implemented this algorithm in C++ language. Finally the output of the two above algorithms will be inputted to table 2.

Table 2. Timetable of the 24 nurses for a four-week review period

Nurse	Week 1							Week 2							Week 3							Week 4						
	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr
1		1	2	2	1	2	1	3	3		3	3	3		1	1		2	2	2	1	2	1		2	1	2	1
2	3		3	3	3	3		2	1		2	1	2	1	2	2		2	1	1	1	2		1	2	1	2	1
3	1	2	1		2	2	1	1	2	2	1	1	2		2	1	2		1	1	2		3	3	3	3		3
4		1	2	1	1	2	2	2	2	2		1	1	1	3	3	3	3			3	2	1		2	1	1	2
5		3	3	3	3	3		1	2		1	2	2	1	1	2	2		2	1	1	1	2	1		2	2	1
6			3	3	3	3	3	2	2	2		1	1	1	1	2		1	2	2	1	1		1	2	1	2	3
7		2	1	1	2	2	1	3	3	3	3		3		2	1	1		2	1	2	1	1		2	2	2	1
8	1		1	2	2	1	2	3	3		3		3	3	1		2	2	2	1	1	1		2	2	2	1	1
9	2	1		2	2	1	1	1	2	1	1	2		2	1	2	1		2	2	1	3	3		3	3	3	
10	2		2	1	2	1	1	2	1	2		2	1	1		3		3	3	3	3	1		2	1	1	2	2
11	2		2	1	1	1	2	1	1	2		2	2	1	1		1	1	2	2	2		3	3		3	3	3
12	3	3			3	3	3	1	1		2	2	2	1	1		2	1	2	2	1		2	1	2	2	1	1
13		2	1	2	1	2	1	3	3	3	3			3	2		1	1	2	1	2	1	2		1	1	2	2
14	1	1		1	2	2	2	1	1		2	2	1	2	3	3		3	3	3		1	1		1	2	2	2
15	2		1	2	1	1	2	2		2	2	1	1	1	3		3	3		3	3	1	2	1		2	1	2
16	1	1		2	1	2	2	2		1	1	2	1	2		2	1	2	1	1	2			3	3	3	3	3
17		1	2	2	2	1	1	2		1	1	2	1	2		3	3		3	3	3	2	1	2		1	1	2
18	1	2		2	1	1	2	1		1	2	1	2	2		1	2	1	1	1	2	3		3	3	3		3
19	2	2	1	1		1	2		3		3	3	3	3	1	2		2	1	2	1		2	1	2	2	1	1
20		3	3	3	3	3			1	1	1	2	2	2	2	1	1		1	2	2	2		2	1	2	1	1
21	1		2	1	2	2	1	1		2	1	1	2	2		3	3	3	3	3		1		2	1	2	1	2
22	3			3	3	3	3	2	1	1		2	1	2	1		2	1	1	2	2	1	1	2	1	2		2
23	3		3	3	3	3		2	2		2	1	1	1		1	2	1	2	1	2	1	2	1		2	1	2
24	1	1		2	1	2	2	3	3	3	3			3		1	2	1	2	1	2	1	2	1		2	1	2

Table 3. Nurses relationship

Nurses with ''bad'' relationship	Nurses with ''neutral'' relationship	Nurses with ''good'' relationship
Nurse 5, 9, 12, 16, 17, 19, 20	Nurse 1, 3, 4, 6, 8, 13, 15	Nurse 2, 7, 10, 11, 14, 18

Table 4. Timetable of the 24 nurses for a four-week review period regarding the relationship between nurses

Nurse	Week 1							Week 2							Week 3							Week 4						
	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr
1		1	2	2	1	2	1	3	3		3	3	3		1	1		2	2	2	1	2	1		2	1	2	1
2	3		3	3	3	3		2	1		2	1	2	1	2	2		2	1	1	1	2		1	2	1	2	1
3	1	2	1		2	2	1	1	2	2	1	1	2		2	1	2		1	1	2		3	3	3	3		3
4		1	2	1	1	2	2	2	2	2		1	1	1	3	3	3	3			3	2	1		2	1	1	2
5		3	*	3	3	*		1	*		1	2	*	1	1	2	*		2	*	*	1	2	*		2	2	1
6			3	3	3	3	3	2	2	2		1	1	1	1	2		1	2	2	1	1		1	2	1	2	3
7		2	1	1	2	2	1	3	3	3	3		3		2	1	1		2	1	2	1	1		2	2	2	1
8	1		1	2	2	1	2	3	3		3		3	3	1		2	2	2	1	1	1		2	2	2	1	1
9		*	2	2	2	1	*	3	2	1	*	3		2	3	*	1		3	2	*	3	3		3	3	*	
10	2		2	1	2	1	1	2	1	2		2	1	1		3		3	3	3	3	1		2	1	1	2	2
11	2		2	1	1	1	2	1	1	2		2	2	1	1		1	1	2	2	2		3	3		3	3	3
12	3	*			*	3	3		*		2	1,*	2	*	*		2	1	*	*	1		*	1	2	*	1	*
13		2	1	2	1	2	1	3	3	3	3			3	2		1	1	2	1	2	1	2		1	1	2	2
14	1	1		1	2	2	2	1	1		2	2	1	2	3	3		3	3	3		1	1		1	2	2	2
15	2		1	2	1	1	2	2		2	2	1	1	1	3		3	3		3	3	1	2	1		2	1	2
16	*	1		2	1	2	*			*,2	*	*	1	*		*	*	2	1	*	*			3	*	*	3	3
17	1		2	*	*	*	1	2		3	*	*	1	*		3	3		3	3	3	2	1	*		1	*	2
18	1	2		2	1	1	2	1		1	2	1	2	2		1	2	1	1	1	2	3		3	3	3		3
19	*	2	1	1		*	2		3		3	3	3	3	*	*		2	1	*	1		*	*	*	*	1	1
20		*	3	*	*	*			*	*	*	*	*	2	2	1	*		*	*	2	2		2	1	*	*	*
21	1		2	1	2	2	1	1		2	1	1	2	2		3	3	3	3	3		1		2	1	2	*	2
22	3			3	3	3	3	2	1	1		2	1	2	1		2	1	1	2	2	1	1	2	1	2		2
23	3		3	3	3	3		2	2		2	1	1	1		1	2	1	2	1	2	1	2	1		2	1	2
24	1	1		2	1	2	2	3	3	3	3			3		1	2	1	2	1	2	1	2	1		2	1	2

Table 5. Nurses' day-off

Nurse	Day	Week 1	Week 2	Week 3	Week 4
1	1	1			
1	3		1	1	1
1	7		2		
2	2	1			1
2	3		1	1	
2	7	1			
3	1				1
3	4	1		1	
3	6				1
3	7		1		1
4	1	1			
4	3		1		
4	4		2	1	2
4	5			1	
4	6			1	
5	1	1			
5	3		1		
5	4			1	1
5	7	1			
6	1	1			
6	2	1			
6	3			1	
6	4		1		
7	1	1			
7	3				1
7	4			1	
7	5		1		
7	7		1		
8	2	1		1	1
8	3		1		
8	5		1		
9	3	1			1
9	4			1	
9	6		1		
9	7				1
10	1			1	
10	2	1			1
10	3			1	
10	4		1		
11	1	1		1	
11	2		1		1
11	4		1		1

Table 5. Continued

Nurse	Day	Week 1	Week 2	Week 3	Week 4
12	1				1
12	2			1	
12	3	1	1		
12	4	1			
13	1			1	
13	2				1
13	3		2	1	1
13	5		1		
13	6		1		
14	3	1	1	1	1
14	7			1	
15	2	1	1	1	
15	4				1
15	5			1	
16	1			1	1
16	2		1		1
16	3	1			
17	1	1		1	
17	1		1		
17	4			1	1
18	1			1	
18	2		1		1
18	3	1			
18	6				1
19	1		1		
19	3		1	1	
20	1	1	1		
20	2				1
20	4			1	
20	7	1			
21	1			1	
21	2	1	1		1
21	7			1	
22	1			1	
22	2	1			
22	3	1			
22	4		1		
22	6				1
23	1			1	
23	2	1			
23	3		1		
23	4				1

Table 5. Continued

Nurse	Day	Week 1	Week 2	Week 3	Week 4
23	4				1
23	7	1			
24	1			1	
24	3	1			
24	4				1
24	5		1		
24	6		1		

4-3- Validation of the proposed approach

In this section in order to validate the proposed model and algorithms some problems with different scales are solved. We solved the problems for 30, 40, 45 and 50 nurses. In all problems the proposed approach is efficient and for instance the relationship between the nurses are shown in table 6 and the results for 40 nurses is presented in table 7.

Table 6. Nurses relationship

Nurses with ''bad'' relationship	Nurses with ''neutral'' relationship	Nurses with ''good'' relationship
Nurse 5, 9, 12, 16, 17, 19, 20, 25, 29, 31, 35, 36, 39	Nurse 1, 3, 4, 6, 8, 13, 15, 26, 30, 37, 38,	Nurse 2, 7, 10, 11, 14, 18, 27, 28, 32, 33, 34, 40

Table 7. Timetable of the 40 nurses for a four-week review period regarding the relationship between nurses

Nurse	Week 1							Week 2							Week 3							Week 4						
	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr
1		2	2	1	3	2	2	3	1		3	2	1		3	3		2	2	2	1	2	1		2	1	2	1
2	1		1	1	2	3		2	3		2	3	2	1	3	1		2	3	1	2	1		1	2	3	2	1
3	3	2	3		1	2	3	1	2	3	1	3	2		1	2	2		3	3	2		1	1	1	3		3
4		1	2	3	3	1	1	1	1	1		2	2	2	3	2	3	2			1	2	3		2	1	3	2
5		*	3	3	1	1		*	1		*	2	2	1	1	*	3		2	*	3	*	2	*		2	2	*
6		1	1	1	1		1	3	3	2	1	3	1	1	2	2		1	3	2	2	1		2	2	3	2	3
7		2	3	1	3	2	2	3	1	1	3	1			2	1	1		3	1	3	1	2	2	1	1		1
8		1		2	3	1	2	3	1	1		2	1		1	3	2	3		2		2	1	2	3	3		1
9		2	*	1	2	3	3	2	*	1	2	1		3	2	*		1	3	2	3	2	*		2	3		*
10	2		2	1	2	1	1	2	1	2		2	1	1	1		3	3	1	1	2	1	2			2	2	1
11		1	3		2		2	2	2	3		2	3	1	1	2	2		1		3	1	3	1	1			3
12	3	*			*	3	3		*		2	3,*	1	*	2		2	1	*	*	1		*	1	2	*	1	*
13		2	1	2	1	2	1	3	3	3	3			3	2		1	1	2	1	2	1	2		1	1	2	2
14	1	1		1	2	2	2	1	1		2	2	1	2	3	3		3	3	3		1	1		1	2	2	2
15	2		1	2	1	1	2	2		2	2	1	1	1	3		1	3		2	3	1	2	1		2	1	2
16	*	1		2	1	2	3	*		*,3	*	*	1	*		*	*	2	1	*	*			3	*	*	3	3
17	1		2	1	*	1	1	*	*	3	*	1	1	*		3	3		3	3	3	2	1	*		1	*	2
18	1	2		2	1	1	2	1		1	2	1	2	2		1	2	1	1	1	2	3		3	3	3		3
19	*	2	1	1		*	2		3		3	3	3	3	*	*		2	1	*	1		*	*	*	*	1	1
20		1	3	1		*	*		*	1	2	3	*	2	*	*	*		*	*	2	2		2	1	*	*	*
21	1		2	1	3	2	1	1		2	1	1	2	2		2	3	3	2	3		1		2	1	2	*	2
22	3			3	2	3	2	2	1	1		2	1	2	3		2	1	1	2	2	1	1	2	1	2		2
23	3		1	3	1	3		2	2		2	1	1	1		1	2	1	2	1	2	1	2	1		2	1	2
24	1	1		2	1	2	1	3	3	3	3		1	1	1	2	2		2	3	2	1	2	1	1	3	1	3

Table 7. Continued

Nurse	Week 1							Week 2							Week 3							Week 4						
	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr	Sa	Su	Mo	Tu	We	Th	Fr
25		1	2	2	1	2	1	3	3		3	3	3		1	1		2	2	2	1	2	1		2	1	2	1
26	3		3	3	3	3		2	1		2	1	2	1	2	2		2	1	1	1	2		1	2	1	2	1
27	1	2	1		2	2	1	1	2	2	1	1	2		2	1	2		1	1	2		3	3	3	3		3
28		1	2	1	1	2	2	2	2	2		1	1	1	3	3	3	3			3	2	1		2	1	1	2
29		3	*	3	3	*		1	*		1	2	*	1	1	2	*		2	*	*	1	2	*		2	2	1
30			3	3	3	3	3	2	2	2		1	1	1	1	2		1	2	2	1	1		1	2	1	2	3
31		2	1	1	2	2	1	3	3	3	3		3		2	1	1		2	1	2	1	1		2	2	2	1
32	1		1	2	2	1	2	3	3		3		3	3	1		2	2	2	1	1	1		2	2	2	1	1
33		*	2	2	2	1	*	3	2	1	*	3		2	3	*	1		3	2	*	3	3		3	3	*	
34	2		2	1	2	1	1	2	1	2		2	1	1		3		3	3	3	3	1		2	1	1	2	2
35	2		2	1	1	1	2	1	1	2		2	2	1	1		1	1	2	2	2		3	3		3	3	3
36	3	*			*	3	3		*		2	1,*	2	*	*		2	1	*	*	1		*	1	2	*	1	*
37		2	1	2	1	2	1	3	3	3	3			3	2		1	1	2	1	2	1	2		1	1	2	2
38	1	1		1	2	2	2	1	1		2	2	1	2	3	3		3	3	3		1	1		1	2	2	2
39	2		1	2	1	1	2	2		2	2	1	1	1	3		3	3		3	3	1	2	1		2	1	2
40	*	1		2	1	2	*			*,2	*	*	1	*		*	*	2	1	*	*			3	*	*	3	3

5- Conclusion

Nurse Rostering Problem (NRP) or the Nurse Scheduling Problem (NSP) is a complex scheduling problem that affects hospital personnel on a daily basis all over the world and is known to be NP-hard. A high quality roster can lead to a more contented and thus more effective workforce. Hence a proper procedure is necessary for this problem. In this paper the graph theory approach is applied to the NRP which is not being used so far in this field. First we propose a mathematical method for the problem and after that by using the graph coloring algorithm (DSATUR) the constraint that nurses with bad relationship should not assign to same shift as satisfied. Another constraint of the problem is that a nurse in training should be assign to a shift that a head nurse is assigned to. For satisfying this constraint another graph concept called bipartite matching is used. An algorithm for the bipartite matching is proposed and nurses are assigned to the shift. Finally the output of the two graph concepts is input to the mathematical model to obtain the final timetable.

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