# Robust inter and intra-cell layouts design model dealing with stochastic dynamic problems 

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#### Abstract

In this paper, a novel quadratic assignment-based mathematical model is developed for concurrent design of robust inter and intra-cell layouts in dynamic stochastic environments of manufacturing systems. In the proposed model, in addition to considering time value of money, the product demands are presumed to be dependent normally distributed random variables with known expectation, variance, and covariance that change from period to period at random. This model is verified and validated by solving a number of different-sized test problems and a real world problem as well as doing sensitivity analysis by using the analysis of variance (ANOVA) technique. The validation process will be ended by investigating the effect of considering dependent product demands and time value of money (interest rate) on the total cost. Dynamic programming and simulated annealing algorithms programmed in Matlab are used to solve the problems. Some conclusions can be summarised as follows: (i) the simulated annealing algorithm has a performance as good as the dynamic programming algorithm from solution quality point of view; (ii) the simulated annealing is a robust algorithm; (iii) different values of the input parameters lead to design of different facility layouts; (iv) total cost of inter and intra-cell layouts is affected by the interest rate and the percentile level; and (v) the proposed model can be used in both of the stochastic and deterministic environments.


Keywords: Simulated annealing, stochastic dynamic, robust cell layout

## 1- Introduction

One of the most critical stages in the design of manufacturing system is the Facility Layout Problem (FLP). Facility layout is the problem of determining the relative location of facilities on the shop floor. In manufacturing systems, a facility can be a work station such as a machine or a group of machines named cell. Regarding the importance of the FLP, it is nessesary to mention that the Material Handling Cost (MHC) forms from twenty to fifty percent of the total manufacturing costs and it can be reduced by at least from ten to thirty percent by an optimal layout design (Tompkins et al., 2003). According to the nature of product demands and time planning horizon, the FLP can be classified into four problems as follows: (i) static (single period) facility layout problem (SFLP) with deterministic and constant flow of materials over a single time period, (ii) dynamic(multi-period) facility layout problem (DFLP) with different deterministic flow of materials in each period, (iii) stochastic static facility layout problem (SSFLP) with stochastic flow of materials over a single time

[^0]period, (iv) Stochastic Dynamic Facility Layout Problem (SDFLP) where product demands are random variables so that their parameters change from period to period. Among the aforementioned problems, the SDFLP is the most comprehensive, realistic and complicated problem.Thus this paper considers the robust layout design approach in the SDFLP.
The MHC is one of the most appropriate measures, which has been widely used to evaluate the efficiency of a facility layout. This measure should be minimised to design an optimal layout.Uncertainties and changes in product demands lead to changes in the flow of materials. Increase in the flow of materials during the transition from the current period to the next period increases the MHC, which in turn leads to an inefficient layout. Therefore, it is necessary to rearrange the facilities in the next period to obtain the optimal layout. The rearrangement of facilities is a costly process. This cost is named the rearrangement cost. Therefore, to avoid the high facility rearrangement costin transition from one period to the next, it is preferable to design just one layout named a robust layout for theentire multi-period time planning horizon. Actually, in each period, the MHC of therobust layout remains near to its optimal (minimum) value in spite of changes in product demadsfrom period to period.The lower deviation from the optimal value of the MHC indicates the better robustness of the layout. The robust layout has some advantagessuch as lack of the rearrangement cost and being flexible (robust) enough to cope with uncertainties in demand of products. However, it suffers from the disadvantage of not necessarily being an optimal layout for each time period.
In general, the FLP having discrete representation and equal-sized facilities assigned to the same number of known locations is usually formulated as the Quadratic Assignment Problem (QAP) model. In discrete representation, the manufacturing cite is split into a quantity of the same-sized facility places.Koopmans and Beckman (1957) formulated the QAP as amathematical model, whichis usedto formulate the SDFLP in this paper.
The QAP is a nonlinear nondeterministic polynomial (NP)-complete combinatorial optimisation problem (COP) (Sahni \& Gonzalez, 1976). Besides, the computational time required for solving the QAP is exponentially proportional to the size of the problem (Foulds, 1983). Therefore, intelligent approaches should be used to solve the large-sized problem rather than the exact methods. Simulated annealing (SA) intelligent approach is one of the promising tools for solving COPs such as the FLP (Alvarenga, Gomes, \& Mestria, 2000). SA algorithm is animitation of physical solids annealing. This algorithm belongs to the class of improvement resolution approaches so that it needs toa known initial solution. SA algorithm consists of two loops namely, the inner loop to search for a neighbouring solution, and the outer loop for decrease the temperature to reduce the probability of accepting the non-improving neighbouring solutions.
The outstanding performance of SA in comparison with genetic algorithm (GA) and tabu search (TS) was concluded in solving a dynamic cell formation problem (Tavakkoli-Moghaddamet al., 2005). This algorithm can not only solve the stochastic and single period inter and intra-cell layout problem as good as the lingo software from quality solution standpoint, but also it can solve the larger problems in a reasonable computation time (Tavakkoli-Moghaddam, Javadi, \& Mirghorbani, 2006). According to Moslemipour et al. (2012), SA algorithm has some advantages as follows: (i) it reduces the computational time and in turn, it has low memory requirement; (ii) accepting the non-improving solution (uphill movement) prevents the algorithm from getting entrapped at a deprived local solution; (iii) the ability of finding global optimal solution; (iv) this algorithm is easy for implementation; (v) it has the convergent property upon executing the large number of iterations; and (vi) SA is more robust and flexible in comparison with other local search methods.In this paper, the SA approach is used for solving the proposed model because of its above-mentioned advantages and the complexity of the model.
Lee et al. (2012) proposed a novel hybrid AC/SA approach having an outstanding performance to solve the SDFLP. Moslemipour et al. (2012) reviewed the SA and other intelligent approaches for solving layout problems, comprehensively.
Rosenblatt and Lee (1987) and also Kouvelis, Kuawarwala and Gutierrez (1992)defined the robustness of a layout as the number of times that the layout drops inside a pre-defined fraction of the optimum solution for dissimilar groups of product demand patterns.Using the robust approach, a single robust layout is designed for the whole time planning horizon so that the total MHC is minimised (Kouvelis \& Kiran, 1991). Montreuil et al. (1993) proposed a robust layout named
holographic or holonic layout where different types of machines are spread over the shop floor to cope with uncertainties in a manufacturing system. For more information about the holonic layout the following papers can be referred (Hsieh, 2009a, , 2009b; Hsieh \& Chiang, 2011). The robust layout design approach is a good method to prevent the shifting cost (Hassan, 1994). Benjaafar and Sheikhzadeh (2000) proposed a robust layout by duplicating the same facilities in the FLP to deal with uncertainties in product demands.The robustness can be an intrinsic property of a layout for example, by replication of the main facilities at the strategic places within the shop floor, which will guarantee a reasonable efficiency for the material handling system during the different production periods (Benjaafar, Heragu, \& Irani, 2002).
Braglia, Simone and Zavanella (2003) designed the most robust layout for a single row FLP by assuming the independent product demands as normally distributed random variables. KulturelKonak,Smith and Norman (2004) considered the most robust layout with minimum region under the total MHC curve over a pre-determined range of uncertainty. Enea, Galante and Panascia (2005) suggested a fuzzy-based model for designinga robust facility layout in the SFLP with multiple product demand scenarios.Braglia,Simone and Zavanella (2005) proved that in the stochastic FLP, the most robust layout is obtained by using the matrix of average flows between facilities.Norman and Smith (2006) proposed a mathematical model to design the most robust layout by considering a large number of independent product demands as random variables with known expected value and variance.Tavakkoli-Moghaddam et al. (2007) proposed a novelformulation to simultaneous design of the optimum intra and inter-cell facility layouts for the SSFLP. Irappa-Basappa and MadhusudananPillai (2008) designed a robust machine layout for the DFLP using the quadratic assignment formulation by considering machine sequence and part handling factor, which represents changes in the attributes of parts from process to process. Balakrishnan and Cheng (2009) considered both of the fixed and rolling planning horizon in the DFLP. They concluded that the algorithms having good performance under condition of fixed planning horizon don't have necessarily good performance in the case of rolling planning horizon.Madhusudanan-Pillai,Irappa-Basappa and Krishna (2011) proposed a SA algorithm to solve their robust layout design model in dynamic environment.
Moslemipour and Lee (2012) designed an optimal machine layout for each period of the SDFLP, which is named as dynamic layout.They considered independent uncertain product demands having normal distribution with known and changeable probability density function (PDF) from current period to the next one. Lee and Moslemipour(2012) proposed a new mathematical model to deal with a dynamic inter-cell layout problem in which the flow of materials is assumed to be a random variable with known expected value. Lee and Moslemipour (2012) developed a novel QAP-based mathematical model for designing the most stable facility layout in the whole time planning horizon of the SDFLP. This layout has the maximalcapability to exhibit a little sensitivity to product demand changes. Forghani, Mohammadi and Ghezavati (2013) proposed a new robust method to deal with the cell formation and the layout design problem by considering stochastic demands. Neghabi, Eshghi and Salmani (2014) developed a novel mathematical model named RABSMODEL along with a two-stage algorithm to design a robust layout in which facilities have flexible dimensions. TavakkoliMoghaddam, Sakhaii and Vatani (2014) proposed a robust optimisation method to design a dynamic cellular manufacturing system (CMS) by incorporating production planning so that processing time of parts is assumed to be a stochastic variable. Hosseini, Khaled and Vadlamani (2014) developed a robust simple hybrid approach by incorporating three meta-heuristic methods including imperialist competitive algorithms, variable neighborhood search, and SA to cope with the DFLP.
In modern manufacturing systems such as the flexible manufacturing system (FMS) and the CMS, machines are grouped into some cells in terms of the philosophy of group technology. Therefore, according to the aforementioned importance of the FLP, an optimal layout of machines inside each cell (intra-cell layout) and an optimal layout of cells on the shop floor (inter-cell layout) should be designed simultaneously. The novelties of this paper as the gaps in the literature are as follows: (i) to propose a QAP-based mathematical model for simultaneous design of robust inter and intra-cell layouts in the SDFLP, (ii) to consider dependent product demands and time value of money. In this model, regarding the SDFLP, the product demands are presumed to be dependent normally distributed random variableswith known expectation, variance, and covariance that change from period to period at random. In fact, despite considering uncertain product demands, the proposed model is free of any uncertain parameters and thereby there is no need to implement the robust optimisation technique. As
mentioned before, the term "robust" refers to the flexibility of the layout to deal with uncertainties and changes in product demands from period to period.
Regarding the normal distribution assumption, it is essential to mention that many real world data naturally follow a normal distribution (Kulturel-Konak et al., 2004). Product demands have also been considered as normally distributed random variables in layout design problem by the following authors (Ji, Yongzhong, \& Haozhao, 2006; Rezazadeh, et al., 2009; Ripon, et al., 2011; TavakkoliMoghaddam et al., 2007; Vitayasak, Pongcharoen, \& Chris Hicks, 2016).

## 2-The proposed model

In this section, the new mathematical model is formulated by considering the following assumptions. Table 1 shows the notations used in the proposed model.

Table1. Notations

| Notation | Description |
| :---: | :--- |
| $K$ | Total quantity of parts |
| $M$ | Total quantity of machines / machine locations. |
| $T$ | Total quantity of periods |
| $M_{c}$ | The quantity of machines / locations of machine inside cell c |
| $C$ | Total quantity of cells $/$ locations of cell |
| $k$ | Part index $(k=1,2, \ldots, K)$ |
| $t$ | Period indicator $(t=1,2, \ldots, T)$ |
| $i, j$ | Machine indices $(i, j=1,2, \ldots, \mathrm{M}) ; i \neq j$ |
| $l, q$ | Machine location indices $(l, q=1,2, \ldots, \mathrm{M}) ; l \neq q$ |
| $c, w$ | Cell indices $(c, w=1,2, \ldots, \mathrm{C}) ; c \neq w$ |
| $u, v$ | Cell location indices $(u, v=1,2, \ldots, \mathrm{C}) ; u \neq v$ |
| $N_{k i}$ | Process number for the process performed on part $k$ by machine $i$ |
| $f_{t i j k}$ | Materials flow between machines $i$ and $j$ in period $t$ created by part $k$ |
| $f_{t i j}$ | Materials flow between machines $i$ and $j$ in period $t$ created by all parts |
| $f_{t c v}$ | Materials flow between cells $c$ and $w$ in period $t$ |
| $D_{t k}$ | Part $k$ demand during period $t$ |
| $B_{k}$ | Part $k$ batch volume |
| $C_{k}$ | Present value of the movement cost per batch for part $k$ |
| $C_{t k}$ | Cost of movements for part $k$ in period $t$ |
| $a_{\text {ilq }}$ | Cost of shifting machine $i$ from location $l$ to location $q$ in period $t$ |
| $a_{t c u v}$ | Fixed cost of shifting cell $c$ from location $u$ to location $v$ in period $t$ |
| $d_{l q}$ | Distance from machine location $l$ to machine location $q$ |
| $d_{u v}$ | Distance from cell location $u$ to cell location $v$ |
| $x_{i l}$ | Decision variable for robust machine (intra-cell) layout problem |
| $x_{c u}$ | Decision variable for dynamic inter-cell layout problem |
| $T C\left(L_{r r m}\right)$ | Total cost of layout $L_{r r m}$ |
| $E()$ | Expectation |
| $V a r()$ | Variance |
| $C o v()$ | Covariance |
| $U T C\left(L_{r m}, p\right)$ | The highest value of $T C(\pi)$ with the percentile value $p$ |
| $I_{r}$ | Interest rate |
| $T_{c}$ | Total part movement and rearrangement costs for cell $c$ |
| $b_{i c}$ | A zero-one variable representing the assignment of machine $i$ to cell $c$ |
| $O F V_{r m}$ | Total cost of the robust machine layout |
| $O F V_{r c}$ | Total cost of inter and intra-cell layouts |

i. Equal-sized machines/cells are assigned to the same number of known machines/cells locations.
ii. The discrete representation of the SDFLP is considered.
iii. Demands of parts are dependent normally distributed random variables with known expected value, variance, and covariance that change from period to period at random.
iv. The confidence level (percentile $p$ ), which represents the decision maker's attitude about uncertainty in product demands, is considered.
v. Time value of money is considered.
vi. The parts are moved in batches between facilities.
vii. The data on number of facilities (machines-cells), number of periods, machine sequence, present value of part movement cost, transfer batch size, distance between facility locations, money interest rate for each period (e.g. year), present value of facility (machine/cell) rearrangement cost, the expected value, variance, and covariance of part demands in each period are known as inputs of the models.
viii. There is no constraint for dimensions and shapes of the shop floor.
ix. Machines can be laid out in any configuration such as rectangular and U-shaped configurations.
x. Cell formation is accomplished in advance so that each cell is formed by a certain number of known machines used for doing known operations on parts.

## 2-1- Robust intra-cell layout design model

Using the robust facility planning approach, a multi-period problem is changed into a single period one. Hence, considering the assumption (i), the following QAP formulation developed by Koopmans and Beckman (1957)is utilised to develop the novel mathematical model for designing inter and intracell layouts in the SDFLP.
Minimise $\sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{l=1}^{M} \sum_{q=1}^{M} f_{i j} d_{l q} x_{i l} x_{j q}$
Subject to:
$\sum_{i=1}^{M} x_{i l}=1 \quad ; \forall l$
$\sum_{l=1}^{M} x_{i l}=1 \quad ; \forall i$
$x_{i l}=\left\{\begin{array}{ccc}1 & \text { if } & \text { facility } i \\ 0 & \text { otherwise }\end{array}\right.$

The equation (1) is a quadratic objective function representing the total MHC. In this equation, $f_{i j}$ symbolises the materials flow between facilities $i$ and $j$. The distance from locations $l$ to location $q$ is indicated by $d_{l q}$. The equation (2) confirms that each facility location has the capability of containing just one facility.The equation (3) states a specified facility can be allocated to precisely one facility location. Equation (4) displays the decision variables $x_{i l}$ as the solution to the problem indicating the place of each facility.

The input parameters of the model are as follows: sequence of machines, batch volume, the present cost of part handling per batch, machine locations distance matrix, cell locations distance matrix, and the expectation and variance of part demands in addition to the covariance of each pairs of part demands in each period. The total cost of the inter and intra-cell layouts is considered as the output of the novel model. This cost should be minimised in order to optimial design of machine and cell layouts.

It is assumed that, $M$ machines are located in $C$ cells such that $\bigcup_{c=1}^{C} c=M$ and $\bigcap_{c=1}^{C} c=\varnothing$ and Cell $c$ contains $M_{c}$ machines in accordance with equations (5) and (6). Equation (7) displays the formulation for computing $f_{t i j k}$, where, the equation $\left|N_{k i}-N_{k j}\right|=1$ denotes two consecutive operations, which are performed on part $k$ using machines $i$ and $j$. The formulation for computing $f_{i j k}$ is shown in equation
(8), which is modified as equation (9) by combining with equation (7). In fact, consistent with equation (8), considering a particular part $k$, the arithmetic average of parts flow in eachperiod $f_{\mathrm{ijk}}$ is regarded as the part flow during the total time planning horizon. The total materials flow between machines $i$ and $j$ created by all parts $\left(f_{i j}\right)$ is computed using equation (10), that can be writtenin the form of equation (11) by combining with equation (9). Lastly, the equation (11) is reordered as equation (12), where $D_{t k}$ is an uncertain variable having normal distribution with expectation $E\left(D_{t k}\right)$ and variance $\operatorname{Var}\left(D_{t k}\right)$. Consequently, $f_{i j}$ is also an uncertain variable with normal distributionhaving the expectation and variance as shown in equations (13) and (14), respectively. Considering the interest rate, the handling cost for part $k$ in time period $t$ is computed by applying the equation (15). Equation (16) shows the total cost of part handling for a known robust machine layout $L_{r m}$ by considering equation (1). According to equation (16), since $f_{i j}$ is an uncertain variable with normal distribution, $T C\left(L_{r m}\right)$ is also a an uncertain variable with normal distribution having the expectation and variance as shown in equations (17) and (18) respectively.

$$
\begin{align*}
& \sum_{i=1}^{M} b_{i c}=M_{c} ; \quad c=1,2, \ldots, C  \tag{5}\\
& b_{i c}=\left\{\begin{array}{ll}
1 & \text { if machine } i \\
0
\end{array} \quad\right. \text { otherwise } \tag{6}
\end{align*} \text { is assigned to cell } c .
$$

$f_{\text {tijk }}= \begin{cases}\frac{D_{t k}}{B_{k}} C_{t k} & \text { if }\left|N_{k i}-N_{k j}\right|=1 \\ 0 & \text { otherwise }\end{cases}$
$f_{i j k}=\frac{\sum_{t=1}^{T} f_{t i j k}}{T}$
$f_{i j k}=\frac{1}{T} \sum_{t=1}^{T} \frac{C_{t k}}{B_{k}} D_{t k}$
$f_{i j}=\sum_{k=1}^{K} f_{i j k}$
$f_{i j}=\sum_{k=1}^{K} \sum_{t=1}^{T} \frac{C_{t k}}{T . B_{k}} D_{t k}$
$f_{i j}=\sum_{t=1}^{T} \sum_{k=1}^{K} \frac{C_{t k}}{T \cdot B_{k}} D_{t k}$
$E\left(f_{i j}\right)=\sum_{t=1}^{T} \sum_{k=1}^{K} \frac{C_{t k}}{T . B_{k}} E\left(D_{t k}\right)$
$\operatorname{Var}\left(f_{i j}\right)=\sum_{t=1}^{T}\left(\sum_{k=1}^{K}\left(\frac{C_{t k}}{T \cdot B_{k}}\right)^{2} \operatorname{Var}\left(D_{t k}\right)+2 \sum_{k=1}^{K} \sum_{k^{\prime}=k+1}^{K} \frac{C_{t{ }^{k}} \cdot C_{t k^{\prime}}}{T^{2} \cdot B_{k} \cdot B_{k^{\prime}}} \operatorname{cov}\left(D_{t k}, D_{t k^{\prime}}\right)\right)$
$c_{t k}=c_{k}\left(1+I_{r}\right)^{t}$
$\operatorname{TC}\left(L_{r m}\right)=\sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{l=1}^{M} \sum_{q=1}^{M} f_{i j} d_{l q} x_{i l} x_{j q}$

$$
\begin{align*}
& E\left(T C\left(L_{r m}\right)\right)=\sum_{i=1}^{M} \sum_{j=1}^{M} E\left(f_{i j}\right) \sum_{l=1}^{M} \sum_{q=1}^{M} d_{l q} x_{i l} x_{j q}  \tag{17}\\
& \operatorname{Var}\left(T C\left(L_{r m}\right)\right)=\sum_{i=1}^{M} \sum_{j=1}^{M} \operatorname{Var}\left(f_{i j}\right)\left(\sum_{l=1}^{M} \sum_{q=1}^{M} d_{l q} x_{i l} x_{j q}\right)^{2} \tag{18}
\end{align*}
$$

For a known robust machine layout $L_{r m}, U T C\left(L_{r m}, p\right)$ is considered as the highest value of the $T C\left(L_{r m}\right)$ with percentile value $p$. Doing so, $U\left(L_{r m}, p\right)$ represented in equation (19) can be minimised rather than minimising $T C\left(L_{r m}\right)$ (Kulturel-Konak et al., 2004; Moslemipour \& Lee, 2012; Norman \& Smith, 2006; Tavakkoli-Moghaddam et al., 2007). Equation (20) is written by utilising equations from (13) to (19). To design the optimum robust machine layout inside cell $c$, the cost function $T_{c r}$ is formulated as equation (21) in accordance with equation (20). Considering $C$ cells in the SDFLP, the total intra-cell layouts cost is computed as equation (22). As a result, the mathematical formulation in order to the intra-cell layouts design can be modelled as equation (22) subject to equations (2), (3), and (4).
$\operatorname{UTC}\left(L_{r m}, p\right)=E\left(T C\left(L_{r m}\right)\right)+Z_{p} \sqrt{\operatorname{Var}\left(T C\left(L_{r m}\right)\right)}$
Minimisation of

$$
\begin{align*}
& O F V_{r m}=\left[\begin{array}{l}
\sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{k=1}^{K} \sum_{t=1}^{T} \frac{C_{k}\left(1+I_{r}\right)^{t}}{T \cdot B_{k}} E\left(D_{t k}\right) \sum_{l=1}^{M} \sum_{q=1}^{M} d_{l q} x_{i l} x_{j q} \\
+Z_{p}\left(\sum_{i=1}^{M} \sum_{j=1}^{M}\left(\sum_{t=1}^{T}\binom{\sum_{k=1}^{K}\left(\frac{C_{k}\left(1+I_{r}\right)^{t}}{T \cdot B_{k}}\right)^{2} \operatorname{Var}\left(D_{t k}\right)+}{2 \sum_{k=1}^{K} \sum_{k^{\prime}=k+1}^{K} \frac{C_{k} \cdot C_{k^{\prime}}\left(1+I_{r}\right)^{2 t}}{T^{2} \cdot B_{k} \cdot B_{k^{\prime}}} \operatorname{cov}\left(D_{t k}, D_{t k^{\prime}}\right)}\right)\right.
\end{array}\right]  \tag{20}\\
& {\left[\sum_{i=1}^{M_{c}} \sum_{j=1}^{M_{c}} \sum_{l=1}^{M_{c}} \sum_{q=1}^{M_{c}} \sum_{k=1}^{K} \sum_{t=1}^{T}\left(\frac{C_{k}\left(1+I_{r}\right)^{t}}{T . B_{k}}\right) E\left(D_{t k}\right) d_{l q} x_{i l} x_{j q}+\right.} \\
& T_{c r}=\left[Z_{p}\left(\sum_{i=1}^{M_{c}} \sum_{j=1}^{M_{c}}\left(\sum_{t=1}^{T}\binom{\sum_{k=1}^{K}\left(\frac{C_{k}\left(1+I_{r}\right)^{t}}{T \cdot B_{k}}\right)^{2} \operatorname{Var}\left(D_{t k}\right)+}{2 \sum_{k=1}^{K} \sum_{k^{\prime}=k+1}^{K} \frac{C_{k} \cdot C_{k^{\prime}}\left(1+I_{r}\right)^{2 t}}{T^{2} \cdot B_{k} \cdot B_{k^{\prime}}} \operatorname{cov}\left(D_{t k}, D_{t k^{\prime}}\right)}\right)\left(\begin{array}{l}
\left.\sum_{l=1}^{M_{c}} \sum_{q=1}^{M_{c}} d_{l q} x_{i l} x_{j q}\right)^{2}
\end{array}\right)^{\frac{1}{2}}\right]\right. \tag{21}
\end{align*}
$$

Minimise $\sum_{c=1}^{C} T_{c r}$
Subject to: Equations (2), (3), and (4).

## 2-2-Robust inter-cell layout design model

In this part, anovel model is developed for designing an inter-cell layout. The totalmaterials flow between cells $c$ and $w$ is computed by utilising equation (23). In this equation, $f_{i j}$ is anuncertain variable and thereby $f_{c w}$ is also an uncertain variable with the expectation and variance as shown in equations (24) and (25), respectively. In equations (24) and (25), the parameters $E\left(f_{i j}\right), \operatorname{Var}\left(f_{i j}\right)$ and $b_{i c}$ are represented in equations (13), (14), and (6), respectively. In the inter-cell layout design process, the cells are regarded as facilities. By doing so, similar to equations (17) and (18), the expected value and variance of the total MHC of the robust cell layout $L_{r c}$ (i.e. $T C\left(L_{r c}\right)$ )are calculated using equations (26) and (27), respectively. Therefore, using the equations (2), (3), (4), (13), (14), (15), (19), (24),
(25), (26), and (27), the mathematical model for the inter-cell layout design can be written as equations from (28) to (31).
$f_{c w}=\sum_{i=1}^{M} \sum_{j=1}^{M} f_{i j} b_{i c} b_{j w}$
$E\left(f_{c w}\right)=\sum_{i=1}^{M} \sum_{j=1}^{M} E\left(f_{i j}\right) b_{i c} b_{j w}$
$\operatorname{Var}\left(f_{c w}\right)=\sum_{i=1}^{M} \sum_{j=1}^{M} \operatorname{Var}\left(f_{i j}\right) b_{i c}{ }^{2} b_{j w}{ }^{2}$
$E\left(T C\left(L_{r c}\right)\right)=\sum_{c=1}^{C} \sum_{w=1}^{C} E\left(f_{c w}\right) \sum_{u=1}^{C} \sum_{v=1}^{C} d_{u v} x_{c u} x_{w v}$
$\operatorname{Var}\left(T C\left(L_{r c}\right)\right)=\sum_{c=1}^{c} \sum_{w=1}^{c} \operatorname{Var}\left(f_{c w}\right)\left(\sum_{u=1}^{c} \sum_{v=1}^{c} d_{u v} x_{c u} x_{w v}\right)^{2}$
Minimisation of

$$
\left[\begin{array}{l}
\sum_{c=1}^{c} \sum_{w=1}^{c} \sum_{i=1}^{M} \sum_{j=1}^{M} \sum_{k=1}^{K} \sum_{t=1}^{T}\left(\frac{C_{k} b_{i c} b_{j w}\left(1+I_{r}\right)^{t}}{T \cdot B_{k}}\right) E\left(D_{t k}\right) d_{u v} x_{c u} x_{w v}+ \\
Z_{p}\left(\sum _ { c = 1 } ^ { c } \sum _ { w = 1 } ^ { c } \sum _ { i = 1 } ^ { M } \sum _ { j = 1 } ^ { M } b _ { i c } ^ { 2 } b _ { j w } ^ { 2 } \left(\begin{array}{l}
\left.\sum_{t=1}^{T}\binom{\sum_{k=1}^{K}\left(\frac{C_{k}\left(1+I_{r}\right)^{t}}{T \cdot B_{k}}\right)^{2} \operatorname{Var}\left(D_{t k}\right)+}{2 \sum_{k=1}^{K} \sum_{k^{\prime}=k+1}^{K} \frac{C_{k} \cdot C_{k^{\prime}}\left(1+I_{r}\right)^{2 t}}{T^{2} \cdot B_{k} \cdot B_{k^{\prime}}} \operatorname{cov}\left(D_{t k}, D_{t k^{\prime}}\right)}\right)
\end{array}\left(\left(\sum_{u=1}^{c} \sum_{v=1}^{c} d_{u v} x_{c u} x_{w v}\right)^{2}\right)^{\frac{1}{2}}\right.\right. \tag{28}
\end{array}\right]
$$

Subject to:
$\sum_{c=1}^{c} x_{c u}=1 \quad ; \forall u$
$\sum_{u=1}^{c} x_{c u}=1 \quad ; \forall c$
$x_{c u}=\left\{\begin{array}{lll}1 & \text { if cell } c \text { is assigned to location u } \\ 0 & \text { otherwise }\end{array}\right.$

## 2-3- Robust inter and intra-cell layouts design model

Finally, the new mathematical model for concurrent design of inter-cell and intra-cell layouts in multi-period uncertain environments of the manufacturing system can be written as follows:

## Minimise $O F V_{r c}=\{$ Intra-cell cost (equation (22)+Inter-cell cost (equation (28) $\}$ (32)

## Subject to:

Equations (2), (3), (4), (5), (6), (29), (30), and (31).

## 3- Computation results and analysis

In this section, in addition to evaluating the performance of the SA algorithm, the proposed model is verified. To this end, first of all,four small-sized test problemsare solved by using dynamic programming (DP) and SA approaches in Sub-section 3-1. Then, in Sub-sections 3-2 and 3-3 a largesized test problem and a real world problemare applied to the proposed model and solved by using the SA algorithm. Finally, In Sub-section 3-4, the sensitivity analysis is perfomed using design of experiment and analysis of variance (ANOVA) techniques.In addition, the effect of considering dependent product demands and varying interest rate on the total cost function of the proposed model
is investigated in Sub-section 3-5.A personal computer with Intel 2.10 GHZ CPU and 3 GB RAM is used to run the SA and the DP algorithms programmed in Matlab.

## 3-1- Solving small-sized test problems

In this section, to evaluate the performance of the SA, four small-sized test problems are solved by using both of the DP exact method and the SA algorithm. If we represent the quantity of machines $M$ and the quantity of time periods $T$ as the ordered pair $(M, T)$, then the Problems 1,2,3, and 4 will be correspond to the ordered pairs $(3,4),(3,5),(4,4)$, and $(4,5)$ respectively.In the problems, the number of cells is assumed to be one ( $\mathrm{C}=1$ ).Rectangular configuration of layouts with recti-linear distance between the locations of facilities is considered.The results are displayed in Tables 2 and 3.On comparison, the SA algorithm has a performance as good as the DP algorithm from solution quality point of view. In addition,SA algorithm is better than the DPalgorithm from computational time standpoint. Therefore, it can be considered as a promising tool for obtaining the best solution to the large-sized dynamic layout problems in reasonable computation time.

Table2.The Results of DP and SA for Problems 1 and 2

| Problem No. | 1 |  | 2 |  |
| :---: | :---: | :---: | :---: | :---: |
| Algorithm | DP | SA | DA | SA |
| Confidence Level |  |  |  |  |
| 0.75 |  |  |  |  |
| 0.85 | 7058400 | 7061400 | 9763100 | 9766100 |
| 0.95 | 7168000 | 7171000 | 9914300 | 9917300 |
| Computational Time (Sec.) | 1.527 | 0.096 | 2.756 | 0.229 |

Table3.The Results of DP and SA for Problems 3 and 4

| Problem No. | 3 |  | 4 |  |
| :---: | :---: | :---: | :---: | :---: |
| Algorithm | DP | SA | DP | SA |
| Confidence Level |  |  |  |  |
| 0.75 | 19796000 | 19796000 | 26557000 | 26557000 |
| 0.85 | 20200000 | 20200000 | 27113600 | 27113600 |
| 0.95 | 20879191 | 20879191 | 28111316 | 28111316 |
| Computational Time (Sec.) | 3.1483 | 0.0298 | 3.3007 | 0.0346 |

## 3-2- Solving a large-sized test problem

To validate the proposed model, alarge-sizedrandomly generated test problem as a numerical example is solved. The test problem includes ten parts, twelve machines grouped into three cells, and ten time periods. The three groups of machines, including ( $1,2,3,4$ ), ( $5,6,7,8$ ) , and ( $9,10,11,12$ ) constitute the cells 1,2 , and 3 respectively.Solving a number of different-sized problems, which are applied to the proposed model, indicates that the DP algorithm cannot solve the SDFLP including five periods and more than ten facilities. Therefore, the above-mentioned problem is solved by using the SA algorithm programmed in Matlab. The initial solution for the SA algorithm is given in Table 4. This solution consists of the initial machine layout within each cell (intra-cell layout) and the initial layout of cells on the shop floor (inter-cell layout). Here, the solution to the robust inter and
intra-cell layout problem is given as a row matrix where each column represents a location, and each element represents a machine/cell number. Three different confidence levels (percentile $p$ ) including $0.75,0.85$, and 0.95 taken from Tavakkoli-Moghaddam et al. (2007) are also considered.

For each of the three above-mentioned confidence levels, the best solutions to the robust inter and intra-cell layout problem obtained by solving the test problem using SA algorithm are displayed in Tables 5 to 7. In fact, using these solutions, the total MHC of inter and intra-cell layouts ( $O F V_{r c}$ ) defined by Eq. (32) is minimised. These results include the best layout of machines within each cell (intra-cell layout) and the best layout of cells on the shop floor (inter-cell layout) along with their corresponding cost function value, the total cost of intra and inter-cell layout ( $O F V_{r c}$ ), and elapsed computation time. The objective function values obtained by running the SA algorithm ten times are evaluated statistically. Considering $p=0.75$, the results obtained from the statistical evaluation, including the worst, mean, best, and standard deviation (Std. Dev.) of the objective function values (OFVs) are given in Table 8. The statistical evaluation shows that the objective function values are pretty close to each other. As a result, the SA algorithm is a robust method and a promising tool to solve the proposed model.Here, the term "robust" refers to the fact that the solutions (i.e. OFVs) obtained by running the SA algorithm ten times for solving the aforesaid problem are close to each other.

Table4. Initial solution (Robust layout)

|  | Intra-cell layout |  |  | Inter-cell layout |
| :---: | :---: | :---: | :---: | :---: |
|  | Cell 1 | Cell 2 | Cell 3 |  |
| Location | 1234 | 1234 | 1234 | 123 |
| Facility | 1234 | 5678 | 9101112 | 123 |

Table5.The best solution (Robust layout; $p=0.75$ )

|  | Intra-cell layout |  |  | Inter-cell layout |
| :---: | :---: | :---: | :---: | :---: |
|  | Cell 1 | Cell 2 | Cell 3 |  |
| Location | 1234 | 1234 | 1234 | 123 |
| Facility | 2341 | 7856 | 1210119 | 213 |
| Cost | 478460 | 767850 | 544300 | Inter-cell cost $=21910580$ |
|  | Intra-cell cost $=1790610$ |  |  |  |
|  | OFV |  |  |  |  |

Table6.The best solution (Robust layout; $p=0.85$ )

|  | Intra-cell layout |  |  | Inter-cell layout |
| :---: | :---: | :---: | :---: | :---: |
|  | Cell 1 | Cell 2 | Cell 3 |  |
| Location | 1234 | 1234 | 1234 | 123 |
| Facility | 1432 | 8765 | 1210119 | 213 |
| Cost | 479950 | 770240 | 545850 | Inter-cell cost $=21972120$ |
|  | Intra-cell cost $=1796040$ |  |  |  |
|  | $O F V_{r c}=23768160$ |  |  |  | Elapsed time $=1.092169$ (seconds) |

Table7.The best solution (Robust layout; $p=0.95$ )

|  | Intra-cell layout |  |  | Inter-cell layout |
| :---: | :---: | :---: | :---: | :---: |
|  | Cell 1 | Cell 2 | Cell 3 |  |
| Location | 1234 | 1234 | 1234 | 123 |
| Facility | 2341 | 7856 | 9111012 | 213 |
| Cost | 482450 | 774270 | 548480 | Inter-cell cost $=22075800$ |
|  | Intra-cell cost $=1805200$ |  |  |  |
|  | OFV |  |  |  |  |

Table8.Statistical evaluation $(p=0.75)$

| Objective |  |  | Function Value ( $O F V_{r c}$ ) |
| :--- | :---: | ---: | ---: |
| Worst | (10 trials $)$ |  |  |
| 2399154023959537 | Best | Std. Dev. |  |

## 3-3- A real world problem( Raytheon Aircraft Company)

In this section, the Raytheon Aircraft Company studied by Krishnan, Cheraghi and Nayak (2006) is used to validate and to show the functionality and application of the proposed model.Inthis company as a real world case, six parts are processed by using 21 equal-sized machines ( $35^{\prime} \mathrm{X} 35^{\prime}$ ), which are grouped into one cell. Part movement cost and machine rearrangement cost are presumed to be $\$ 3.75 / f$ oot and zero respectively. The time planning horizon includes 5 time periods (years). An aisle space of 10 feet is considered around each machine and Euclidean distance between centres of machines is considered in the case study. The data on yearly part demands and Machine sequence has been given by Krishnan, Cheraghi and Nayak (2006). As mentioned, the proposed model deals with the SDFLP where the expectation, variance and covariance of part demands are known in each period. Thus, to apply the proposed model to the deterministic aforementioned real case, the data on yearly part demands are considered as the expectation of the part demands. Since there is no data on variance and covariance of part demands, a fifty percent percentile $p$ equivalent of $z_{p}=0$ is regarded. Doing so,the second term of the proposed model given in Eq. (32) is ignored. The data of the problem are applied to the proposed model and it is solved by using the SA algorithm. Table 9 shows the results of the proposed method and that of the previous one including the cost associated with each period and the total cost over the whole time planning horizon. As shown in Table 9, the obtained robust machine layout leads to $7.35 \%$ improvement with respect to the previous one proposed by (Krishnan et al., 2006).

Table9.The Results of Real Case Study

| Approach | Krishnan et al. <br> $(2006)$ | Proposed model <br> \&SA algorithm | Percentage <br> Savings |
| :---: | :---: | :---: | :---: |
| 2002 | $\$ 194,638.76$ | $\$ 165,816$ | $14.80 \%$ |
| 2003 | $\$ 211,428.07$ | $\$ 200,324$ | $5.25 \%$ |
| 2004 | $\$ 347,675.62$ | $\$ 343,410$ | $1.22 \%$ |
| 2005 | $\$ 464,675.32$ | $\$ 480,124$ | $3.20 \%$ |
| 2006 | $\$ 560,878.42$ | $\$ 458,821$ | $18.20 \%$ |
| Total Cost | $\$ 1,779,296.19$ | $\$ 1,648,495$ | $7.35 \%$ |

## 3-4- Sensitivity analysis

In this section, to investigate more about the behaviour of the proposed model, sensitivity analysis is carried out.Besides, using the sensitivity analysis, the inputs of the model are ranked in terms of the degree of their impact on the output of the model (objective function). Sensitivity of the output of the proposed robust layout design model with respect to the input parameters including expectation of materials flow, variance of materials flow, and confidence level is investigated by using one-way analysis of variance (ANOVA) technique.Using this technique, the null and alternative hypotheses are usually tested by using the F-test. The null hypothesis states that the means amongst two or more groups are equal and the alternative one indicates that at least two means are different. In ANOVA, it is assumed that the mean of the model outputs for each group is normally distributed random variable with approximately the same variance. Considering the assumptions, the F-value is statistically important at P -value < 0.05 and the null hypothesis is rejected (Sharma, 1996). In ANOVA, an input and an output of a model are named as a "factor" and a "response variable" respectively (Neter, Kutner, \& Nachtsheim, 1996).The factors are ranked according to the F-values (Carlucci, Napolitano, Girolami, \& Monteleone, 1999). Inputs with higher F-values are more sensitive factors, which have more effects on the output of a model.The aim of one-way ANOVA is to realise whether data from several groups have the same mean.
To perform the sensitivity analysis, 100 randomly generated test problems are applied to the proposed model in three different cases, namely Case E, Case V, and Case P, which are corresponding to investigate sensitivity of the objective function of the model with respect to expectation of materials flow (matrix $E$ ), variance of materials flow (matrix $V$ ) and confidence (percentile) level ( $p$ ) respectively. The input data are as follows: For each test problem, the expectation and variance of part demands ( $E$ and $V$ ) are randomly generated with uniform distribution so that $E \in(1000,10000)$ and $V \in(1000,3000)$. Besides, the number of cells, the number of machines and the number of periods are one, six and three respectively ( $C=1, M=6, T=3$ ). For simplicity and without losing the generality, independent part demands is considered. It is necessary to mention that the effect of assuming dependent part demands and time value of money (interest rate) on the total cost of the proposed model is investigated in sub-section 3-5.
In the case E , matrix $E$ is changed by $E^{\prime}=E+r^{*} E$, whereas matrix $V$ and confidence level $p$ remain unchanged. Considering nine different values of $r \in A=\{0.1,0.2,0.3,0.4,0.5,0.6,0.7,0.8$, $0.9\}$ leads to generate nine different matrices including $E_{1}, E_{2}, \ldots, E_{9}$. Each of the 100 test problems is solved by considering the nine different matrices so that the optimal value of the objective function of the proposed model corresponds to each matrix is obtained. Similarly, in the Case V, matrix $V$ is changed by $V^{\prime}=V+r^{*} V$,whereas matrix $E$ and confidence level $p$ remain unchanged. Considering the nine aforementioned values of $r$ leads to generate nine different matrices including $V_{1}, V_{2}, \ldots, V_{9}$.. Each of the 100 test problems is solved by considering the nine different matrices so that the optimal value of the objective function corresponds to each matrix is obtained. Finally, in the case P, the confidence level $p$ is set to each element in $A$ while matrices $E$ and $V$ remain unchanged. Likewise the two former cases, each test problem is solved for each of the nine different $p$ values so that the optimal value of the objective function corresponds to each $p$ is obtained. In fact, each case includes nine groups (populations) and each group contains a hundred samples of objective function values. Each case, group, and sample is denoted by $k, g$, $s$ respectively, where ( $k=\mathrm{E}, \mathrm{V}, \mathrm{P}$ ) , $(g=1,2, \ldots, 9$ ), and $(s=1,2, \ldots, 100)$. Mean of samples within group $g$ in case $k$ is represented by $\mu_{g}^{k}$. As mentioned, the condition of having normal distribution for the mean of each group is necessary for the ANOVA technique. To meet the condition, 100 randomly generated test problems is considered. This is due to the central limit theorem (CLT), which states that the average of sufficiently large number (say, bigger than 30) of independent random variables follows normal distribution (Hogg \& Ledolter, 1992).

In fact, the aim of this section is to test the hypothesis given in table 10.To this end, using Matlab software, the ANOVA technique is applied to the results of the randomly generated test problems for testing the aforementioned hypothesis. The results of the ANOVA technique is given in table 11 . According to the results including $F$-values and $P$-values, the null hypothesis $H_{0}$ is rejected. In other words, as expected, different values of input parameters containing expectation of product demands,
variance of product demands, and confidence level lead to design of different facility layouts. Therefore, the prposed model is valid. As mentioned, the input with higher $F$-value is more sensitive parameter. According to $F$-values in table 11, the expectation of product demand and the variance of product demand are the most sensitive and the least sensitive parameters respectively. This is because of $\left(F_{E}=8.70\right) \geq\left(F_{P}=2.99\right) \geq\left(F_{V}=0.35\right)$, where $F_{E}, F_{V}$, and $F_{P}$ are $F$ - values in the cases E, V, and P respectively. Experimentally, we concluded that changes in the parameters including number of machines, number of periods, confidence level, the range in which the expectation and variance of product demands are randomly generated, can change the sensitivity ranking of the inputs studied in this section.

Table10. Hypotheses needed for sensitivity analysis using ANOVA

| Case | Case Description | Hypothesis |
| :---: | :---: | :---: |
|  | $E^{\prime}=E+r^{*} E$ | $H_{0}^{E}: \mu_{1}^{E}=\mu_{2}^{E}=\ldots .=\mu_{9}^{E}$ |
| E | $V^{\prime}=V$ |  |
|  | $p=0.75$ | $H_{1}^{E}:$ At least two means are different |
|  | $E^{\prime}=E$ | $H_{0}^{V}: \mu_{1}^{V}=\mu_{2}^{V}=\ldots=\mu_{9}^{V}$ |
| V | $V^{\prime}=V+r^{*} V$ |  |
|  | $p=0.75$ | $H_{1}^{V}:$ At least two means are different |
|  | $E^{\prime}=E$ | $H_{0}^{P}: \mu_{1}^{P}=\mu_{2}^{P}=\ldots .=\mu_{9}^{P}$ |
| P | $V^{\prime}=V$ |  |
| $p=r$ | $H_{1}^{P}:$ At least two means are different |  |

Table11. Results of ANOVA

| Case | Source | Sums of Squares (SS) | Degrees of Freedom (df) | Mean Squares (MS) | $\begin{gathered} \mathrm{F}= \\ \mathrm{SS} / \\ \mathrm{df} \end{gathered}$ | $\begin{gathered} \text { Prob > F } \\ (\mathrm{P}- \\ \text { Value }) \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| E | Columns | $1.232 \mathrm{e}+017$ | 8 | $1.54 \mathrm{e}+016$ | 8.70 | 2.013e-10 |
|  | Error | $1.574 \mathrm{e}+018$ | 891 | $1.77 \mathrm{e}+015$ |  |  |
|  | Total | $1.697 \mathrm{e}+018$ | 899 |  |  |  |
| V | Columns | $2.904 \mathrm{e}+015$ | 8 | $3.63 \mathrm{e}+014$ | 0.35 | 0.9305 |
|  | Error | $9.344 \mathrm{e}+017$ | 891 | $10.48 \mathrm{e}+014$ |  |  |
|  | Total | $12.24 \mathrm{e}+017$ | 899 |  |  |  |
| P | Columns | $1.506 \mathrm{e}+016$ | 8 | $1.88 \mathrm{e}+015$ | 2.99 | 0.0034 |
|  | Error | $5.598 \mathrm{e}+017$ | 891 | $6.28 \mathrm{e}+014$ |  |  |
|  | Total | $5.749 \mathrm{e}+017$ | 899 |  |  |  |

## 3-5-The effect of demands correlation and interest rate on total cost

In this section, the effect of assuming dependent part demands and time value of money (interest rate) on the total cost of the proposed model is investigated. To this end, a numerical example with input data given in table 12 is applied to the model. For the known solution [123], the values of the objective function of the proposed model is calculated by considering different $p$ percentile levels in the three following cases: (i) independent demands with no interest rate, (ii) dependent demands with no interest rate, (iii) independent demands with non-zero interest rate. Regarding other data,this problem includes two periods and three equal-sized machines placed in a line with a unit distance between each two consecutive ones. For each part, transfer batch size and movement cost are assumed to be fifty and five respectively. The results are shown in table 13.

Table12. Example for analysing demands correlation and interest rate

| Part <br> Number | Variance- Covariance Matrix |  | Expectation of part <br> demand |  | Machine <br> sequence |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 1 | 2 | 3 | Period 1 | Period 2 |  |
| 1 | 10,000 | 640 | 4000 | 1000 | 1500 | $1 \rightarrow 2 \rightarrow 3$ |
| 2 |  | 100 | 4000 | 10,000 | 15,000 | $2 \rightarrow 3$ |
| 3 |  |  | 2500 | 5,000 | 7500 | $1 \rightarrow 2$ |
| Machine relocating cost $=1000$ |  |  |  |  |  |  |

Table13. Total cost for three cases

| $P$ <br> Case | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.9 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| i | 3763.6 | 3857.1 | 3926.9 | 3983.8 | 4009 | 4094.3 | 4151.3 | 4225 | 4314.4 |
| ii | 3649.6 | 3782.9 | 3879.9 | 3961.8 | 4037.5 | 4116.3 | 4198.1 | 4295.1 | 4428.4 |
| iii | 4344.7 | 4471 | 4564 | 4640.3 | 4712.5 | 4787.4 | 4865.1 | 4957.9 | 5085.3 |

Using the results, the curve of the total cost with respect to confidence level is plotted in figure1. The figure indicates that a non zero interest rate leads to increase in the total cost over the range of uncertainty. As shown in the figure, considering $50 \%$ percentile level $(p=0.5)$, the cost function has the same value in both cases of independent and dependent demands, because this percentile level, which is equivalent of $z_{p}=0$, leads to ignoring the second term of the objective function of the proposed model given in equation (32). According to the equation, the second term of the objective functions is variance of MHC, which is function of demands correlation indicated by covariance. Therefore, by ignoring this term, demands correlation does not affect the total cost. In other words, if the user defined percentile level is equal to 0.5 , independency or dependency of demands does not affect the total cost. It is necessary to state that, in equation (32), in the case of independent and dependent demands, the term of covariance is zero and non-zero respectively. Besides, the total cost is decreased for $p<0.5$ (equivalent of $z_{p}<0$ ) and it is increased for $p>0.5$ (equivalent of $z_{p}>0$ ) percentile levels by considering dependent demands.


Figure1.Demands correlation and time value of money

## 4- Conclusion

This paper proposed a new nonlinear QAP-based mathematical model for concurrent design of robust inter and intra-cell layouts in uncertain dynamic (multi-period) environments of manufacturing systems. In the proposed model, in addition to considering time value of money, the product demands have been presumed to be dependent normally distributed random variables with known expectation, variance, and covariance that change from period to period at random. This model has beenverified and validated by solving a number of different-sized test problems and doing sensitivity analysis by using the ANOVA technique.Since the proposed model is an NP-Complete COP, SA intelligent approach has been used for solving the problems. To evaluate the performance of the SA algorithm, four small-sized test problems have been solved using both of the DP and SA algorithms.The validation process has been ended by investigatingthe effect of assuming dependent part demands and time value of money (interest rate) on total cost.The obtained conclusions can be summarised as follows: (i) the SA algorithm has a performance as good as the DP algorithm from solution quality point of view; (ii) SA algorithm is better than the DP algorithm from computational time standpoint;(iii) according to the statistical evaluation, the objective function values are pretty close to each other and thereforethe SA is a robust algorithm; (iv) sensitivity analysis indicated that different values of input parameters containing expectation of product demands, variance of product demands, and confidence level lead to design of different facility layouts;(v)the expectation and the variance of product demands are the most sensitive and the least sensitive parameters respectively. However, changes in the parameters including the number of machines, the number of periods, the confidence level, the range in which the expectation and variance of product demands are randomly generated, can change the sensitivity level of the inputs; (vi) considering nonzero interest rate leads to increase in the total cost over the range of uncertainty; (vii) the total cost is decreased for $p<0.5$ (equivalent of $z_{p}<0$ ) and it is increased for $p>0.5$ (equivalent of $z_{p}>0$ ) percentile levels by considering dependent demands;(viii) Regarding the application of the proposed model,it can be used in both of the stochastic and deterministic environments. The real world problemstudiedin Sub-section 3-3 is an example of a deterministic case and the problems solved in other sub-sections are samples of the stochastic case;(ix) In addition, since the proposed model has been developed based on the QAP formulation, it can be applied to any manufacturing systems, particularly the modern ones such as cellular and flexible manufacturing systems having equal-sized facilities.The Raytheon Aircraft Company discussed in Sub-section 3-3 and the Vought Aerospace Company in Dallas, Texas are two real world examples of such systems (Groover, 2008). Finally, the following works can be taken into consideration in the future researches:(i) concurrent design of a dynamic inter and intra-cell layout so that the best layout of each period is found; (ii)considering some constraints such as unequal-sized machines/cells, adding and removing machines in different periods, closeness ratio, aisles, routing flexibility, and budget constraint for the total cost.

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