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An integrated production and preventive maintenance planning model with imperfect maintenance in multi-state system

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Abstract
Production planning and maintenance are two important problems in manufacturing systems. Despite the relationship exists between these two problems due to sudden failures and production capacity occupied by maintenance activities, each of these problems planned separately and as a result program and model efficiencies reduce in the real world. The aim of integrated production and maintenance planning models is to plan production and maintenance simultaneously due to the interaction of these two programs on one another. In this study, an integrated model is provided for planning production and preventive maintenance in a multi-state system. This model can be used to determine the production planning variables such as time to setup, the levels of production, inventory and shortages simultaneously with maintenance planning variables such as time and type of preventive maintenance. This research considers the concept of imperfect maintenance in modeling the preventive maintenance variable. At the end, a numerical example is used to assess the performance of the model. It is shown that using proposed model with two type of preventive maintenance will reduce the total cost of integrated problem.
Keywords: Production planning, preventive maintenance, multi-state system, integrated model

1-Introduction

Production Planning is an activity that responds to the goals of production (customer demands) by making maximum use of existing capacities in a specific period of time (planning horizon) (Karimi, Fatemi Ghomi and Wilson, 2003). Based on the length of planning horizon, production planning divided into three levels: strategic planning level, tactical planning level and operational planning level. In long term planning with duration of one year or more, the aggregate demand is predicted and the overall decisions on how to design the production system, classification of goods, materials and required equipments are taken. In medium term planning with duration of one month or more, decisions associated with material requirement planning (MRP), level of production or lot size (LS) and general maintenance policy are made. In short term planning with duration of one week or less, the daily scheduling of operations and preventive maintenance activities could be taken according to medium term plans and customer's demand (Liu, Wang and Peng, 2015).

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Production planning at the tactical level can be studied on the basis of three factors: number of products, system capacity and type of demand. Depending on demand type, existing models can be categorized into two classes: economic production quantity (EPQ) models with fixed demand at each period and lot sizing models with certain but variable demand at each period (Gehan, Castanier and Lemoine, 2014).

Maintenance is a set of activities to improve or maintain the safety performance, reliability and availability of a single component or an entire production system in a given period of time (Khoshalhan and Cheraghali Khani, 2012). There are three types of maintenance activities: preventive, corrective and emergency maintenance. Preventive maintenance performed as a planned activity such as setup, inspection, overhaul and lubrication to reduce system downtime or prevent the run-time failures, while corrective maintenance is performed at a state of failure occurrence in a form of activities like repair, replacement and equipment recovery (Ashayeri, Teelen and Selenj, 1996), (Najid, Alaoui-Selsouli and Mohafid, 2011).

Although there is a connection between production and maintenance planning, decisions are made separately due to lack of communication and conflicting goals between these two programs (Najid, Alaoui-Selsouli and Mohafid, 2011), (Weinstein and Chung, 1999). Generally in failure-prone systems, production systems with expensive equipments and just-in-time (JIT) production systems failure occurrence, improper maintenance policy and system breakdown could cause failure to produce, loss of capacity and time, increase defect in product's quality and impose additional and unnecessary costs such as downtime and rescheduling costs (Aghezzaf and Najid, 2008),(Sitompul and Aghezzaf, 2001). In order to solve these problems and reduce the production planning errors, integrating the production planning problem with maintenance problem has been interested in early years. In other words the aim of integrated production and maintenance planning problem is to use available production capacity efficiently, optimize the efficiency of the machines, meet the demand closely and hence to reduce the total production and maintenance cost and plan precisely (Yildirim and Ghazi Nezami, 2014).

The remainder of the paper is formed as follows. A literature review is presented in Section 2. In Section 3 the proposed model is described. Computational results are addressed in Section 4. Discussion is added in Section 5 and finally the conclusion and future suggestions would be presented in Section 6.

2- Literature Review

Najid et al. (2011) have classified the integrated production and maintenance planning problem into five different levels; It could be updated into these six levels:

- Stochastic models; in which the machine state or the level of system buffer is modeled by Markov chain. The topic of control in stochastic production systems is a subset of this area in which the rate of production and preventive maintenance are determined along with minimizing the total cost of production, inventory, backlog and maintenance.
- Production system with inventory buffer models; in which the time of implementing the preventive maintenance and the level of buffer inventory is determined in order to satisfy the demand when the preventive maintenance is being carried out.
- Preventive maintenance policy with production quality control models; in which the control limits and the condition of preventive maintenance are determined by considering the time when conforming or nonconforming products are produced.
- Continuous-time models with an infinite time horizon; in which the lot size, the level of inventory and the time of implementing the preventive maintenance are determined with considering the certain or stochastic demand and failure effects in a economic manufacturing quantity (EMQ) system.
- Discrete-time models with a finite time horizon in operational (scheduling) level: in which the sequence of production activities and preventive maintenance are determined in short term.
- Discrete-time models with a finite time horizon in tactical level: in which the lot size and time of implementing the preventive maintenance are determined in long term.

Our research will be included in the final field. Models in this area according to their production planning model could be categorized into three fields: models with multi-item capacitated lot sizing problem (MICLSP), models with aggregate production planning problem (APP) and compilation, qualitative and case studies.

Maybe the first study in the area of integrated models with multi-item capacitated lot sizing problem is the work of Ashayeri et al. (1996). They proposed a model for a system that contains the number of production lines which can be replaced. Each line has its specific number of machines that one of them has a bottleneck feature. Also they used conditional maintenance policy to model the maintenance problem. In the end they presented a special branch and bound procedure to find a feasible solution in a reasonable amount of time (Ashayeri, Teelen and Selenj, 1996). Pixopoulou & Papageorgiou (2004) extended the model proposed by Ashayeri et al. by adding hazard rate to the variables of the problem and using run-based preventive maintenance in modeling the system. Aghezzaf et al. (2007) introduced a procedure to model the integrated problem that has been referenced in many studies as the most important and basic research in the area of integrated models with MICLSP problem. They used an available production capacity concept to link the production and maintenance problem; in this way that either corrective or periodic preventive maintenance consumes a special amount of nominal production capacity of a single production line. For solving the problem they proposed an iterative algorithm to numerate the length of preventive maintenance variable and solve the remaining lot sizing problem by exact or heuristic methods. Aghezzaf & Najid (2008) extended the work of Aghezzaf et al (2007) by adding parallel production line to the system, studying the periodical and general preventive maintenance and solving the problem by a heuristic procedure based on Lagrangean relaxation method. Alaoui et al. (2010) discussed some Lagrangean heuristic methods to solve and find the upper bounds of the solution in a conference paper. Later Alaoui-Selsouli et al. (2012) published their work in a journal paper. Najid et al. (2011) extended the base model by adding lost sale concept to the production problem and using time window concept to model the periodic preventive maintenance. Lu & Zhang (2011) added reliability constraint to the base model. They used run-based preventive maintenance to model the problem and deleted corrective maintenance activities. Lu et al. (2013) extended their works in Lu & Zhang (2011) by discussing a three stage heuristic procedure for solving the large scale integrated problem. Nourelfath & Chatelet (2012) proposed a joint production and maintenance model by discussing dependence and common cause failures between machines. Fitouhi & Nourelfath (2012) used noncyclical preventive maintenance policy to model the integrated problem in a production system which contains only a single machine. Their approach to calculate the available production capacity was unique since they used effective age concept to calculate the total time and total cost of maintenance. Limere et al. (2012) corrected the base model of Aghezzaf et al. (2007) by adding a binary variable for not implementing the corrective maintenance in the periods which there is no production activity.

Multi-State systems are one of the recent emerging fields in production and reliability areas. One of the conditions which the system is called multi-state is when the system contains of several machines with general configuration (such as series, parallel, series-parallel, network and etc) that they have a cumulative effect on system's performance (Machani and Nourelfath, 2010). The first study to discuss the multi-state system in integrated problem with MICLSP is the conference paper of Machani & Nourelfath (2010). They used the approach which is introduced by Fitouhi & Nourelfath (2012) and they proposed the heuristic procedure based on Genetic Algorithm (GA) to solve the problem in large scale environment. After that Nourelfath et al. (2010) published their work in a journal paper by adding an exact algorithm for solving the problem in small scale cases (Nourelfath, Fitouhi and Machani, 2010). Machani & Nourelfath (2012) extended their work by using Variable Neighborhood Search Algorithm (VNS) to approximately solve the problem. Fitouhi & Nourelfath (2014) proposed a model for a system with general preventive maintenance policy that preventive maintenance activities, instead of beginning, can be done during or at the end of production period. They also used Simulated Annealing Algorithm (SA) to solve the model in large scale cases.

One of three researches on imperfect preventive maintenance is the work of Wang (2013). He added the imperfect maintenance concept to base model of Aghezzaf et al (2007). His model consists of imperfect preventive maintenance variable in addition to perfect preventive maintenance variable (for lowering the hazard rate in long term) and corrective maintenance. Yildirim & Ghazi Nezami (2014) implemented a unique approach to model the integrated problem with imperfect preventive maintenance in a single machine production system. They studied the impact of machine performance degradation on processing times and also discussed reliability and energy consumption in their model. Aghezzaf et al. (2016) used hybrid hazard rate function to pattern the imperfect preventive maintenance in a single machine production system.

Yalaoui et al. (2014) developed some exact and heuristic algorithms for improving the speed and quality of the obtained solutions in a production system with parallel lines. They also modeled the system by considering the length of periodic preventive maintenance as a constant parameter. Zhao et al. (2014) used operation-dependent failure (ODF) concept to model the integrated problem with unequal time periods. They also proposed an iterative method to approximately solve the model. Gehan et al. (2014) focused on feasibility of the production plan in the integrated model; hence they added feasibility constraint into the model and didn't consider the corrective maintenance in their problem. They also added setup times into calculating the production times. The second research on considering the setup time in modeling the integrated problem is the work of Liu et al. (2015). They used delay-time based preventive maintenance concept in modeling the maintenance problem and added unqualified products to the production problem.

For models with aggregate production planning problem, we refer to the works of Weinstein & Chung (1999), Sitompul & Aghezzaf (2011), Portioli-Staudacher & Tantardini (2012), Khoshalhan & Cheraghali Khani (2012) and Mehdizadeh & Atashi Abkenar (2014).

For compilation, qualitative and case studies, we refer to the works of Rishel & Christy (1996), Pistikopoulos et al. (2001), Goel et al. (2003), Nikolopoulos et al. (2003), Bergeron et al. (2009), Powell & Rodseth (2013), Haoues et al. (2013), Wolter & Helber (2013), Cazanas & Sobrino (2014), Macchi et al. (2014),Liu et al. (2014) and Nourelfath et al. (2016).

Although vast amount of studies focused on modeling the perfect preventive maintenance in integrated problem, there are little existing studies dealt with imperfect preventive maintenance (especially non in multi-state system). Ahead study provides an integrated mixed integer model for planning production and preventive maintenance in a multi-state system. This model considers the concept of imperfect maintenance in modeling the preventive maintenance variable.

3- The mathematical model

3-1- Problem description

A production process consists of set of components (machines) that arranged according to a given configuration is considered. We suppose the production system is a generic multi-state system (MSS) with general configuration (such as series, parallel, series-parallel, network and etc).The system has different levels of performance from complete total performance to complete failure since each component j has a binary state (i.e. either good or failed) and its own nominal performance rate G_j. This system should produce set of P items (products) during a given planning horizon H including T periods. Each production time period t has a fixed and equal length L that could be divided into several maintenance sub-period m with fixed length τ for performing the preventive maintenance.

The maintenance policy is based on implementing the corrective and preventive maintenance on each component. Corrective maintenance is performed as a minimal repair whenever an unplanned failure occurs to change the state of a component to "as bad as old". Preventive maintenance is carried out noncyclical and divided into two type of perfect and imperfect. The imperfect preventive maintenance is performed as a planned repair to change the state of a component to "part of as good as new"; whereas the perfect preventive maintenance is performed as a planned overhaul to change the state of a component to "as good as new".

Other main assumptions of the problem are listed as follows:

- States of the system are considered independent from each other.
- Nominal performance rate of the Components are assumed independent from type of item.
- No economical, stochastic and structural dependency are considered for components.
- Failure rate of each component is fixed and calculated via failure probability distribution.
- Each component has expected number of failures that conforms the non homogeneous Poisson process in any time interval.

- Implementation of Preventive maintenance activities takes place during each maintenance sub-period.
- Shortage of the items is considered as lost sale.
- No initial stock or lost sale is assumed.

3-2- Nomenclature

Indices

Q : set of items (products).
J : set of components (machines).
A : set of production planning periods.
B : set of maintenance planning sub-periods.
p : item (product) index.
j : component (machine) index.
t : production planning period index.
m : maintenance planning sub-period index.

Model parameters

d_{pt}: Demand of item p to be satisfied at the end of period t.
H : Time horizon.
L : Length of production planning period t.
τ : Length of maintenance planning sub-period m.
τ^{tm} : The m^{th} maintenance planning sub-period of the production planning period t.
a_j^{tm} : Effective age function of component j at the end of maintenance planning sub-period
τ^{tm} : The m^{th} maintenance planning sub-period of the production planning period t.
A_j^t : Availability of the component j during the production planning period t.
M_j^{tm} : Expected number of failures for component j during the maintenance planning sub-period τ^{tm}.
G_j : Nominal production rate of component j.
G_{MSS}^t : MSS total available production capacity during the period t.
$r_j(.)$: Hazard function of component j.
$f_j(.)$: Failure probability distribution of component j.
α : Age function recovery percentage (age reduction) of component j under imperfect preventive maintenance.
TMR_j : Minimal repair time for component j.
TPR_j : Imperfect preventive maintenance time for component j.
TOR_j : Perfect preventive maintenance time for component j.
CMR_j : Minimal repair cost for component j.
CPR_j : Imperfect preventive maintenance cost for component j.
COR_j : Perfect preventive maintenance cost for component j.
π_{pt} : Cost of producing one unit of item p in period t.
Set_{pt} : Fixed setup cost of producing item p in period t.
h_{pt} : Inventory holding cost per unit of item p by the end of period t.
b_{pt} : Lost sale cost per unit of item p by the end of period t.

Decision Variables

x_{pt} : Quantity of item p to be produced in period t.

y_{pt} : Binary variable, which is equal to1 if the setup of item p occurs at the end of period t, and 0 otherwise.

I_{pt} : Inventory level of item p at the end of period t.

B_{pt} : Lost sale level of item p at the end of period t.

z_j^{tm} : Binary variable, equal to 1 if an imperfect preventive maintenance is carried out on component j at the beginning of the maintenance planning sub-period τ^{tm}, and 0 otherwise.

δ_j^{tm} : Binary variable, equal to 1 if a perfect preventive maintenance is carried out on component j at the beginning of the maintenance planning sub-period τ^{tm}, and 0 otherwise.

3-3- The model

Each production planning period has a fixed length of L which is divided into m equal maintenance planning sub-period of τ^{tm}. The length τ^{tm} could be calculated as follow:

$$\tau^{tm} = \frac{L}{M} = \tau \qquad \forall t \in A, m \in B \tag{1}$$

Beside the hazard function of component j at the time x is obtained from equation (2):

$$r_j(x) = \frac{f_j(\mathrm{x})}{1 - F_j(x)} \qquad \forall j \in J \tag{2}$$

Now in equation (3), the effective age function of component j during the τ^{tm} sub-period is calculated as regard as implementing the perfect or imperfect preventive maintenance:

$$a_j^{tm} = \begin{cases} (1-\delta_j^{tm})(1-\alpha \mathrm{z}_j^{tm}) & if \;\; j = 1,2,...,\mathrm{n}, \; t = 1, \; m = 1 \\ (1-\delta_j^{tm})(1-\alpha \mathrm{z}_j^{tm})(a_j^{t(m-1)} + \tau) & for \; j = 1,2,...,n, \; t = 1,2,...,T, \; m = 2,3,...,M \\ (1-\delta_j^{tm})(1-\alpha \mathrm{z}_j^{tm})(a_j^{(t-1)M} + \tau) & for \; j = 1,2,...,n, \; t = 2,3,...,T, \; m = 1 \end{cases} \tag{3}$$

Given the fact that expected number of failures for each component in a given time interval follows the non homogeneous Poisson process and by adding the (2) and (3) equations, the expected number of failures for component j during the τ^{tm} sub-period considered as follow:

$$M_j^{tm} = \int_{a_j^{tm}}^{a_j^{tm}+\tau} r_j(x)dx \qquad \forall j \in J, t \in A, m \in B \tag{4}$$

Hence the total maintenance cost resulted as below:

$$MC = \sum_{j=1}^{n} \sum_{t=1}^{T} \sum_{m=1}^{M} \left(CMR_j M_j^{tm} + CPR_j z_j^{tm} + COR_j \delta_j^{tm} \right) \tag{5}$$

For computing the MSS total available production capacity during the period t, we need to determine the value of availability for each component in every production planning period. These values could be obtained by the same way as the total maintenance cost is calculated:

$$A_j^t = \frac{L - \sum_{m=1}^{M}\left(TMR_j M_j^{tm} + TPR_j z_j^{tm} + TOR_j \delta_j^{tm}\right)}{L} \quad \forall j \in J, t \in A \tag{6}$$

As a result, by knowing the MSS configuration and using the Universal Generating Function (UGF), which was developed by Levitin (2005). The MSS total available production capacity is resulted as follow:

$$G_{MSS}^t = \sum_{j=1}^{J} G_j A_j^t \qquad \forall t \in A \tag{7}$$

Finally the integrated production and preventive maintenance planning model is described as below:

$$Min \;\; z = \sum_{p=1}^{P}\sum_{t=1}^{T}\left(h_{pt} I_{pt} + b_{pt} B_{pt} + \pi_{pt} x_{pt} + Set_{pt} y_{pt}\right) + MC \tag{8}$$

$S.t.$

$$x_{pt} - I_{pt} + I_{p(t-1)} + B_{pt} = d_{pt} \qquad \forall p \in Q, t \in A \tag{9}$$

$$x_{pt} \le \left(\sum_{t=1}^{T} d_{pt}\right) y_{pt} \qquad \forall p \in Q, t \in A \tag{10}$$

$$\sum_{p=1}^{P} x_{pt} \le G_{MSS}^t L \qquad \forall t \in A \tag{11}$$

$$z_j^{tm} + \delta_j^{tm} \le 1 \qquad \forall j \in J, t \in A, m \in B \tag{12}$$

$$B_{p0} = 0 \;\; \& \;\; I_{p0} = 0 \qquad \forall p \in Q \tag{13}$$

$$x_{pt}, I_{pt}, B_{pt} \ge 0 \qquad \forall p \in Q, t \in A \tag{14}$$

$$y_{pt} = 0 \; or \; 1 \qquad \forall p \in Q, t \in A \tag{15}$$

$$z_j^{tm}, \delta_j^{tm} = 0 \; or \; 1 \qquad \forall j \in J, t \in A, m \in B \tag{16}$$

Equation (8) represents the model objective function, which is the sum of inventory holding, shortage, production, setup and maintenance costs.

The first constraint (9) is the production planning balance constraint which relates the inventory and lost sale levels to the demand and production level. Constraint (10) shows the relation between production level and setup variables. Equation (11) is the total available production capacity constraint; it dictates that the production level of all items in period t should not exceed the amount of total available production capacity in that same period. Fourth constraint (12) corresponds to the preventive maintenance variables and shows that no imperfect and perfect preventive maintenances could be performed simultaneously on a same component during any sub-period. Constraint (13) shows that there is no initial stock or lost sale. Constraints (14) to (16) show the non-negativity and binary aspect of the model decision variables.

4- Numerical Example

4-1- Problem Data

A multi-state production system that contains 2 binary state parallel components is considered. This system produces 2 items in 4 months. The production planning period determined monthly (T=4), hence the planning horizon is 4 (H=4). Each production planning period is divided into 3 ten day sub-period maintenance planning period, so the length of maintenance planning sub-period is 0.33 (M=3).

The characteristic and the lifetime distribution of system's components along with the cost and duration of each maintenance activity are shown in (Table 1).

Table 1. Characteristics and maintenance activity requirements for system components

Comp. j	CPR_j (\$)	CMR_j (\$)	COR_j (\$)	TPR_j (week)	TMR_j (week)	TOR_j (week)	Lifetime distribution	G_j (item/month)
1	2500	5000	5000	0.09	0.05	0.18	Weibull (2,2)	105
2	2000	5100	4000	0.08	0.04	0.16	Weibull (2,2)	110

The demand of each item is estimated for 4 production period in (Table 2).

Table 2. Demand of items

	Item	t=1	t=2	t=3	t=4
d_{pt} (item)	1	95	93	90	95
	2	80	84	87	82

(Table 3) shows the inventory, shortage, setup and production costs of each item. Costs are the same for all production periods.

Table 3. Inventory, shortage, setup and production costs of items

Item	h_{pt} (\$)	b_{pt} (\$)	Set_{pt} (\$)	π_{pt} (\$)
1	40	150	1000	100
2	40	150	1000	100

Finally age function recovery percentage is considered 50% (α=0.5).

4-2- Linearization

Due to system configuration and Universal Generating Function (UGF), MSS total available production capacity of the example is calculated from equation (17).

$$G_{MSS}^{t} = (105+110)A_1^t A_2^t + 105A_1^t\left(1-A_2^t\right) + 110\left(1-A_1^t\right)A_2^t \qquad \forall t \in A \tag{17}$$

By expanding the equation (17), linear equation (18) is achieved.

$$G_{MSS}^{t} = 105A_1^t + 110A_2^t \qquad \forall t \in A \tag{18}$$

On the other hand due to lifetime distribution of components, the expected number of failures for each component is achieved through equation (19).

$$M_j^{tm} = \int_{a_j^{tm}}^{a_j^{tm}+\tau} \frac{x}{2} dx = \frac{(a_j^{tm}+\tau)^2 - (a_j^{tm})^2}{4} \qquad \forall j \in J, t \in A, m \in B \tag{19}$$

And same way, by expanding the equation, linear equation (20) is resulted.

$$M_j^{tm} = \frac{2\tau.a_j^{tm} + \tau^2}{4} \qquad \forall j \in J, t \in A, m \in B \tag{20}$$

For linearizing the effective age equations, we investigate the possible condition for every sub-equation. For instance, all the possible states for second sub-equation of equation (3) are shown below;

$$a_j^{tm} = \begin{cases} 0 & \delta_j^{tm} = 1,\ z_j^{tm} = 1 \\ 0 & \delta_j^{tm} = 1,\ z_j^{tm} = 0 \\ (1-\alpha).(a_j^{t(m-1)} + \tau) & \delta_j^{tm} = 0,\ z_j^{tm} = 1 \\ (a_j^{t(m-1)} + \tau) & \delta_j^{tm} = 0,\ z_j^{tm} = 0 \end{cases} \qquad \forall\, j \in J,\ t \in A,\ m \in B - \{1\} \tag{21}$$

We can summarize the states into quadratic equation (22);

$$a_j^{tm} \ge -\delta_j^{tm}.\overline{M} + (a_j^{t(m-1)} + \tau) - \alpha\, z_j^{tm}\,(a_j^{t(m-1)} + \tau) \qquad \forall\, j \in J,\ t \in A,\ m \in B - \{1\} \tag{22}$$

In which $\overline{M}$ is a big number. Equation (22) is nonlinear too. Now by defining variable V_j^{tm}, the nonlinear second sub-equation of equation (3) is turned into set of linear equations (24).

$$V_j^{tm} = z_j^{tm}\,(a_j^{t(m-1)} + \tau) \tag{23}$$

$$\begin{aligned} & a_j^{tm} \ge -\delta_j^{tm}.\overline{M} + (a_j^{t(m-1)} + \tau) - \alpha V_j^{tm} && \forall\, j \in J,\ t \in A,\ m \in B - \{1\} \\ & V_j^{tm} \le z_j^{tm}.\overline{M} && \forall\, j \in J,\ t \in A,\ m \in B - \{1\} \\ & V_j^{tm} \le (a_j^{t(m-1)} + \tau) && \forall\, j \in J,\ t \in A,\ m \in B - \{1\} \end{aligned} \tag{24}$$

The same procedure is applied for first and third sub-equations of equation (3). Finally by adding mentioned equations to the model (10)-(20), the linearized integrated robust model is resulted as below:

$$Min\ \ z = \sum_{p=1}^{P} \sum_{t=1}^{T} \left(h_{pt} I_{pt} + b_{pt} B_{pt} + \pi_{pt} x_{pt} + Set_{pt} y_{pt} \right) + MC \tag{8}$$

$S.t.$

$(9)-(16)$

$$A_j^t = \frac{L - \sum_{m=1}^{M} \left(TMR_j M_j^{tm} + TPR_j z_j^{tm} + TOR_j \delta_j^{tm} \right)}{L} \qquad \forall j \in J, t \in A \tag{6}$$

$$G_{MSS}^t = 105 A_1^t + 110 A_2^t \qquad \forall t \in A \tag{18}$$

$$M_j^{tm} = \frac{2\tau . a_j^{tm} + \tau^2}{4} \qquad \forall j \in J, t \in A, m \in B \qquad (20)$$

$$V_j^{tm} \leq z_j^{tm} . \overline{M} \qquad \forall\, j \in J,\, t \in A,\; m \in B - \{1\}$$

$$V_j^{tm} \leq (a_j^{t(m-1)} + \tau) \qquad \forall\, j \in J,\, t \in A,\; m \in B - \{1\} \qquad (24)$$

$$a_j^{tm} \geq -\delta_j^{tm} . \overline{M} + (a_j^{t(m-1)} + \tau) - \alpha V_j^{tm} \qquad \forall\, j \in J,\, t \in A,\; m \in B - \{1\}$$

$$a_j^{tm} \geq -\delta_j^{tm} . \overline{M} + (1 - \alpha\, z_j^{tm}) \qquad \forall\, j \in J, t = 1, m = 1 \qquad (25)$$

$$V_j^{tm} \leq z_j^{tm} . \overline{M} \qquad \forall\, j \in J,\, t \in A - \{1\},\, m \in B$$

$$V_j^{tm} \leq (a_j^{(t-1)M} + \tau) \qquad \forall\, j \in J,\, t \in A - \{1\},\, m \in B \qquad (26)$$

$$a_j^{tm} \geq -\delta_j^{tm} . \overline{M} + (a_j^{(t-1)M} + \tau) - \alpha V_j^{tm} \qquad \forall\, j \in J,\, t \in A - \{1\},\, m \in B$$

$$a_j^{tm}, A_j^t, G_{MSS}^t \geq 0 \qquad \forall j \in J, t \in A, m \in B \qquad (27)$$

As seen, effective age function (a_j^{tm}), availability (A_j^t) and MSS total available production capacity (G_{MSS}^t) are defined as additional variables and their related equations as additional constraints to retain the model integrality and also to enable solving the model through GAMS software. It must be mentioned that if the lifetime distribution of components follows the exponential or Weibull (α,β) (with β parameter smaller than 2) distributions, linear solvers (e.g. CPLEX) can be used to solve the model; otherwise nonlinear solvers (e.g. BARON) are suggested for solving the model.

4-3- Computational Results

The linearized model is solved by GAMS software using CPLEX solver on an i5 3.30 GHz processor. Results are shown in (Table 4 – 7).

Table 4. Imperfect and perfect preventive maintenance variables

	Component	Period	m=1	m=2	m=3
(z_j^{tm},δ_j^{tm})	1	1	(0,0)	(0,0)	(0,1)
		2	(0,0)	(0,0)	(0,0)
		3	(0,0)	(0,1)	(0,0)
		4	(0,0)	(0,0)	(0,0)
	2	1	(0,0)	(0,0)	(0,1)
		2	(0,0)	(0,0)	(0,0)
		3	(0,0)	(0,1)	(0,0)
		4	(0,0)	(0,0)	(0,0)

Table 5. Production level of items

	Item	t=1	t=2	t=3	t=4
x_{pt} (item)	1	95	93	90	95
	2	79	86	85	82

Table 6. Inventory and shortage level of items

	Item	t=1	t=2	t=3	t=4
(I_{pt},B_{pt}) (item)	1	(0,0)	(0,0)	(0,0)	(0,0)
	2	(0,1)	(2,0)	(0,0)	(0,0)

Table 7. Effective age function of components

	Component	Period	m=1	m=2	m=3
a_j^{tm}(item)	1	1	1	1.33	0
		2	0.33	0.66	0.99
		3	1.32	0	0.33
		4	0.66	0.99	1.32
	2	1	1	1.33	0
		2	0.33	0.66	0.99
		3	1.32	0	0.33
		4	0.66	0.99	1.32

According to (Table 5) model decided to produces each item in all production periods. So the value of y_{pt} variables for each item in each production period is equal to 1.

The MSS total available production capacity in first period is 174, in second period is 211.06, in third period is 175.09 and in fourth period is 209.48. Total maintenance cost is $36181.52 and total cost of integrated model is $114911.52.

5- Discussion

One of the important parameters of the presented model is the age function recovery percentage parameter (α). For the small values of α, the preventive maintenance is too imperfect and the cost of corrective maintenance increases. Hence the model decides the preventive maintenance to be performed perfectly. Otherwise for large values of α, model turns to employ imperfect preventive maintenance. Sensitivity analysis for α is presented in (Table 8).

Table 8. Sensitivity analysis for age function recovery percentage parameter

α (%)	Maintenance cost ($)	Total integrated cost ($)
10	36181.52	114911.52
20	36181.52	114911.52
30	36181.52	114911.52
40	36181.52	114911.52
50	36181.52	114911.52
60	36181.52	114845.62
70	34832.90	113432.90
80	31814.44	110414.44
90	29083.64	107683.64
100	26665.43	105265.43

In another aspect, the efficiency of the model could be compared with integrated models with only one type of perfect or imperfect preventive maintenance. Result of comparing the total integrated costs is shown in (Table 9). It could be derived that for α equal to 0.59 and lesser, both the main model and the model with only perfect preventive maintenance have lower costs; meanwhile from values equal to 0.7 and greater, both the main model and the model with only imperfect preventive maintenance have lower costs. In critical area, between 0.6 up to 0.67, the main model has the lowest total cost. Totally it can be concluded that the represented main model obtains the lowest cost for all the values of α.

Table 9. Total integrated cost comparisons for different types of preventive maintenance

α (%)	Total integrated cost for the main model ($)	Total integrated cost for model with imperfect maintenance ($)	Total integrated cost for model with perfect maintenance ($)
10	114911.52	138171.19	114911.52
20	114911.52	135810.47	114911.52
30	114911.52	130144.97	114911.52
40	114911.52	124951.83	114911.52
50	114911.52	120503.41	114911.52
60	114845.62	116703.39	114911.52
70	113432.90	113432.90	114911.52
80	110414.44	110414.44	114911.52
90	107683.64	107683.64	114911.52
100	105265.43	105265.43	114911.52

5-Conclusion

This study introduces an integrated production and preventive maintenance planning model for a multi-state system. The purpose of the model is to determine the level of inventory, shortage and the quantity of the items to be produced along with the time and type of preventive maintenance activities. The main contributions of this study are listed as follows:

- Considering imperfect preventive maintenance in multi-state system with general configuration for integrated model
- Empowering the model to decide between perfect or imperfect preventive maintenance to be performed

Numerical example shows that the proposed model can obtain lowest integrated cost in comparison with similar models using only one type of preventive maintenance strategy. Future works could be reformulating the model for other parameter uncertainties or robust optimization procedures. Using the cyclical, hybrid hazard rate or another imperfect preventive maintenance policy would be an idea for extending the model. Also there could be a study to extend the model for dealing with the case of systems with dependant components.

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