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Coordination and profit sharing in a two-level supply chain under periodic review inventory policy with delay in payments contract

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Abstract

In this paper, a coordination model has been investigated for a two-level supply chain (SC) consisting of one retailer and one supplier under periodic review inventory system. The review period and the retailer's safety factor are assumed to be decision variables. The retailer faces stochastic demand following a normal distribution with known mean and variance. Moreover, it is assumed that unmet demand will be backordered. Firstly, the investigated SC is modeled under the decentralized and centralized decision-making structures, afterwards, a coordination mechanism based on delay in payments is proposed for transition from the decentralized to centralized model. To fairly share the surplus profit obtained by coordination, a profit sharing strategy is developed which is based on the bargaining power of the two SC members. Finally, a set of numerical experiments and sensitivity analysis are carried out. Numerical examples indicate that the proposed delay in payments contract can achieve channel coordination and the whole SC cost will decrease under the coordination model while the costs of neither retailer nor the supplier will increase.

Keywords: Supply chain coordination, periodic review inventory system, delay in payments, profit sharing

1- Introduction

Nowadays, competition among supply chains has been replaced with competition among individual enterprises. In traditional business environments, each SC member acts as an independent economical entity which is called decentralized structure. In such a situation, each SC member tries to maximize its own profit regardless of other SC actors. This case leads to an inefficient SC. Conversely, under the centralized decision-making all SC members try to maximize their profits according to the whole SC viewpoint. Although all members in the centralized system are managed by one economical entity, it may not be realistic as it might incur losses for some SC members. To resolve the conflicts of interest under the decentralized model, an appropriate coordination mechanism should be used.

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Coordination mechanisms can guarantee the participation of all SC actors in the coordination plan (Chaharsooghi et al., 2011; Mokhlesian and Zegordi, 2015; Heydari and Norouzinasab, 2016).

Different contracts as coordination mechanisms have been broadly exerted in the literature. These contracts are classified on the basis of quantity, time, quality, and price that share the risk from different sources of uncertainty (time, demand, and price) between the SC members (Giannoccaro and Pontrandolfo, 2004). The most popular contracts are revenue sharing (Giannoccaro and Pontrandolfo, 2004; Cachon and Lariviere, 2005; Linh and Hong, 2009), quantity discount (Munson and Rosenblatt, 2001; Li and Liu, 2006; Heydari, 2014), return policies (Ding and Chen, 2008;Xiong et al.,2011; Heydari et al., 2016), quantity flexibility (Tsay, 1999;Chung et al., 2014), sales rebate (Talor, 2002; Heydariand Asl-Najafi, 2016) and delay in payments(Chaharsooghi and Heydari, 2009;Heydari, 2014).According to (Chaharsooghi and Heydari, 2009) contracts can be appropriately used for coordinating supply chains. Among various contracts, delay in payments of contracts play an important role in the modern business environment. Based on this contract, any increases in the retailer's costs will be compensated by the supplier as long as the retailer makes globally optimal decisions from the entire SC point of view. Also, the retailer can benefit from returns on investment during the period of credit (Gao et al., 2014).

Due to the crucial decisions in the supply chains such as replenishment, reorder point, order quantity, protection interval and so on (Chaharsooghi et al., 2011;Johari et al. 2016), many researchers have been vastly paid attention to coordinate SC decisions. The current paper investigates supply chain coordination (SSC)in order to coordinate SC decisions under periodic review inventory system. The periodic review inventory models can often be used in managing inventory cases such as small retail stores, pharmacies, and grocery stores (Annadurai&Uthayakumar, 2010).For example, due to the high number of drugs in pharmacies, the inventory level is reviewed every T units of time and an enough stock is ordered up to the order-up-to level R. Pharmacies' ordering decisions impact on both upstream and SC costs. By applying delay in payments contract as a coordination mechanism, the upstream offered the credit option to the downstream and it can decrease cost in the SC. Although a handful of studies have been conducted on the coordination of periodic review inventory systems within SC, the delay in payments contract as a coordination mechanism has not yet been developed for coordinating these systems.

The current study contributes to the literature by applying the delay in payments contract as an incentive mechanism for coordinating supply chain under periodic review inventory system. To this end, a two-level supply chain consisting of one retailer and one supplier with one type of product is considered. The retailer faces stochastic demand following a normal distribution and uses a periodic review order-up-to level inventory system(T, R). The review period and safety factor arethe retailer's decision variables. The stock out is considered to occur for both the retailer and supplier. The decisions made by the retailer (i.e., review period and safety factor) not only impacts on his/her own inventory costs, but also influences the whole SC inventory costs. Therefore, coordinating these primary decisions throughout the SC is of high importance. Firstly, the decentralized and the centralized models are developed and optimal values of decision variables are calculated. Then, a delay in payments contract is proposed to coordinate the investigated SC. Finally, a profit sharing strategy is developed based on the two SC members' bargaining power. In addition, a set of numerical experiments and sensitivity analysis are carried out to evaluate the performance of the proposed models. The results indicate that the investigated delay in payments contract is capable of reducing the whole SC costs while the developed coordination scheme is mutually beneficial.

The rest of this article is arranged as follows. In the next section, a literature review of inventory control policies and SC coordination is given. Problem definition, the notation and assumptions are introduced in section 3. Section 4 presents the mathematical model for decentralized, centralized and coordination models and solution procedures. Section 5 contains numerical experiments and sensitivity analysis. Conclusions and future researches are provided in section 6.

2- Literature review

In this section, the related literature on delay in payments contract and periodic review inventory models is discussed and then the contributions of this study are expressed.

Incentive schemes guarantee that all SC members participate in the coordination model. By using delay in payments contracts as a coordination mechanism, the supplier by offering the credit period persuades the retailer to participate in the coordination model. Jaber and Osman (2006) considered delay in payments mechanism in a two-level supply chain. In the investigated model, the length of credit option was assumed as a decision variable. Chaharsooghi and Heydari (2009) proposed a delay in payments contract for the joint determination of order quantity and reorder point. Moreover, they developed a profit sharing strategy based on the members' bargaining power. Duan et al. (2012) and Wu and Zhao (2014) used delay in payments contracts in a two-level supply chain for fixed lifetime products. The credit option as a coordination mechanism for replenishment decisions supposing truckload limitations was applied by Heydari (2014). The author considered different rates of return for two SC members to invest during the period of credit.

A group of studies related to periodic review inventory models, Ouyang and Chuang (2000) proposed a periodic review inventory model in which the lead time and the review period were assumed as decision variables. In the proposed model, a service level constraint was considered Instead of stock-out term in the objective function. Ouyang et al. (2007) developed a periodic review inventory model in two different cases: (1) the protection interval demand followed a normal distribution (2) the protection interval demand followed a distribution free. They employed lost-sales rate reduction in their study. Also target inventory level, length of a review period, and fraction of the shortage that will be lost were assumed as decision variables. Annadurai and Uthayakumar (2010) developed a probabilistic inventory model that the lost sale rate was reduced by more investment. The review period, lead time safety factor, and lost-sales rate were supposed as decision variables.

All above mentioned papers were done in a single-echelon inventory system. There are few articles in multi-echelon periodic review inventory systems. Matta and Sinha (1991) considered a two-level periodic review inventory model with stochastic demand followed a normal distribution function. Kanchanasuntorn and Techanitisawad (2006) developed a two-echelon inventory–distribution system with periodic review policy for fixed-life perishable products. They considered stock-out for the retailer in their work. Hsu and Lee (2009) considered an integrated inventory model for a two-level supply chain with single manufacturer and multiple retailers. In the proposed model, the decisions of replenishment and lead-time reduction were investigated. Lin (2010) developed an integrated supplier-retailer inventory problem with stochastic demand. In this study, Length of the protection interval, the backorder price discount, the numbers of shipments from the supplier to the retailer per production run and the lead time were assumed as control variables. All mentioned multi-level periodic review inventory models are expanded in an integrated supply chain. Taking a different methodology, Nematollahi et al. (2016) developed a coordination model for a two- level pharmaceutical supply chain under periodic review inventory policy. In their proposed model, coordination model was considered in two different scenarios, economic coordinative decision-making and social coordinative decision-making. Johari et al. (2016) proposed a coordination model in a manufacturer-retailer chain under periodic review inventory system. They used quantity discount contract as a mechanism of coordination. In their proposed model, review period (T), order-up-to-level (R) and the number of shipments from manufacturer to retailer per production run (n) were considered as decision variables.

In addition, most of the previously coordination models have focused on the continuous review inventory system, such as Jaber and Osman, 2004;Chaharsooghi and Heydari, 2009; Chaharsooghi et al., 2011…,just a handful of studies have been conducted on the coordination of the periodic review inventory systems within supply chain as mentioned above. As a result, in this paper, the delay in payments contract as a coordination mechanism is developed to coordinate the periodic review inventory decisions in addition to the length of the credit period. To fairly share the surplus profit obtained by applying coordination scheme, a profit sharing strategy based on the bargaining power of the two SC members is developed.

3- Problem definition

This paper investigates a two-level supply chain consisting of one retailer and one supplier with one type of product. The customer's demand follows a normal distribution with known mean and variance. The stock out at two levels is backlogged and in the next period must be answered. The proposed supply chain is shown in figure 1. The retailer uses a periodic review inventory policy (T, R). For minimizing the supplier's inventory cost, the supplier's review period is supposed as an integer multiple of the retailer's review period (mT) where m=1,2,…(Jaber and Osman, 2006). Following assumptions are considered in the current study:

1. The inventory level is reviewed every T units of time. A sufficient quantity is ordered up to the order-up-to level$R$, and the ordering quantity will be arrived after Lunits of time.
2. The length of the lead time L does not exceed an inventory cycle time T. $L \leq T$
3. Demand during the protection period $(T + L)$ has a normal distribution with mean $D(T +$ $L$and variance $\sigma 2T+L$
4. The supplier's review period is an integer multiple of the retailer's review period(mT).
5. The order-up-to level R_r for the retailer is equal to the sum of retailer's expected demand during the protection period $D(T + L)$ and the safety stock (SS_r), in which $SS_r = k_r \times$ (standard deviation of protection interval demand) and consequently $R_r = D(T + L) + k_r \sigma \sqrt{T + L}$
6. The order-up-to level R_s for the supplier is equal to the sum of supplier's expected demand during period mTD and the safety stock (SS_s), in which $SS_s = k_s \times$ (standard deviation of protection interval demand).
   $R_s = mTD + k_s \sigma \sqrt{mT}$
7. The supplier lead time is supposed to be zero.

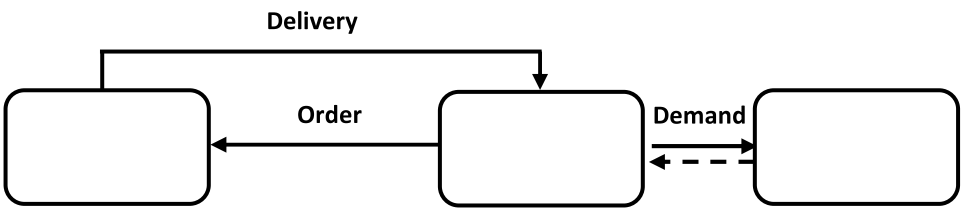

Fig. 1. The investigated two-level supply chain model

The models are developed according to the following notations and assumptions.

3-1- Notations

Parameters

D	The retailer's expected demand per year
L	Length of lead time
x	The retailer's demand during the protection period $(T + L)$, which has a normal distribution with mean $D(T + L)$ and variance $\sigma^2(T + L)$
y	The supplier's demand during period (mT), the demand has a normal distribution with mean mTD and variance $mT\sigma^2$
R_r	Order-up-to level for retailer
R_s	Order-up-to level for supplier
A_r	Unit ordering cost per replenishment for retailer
A_s	Unit ordering cost per replenishment for supplier
h_r	Unit inventory holding cost per year for retailer
h_s	Unit inventory holding cost per year for supplier
π_r	Shortage cost per unit for retailer
$\imath$	Shortage cost per unit for supplier
F_r	Fixed transportation cost for retailer
k_s	Safety factor for supplier,$k_s \geq 0$

β	Percentage of holding costs are due to the investment cost
α	The bargaining power of retailer
m	The number of supplier replenishment cycles as a multiple of retailer review period that is a positive integer$m \geq 1$
Decision variables	
T	Length of a review period
k_r	Safety factor for retailer,$k_r \geq 0$
CT	Length of credit option

4- Model formulation and solution procedures

In this section, firstly the decentralized case is developed. Under the decentralized decision-making, each supply chain member makes decisions individually and tries to minimize its own cost function regardless of the other member. Then, the centralized case is considered. In the centralized decision-making, supply chain is considered as integrated SC. Optimal values of decision variables are calculated at each case. Finally, the delay in payments contract as an incentive scheme is developed. Coordination case is based on the joint decision-making in a decentralized structure. Finally, a profit sharing strategy is developed based on the two SC members' bargaining power and the proposed models are evaluated.

4-1- Decentralized model

In this case, each SC member tries to minimize its own cost regardless of other SC members. Therefore, we model two different inventory problems for the retailer and supplier and obtain optimal solutions for the members.
The retailer uses a periodic review inventory policy (T, R), in which review period T, and the safety factor k_r, are decision variables. Figure 2 shows the inventory level for the supplier and the retailer. As can be seen in figure 2, review period T is considered as the time between the arrivals of two successive orders. The expected net inventorylevel at the beginning of the period is $R_r - DL$, and the expected net inventory level at the end of the period is $R_r - D(T + L)$. Therefore, the expected average inventory level is equal to:

$$\frac{(R_r - DL) + \left(R_r - D(T + L)\right)}{2} = R_r - DL - \frac{DT}{2} \tag{1}$$

Now, putting value of R_r into Eq. (1) will lead to the expected average inventory level as $\frac{DT}{2} + k_r\sigma\sqrt{T + L}$.
In this study, it is assumed that if the customer's demand cannot be met by the retailer immediately, the order is backlogged. The expected stock-out per replenishment cycle can be expressed as:

$$E(x - R_r)^+ = \int_{R_r}^{\infty} (x - R_r)\, f_x(x)dx = \int_{k_r}^{\infty} \sigma\sqrt{T + L}\,(Z - K_r) f_z(z)dz = \sigma\sqrt{T + L}G(K_r) > 0 \tag{2}$$

Where $G(K_r) \equiv \varphi(K_r) - K_r[1 - \emptyset(K_r)]$, $\varphi(K_r)$ and $\emptyset(K_r)$denote the Standard normal p.d.f and c.d.f, respectively.
The retailer’s inventory costs consist of ordering, inventory holding, stock-out, and fixed transportation costs. The expected total retailer’s cost function can be calculated as:

$$TC_r^d(T, R_r) = \frac{A_r}{T} + h_r\left(\frac{DT}{2} + k_r\sigma\sqrt{T + L}\right) + \frac{\pi_r}{T}E(x - R_r)^+ + \frac{F_r}{T} \tag{3}$$

Putting $E(x - R_r)^+$ from equation (2) into equation (3), we get

$$TC_r^d(T^d, k_r^d) = \frac{A_r}{T} + h_r\left(\frac{DT}{2} + k_r\sigma\sqrt{T+L}\right) + \frac{\pi_r}{T}\sigma\sqrt{T+L}G(k_r) + \frac{F_r}{T} \quad (4)$$

In which the first term denotes the ordering cost, the second term denotes the inventory holding cost, the third term denotes the shortage cost and the last term denotes fixed transportation cost.

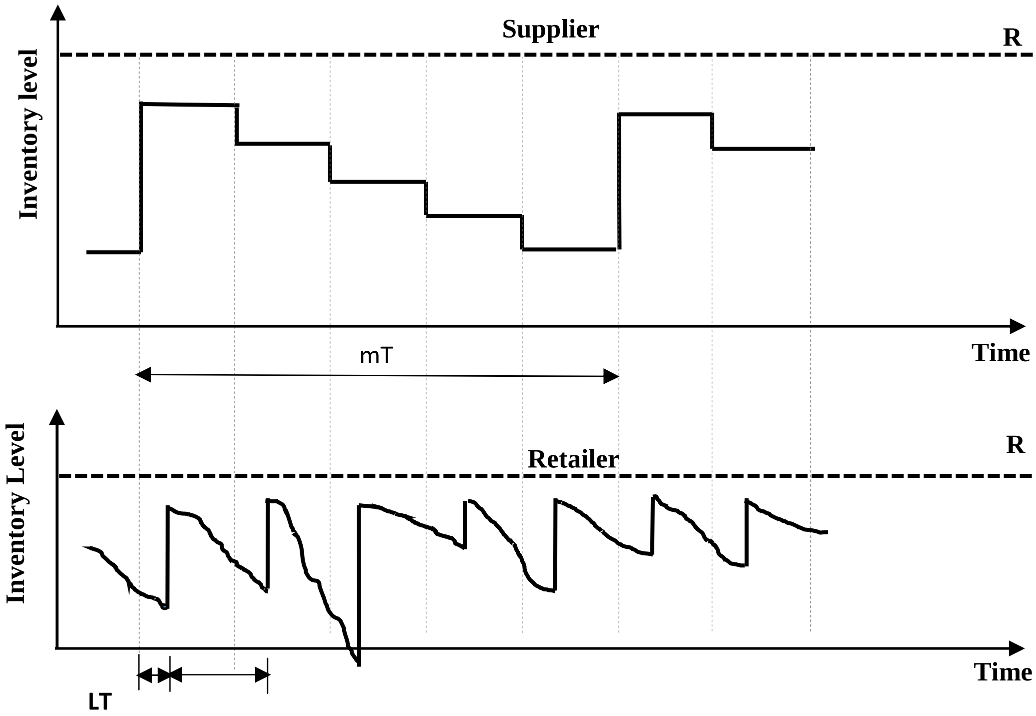

Fig. 2. Inventory level for supplier and retailer

Proposition 1: The retailer's cost function is convex with respect to T^d andk_r^d.

Proof: see Appendix A for detailed proof.
To minimize the retailer's cost function, taking the first order partial derivatives of $TC_r^d(T^d, k_r^d)$ with respect to $T^d$and k_r^d gives:

$$\frac{\partial TC_r^d}{\partial T^d} = \frac{-(A_r + F_r)}{T^2} + h_r\left(\frac{D}{2} + \frac{\sigma k_r}{2\sqrt{T+L}}\right) + \pi_r\sigma G(k_r)\left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2}\right) \quad (5)$$

$$\frac{\partial TC_r^d}{\partial k_r^d} = h_r\sigma\sqrt{T+L} + \frac{\pi_r\sigma\sqrt{T+L}[\emptyset(k_r) - 1]}{T} \quad (6)$$

By setting equations (5) and (6) equal to zero, we obtain

$$\frac{A_r + F_r}{T^2} + \frac{\pi_r\sigma G(k_r)\sqrt{T+L}}{T^2} = h_r\left(\frac{D}{2} + \frac{\sigma k_r}{2\sqrt{T+L}}\right) + \frac{\pi_r\sigma G(k_r)}{2T\sqrt{T+L}} \quad (7)$$

and:

$$1 - \emptyset(k_r) = \frac{h_r T}{\pi_r} \quad (8)$$

The following algorithm1 is used to find the optimal solution of T^d and k_r^d.

Algorithm1

Step1:	Get T_2, which is calculated based on Eq. (7), as follows: $T_2 = \sqrt{\dfrac{A_r + F_r + \pi_r \sigma G(k_r)\sqrt{T+L}}{h_r\left(\frac{D}{2} + \frac{\sigma k_r}{2\sqrt{T+L}}\right) + \frac{\pi_r \sigma G(k_r)}{2T\sqrt{T+L}}}}$ (I)
Step2:	Set $k_r = 0$.
Step3:	Find T through step 3-1 to step 3-4 as follows:
Step3-1:	Start with $T = 0.0001$.
Step3-2:	Calculate T_2 using Eq. (I).
Step3-3:	Find value $T_2 - T$.
Step3-4:	If $T_2 - T \leq \varepsilon$ then substitute $T = T_2$ and go to step 4, otherwise substitute $T = T_2$and go to step 3-2.
Step4:	Utilizing T determines k_r using Eq. (8).
Step5:	If two successive T and k_r are less than εsimultaneously, then T and k_r get their optimum values. Otherwise, go to step 3.

By applying Algorithm1, the optimal solution of $T^d$and k_r^d for the retailer under the decentralized decision-making model will be obtained. The supplier also must solve its own problem separately. For minimizing the supplier's inventory cost, the supplier's review period is considered as an integer multiple of the retailer's review period mT. The supplier's costs consist of ordering, inventory holding and stock-out costs.

As illustrated in figure 2, the supplier's average inventory level can be calculated as:

$$\frac{T\big((m-1)DT + (m-2)DT + \cdots + DT\big)}{mT} + \int_0^{R_s} (R_s - y) f_y(y)dy = \frac{DT^2(m-1)m}{2mT} + k_s\sigma\sqrt{mT}$$
$$= \frac{DT(m-1)}{2} + k_s\sigma\sqrt{mT} \quad (9)$$

In which, the first term is the average inventory and the second term is the safety stock. At the supplier's site, it is assumed that the unsatisfied demand will be backlogged. The expected supplier's stock-out per replenishment cycle can be expressed as:

$$E(y - R_s)^+ = \int_{R_s}^{\infty} (y - R_s)\, f_y(y)dy = \int_k^{\infty} \sigma\sqrt{mT}\,(Z - K) f_z(z)dz = \sigma\sqrt{mT}G(k_s) > 0 \quad (10)$$

Therefore, the expected supplier's cost function can be approximated as:

$$TC_s^d = \frac{A_s}{mT} + h_s\left(\frac{DT(m-1)}{2} + k_s\sigma\sqrt{mT}\right) + \frac{\pi_s}{mT} E(y - R_s)^+ \quad (11)$$

Putting $E(y - R_s)^+$ calculated in Eq. (10) into Eq. (11), we get:

$$TC_s^d = \frac{A_s}{mT} + h_s\left(\frac{DT(m-1)}{2} + k_s\sigma\sqrt{mT}\right) + \frac{\pi_s}{mT}\sigma\sqrt{mT}G(k_s) \quad (12)$$

In which the first term denotes the ordering cost, the second and third terms denote inventory holding cost and shortage cost, respectively. The supplier's cost function depends on the variable T, which was determined by the retailer. In the following, the centralized decision-making structure is investigated in which the value of review period is jointly determined.

4-2- Centralized model

In the centralized case, all SC members try to minimize their cost according to the whole SC viewpoint, accordingly the SC expected cost functions is the sum of the retailer's and supplier's costs functions.

$$TC^{C}_{chain}(T^{c},k^{c}_{r}) = TC^{d}_{r} + TC^{d}_{s}$$
$$= \frac{A_r}{T} + h_r\left(\frac{DT}{2} + k_r\sigma\sqrt{T+L}\right) + \frac{\pi_r}{T}\sigma\sqrt{T+L}G(k_r) + \frac{F_r}{T} + \frac{A_s}{mT} + h_s\left(\frac{DT(m-1)}{2} + k_s\sigma\sqrt{mT}\right) + \frac{\pi_s}{mT}\sigma\sqrt{mT}G(k_s) \quad (13)$$

Proposition 2: The centralized cost function is convex with respect to T^c andk^c_r.
Proof: see Appendix B for detailed proof.
To minimize the SC cost function, taking the first order partial derivatives of $TC^{C}_{chain}(T^{c},k^{c}_{r})$ with respect to T^c and k^c_r gives:

$$\frac{\partial TC^{C}_{chain}}{\partial T^{c}} = \frac{-(A_r+F_r)}{T^2} + h_r\left(\frac{D}{2} + \frac{\sigma k_r}{2\sqrt{T+L}}\right) + \pi_r\sigma G(k_r)\left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2}\right) - \frac{A_s}{mT^2} + h_s\left(\frac{D(m-1)}{2} + \frac{k_s\sigma}{2\sqrt{mT}}\right) + \pi_s\sigma G(k_s)\left(\frac{1}{2T\sqrt{mT}} - \frac{\sqrt{mT}}{mT^2}\right) \quad (14)$$

$$\frac{\partial TC^{C}_{chain}}{\partial k^{c}_{r}} = h_r\sigma\sqrt{T+L} + \frac{\pi_r\sigma\sqrt{T+L}[\emptyset(k_r)-1]}{T} \quad (15)$$

By setting equations (14) and (15) equal to zero, we obtain

$$\frac{A_r+F_r}{T^2} + \frac{\pi_r\sigma G(k_r)\sqrt{T+L}}{T^2} + \frac{A_s}{mT^2} + + \frac{\pi_s\sigma G(k_s)\sqrt{mT}}{mT^2} = h_r\left(\frac{D}{2} + \frac{\sigma k_r}{2\sqrt{T+L}}\right) + \frac{\pi_r\sigma G(k_r)}{2T\sqrt{T+L}} + h_s\left(\frac{D(m-1)}{2} + \frac{k_s\sigma}{2\sqrt{mT}}\right) + \frac{\pi_s\sigma G(k_s)}{2T\sqrt{mT}} \quad (16)$$

And

$$1 - \emptyset(k_r) = \frac{h_r T}{\pi_r} \quad (17)$$

The following Algorithm 2 is used to find the optimal solution of T^c andk^c_r.

Algorithm 2:

Step1	Get T_2 based on Eq. (16) as follows: $$T_2 = \sqrt{\frac{m\left(A_r+F_r+\pi_r\sigma G(k_r)\sqrt{T+L}\right)+A_s+\pi_s\sigma G(k_s)\sqrt{mT}}{m\left(\left(h_r\left(\frac{D}{2}+\frac{\sigma k_r}{2\sqrt{T+L}}\right)+\frac{\pi_r\sigma G(k_r)}{2T\sqrt{T+L}}+h_s\left(\frac{D(m-1)}{2}+\frac{k_s\sigma}{2\sqrt{mT}}\right)+\frac{\pi_s\sigma G(k_s)}{2T\sqrt{mT}}\right)\right)}} \quad (\Pi)$$
Step2:	Set $k_r = 0$.
Step3:	Find T through step 3-1 to step 3-4 as follows:
Step3-1:	Start with $T = 0.0001$.
Step3-2:	Calculate T_2 using Eq. (Π).
Step3-3:	Find value $T_2 - T$.
Step3-4:	If $T_2 - T \le \varepsilon$ then substitute $T = T_2$ and go to step 4. Otherwise, substitute $T = T_2$and go to step 3-2.
Step4:	Utilizing T determines $\emptyset(k_r)$ from the Eq. (17).
Step5:	If two successive T and k_r are less than εsimultaneously, then T and k_r gets their optimum values. Otherwise, go to step 3.

Optimum values of $T^c$and k^c_r can be calculated by using the proposed algorithm 2. The centralized solution can be considered as a benchmark for the coordination model. Due to the optimization of the whole SC in the centralized model, entire SC costs are less than in decentralized model (Heydari, 2014; Chaharsooghi and Heydari, 2009).

4-3- Coordination model based on delay in payments scheme

A coordination model has two main objectives, (1) to increase the SC profit up to the centralized chain's profit, and (2) to share the surplus profit among the SC members (Giannoccaro and Pontrandolfo, 2004). By applying incentive schemes in the coordination models, SC members will be motivated to follow globally optimum decisions for the entire SC (Nematollahi et al., 2016). In this section, a coordination model based on the delay in payments contract is developed. Under such a contract, the supplier persuades the retailer to participate in the coordination model by offering the credit period and any increases in the retailer's costs will be compensated by the supplier.

4-3-1- Incentive scheme based on delay in payments

Incentive schemes guarantee that all SC members participate in the coordination model. Similar to Chaharsooghi and Heydari (2009), in this section, we define a set of operational coefficients for decision variables (i.e., T and k_r) to decrease the cost of chain down to the centralized case. The review period coefficient k_T to achieve channel coordination is:

$$k_T = \frac{T^c}{T^d}$$

The operational coefficient for the variable k_r is defined as follows:

$$k_k = \frac{k_r^c}{k_r^d}$$

By applying $\mathrm{k_T}$, $\mathrm{k_k}$, the retailer's review period shifts from T^d to $T^c = k_T T^d$ and the retailer's safety factor shifts from k_r^d to $k_r^c = k_k k_r^d$. The use of these coefficients in the decentralized SC will lead to channel coordination, but the retailer' cost will increase. In this section, an incentive scheme based on a credit option is developed. In this contract, the supplier persuades the retailer to participate in the coordination model by offering the credit period and any increases in the retailer's costs will be compensated by the supplier.

The investment cost is one main cost of inventories that to base on the time value of money. At the retailer's site, we assume that β% of inventory holding costs are due to the investment cost. As shown in figure 3, the retailer's unit inventory holding cost in credit time is decreased by the coefficient $1-\beta$. So by applying a credit option, the retailer's inventory costs can be reduced and the extra costs can be compensated.

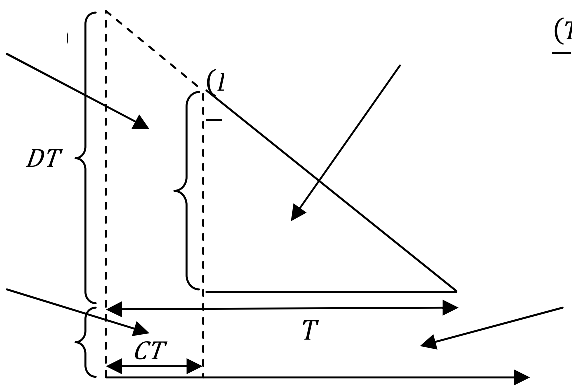

Fig. 3. One average replenishment cycle of the retailer and the area of each part

By applying the credit option, the retailer's inventory holding costs decreases in the credit time. As shown in figure 3, the retailer's inventory holding costs are decreased by $\beta\%$ in credit time. The expected cycle inventory holding cost changes from $h_r\frac{DT}{2}$ to $h_r\left(\frac{D(T^c-CT)^2}{2T^c}+(1-\beta)\left(D-\frac{D.CT}{2T^c}\right)CT\right)$ and the expected safety stock cost changed from $h_r\left(K^c\sigma\sqrt{T^c+L}\right)$ to $h_r\left(K^c\sigma\sqrt{T^c+L}\left(1-\frac{\beta.CT}{T^c}\right)\right)$. Therefore, the retailer's cost function after participating in the coordination plan will be:

$$TC_r^{co}(T^{**},k_r^{**},CT) = \frac{A_r}{T^c} + h_r\left(\frac{D(T^c-CT)^2}{2T^c}+(1-\beta)\left(D-\frac{DCT}{2T^c}\right)CT+K^c\sigma\sqrt{T^c+L}\left(1-\frac{\beta.CT}{T^c}\right)\right) + \frac{\pi_r}{T^c}\sigma\sqrt{T^c+L}G(k^c)+\frac{F_r}{T^c} \quad (18)$$

In which the first term denotes the ordering cost, the second term shows the inventory holding cost, the third term represents the shortage cost and the last term denotes the fixed transportation cost. Therefore, the supplier' cost function will be:

$$TC_s^{co}(T^{**},k_r^{**},CT) = \frac{A_s}{mT^c}+h_s\left(\frac{DT^c(m-1)}{2}+k_s\sigma\sqrt{mT^c}\right)+\frac{\pi_s}{mT^c}\sigma\sqrt{mT^c}G(k_s) + \frac{\beta h_r}{T^c}\left[CT\left(DT^c-\frac{1}{2}DCT+k_r^c\sigma\sqrt{T^c+L}\right)\right] \quad (19)$$

In equation (19), the first term shows the ordering cost, the second term represents the inventory holding cost, the third term denotes the shortage cost and the last term shows the additional cost due to the applying the delay in payments contract. According to figure 3, to calculate the last term, the surface under the retailer's inventory level curve in the interval $[0,CT]$ is calculated and then the obtained value is multiplied by the investment cost factor, the retailer's inventory holing cost and by the number of the retailer's replenishment cycles in each year(i.e.,$\frac{\beta h_r}{T^c}$).

Retailer participates in the coordination model, when its cost does not exceed that before participating; thus, we have:

$TC_r^{co}(T^c,k_r^c,CT) \leq TC_r^d(T^d,k_r^d)$

Therefore, the minimum amount of credit period that guarantees retailer's participation in the coordination model can be calculated as:

$$CT_{min} = \frac{\left[Dk_T T^d + k_k k^d\sigma\sqrt{k_T T^d+L}\right]}{D} - \sqrt{\frac{2A}{D}} \quad (20)$$

Where

$$A = \frac{\left[Dk_T T^d + k_k k_r^d \sigma\sqrt{k_T T^d + L}\right]^2}{2D} + \frac{k_T T^d}{\beta h_r}\left[(A_r + F_r)\left(\frac{1}{T^d} - \frac{1}{k_T T^d}\right) + h_r\left(\frac{DT^d}{2} + k_r^d \sigma\sqrt{T^d + L} - \frac{Dk_T T}{2} - k_k k_r^d \sigma\sqrt{k_T T^d + L}\right) + \frac{\pi_r}{T^d}\sigma G(k_r^d)\sqrt{T^d + L} - \frac{\pi_r}{k_T T^d}\sigma\sqrt{k_T T^d + L}G(k_k k_r^d)\right] \tag{21}$$

Supplier participates in the coordination model if its costs after applying the delay in payments do not exceed its costs under the decentralized model; thus, we have:

$$TC_s^{co}(T,k,CT) \leq TC_s^d$$

Therefore, the maximum credit period that guarantees supplier's participation in the coordination scheme can be calculated as:

$$CT_{max} = \frac{\left[Dk_T T^d + k_k k^d \sigma\sqrt{k_T T^d + L}\right]}{D} - \sqrt{\frac{2M}{D}} \tag{22}$$

Where

$$M = \frac{\left[Dk_T T^d + k_k k^d \sigma\sqrt{k_T T^d + L}\right]^2}{2D} - \frac{k_T T^d}{\beta h_r}\left[A_s\left(\frac{1}{mT^d} - \frac{1}{mk_T T^d}\right) + h_s\left(\frac{DT^d(m-1)}{2}(1 - k_T) + k_s\sigma\left(\sqrt{mT^d} - \sqrt{mk_T T^d}\right)\right) + \pi_s\left(\frac{\sigma\sqrt{mT^d}G(k_s)}{mT^d} - \frac{\sigma\sqrt{mk_T T^d}G(k_s)}{mk_T T^d}\right)\right] \tag{23}$$

Each amount of credit time in this interval $[CT_{min}, CT_{max}]$ can lead to channel coordination. At CT_{min}, all coordination benefits will be achieved by the supplier, and at CT_{max} all coordination benefits will be obtained by the retailer. In the following, it is presented a profit sharing strategy to calculate the exact credit time.

4-3-2- Profit sharing strategy

The surplus benefit obtained by a coordination mechanism should fairly share among the SC members. Accordingly, in the following, a profit sharing strategy based on the bargaining power of the two SC members is developed. We define the retailer' bargaining power against the supplier asα, and therefore the supplier's bargaining power will be $1 - \alpha$.

The amount of reduced SC cost resulting by using the coordinated model can be calculated as:

$$\Delta TC = TC_{chain}^{d}\left(T^d, k_r^d\right) - TC_{chain}^{c}(T^c, k_r^c)$$
$$= \left(\frac{A_r}{T^d} + h_r\left(\frac{DT^d}{2} + k_r\sigma\sqrt{T^d + L}\right) + \frac{\pi_r}{T^d}\sigma\sqrt{T^d + L}G(k_r) + \frac{F_r}{T^d} + \frac{A_s}{mT^d} + h_s\left(\frac{DT^d(m-1)}{2} + k_s\sigma\sqrt{mT^d}\right) + \frac{\pi_s}{mT^d}\sigma\sqrt{mT^d}G(k_s)\right)$$
$$- \left(\frac{A_r}{T^c} + h_r\left(\frac{DT^c}{2} + k_r\sigma\sqrt{T^c + L}\right) + \frac{\pi_r}{T^c}\sigma\sqrt{T^c + L}G(k_r) + \frac{F_r}{T^c} + \frac{A_s}{mT^c} + h_s\left(\frac{DT^c(m-1)}{2} + k_s\sigma\sqrt{mT^c}\right) + \frac{\pi_s}{mT^c}\sigma\sqrt{mT^c}G(k_s)\right) \quad (24)$$

By considering the retailer's bargaining power (i.e., α), it is expected that $\alpha\%$ of the amount of reduced SC cost will be transferred to the retailer, we have:

$$TC_r^{co}(T^c, k_r^c, CT) = TC_r^d\left(T^d, k_r^d\right) - \alpha\Delta TC \quad (25)$$

Putting equations (4) and (18) into Eq. (25), we get:

$$\alpha\Delta TC = TC_r^d(T^d, k_r^d) - TC_r^{co}(T^c, k_r^c, CT)$$
$$= (A_r + F_r)\left(\frac{1}{T^d} - \frac{1}{T^c}\right) + h_r\left(\frac{DT^d}{2} + k_r\sigma\sqrt{T^d + L}\right) + \frac{\pi_r}{T^d}\sigma\sqrt{T^d + L}G(k_r)$$
$$- h_r\left(\frac{D(T^c - CT)^2}{2T^c} + (1-\beta)\left(D - \frac{D.CT}{2T^c}\right).CT + K^c\sigma\sqrt{T^c + L}\left(1 - \frac{\beta.CT}{T^c}\right)\right)$$
$$- \frac{\pi_r}{T^c}\sigma\sqrt{T^c + L}G(k^c) \quad (26)$$

By some simplifying the above equation, we have:

$$CT = \frac{\left[Dk_T T^d + k_k k^d\sigma\sqrt{k_T T^d + L}\right]}{D} \pm \sqrt{\frac{2A}{D} - \frac{2k_T T^d\alpha\Delta TC}{D\beta h_r}} \quad (27)$$

The obtained credit time is acceptable if and only if it be in the interval $[CT_{min}, CT_{max}]$. Using the calculated CT as the credit time, the coordination plan will benefit both members based on their bargaining power.

5-Numerical experiments

In this section, by using a set of numerical experiments, the performance of the proposed coordination model is evaluated. To this end, the results of decentralized, centralized and coordination models are obtained. In addition, a set of sensitivity analysis with respect to the primary parameters are conducted. Table 1 displays the data for the five test problems. We try to cover a wide range of parameters in the numerical examples.

Table1. Five test problem parameters

	Test problem 1	Test problem 2	Test problem 3	Test problem 4	Test problem 5
$\mathbf{D}$	5000	6000	9000	10000	11000
$\mathbf{L(day)}$	1	6	15	35	20
$\mathbf{m}$	5	5	2	2	2
$\mathbf{A_r}$	300	600	900	800	1100
$\mathbf{A_s}$	300	600	500	500	500
$\mathbf{h_r}$	5	5	5	5	4
$\mathbf{h_s}$	2	2	4	5	4
$\boldsymbol{\pi_r}$	3	4	3	3	3
$\boldsymbol{\pi_s}$	1	1	2	1	2
$\mathbf{F_r}$	50	50	60	100	100
$\mathbf{k_s}$	2	1.2	1	0.9	0.8
$\boldsymbol{\sigma}$	1000	2000	2000	2500	2800
$\boldsymbol{\beta}$	0.8	0.5	0.6	0.7	0.8
$\boldsymbol{\alpha}$	0.3	0.5	0.7	0.6	0.4

By running the model in mathematical software, the results of decision variables and the cost functions in the decentralized, centralized, and coordinated models are calculated. As illustrated in Table 2, by comparing the results in the decentralized and centralized models, it is observed that the centralized model decreases the cost of whole supply chain and the length of period review, also increases the value of safety factor in all test problems. However, the retailer’s cost increases by shifting from the decentralized to the centralized mode. As shown in Table 2, parameters of the coordination model are calculated. The results indicate that the interval $[CT^{min}, CT^{max}]$ is a non-empty interval and therefore channel coordination is accessible in all test problems. Moreover, using the Profit sharing strategy, the exact value of CT is obtained on the basis of the bargaining power of SC members. By using the obtained credit option in the coordination model, the retailer's and supplier's costs are less than those ones in the decentralized mode. Therefore, applying the credit period guarantees that both members participate in coordination model. Also Table 2 shows improvement of coordination model versus the decentralized model in all numerical experiments.

Table 2. Results of running decentralized, centralized, and coordinated models

	Test problem 1	**Test problem 2**	**Test problem 3**	**Test problem 4**	**Test problem 5**
Decentralized SC					
T	62.16	78.4	90.67	95.08	99.97
k_r	0.57	0.62	0.22	0.166	0.34
TC_r^d	6667	12153	14518	16731	17017
TC_s^d	7459	10797	11589	15940	14480
TC_{chain}^d	14126	22950	26107	32671	31497
Centralized SC					
T	37.85	47.4	62.05	55.33	65.07
k_r	0.94	0.98	0.57	0.66	0.71
TC_r^c	7085	12741	14962	17592	17687
TC_s^c	5545	8047	9767	12089	11802
TC_{chain}^c	12630	20788	24729	29681	29490
Coordination SC					
$\boldsymbol{T}$	37.85	47.4	62.05	55.33	65.07
$\boldsymbol{k_r}$	0.94	0.98	0.57	0.66	0.71
TC_r^{co}	6218	11073	13553	14937	16214
TC_s^{co}	6412	9715	11176	14744	13275
$\boldsymbol{TC_{chain}^{co}}$	12630	20788	24729	29681	29490
Coordination parameter (day) (year = 365 days)					
CT_{min}	5	7.57	4,6	6.02	4.78
CT_{max}	28.79	44.73	20.96	31.92	20.85
CT	10.92	23.63	15.64	20.31	10.84
Improvement%	10.59	9.42	5.28	9.15	6.37

In the following, a set of sensitivity analysis with respect to the demand uncertainties and the members' costs are conducted. Using the data of numerical example 2, the results of running the model for various values of $\sigma$are shown in Table 3. We change the value of σbetweenσ -%40 and σ+%40. For the all values of σ, the SC cost in the coordination model is less than the decentralized model.

Table 3. Results of sensitivity analysis with respect to the demand uncertainties

	σ-%40	σ -%30	σ -%20	σ -%10	σ	σ +%10	σ +%20	σ +%30	σ +%40
$\boldsymbol{\sigma}$	1200	1400	1600	1800	2000	2200	2400	2600	2800
$\boldsymbol{TC^{d}_{chain}}$	18464	19604	20700	21819	22950	24095	25237	26388	27558
$\boldsymbol{TC^{co}_{chain}}$	16901	17898	18853	19820	20788	21758	22719	23677	24632
$\boldsymbol{\Delta TC}$	1563	1706	1848	1999	2162	2337	2518	2711	2926
Improvement%	8.47	8.70	8.92	9.16	9.42	9.70	9.98	10.27	10.62

As shown in figure 4, by increasing the value of σ, the proposed coordination model has better performance compared to the decentralized model.Moreover, in the high values of uncertainty the difference between the supply chain's cost in both decentralized and coordinated models increases. Therefore, applying the coordination model is of high significance under demand uncertainty.

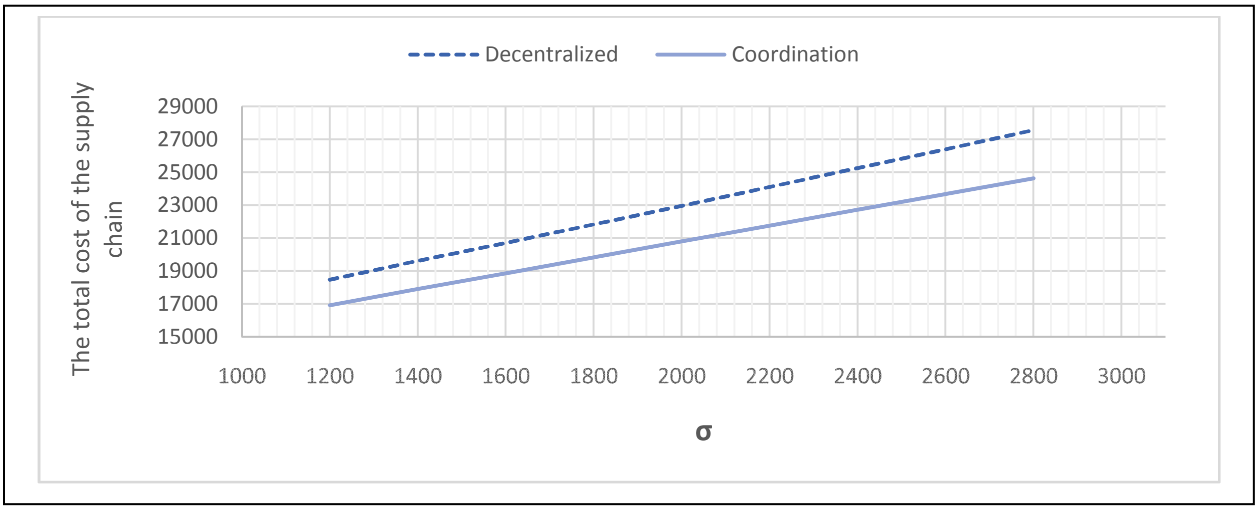

Fig. 4. The supply chain's cost in the decentralized and coordination modes by increasing σ

According to the numerical example 2, the changes of SC members' cost in the coordination and decentralized models under various values of σ are shown in figure 5. As σ increases the cost function of two members improve. In the other words, by increasing σ, difference between the profitability under the coordination and decentralized models increases. Accordingly, it can be concluded that the developed coordination model is of great importance under demand uncertainty. It is noteworthy that the improvement index is calculated as $\left(\frac{TC^{d}_{chain}-TC^{co}_{chain}}{TC^{d}_{chain}}\right)*100$.

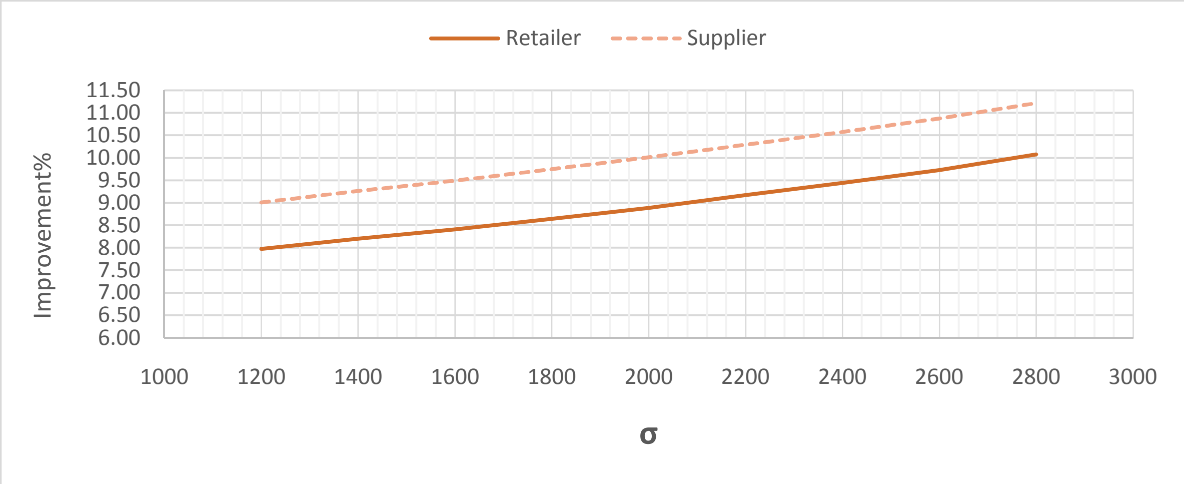

Fig. 5. Improvement the retailer's and supplier's cost in the decentralized and coordination modes

Figure 6, indicates the changes of SC members' cost after applying the coordination model under various values of the retailer's shortage cost in the numerical example 1. As shown in figure 6, by increasing π_r, the profitability of the model for both members will decrease. In the other words, the members' profitability is high in the low values of π_r.Moreover, for the various values of π_r, improvement of the supplier's cost will be more than the retailer.

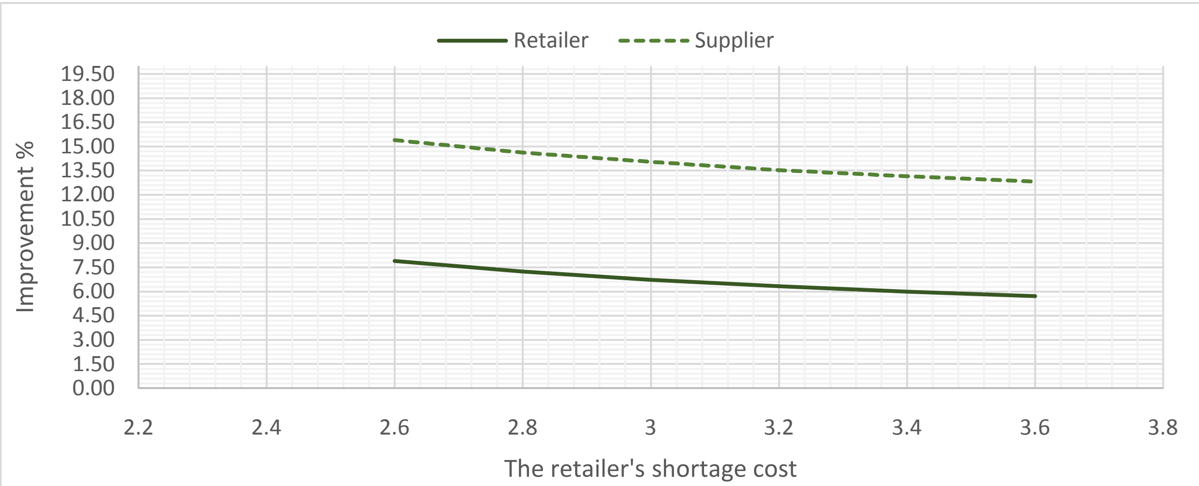

Fig. 6. The improvement of the retailer's shortage cost on the retailer's and supplier's cost

According to the numerical example 1, the improvement of both SC members after applying the coordination model under various values ofh_r are shown in figure 7. As h_r increases the profitability of the model for both members decreases. Therefore, in the high values of h_r, the SC members prefer to hold less inventory.

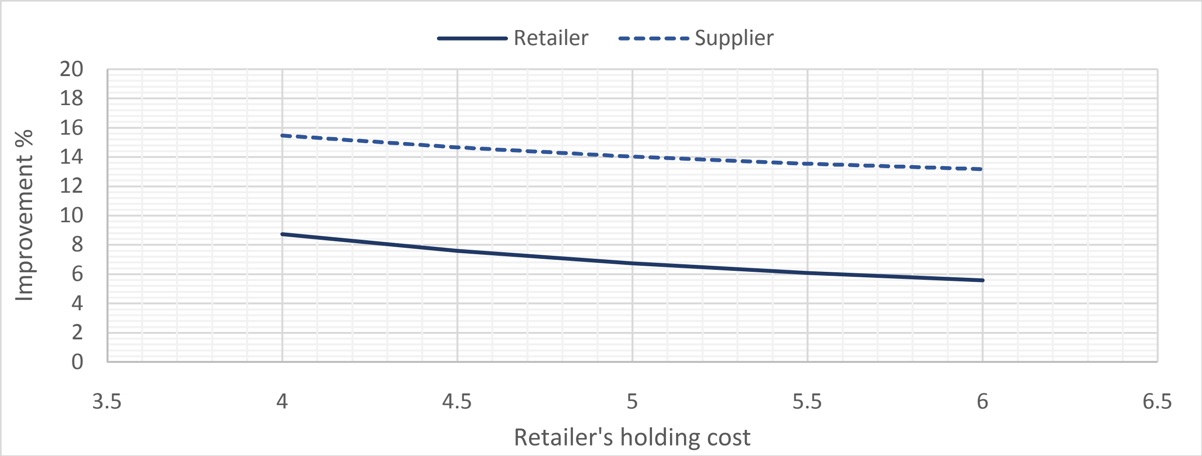

Fig. 7. The improvement of the retailer's holding cost on the retailer's and supplier's cost

Table 4 shows the results of sensitivity analysis with respect to 10% increase and decrease the retailer's and supplier's holding cost in numerical example 2. As shown in Table 4, increasing the retailer's holding cost reduces the value of CT in most cases and this is undesirable for the retailer, also increasing the supplier's holding cost leads to increase resulting CT in most cases which is undesirable for the supplier. So the coordination model encourages SC members to keep down their holing cost.

Table 4. The impact of the holding cost on credit option

	Retailer		Supplier	
	Changes on CT (%)		Changes on CT (%)	
	Under estimate h_r	Over estimate h_r	Under estimate h_s	Over estimate h_s
Test problem 1	1.93	-1.43	-1.7	1.78
Test problem 2	3.27	-2.22	-3.8	4.08
Test problem 3	2.61	-1.3	-3.01	3.27
Test problem 4	0.53	0.7	-3.27	3.48
Test problem 5	1.28	-0.77	-1.84	1.96

Table 6, indicates the values of credit option with respect to 10% increase and decrease β in the five test problems. As shown in Table 6, by increasing β, credit option decreases in the five test problems, which is expected and this is undesirable for the retailer.

Table 6. The impact of various values of β on credit option

	The values of CT		
	underestimateβ		overestimateβ
Test problem 1	12.68	10.92	9.59
Test problem 2	30.92	23.63	19.17
Test problem 3	19.22	15.64	13.19
Test problem 4	24.35	20.31	17.44
Test problem 5	12.5	10.84	9.57

6-Conclusion

In this paper, a coordination model based on the delay in payments contract for a two-level supply chain has been developed. The customer's demand followed a normally distributed function and backorder shortage was allowable in the system. The retailer used a periodic review inventory model and decided simultaneous on the review period and the safety factor. Firstly, the investigated SC was modeled under the decentralized and centralized decision-making structure, afterwards, a delay in payments contract as an incentive mechanism was proposed to guarantee the retailer's participates in the coordination plan. The main contribution of this paper is to coordinate the review period and safety factor simultaneously in a supplier-retailer SC under periodic review inventory system. Although a handful of studies have been conducted on coordination of periodic review inventory systems within SC, the delay in payments contract as a coordination mechanism has not yet been developed for coordinating these systems. The length of credit option is assumed to be decision variable in current study. A profit sharing strategy to fairly share the surplus profit obtained by coordination was considered according to the bargaining power of the two SC members and the exact value ofCT is calculated.By running the models, the results of decision variables and the cost functions in the decentralized, centralized, and coordinated models were calculated. The results indicate that changes of the members' holding cost impact on the credit time. However, the proposed coordination contract is capable of coordinating the SC under various values of holding cost. Also, in the high demand uncertainty (i.e., by increasing σ), the results indicate that the proposed coordination model performed quite well and resolved the conflict of interests.

Managerial implications from the proposed model can be summarized as: (1) In a two-level supply chain under periodic review inventory policy, the centralized decision making decreased the whole SC cost but increased the retailer’s cost. By using the delay in payments contract as a coordination mechanism, the whole SC cost and the retailer and supplier’s cost were less than those under the decentralized decision making. Therefore, the proposed delay in payments contract guaranteed that both SC members would participate in the coordination model. (2) Based on the sensitivity analyses, the SC members’ holding cost impact on the value of CT. As by increasing the retailer’s holding cost reduces the value of CT in most cases and this is undesirable for the retailer, also increasing the supplier’s holding cost leads to increase in resulting CT in most cases which is undesirable for the supplier. So the coordination model encourages SC members to keep down their holing cost.(3) The results indicate that by increasing the value of σ, in the high demand uncertainty, the difference between the profitability of the coordinated and decentralized models increases. Therefore, applying the coordination model is of high significance under high demand uncertainty. (4) Results of sensitivity analysis indicate that the members’ profitability is high in the low values of π_r. Moreover, for the various values of π_r, improvement of the supplier’s cost will be more than the retailer.

As future research, it is suggested to assume the number of supplier replenishment cycles and the lead time as decision variables that the length of lead time can shorten at an extra crashing cost. In addition, this model can apply lost-sale inventory models.

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Appendix A

Convexity of the retailer expected annual cost function in the decentralized model

Proof: To prove convexity, it is essential to compute the Hessian matrix of the retailer expected annual cost function with respect to $k_r$and T. We have:

$$H = \begin{pmatrix} \frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial {k_r}^2} & \frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial k \partial T} \\ \frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial T \partial k} & \frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial T^2} \end{pmatrix}$$

$$\frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial {k_r}^2} = \frac{\pi_r \sigma (T+L)^{1/2} f(k)}{T} = |H_{11}| > 0$$

$$\frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial T^2} = \frac{2(A_r + F_r)}{T^3} + \pi_r \sigma G(k_r) \left(\frac{2\sqrt{T+L}}{T^3} - \frac{1}{T^2\sqrt{T+L}} - \frac{1}{4T(T+L)^{3/2}} \right) - \frac{h_r k_r \sigma}{4(T+L)^{3/2}}$$

$$\frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial T \partial k} = \frac{\partial^2 TC_r^d(T^d, k_r^d)}{\partial k \partial T} = \frac{h_r \sigma}{2\sqrt{T+L}} + \pi_r \sigma \left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2} \right) [\emptyset(k_r) - 1]$$

$$|H_{22}| = \frac{\partial^2 TC_r^d\left(T^d, k_r^d\right)}{\partial T^2} \times \frac{\partial^2 TC_r^d\left(T^d, k_r^d\right)}{\partial {k_r}^2} - \frac{\partial^2 TC_r^d\left(T^d, k_r^d\right)}{\partial T \partial k} \times \frac{\partial^2 TC_r^d\left(T^d, k_r^d\right)}{\partial k \partial T}$$

$$= \left[\frac{2(A_r + F_r)}{T^3} + \pi_r \sigma G(k_r)\left(\frac{2\sqrt{T+L}}{T^3} - \frac{1}{T^2\sqrt{T+L}} - \frac{1}{4T(T+L)^{3/2}}\right) - \frac{h_r k_r \sigma}{4(T+L)^{3/2}}\right] \times \left[\frac{\pi_r \sigma \sqrt{T+L} f(k)}{T}\right]$$

$$- \left[\frac{h_r \sigma}{2\sqrt{T+L}} + \pi_r \sigma \left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2}\right)[\emptyset(k_r) - 1]\right]^2$$

$$= \frac{2(A_r + F_r)\pi_r \sigma \sqrt{T+L} f(k)}{T^4} - \frac{h_r k_r \sigma^2 \pi_r f(k)}{4T(T+L)} - \frac{\pi_r^2 \sigma^2 G(k_r) f(k)}{T^3} - \frac{\pi_r^2 \sigma^2 G(k_r) f(k)}{4T^2(T+L)} + \frac{\pi_r^2 \sigma^2 G(k_r) f(k) 2\sqrt{T+L}}{T^4} - \frac{h_r^2 \sigma^2}{4(T+L)} - \frac{\pi_r^2 \sigma^2 (T+L)[\emptyset(k_r) - 1]^2}{T^4} - \frac{\pi_r^2 \sigma^2 [\emptyset(k_r) - 1]^2}{4T^2(T+L)} + \frac{h_r \sigma^2 \pi_r [\emptyset(k_r) - 1]}{T^2} - \frac{h_r \sigma^2 \pi_r [\emptyset(k_r) - 1]}{4T(T+L)} + \frac{\pi_r^2 \sigma^2 [\emptyset(k_r) - 1]^2}{T^3} > 0$$

By calculating the above Hessian matrix can be observed that the first principal minor Hessian has a positive value. Under the condition of problem, the second principal minor is positive. The problem is tested with the various numerical examples which cover a wide range of reasonable parameters and it is observed that it has a positive value for all numerical examples.

Appendix B

Convexity of supply chain cost function in the centralized model

To prove convexity of supply chain cost function in the centralized model, the Hessian matrix is calculated with respect to k_r and T as follows:

$$H = \begin{pmatrix} \frac{\partial^2 TC_{chain}^C(T^c, k_r^c)}{\partial {k_r}^2} & \frac{\partial^2 TC_{chain}^C(T^c, k_r^c)}{\partial k_r \partial T} \\ \frac{\partial^2 TC_{chain}^C(T^c, k_r^c)}{\partial T \partial k_r} & \frac{\partial^2 TC_{chain}^C(T^c, k_r^c)}{\partial T^2} \end{pmatrix}$$

$$\frac{\partial^2 TC_{chain}^C(T^c, k_r^c)}{\partial {k_r}^2} = \frac{\pi_r \sigma (T+L)^{1/2} f(k_r)}{T} = |H_{11}| > 0$$

$$\frac{\partial^2 TC_{chain}^C(T^c, k_r^c)}{\partial T^2} = \frac{2(A_r + F_r)}{T^3} + \pi_r \sigma G(k_r)\left(\frac{2\sqrt{T+L}}{T^3} - \frac{1}{T^2\sqrt{T+L}} - \frac{1}{4T(T+L)^{3/2}}\right) - \frac{h_r k_r \sigma}{4(T+L)^{3/2}} + \frac{2A_s}{mT^3} - \frac{h_s k_s \sigma}{4T\sqrt{mT}} + \pi_s \sigma G(k_s)\left(\frac{-1}{4T\sqrt{mT}} + \frac{1}{T^2\sqrt{mT}}\right)$$

$$\frac{\partial^2 TC^C_{chain}(T^c,k^c_r)}{\partial T \partial k} = \frac{\partial^2 TC^C_{chain}(T^c,k^c_r)}{\partial k \partial T} = \frac{h_r\sigma}{2\sqrt{T+L}} + \pi_r\sigma\left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2}\right)[\emptyset(k_r)-1]$$

$$|H_{22}| = \frac{\partial^2 \mathrm{TC}^C_{chain}(T^c,k^c_r)}{\partial T^2} \times \frac{\partial^2 \mathrm{TC}^C_{chain}(T^c,k^c_r)}{\partial {k_r}^2} - \frac{\partial^2 \mathrm{TC}^C_{chain}(T^c,k^c_r)}{\partial T \partial k} \times \frac{\partial^2 \mathrm{TC}^C_{chain}(T^c,k^c_r)}{\partial k \partial T}$$

$$= \left[\frac{2(A_r+F_r)}{T^3} + \pi_r\sigma G(k_r)\left(\frac{2\sqrt{T+L}}{T^3} - \frac{1}{T^2\sqrt{T+L}} - \frac{1}{4T(T+L)^{3/2}}\right) - \frac{h_r k_r \sigma}{4(T+L)^{3/2}} + \frac{2A_s}{mT^3} - \frac{h_s k_s \sigma}{4T\sqrt{mT}} + \pi_s\sigma G(k_s)\left(\frac{-1}{4T\sqrt{mT}} + \frac{1}{T^2\sqrt{mT}}\right)\right] \times \left[\frac{\pi_r\sigma\sqrt{T+L}f(k_r)}{T}\right] - \left[\frac{h_r\sigma}{2\sqrt{T+L}} + \pi_r\sigma\left(\frac{1}{2T\sqrt{T+L}} - \frac{\sqrt{T+L}}{T^2}\right)[\emptyset(k_r)-1]\right]^2$$

$$= \frac{2(A_r+F_r)\pi_r\sigma\sqrt{T+L}f(k_r)}{T^4} - \frac{h_r k_r \sigma^2 \pi_r f(k_r)}{4T(T+L)} - \frac{\pi_r^2\sigma^2 G(k_r)f(k_r)}{T^3} - \frac{\pi_r^2\sigma^2 G(k_r)f(k_r)}{4T^2(T+L)} + \frac{2\pi_r^2\sigma^2 G(k_r)f(k_r)\sqrt{T+L}}{T^4} + \frac{2\mathrm{A_s}\pi_r\sigma\sqrt{T+L}f(k_r)}{mT^4} - \frac{\mathrm{h_s k_s}\sigma^2\pi_r\sqrt{T+L}f(k_r)}{4T^2\sqrt{mT}} - \frac{\pi_s\pi_r\sigma^2 G(k_s)f(k_r)\sqrt{T+L}}{4T^2\sqrt{mT}} + \frac{\pi_s\pi_r\sigma^2 G(k_s)f(k_r)\sqrt{T+L}}{T^3\sqrt{mT}} - \frac{h_r^2\sigma^2}{4(T+L)} - \frac{\pi_r^2\sigma^2(T+L)[\emptyset(k_r)-1]^2}{T^4} - \frac{\pi_r^2\sigma^2[\emptyset(k_r)-1]^2}{4T^2(T+L)} + \frac{h_r\sigma^2\pi_r[\emptyset(k_r)-1]}{T^2} - \frac{h_r\sigma^2\pi_r[\emptyset(k_r)-1]}{4T(T+L)} + \frac{\pi_r^2\sigma^2[\emptyset(k_r)-1]^2}{T^3} > 0$$

The first principal minor Hessian has a positive value. Under the condition of problem, the second principal minor is positive. The problem is tested with the various numerical examples which cover a wide range of reasonable parameters and it is observed that it has a positive value for all numerical examples.