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A decentralized multi-level leader-follower game for network design of a competitive supply chain

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Abstract

This paper develops a decentralized leader-follower game for network design of a competitive supply chain problem in which a new chain as the leader enters market with one existing supply chain as a follower. Both chains produce an identical product, customers' demand is inelastic and customer utility function is based on Huff gravity-based model. The leader wants to shape his network and set assignments where the follower will show reactions by changing her networks in a sequential manner. Multi-level mixed integer nonlinear programming model is used to model the problem. Each chain can enter the market in centralized; decentralized or cooperative modes. Enumeration method is applied to solve the problem by the help of Stackelberg equilibrium concept. Finally, some numerical examples are used to explore the algorithm and different mode structures affect the equilibrium solution.

Keywords: Competitive supply chain network design, leader - follower game, bi-level programming model, multi-level programming model

1- Introduction

Today's fierce competition and varied customers` expectations are changing and promoting the competition from "firms against firms" to "supply chains versus supply chains". Monopoly competition is being gotten away and replaced by duopoly, oligopoly or pure competition types. Developing countries are trying to omit monopoly to enroll in the World Trade Organization (WTO) and a lot of markets with just one existing rival are available for the investors who want to enter the markets, design their chains, and maximize their profits and market shares. So the investors have the opportunity to design their supply chains (SCs) but on the other hand, the existing rivals encounter with the risk of newcomers and lose their markets, they may show reactions to get back to their markets. In this environment, the investors encounter with the questions like: what is their equilibrium network design? How can they consider the reactions of the existing rivals? How much market shares can they obtain? What is their best structure by considering the structure of the existing rival? This paper intends to answer the questions and help the newcomers to design their networks by considering the reactions of the existing rivals.

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Deloitte Consulting (1999), reports that the future competition will be SC against SC instead of firms against firms. As an example of this type of competition, Xiao and Yang (2008) mentioned Microsoft (software supplier) and HTC (device manufacturer) to constitute a SC that competes with the SC constitute of Symbian (software supplier) and Nokia (device manufacturer). A lot of studies show the importance of competition in SC (Farahani et al. 2014) and emphasize that this matter should be considered in all stages of supply chain management including supply chain network design (SCND) that is related to designing the physical structure of a SC.SCND includes three levels of decisions such as:1) strategic level (that is related to long term decisions like location decision), 2) tactical level (like inventory decision) and 3) operational level (like transportation).

SC is a network of suppliers, manufacturers, warehouses and retailers organized to produce and distribute merchandise at the right quantities, locations, and time in order to minimize total costs while satisfying service level requirements (Simchi-Levi et al. 2003). According to this definition there are many independent business entities that try to minimize their cost or maximize their profit. Designing the physical structure of a SC is called SCND that has a tremendous effect on performance and cost reduction of a SC.Shen 2007; Meixell and Gargeya 2005; Beamon 1998 did a review on (SCND) problem also SCND has a tremendous body of literature (Altiparmark et al. 2006; Torabi and Hassini 2008; Pishvaee and Rabbani 2011; Shankar et al. 2013; Badri et al. 2013,Özceylan et al. 2014,Vahdani and Mohamadi 2015,Yang et al 2015, Ardalan et al. 2016,Keyvanshokooh et al. 2016, Özceylan et al. 2016,Jeihoonian et al. 2017, Varsei and Polyakovskiy 2017) that do not consider competition effect on optimization problem which leads to the solution , not an optimum answer.Farahani et al. 2014 have done a review on Competitive SCND (CSCND) and mentioned that considering the impact of competitive markets in designing the network structure of a chain can improve its future competitiveness.

Number of existing SCs in the market, their reactions to the newcomers and also customer utility function and customer demands are the factors that should be considered in CSCND problem. According to the existing rivals, different games can be taken place as monopoly (when there is no rival), duopoly (when there isonly one rival) and oligopoly (when there is more than one rival). Customer utility function is another consideration that is categorized mostly into random utility function and deterministic one. Hotelling (1929) is the first one who introduced deterministic utility model in which customers only visit a facility which gives them the highest utility, in random utility function each facility has a certain chance to be chosen by the customers. HuffGravity-based model, introduced by Huff (1964, 1966), is the most used random utility model in the literature; in this model, the probability that a customer patronizes a facility is proportional to its attractiveness and inversely proportional to the attractiveness of the all existing facilities.

Competition is classified into three groups in the literature: **1)** static competition: in this type of competition there is no rival in the market or does not show any reactions to the new entrance. One can see Berman and Krass (1998), Aboolian et al. (2007), Revelle et al. (2007) and Aboolian et al. (2007). Plastria (2001) has done a review of this kind of competition. **2)** Dynamic competition: this competition is usually carried out between rivals on operational characteristics like price or service levels in which they can adjust the competitive characters simultaneously. This type of competition is usually modeled by unconstraint optimization models and solved by differential systems to reach the Nash equilibrium. One can see Xiao and Yang (2008), Zhang (2006), Godinho and Dias (2013), Godinho and Dias (2010), Sinha and Sarmah (2010), Friesz et al. (2011), Jain et al. (2014), Chen et al. (2015), Nagurney et al. (2015), Santibanez-Gonzalez and Diabat (2016), Hjaila et al. (2017) and Lipan et al (2017). **3)** Stackelberg competition: in this type of competition the rivals show reaction toward strategic characteristics usually in sequential manner.It is also known by competition with foresight or leader-follower game and is modeled by bi-level or multi-level programming. One can see Drezner and Drezner (1998), Plastria and Vanhaverbeke (2008), Kucukaydın et al. (2011), Kucukaydın et al. (2012), Zhang and Liu L.P (2013), Yue and You (2014), Zhu (2015), Drezner et al. (2015), Yang et al. (2015), Hjaila et al. (2016), Aydin et al. (2016) and Genc and Giovanni (2017). Eiselt and Laporte (1997) and Krass and Pesch(2012) have done a review of this kind of competition.

There are three different competition types in the SC context: **1)** horizontal competition: competition between the firms of one tier of a SC. One can see Nagurney et al. (2002), Dong et al. (2004), Cruz (2008),

Zhang and Zhou (2012), Qiang et al (2013); Huseh (2015), Qiang (2015), Li and Nagurney (2016) and Nagurney et al. (2016); **2)** vertical competition: competition between the firms of different tiers of a SC. One can see Bernstein and Federgruen (2005), Anderson and Bao (2010), Chen et al. (2013), Wu (2013), Zhao and Wang (2015), Zhang et al. (2015), Bai et al. (2016), Bo and Li (2016), Li et al. (2016), Huang et al. (2016), Wang et al. (2017), Genc and Giovanni (2017) and Chaeb and Rasti-Barzoki (2017); **3)** SC versus SC: competition between SCs. One can see Boyaci and Gallego (2004), Xiao and Yang (2008), Zhang (2006), Li et al. (2013) and Chung and Kwon (2016).

A little work has been done on CSCND problem; Rezapour and Farahani (2010) developed a model for CSCND by dynamic competition in price dependent market.Rezapour et al. (2011) proposed a model for duopolistic CSCND with sequential acting and variable delivered price. Rezapour and Farahani (2014) proposed a bi-level model for competitive SCND in the market under stochastic price and service level, the inner level determines equilibrium retail price and service level and the outer level determines the network structure. Rezapour et al. (2014) presented a bi-level model for competitive SCND in the market where demand is elastic with respect to price and distance and customer behavior is probabilistic based on these factors. Rezapour et al. (2015) developed a closed-loop CSCND problem in price-dependent market demand by an existing rival chain. Fallah et al. (2015) developed a closed-loop CSCND and investigated the impact of simultaneousness and Stackelberg competitions between the chains. Fahimi et al. (2017) developed a simultaneous CSCND in which two SCs simultaneously enter virgin market and shape their network in centralized, decentralized and cooperative modes along with their combinations.

1-1- Contribution of this paper

In this paper, we present a CSCND problem in which a new SC is going to enter a market where an existing SC will show reactions to his entrance by time sequence so the chains can encounter with a leader-follower game in which the new chain is the leader and the existing one is the follower. Both chains are in decentralized mode and have two levels named by plant and distribution center (DC)and also they produce an identical product. Customer utility function is based on Huff gravity rule model and customer demand is inelastic. Follower reactions are based on the location variables of the plant and DC level and she can open a predetermined number of plants and DCs to get back to the market. On the other hand, similar to Fahimi et al 2017, we consider three main different approaches: centralized versus centralized, decentralized versus decentralized and cooperative versus cooperative models and their possible combination like centralized versus cooperative model. Multi-level mixed integer nonlinear programming is used to model the described environment. Based on our knowledge, there is no solution algorithm in the literature to solve the problem so we construct an algorithm to solve the model based on enumeration method and Stackelberg equilibrium concept. To the best of our knowledge such a multi-level mixed inter nonlinear programming model for CSCND problem has not appeared previously in the literature. Table 1 highlights the contributions of this paper in relation with the most recent and related literature.

Table 1. Literature review

authors	Customer demand		Customer utility function		Competition type		Competition mode			Competition characteristic		Kinds of competition		Solution method
	elastic	inelastic	Hoteling based	Huff based	dynamic	Stackelberg	centralized	Decentralized	cooperative	operational	strategic	duopoly	oligopoly	
Rezapour and Farahani (2010)	✓		✓		✓		✓			✓		✓		Modified projection method
Rezapour et al. (2011)	✓		✓			✓	✓			✓		✓		Simulated annulling and branch and bound
Rezapour and Farahani (2014)	✓		✓		✓		✓			✓		✓		Fractional programming
Rezapour et al. (2014)	✓			✓	✓		✓			✓		✓		Exact and metaheuristic algorithms
Rezapour et al. (2015)	✓		✓		✓		✓			✓		✓		Bi level and modified projection method
Fallah et al. (2015)	✓		✓		✓	✓	✓			✓		✓		GAMS
Fahimi et al. (2017)		✓		✓	✓		✓	✓	✓		✓	✓		Modified projection method
This research		✓		✓		✓	✓	✓	✓		✓	✓		Enumeration method

The rest of this paper is organized as follows: Section 2 is problem definition, Section 3 describes our solution approach, Section 4 is computational result and Section 5 is conclusion.

2- Problem definition

This paper discusses a problem faced by an incoming SC (named by SC1), including plant and DC levels, entering a market with one existing rival chain (named by SC2) and wants to design his network, set the location of his plants and DCs to maximize his market shares and minimize his costs (Fig 1). On the other hand, SC2 will show reactions by time sequence to open a predetermined number of plants and DCs to get back to the market, so a leader-follower game will happen between the chains in which SC1 is the leader and SC2 is the follower. Customer utility function is based on Huff gravity rule model and customer demand is inelastic where both chains are in decentralized mode and produce an identical product. Now consider there is e customer points indexed by k and the leader has m candidate location to open P DCs, if he opens a DC in site j with random attractiveness Q_j and Euclidean distance d_{jk}^2 from customer k, according to Huff gravity based rule, the utility of this facility for customer k is given by $\frac{Q_j}{d_{jk}^2}$ utilizing the gravity based rule the total utility of the leader for customer and all the opened DCs are given by $\sum_j \frac{Q_j}{d_{jk}^2}$, similarly if the follower has m' DCs indexed by j' and m'' potential DCs indexed by j'' with random attractiveness $W_{j'}, A_{j''}$ and Euclidean distance $d_{j'k}^2, d_{j''k}^2$ from customer k, the total utility of the follower for customer k based on existing and newly opened DCs is $\sum_{j=1}^{m'} \frac{W_{j'}}{d_{j'k}^2} + \sum_{j=1}^{m''} \frac{A_{j''}}{d_{j''k}^2}$ .Hence the

probability p_{jk} that customer k visit facility j of the leader(based on all opened DCs) is expressed as $p_{jk} = \frac{\frac{Q_j}{d_{jk}^2}}{\sum_{j=1}^{m} \frac{Q_j}{d_{jk}^2} y_j + \sum_{j=1}^{m} \frac{W_{j'}}{d_{j'k}^2} + \sum_{j=1}^{m} \frac{A_{j''}}{d_{j''k}^2}}$,Therefore if the demand of customer k is equal to D_k then the revenue of the leader is as follow $\sum_j \sum_k D_k p_{jk}$, the total revenue of the follower can be archived in similar fashion. Now we can describe different modeling approaches assumption, parameters and decision variables as follows:

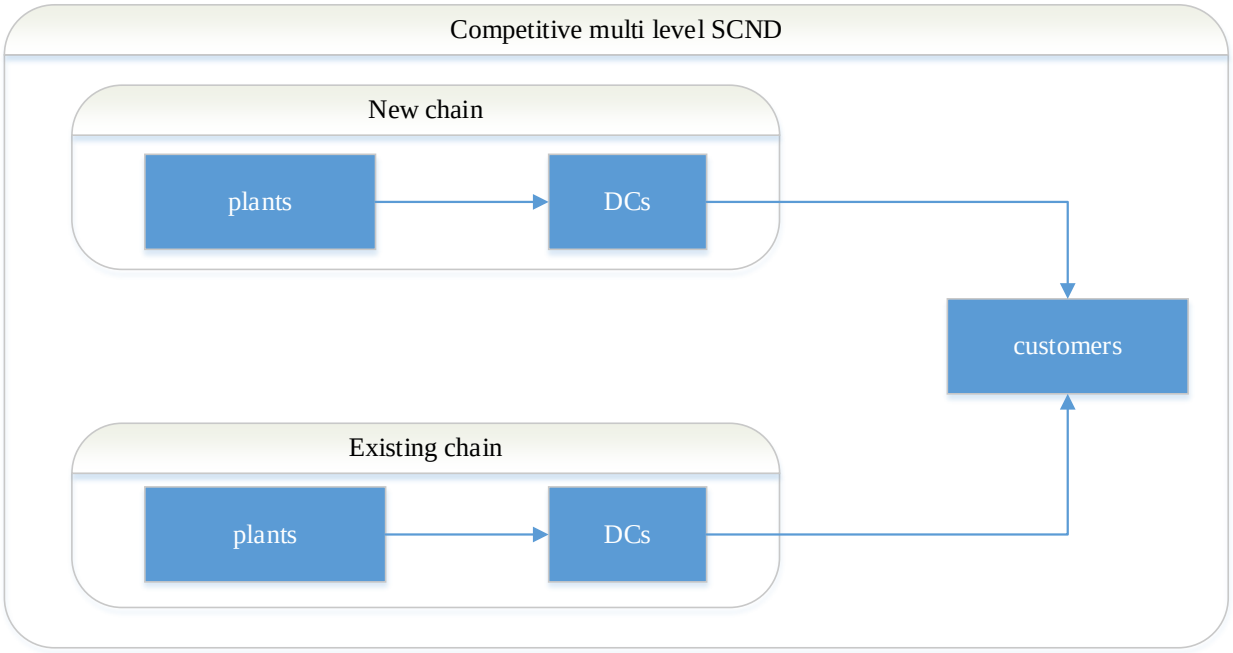

Figure 1. Competitive multi-level SCND

Assumptions:

- ✓ The candidate locations of the leader's plants are known in advance.
- ✓ The candidate locations of the leader's DCs are known in advance.
- ✓ The candidate locations of the followers` plants are known in advance.
- ✓ The candidate locations of the followers` DCs are known in advance.
- ✓ Structures of existing chains are known in advance.
- ✓ There are no common facilities between the chains
- ✓ The demand of each customer market is concentrated on discrete point

Indexes:

i	Index of candidate location of plants for the leader
j	Index of candidate location of DCs for the leader
i'	Index of plant location of the follower

j' Index of DC location of the follower

k Index of customer`s location

i'' Index of candidate`s location of plants for the follower

j'' Index of candidate location of DCs for the follower

Parameters:

f_i Fixed cost of opening a plant for the leader on location i

f_j Fixed cost of opening a DC for the leader on location j

$f_{i''}$ Fixed cost of opening a plant for the follower on location i''

$f_{j''}$ Fixed cost of opening a DC for the follower on location j''

s_i Unit production cost for the leader in plant i

$s_{i'}$ Unit production cost for the follower in plant i

$s_{i''}$ Unit production cost for the follower in plant i''

c_{ij} Unit transportation cost between plant i and DC j for the leader

$c_{ij'}$ Unit transportation cost between plant i' and DC j' for the follower

$c_{i'j''}$ Unit transportation cost between plant i' and DC j'' for the follower

$c_{j''k}$ Unit transportation cost between DC j'' and customer k for the follower

$c_{i''j'}$ Unit transportation cost between plant i'' and DC j' for the follower

$c_{i''j''}$ Unit transportation cost between plant i'' and DC j'' for the follower

c_j Unit attractiveness cost at DC j for the leader

d_{jk} Euclidian distance between DC j and customer k

$d_{j'k}$ Euclidian distance between DC j' and customer k

$d_{j''k}$ Euclidian distance between DC j'' and customer k

D_k Demand of customer k

Cap_i Capacity of plant i for the leader

Cap_j Capacity of DC j for the leader

$Cap_{i'}$ Capacity of plant i' for the follower

$Cap_{j'}$ Capacity of plant j' for the follower

$Cap_{i''}$ Capacity of plant i'' for the follower

$Cap_{j''}$ Capacity of plant j'' for the follower

$c_{j''}$ Unit attractiveness cost at DC j'' for the follower

$W_{j'}$ Attractiveness level of follower's DC at location j'

Q_j Attractiveness level of leader's DC at location j

$A_{j''}$ Attractiveness level of follower's DC at location j''

Decision variables:

y_i $\begin{cases}1 \text{ if the leader opens a plant at location i} \\ 0 \text{ else}\end{cases}$

y_j $\begin{cases}1 \text{ if the leader opens a DC at location j} \\ 0 \text{ else}\end{cases}$

$y_{i''}$ $\begin{cases}1 \text{ if the leader opens a plant at location i}'' \\ 0 \text{ else}\end{cases}$

$y_{j''}$ $\begin{cases}1 \text{ if the leader opens a DC at location j}'' \\ 0 \text{ else}\end{cases}$

x_{ij} Quantity of product shipped from plant i to DC j for the leader

x_{jk} Quantity of product shipped from DC j to customer k for the leader

$x_{i'j'}$ Quantity of product shipped from plant i' to DC j' for the follower

$x_{j'k}$ Quantity of product shipped from DC j' to customer k for the follower

$x_{i'j''}$ Quantity of product shipped from plant i' to DC j'' for the follower

$x_{j''k}$ Quantity of product shipped from DC j'' to customer k for the follower

$x_{i''j'}$ Quantity of product shipped from plant i'' to DC j' for the follower

$x_{i''j''}$ Quantity of product shipped from plant i'' to DC j'' for the follower

Now the multi-level mixed integer nonlinear problem in decentralized mode can be formulated as follows:

$$P_{DC}^{L}: \max Z_1 = \sum_{k=1}^{e}\sum_{j=1}^{m} D_k \frac{\frac{Q_j}{d_{jk}^2} y_j}{\sum_{j=1}^{m}\frac{Q_j}{d_{jk}^2} y_j + \sum_{j=1}^{m}\frac{W_{j'}}{d_{j'k}^2} + \sum_{j=1}^{m}\frac{A_{j''}}{d_{j''k}^2} y_{j''}} - \sum_{j=1}^{m} c_j Q_j - \sum_{j=1}^{m} f_j y_j \quad (1)$$

$s.t$

$$x_{jk} = D_k P_{jk} \qquad \forall j,k \qquad (2)$$

$$\sum_{j=1}^{m} y_j = P \qquad (3)$$

$$\sum_{k=1}^{e} x_{jk} \le Cap_j y_j \qquad \forall j \qquad (4)$$

$$x_{jk} \geq 0 \ , y_j \in \{0,1\} \qquad \forall \mathrm{j},k \qquad (5)$$

The objective function of the leader maximizes total revenue including revenue captured by opened DCs mines fixed cost of opening each DC and attractiveness cost associated with opening them, constraint 2 ensures that each DC should satisfy the amount of demands associated with it, constraint 3 ensures that only P DCs are opened, constraint 4 ensures that only opened DCs can satisfy customer demands up to their capacity and constraint 5 is related to binary and non-negativity restriction on the corresponding decision variables.

$$P^{L}_{Plant}: \min Z_2 = \sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m} s_i x_{ij} \qquad (6)$$

$s.t$

$$\sum_{j=1}^{m} x_{ij} \leq Cap_i y_i \qquad \forall \mathrm{i} \qquad (7)$$

$$\sum_{i=1}^{n} y_i = q \qquad (8)$$

$$\sum_{i=1}^{n} x_{ij} = \sum_{k=1}^{e} x_{jk} \qquad \forall \mathrm{j} \qquad (9)$$

$$x_{ij} \geq 0 \ , y_i \in \{0,1\} \qquad (10)$$

Constraint 7 is the plant objective function of the new chain that includes the cost of opening plants, production cost at each plant and transportation cost between plants and DCs, constraint 7 ensures that each opened plant can fulfill DCs demand up to its capacity, constraint 8 ensures that only P plants are opened, constraint 9 ensures flow balance and constraint 10 is related to binary and non-negativity restriction on the corresponding decision variables.

$$P^{F}_{DC}: \max Z_3 = \sum_{k=1}^{e} D_k \frac{\sum_{j=1}^{m} \frac{W_{j'}}{d^2_{j'k}} + \sum_{j=1}^{m} \frac{A_{j''}}{d^2_{j''k}} y_{j''}}{\sum_{j=1}^{m} \frac{Q_j}{d^2_{jk}} y_j + \sum_{j=1}^{m} \frac{W_{j'}}{d^2_{j'k}} + \sum_{j=1}^{m} \frac{A_{j''}}{d^2_{j''k}} y_{j''}} - \sum_{j''=1}^{m''} c_{j''} A_{j''} - \sum_{j''=1}^{m''} f_{j''} y_{j''} \qquad (11)$$

$s.t$

$$x_{j'k} = D_k P_{j'k} \qquad \forall \mathrm{j}',\mathrm{k} \qquad (12)$$

$$x_{j''k} = D_k P_{j''k} \qquad \forall \mathrm{j}'',\mathrm{k} \qquad (13)$$

$$\sum_{k=1}^{e} x_{j'k} \le Cap_{j'} \qquad \forall j' \qquad (14)$$

$$\sum_{k=1}^{e} x_{j''k} \le Cap_{j''} y_{j''} \qquad \forall j'' \qquad (15)$$

$$\sum_{j''=1}^{m} y_{j''} = P'' \qquad (16)$$

$$x_{j'k}, x_{j''k} \ge 0 \ , y_{j''} \in \{0,1\} \qquad (17)$$

The objective function of the follower maximizes total revenue including revenue captured by newly opened and existing DCs mines, fixed cost of opening each DC and attractiveness cost associated with opening them, constraint 12,13 ensure that each DC should satisfy the amount of demands associated with it, constraint 14,15 ensure that only opened DCs can satisfy customer demands up to their capacity, constraint 16 ensures that only P'' DCs are opened and constraint 17 is related to binary and non-negativity restriction on the corresponding decision variables.

$$P^{F}_{Plant}: \min Z_4 = \sum_{i''=1}^{n''} f_{i''} y_{i''} + \sum_{i'=1}^{n'} \sum_{j'=1}^{m'} c_{i'j'} x_{i'j'} + \sum_{i'=1}^{n'} \sum_{j''=1}^{m''} c_{i'j''} x_{i'j''} + \qquad (18)$$
$$\sum_{i''=1}^{n''} \sum_{j'=1}^{m'} c_{i'j'} x_{i'j'} + \sum_{i''=1}^{n''} \sum_{j''=1}^{m''} c_{i'j''} x_{i'j''} + \sum_{i'=1}^{n'} \sum_{j'=1}^{m'} s_{i'} x_{i'j'} +$$
$$\sum_{i'=1}^{n'} \sum_{j''=1}^{m''} s_{i'} x_{i'j''} + \sum_{i''=1}^{n''} \sum_{j'=1}^{m'} s_{i'} x_{i'j'} + \sum_{i''=1}^{n''} \sum_{j''=1}^{m''} s_{i''} x_{i'j''}$$

$s.t$

$$\sum_{i'=1}^{n'} x_{i'j'} + \sum_{i''=1}^{n''} x_{i''j'} = \sum_{k=1}^{e} x_{j'k} \qquad \forall j' \qquad (19)$$

$$\sum_{i'=1}^{n'} x_{i'j''} + \sum_{i''=1}^{n''} x_{i''j''} = \sum_{k=1}^{e} x_{j''k} \qquad \forall j'' \qquad (20)$$

$$\sum_{j'=1}^{m'} x_{i'j'} + \sum_{j''=1}^{m''} x_{i'j''} \le Cap_{i'} \qquad \forall i' \qquad (21)$$

$$\sum_{j'=1}^{m'} x_{i''j'} + \sum_{j''=1}^{m''} x_{i''j''} \le Cap_{i''} y_{i''} \qquad \forall i'' \qquad (22)$$

$$\sum_{i''=1}^{n''} y_{i''} = q'' \qquad (23)$$

$$x_{i'j'}, x_{i'j''}, x_{i''j'}, x_{i''j''} \geq 0 \ , \ y_{i''} \in \{0,1\} \tag{24}$$

Constraint 18 is the plant objective function of the existing chain that includes cost of opening plants, production cost at each plant and transportation cost between plants and DCs, constraint 19, 20 ensure flow balance, constraint 21, 22 ensure that each plant (existing and opened one) can fulfill DCs demand up to its capacity, constraint 23 limits the number of opened plants and constraint 24 is related to binary and non-negativity restriction on the corresponding decision variables.

3- Solution approach

In this section, our solution approach towards the multi-level CSCND problem is described. According to the fact that Huff gravity model makes the model nonlinear, we encountered multi-level mixed integer nonlinear optimization problem that has no easy way to solve, therefore, for tackling this complexity, we use equilibrium concept and enumeration method to solve the leader-follower multilevel models.
With respect to Huff gravity model of the SCs it is clear that the only joining factor is location variables of the DCs $y_j, y_{j''}$, also these variables make the model nonlinear so if we can fix them, the rests are linear.

$$\frac{\frac{Q_j}{d_{jk}^2} y_j}{\sum_{j=1}^{m} \frac{Q_j}{d_{jk}^2} y_j + \sum_{j=1}^{m} \frac{W_{j'}}{d_{j'k}^2} + \sum_{j=1}^{m} \frac{A_{j''}}{d_{j''k}^2} y_{j''}}, \frac{\sum_{j=1}^{m} \frac{W_{j'}}{d_{j'k}^2} + \sum_{j=1}^{m} \frac{A_{j''}}{d_{j''k}^2} y_{j''}}{\sum_{j=1}^{m} \frac{Q_j}{d_{jk}^2} y_j + \sum_{j=1}^{m} \frac{W_{j'}}{d_{j'k}^2} + \sum_{j=1}^{m} \frac{A_{j''}}{d_{j''k}^2} y_{j''}} \tag{25}$$

On the other hand,the number of opened DCs in each chain, P, P'' , are known beforehand, so we can define possible strategies by considering the combination of predetermined number of opened DCs from all the possible locations m, m'' . To clarify it more , consider a situation in which SC1 wants to open 2 DCs from 5 potential locations $\binom{5}{2}$ and SC2 considers to open 1 DCs from 4 potential locations $\binom{4}{1}$ so SC1 and SC2 have 10,4 pure strategies and also they encountered with $\binom{5}{2} * \binom{4}{1} = 40$ possible scenarios.

On the other hand, Stackelberg games are composed of a leader and a follower. In this kind of game the leader first makes his decision and then, by knowing the leader's decision, the follower will make her decision to optimize her objective function. The most used solution approaches for this kind of problem is based on KKT condition to convert the bi-level problem into a single one. This process will increase the non-convexity of the problem case of Lagrange terms that include the original problem and make it very hard be solved even for the small size problems [refer to Colson et al., 2007, for more details]. Also this game is solved in the literature by backward deduction approach (Kreps, 1990) which leads to the equilibrium Rasmusen (2007). In this way, tentative solution of the follower goes into the leader model so the leader can calculate his decisive equilibrium solution and this solution goes back to the follower's model where she can calculate her decisive equilibrium solution.
With the aim of constructing different scenarios and using Stackelberg equilibrium concept, we construct our algorithm steps as follows:

1) Initialize the strategies for the DCs:
1.1) Construct an empty bi-matrix by considering any possible combinations of the DCs location variables.
2) Calculate flows, assignments and opened plants the SCs in the defied strategies:
2.1) Use any commercial optimization software for this purpose.
3) Find the optimum solution of the players.

3.1) Fill up the empty bi-matrix by the given solutions from the previous step and finding pure or mixed Stackelberg strategy (ies).

In step one the possible scenarios are considered and an empty bi-matrix is shaped then in the second step, we can calculate the best response of the SCs. With respect to the fact that now we encountered with linear programming model this step can easily be done by any commercial optimization software then in the last step the bi-matrix is filled and according to Stackelberg concept, the equilibrium solution(s) is selected. Now we can go to the computational results.

4- Computational result

In this section, some numerical examples in decentralized mode are presented in Section 4-1, Section 4-2 explores the different competition modes through one numerical example and Section 4-3 discusses the impact of different scenarios on the objective functions of the players.

4-1- Numerical examples in decentralized mode

As there is no benchmark problem on the proposed CSCND in the literature. We therefore generate 7 random instances to test the proposed algorithm through run time. We consider demand points from $k \in \{50,70,100\}$, the number of candidate facility for plants as $i, i'' \in \{2,3,4,5,6,7\}$, and DCs as $j, j', j'' \in \{3,4,5,6,7\}$. Every instance is shown by $\left(i', j', \begin{pmatrix} n' \\ q^{(2)} \end{pmatrix}, \begin{pmatrix} m'' \\ P^{(2)} \end{pmatrix}, \begin{pmatrix} n \\ q^{(1)} \end{pmatrix} \begin{pmatrix} m \\ P^{(1)} \end{pmatrix}, k \right)$ which represents the number of operating plants i' and DCs j' and the number of opened facilities from potential facilities of the SC2 for plant $\begin{pmatrix} n' \\ q^{(2)} \end{pmatrix}$ and DC level $\begin{pmatrix} m'' \\ P^{(2)} \end{pmatrix}$ and the number of opened facilities from potential facilities of the SC2 for plant $\begin{pmatrix} n \\ q^{(1)} \end{pmatrix}$ and DC level $\begin{pmatrix} m \\ P^{(1)} \end{pmatrix}$ and the number of available customers k; $\begin{pmatrix} m'' \\ P^{(2)} \end{pmatrix}$ and $\begin{pmatrix} m \\ P^{(1)} \end{pmatrix}$ show the potential pure strategies of each chain; and $\begin{pmatrix} m'' \\ P^{(2)} \end{pmatrix}.\begin{pmatrix} m \\ P^{(1)} \end{pmatrix}$ is the whole available strategies of Stackelberg CSCND problem The required parameters are generated according to table 2. The proposed algorithm was implemented in Matlab 2014a and carried out on a Pentium dual-core 2.6 GHz with 2 GB RAM, table 3 shows the results, according to the obtained results it is observable that solution time of the algorithm is increased exponentially so proposing a Meta heuristic algorithm can be a good idea for handling large scale problems but for small and medium scale problem, the proposed algorithm can reach equilibrium solution in a reasonable time.

Table 2. Required parameters

$f_i \sim u(20000,35000)$	$c_{i''j''} \sim u(20,35)$	$x_j \sim u(0,100)$	$Cap_{i'} \sim u(1000000,1200000)$
$f_j \sim u(15000,25000)$	$c_{i''j'} \sim u(20,35)$	$y_k \sim u(0,100)$	$Cap_{j'} \sim u(1000000,1200000)$
$f_{i''} \sim u(20000,35000)$	$s_{i''} \sim u(25,30)$	$y_{j'} \sim u(0,100)$	$Cap_{i''} \sim u(25000,26000)$
$f_{j''} \sim u(15000,25000)$	$s_{i'} \sim u(25,30)$	$y_{j''} \sim u(0,100)$	$Cap_{j''} \sim u(1000000,1200000)$
$c_{ij} \sim u(15,25)$	$c_j \sim u(10000,19000)$	$y_j \sim u(0,100)$	$c_{j''} \sim u(10000,19000)$
$c_{i'j'} \sim u(20,35)$	$x_k \sim u(0,100)$	$D_k \sim u(100000,120000)$	$W_{j'} \sim u(1,5)$
$s_i \sim u(25,30)$	$x_{j'} \sim u(0,100)$	$Cap_i \sim u(1000000,1200000)$	$Q_j \sim u(1,5)$
$c_{i'j''} \sim u(20,35)$	$x_{j''} \sim u(0,100)$	$Cap_j \sim u(1000000,1200000)$	$A_{j''} \sim u(1,5)$

Table 3. Numerical instances

	Examples $\left(i', j', \binom{n'}{q^{(2)}}, \binom{m''}{P^{(2)}}, \binom{n}{q^{(1)}} \binom{m}{P^{(1)}}, k\right)$		Pure strategies of the chains	Opened DCs	objDC(profit)	OBJ plant(cost)	objSC(cost)	CPU (Seconds) Average time(10 runs)
1	$\left(1,1,\binom{5}{1},\binom{5}{2},\binom{5}{1},\binom{5}{3},100\right)$	SC1	10	(1,3,5)	3513696	21037891	17524195	1536
		SC2	10	(2,4,5)	4969649	147015934	142046285	
2	$\left(1,1,\binom{5}{2},\binom{5}{1},\binom{5}{3},\binom{6}{2},100\right)$	SC1	15	(4,6)	3798525	15460816	11662291	1107
		SC2	5	(5)	5720236	175076818	169356582	
3	$\left(1,1,\binom{5}{2},\binom{5}{1},\binom{4}{1},\binom{5}{2},100\right)$	SC1	10	(2,3)	3977356	10934662	6957306	417
		SC2	5	(4)	4637681	13697246	9059565	
4	$\left(1,1,\binom{3}{2},\binom{3}{1},\binom{7}{2},\binom{7}{2},70\right)$	SC1	21	(4,7)	2179635	19573648	17394013	206
		SC2	3	(1)	3953647	15684529	11730882	
5	$\left(1,1,\binom{7}{2},\binom{3}{1},\binom{5}{1},\binom{7}{4},70\right)$	SC1	35	(1,3,5,6)	3223754	20472384	17248630	855
		SC2	3	(3)	3646549	16704935	13058386	
6	$\left(1,1,\binom{4}{1},\binom{4}{1},\binom{7}{2},\binom{4}{2},50\right)$	SC1	6	(3,4)	2239462	19138469	16899007	20
		SC2	4	(4)	1988235	10361527	8373292	
7	$\left(1,1,\binom{2}{1},\binom{3}{1},\binom{2}{1},\binom{3}{2},50\right)$	SC1	3	(1,3)	2447932	21327861	18879929	7
		SC2	3	(3)	1723647	11398455	9674808	

4-2- A numerical example in different modes

In this part, an example is described to illustrate the proposed problem in different competitive modes and clarify the solution approach. Imagine a new comer (SC1) that wants to enter the market in which there is a rival (SC2) by two DCs and one plan in the market. The new comer has three candidate locations for opening plants and five candidate locations for opening DCs and wants to open one plant and two DCs to capture ten available markets. The existing rival will show reaction to his entrance by opening one DC through three candidate locations and one plant through two candidate locations. As explained in the solution approach the leader has $\binom{5}{2}=10$ different strategies and the follower has $\binom{3}{1}=3$ different strategies that should be considered to construct the bi-matrix and find the equilibrium solution. Table 2 is used to generate required parameters. Structural mode of each chain is mainly categorizes into three classes as follows:

Decentralized mode: this mode is categorized into 2 kinds based on the dominant player, if the DC level is the dominant player in each SC then only objective function of the DCs is used to find the equilibrium solution. Table 4 shows the objective function of them. The equilibrium solution in this mode is when the leader opens DC 3 and DC 4 and the follower open DC 3.The corresponding DC's objective function for this solution is 144669 for the leader and 524677 for the follower. In addition, in SC1, the plant's objective function is 11331735 and total SC cost is 11,187,066 similar to the existing plant objective function which is 31096780 and total SC cost for the existing is 30572103.Also if the plants are the dominant players another equilibrium solution based on their best response matrix is achieved as shown in the table 4.

Centralized mode: Two kinds of structures are assumed here, 2 player modes and 4 player modes.
2 player modes: if each SC has one owner this leads to 2 player- game that each owner should optimize the following bi-level mixed integer nonlinear programming problem:

$$P^{L}_{CenSC}: \max Z_5 = \sum_{k=1}^{e}\sum_{j=1}^{m} D_k \frac{\frac{Q_j}{d^2_{jk}} y_j}{\sum_{j=1}^{m}\frac{Q_j}{d^2_{jk}} y_j + \sum_{j=1}^{m}\frac{W_{j'}}{d^2_{j'k}} + \sum_{j=1}^{m}\frac{A_{j''}}{d^2_{j''k}} y_{j''}} - \left(\sum_{j=1}^{m} c_j Q_j + \sum_{j=1}^{m} f_j y_j\right) - \left(\sum_{i=1}^{n} f_i y_i + \sum_{i=1}^{n}\sum_{j=1}^{m} c_{ij} x_{ij} + \sum_{i=1}^{n}\sum_{j=1}^{m} s_i x_{ij}\right) \quad (26)$$

$s.t$
(2-5)
(7-10)

The second level of the model is as follows.

$$P^{F}_{CenSC}: \max Z_6 = \sum_{k=1}^{e} D_k \frac{\sum_{j=1}^{m} \frac{W_{j'}}{d^2_{j'k}} + \sum_{j=1}^{m} \frac{A_{j''}}{d^2_{j''k}} y_{j''}}{\sum_{j=1}^{m} \frac{Q_j}{d^2_{jk}} y_j + \sum_{j=1}^{m} \frac{W_{j'}}{d^2_{j'k}} + \sum_{j=1}^{m} \frac{A_{j''}}{d^2_{j''k}} y_{j''}} - \left(\sum_{j''=1}^{m''} c_{j''} A_{j''} + \sum_{j''=1}^{m''} f_{j''} y_{j''} \right) -$$

$$\left(\begin{array}{l} \sum_{i''=1}^{n''} f_{i''} y_{i''} + \sum_{i'=1}^{n'} \sum_{j'=1}^{m'} c_{i'j'} x_{i'j'} + \sum_{i'=1}^{n'} \sum_{j''=1}^{m''} c_{i'j''} x_{i'j''} + \\ \sum_{i''=1}^{n''} \sum_{j'=1}^{m'} c_{i'j'} x_{i'j'} + \sum_{i''=1}^{n''} \sum_{j''=1}^{m''} c_{i'j''} x_{i'j''} + \sum_{i'=1}^{n'} \sum_{j'=1}^{m'} s_{i'} x_{i'j'} + \\ \sum_{i'=1}^{n'} \sum_{j''=1}^{m''} s_{i'} x_{i'j''} + \sum_{i''=1}^{n''} \sum_{j'=1}^{m'} s_{i'} x_{i'j'} + \sum_{i''=1}^{n''} \sum_{j''=1}^{m''} s_{i''} x_{i'j''} \end{array} \right) \quad (27)$$

s.t

(12-17)

(19-24)

Terms (26) and (27) represent the objective functions of leader and the follower that are the summations of objective functions of their plants and DCs.

4 player mode: this situation is taken place in the case of each tier that has an independent owner. In this structure, sum of DC's and plant objective function will specify the equilibrium solution, these two different models should be considered simultaneously by the same dimension and direction so this considerations lead the centralized mode to a multi objective decision making problem that can be solved by some methods, Ehrgott and Gandibleux (2000). A method which combines the global criterion method and simple additive weighting are used here and the resultant model is as follows:

$$P^{L}_{CO}: \min Z_5 = \lambda \left(\sqrt[P]{\left(\frac{(Z_1) - Z^*_1}{Z^*_1}\right)^P} \right) + (1-\lambda) \sqrt[p]{\left(\frac{(Z_2) - Z^*_2}{Z^*_2}\right)^p} \quad (28)$$

s.t

(2-5)

(7-10)

$$P^{L}_{CO}: \min Z_6 = \lambda \left(\sqrt[P]{\left(\frac{(Z_3) - Z^*_3}{Z^*_3}\right)^P} \right) + (1-\lambda) \left(\sqrt[P]{\left(\frac{(Z_4) - Z^*_4}{Z^*_4}\right)^P} \right) \quad (29)$$

s.t

(12-17)

(19-24)

It is worth noting that in this case λ should be equal to 0.5 to put an equal emphesis onobjective function of plants and DCs. The results are shown in table 4.

Cooperative mode: a weighted sum of DC's and plant objective function will make the cooperative CSCND and the centralized mode be a special case of this one when both weights are similar, in this mode the cooperative solution by different willing of cooperation that is defined as the weighing factor, λ is related to objective function of the DC and $1-\lambda$ is related to achieved plant objective function. Table 4 shows total computational results for different approaches. It is worth noting that $0 \le \lambda \le 1$, is assumed that λ belongs to $\lambda \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$.

Mixed structure: there are 17 difftent games derived from this three baisc modes;when the chain does not have the same structute as follows:

- Two players centralized VS. two players centralized
- Two players centralized VS. four players centralized
- Four players centralized VS. two players centralized
- Four players centralized VS. four players centralized
- Decentralized based on DC VS. decentralized based on DC
- Decentralized based on DC VS. decentralized based on plant
- Decentralized based on plant VS. decentralized based on DC
- Decentralized based on plant VS. decentralized based on plant
- Cooperative VS. cooperative
- Cooperative VS. two players centralized
- Cooperative VS. four players centralized
- Cooperative VS. decentralized based on DC
- Cooperative VS. decentralized based on plant
- Two players centralized VS. Cooperative
- Four players centralized VS. cooperative
- Decentralized based on DC VS. cooperative
- Decentralized based on plant VS. cooperative

Tables 5 to12 show the results of thsese competition modes.

Table 4. Main games results

			Equilibrium strategy	objDC(profit)	OBJ plant(cost)	objSC(cost)
Decentralized	DC VS DC	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	P VS P	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	P VS DC	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	DC VS P	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
Centralized	4 players	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	2 players	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	one VS two	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	two VS one	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
Cooperative	λ					
	0.1	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.2	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.3	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.4	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.5	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.6	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.7	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.8	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.9	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103

Table 5. Cooperative VS one player Centralized

			Equilibrium strategy	objDC(profit)	OBJ plant(cost)	objSC(cost)
Cooperative VS one player Centralized	λ					
	0.1	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.2	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.3	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.4	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.5	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.6	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.7	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.8	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.9	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103

Table 6. Cooperative VS two player Centralized

			Equilibrium strategy	objDC(profit)	OBJ plant(cost)	objSC(cost)
Cooperative VS two player Centralized	λ					
	0.1	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.2	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.3	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.4	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.5	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.6	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.7	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.8	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.9	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103

Table 7. Cooperative VS Decentralized (based on the plant)

			Equilibrium strategy	objDC(profit)	OBJ plant(cost)	objSC(cost)
Cooperative VS Decentralized(based on the plant)	λ					
	0.1	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.2	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.3	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.4	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.5	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.6	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.7	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.8	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.9	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103

Table 8. Cooperative VS Decentralized (based on the DC)

			Equilibrium strategy	objDC(profit)	OBJ plant(cost)	objSC(cost)
Cooperative VS Decentralized(based on the DC)	λ					
	0.1	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.2	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.3	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.4	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.5	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.6	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.7	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.8	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.9	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103

Table 9. One player centralized VS Cooperative

			Equilibrium strategy	objDC(profit)	OBJ plant(cost)	objSC(cost)
one players centralized VS Cooperative	λ					
	0.1	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.2	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.3	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.4	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.5	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.6	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.7	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.8	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.9	Leader	(2,4)	-51540.1	6627231	6678771
		followe	3	721224.9	41641209	40919984

Table 10. Two players centralized VS cooperative

			Equilibrium strategy	objDC(profit)	OBJ plant(cost)	objSC(cost)
two players centralized VS cooperative	λ					
	0.1	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.2	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.3	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.4	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.5	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.6	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.7	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.8	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.9	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103

Table 11. Decentralized based on DC VS cooperative

			Equilibrium strategy	objDC(profit)	OBJ plant(cost)	objSC(cost)
Decentralized based on DC VS cooperative	λ					
	0.1	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.2	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.3	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.4	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.5	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.6	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.7	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.8	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103
	0.9	Leader	(3,4)	144668.7	11331734	11187066
		follower	3	524676.5	31096780	30572103

Table 12. Decentralized based on plant VS cooperative

			Equilibrium strategy	objDC(profit)	OBJ plant(cost)	objSC(cost)
Decentralized based on plant VS cooperative	λ					
	0.1	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.2	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.3	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.4	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.5	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.6	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.7	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.8	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984
	0.9	Leader	(2,4)	-51540.1	6627231	6678771
		follower	3	721224.9	41641209	40919984

4-3- Managerial insights

Tables 4 to 12 present the equilibrium strategies, DC, plant and SC objective functions over the proposed games between the leader and the follower. The amount of power of DC and plant specifies the game and influences the equilibrium strategy, with respect to this factor,the follower has one pure strategy,opens a DC on the third location, shown by (3). On the other side the leader with respect to his dominant player, different strategies are obtained for example, when DC is the dominant player, opening DCs onthird and forth location , shown by (3,4), is the equlibruim strategy that leads to 144668.7 unit profits for the DC, 11331734 unit costs for the plant and 11187066 unit costs for the SC correspondingly 524676.5, 31096780, 30572103 are the related results for the follower butif the plant is the dominant player in the leader's chain, opening DCs in second and forth locations, shown by (2,4) is the equlibrim strategy and results in-51540.1 unit profits for the DC, 6627231 unit costs for the plant and 6678771 unit costs for the SC corespondingly 721224.9, 41641209, 40919984 are the related results for the follower. By comparing these two different situations it is obvious that if the the leader lets the plant be the dominant player, the tolal SC costs can be improved by 40.3% and plant costs improved by 41.5% but DC profits decreased by 135.62% also in this structure total SC costs for the follower increased by 33.84%, plant's cost increased by 33.9% and DC's profits increased by 37.46% therefor, the best strategy that leads to the best total SC costs for the leader is (2,4) but in this strategy clearly DC has no attention to play the game and no network will be shaped unless a good mechanism isconstructed to divide the amount of profits that the SC gained between plant and DC to get enough motivations to the DC to play its role, on the other side this strategy imposed more costs to the follower that is also beneficial for the leader's chain.

5- Conclusion

This paper developed a multi-level mixed integer nonlinear programing model for decentralized leader-follower network design of competitive supply chain problem in which one SC as the leader enters market with one existing SC then reacts to his entrance as the follower therefor a Stackelberg game will be taken place between the chains.Each SC has two independent entities and each entity has its objective function and some related constraints try to optimize it.The problem is tackeld by 3 main approches as:centralized ,decentralized and cooperative mode, also different games considered by combination of this three baisc modes are considered. A solution algorithm based on Stackeleberg concept and enumeration method is developed to solve the problem. Finally, some numerical examples are used to illustrate the problem and proposed algorithm in each competitive mode. This work can be enhanced by considering green, sustainable, risky or agile considerations. Also using stocastic programming, robust optimization or closed-loop CSCND can be considered for the future research.

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