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Vehicle Routing Problem in Competitive Environment: Two-Person Nonzero Sum Game Approach

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Abstract
Vehicle routing problem is one of the most important issues in transportation. Among VRP problems, the competitive VRP is more important because there is a tough competition between distributors and retailers. In this study we introduced new method for VRP in competitive environment. In these methods Two-Person Nonzero Sum games are defined to choose equilibrium solution. Therefore, revenue given in each route is different. In this paper, two distributors has been considered in a city with a set of customers and the best route with maximum revenue has been determined. First we introduced the Hawk-Dove procedure for the VRP problem and then by using Nash bargaining model the equilibrium strategy of the game is calculated. The result of this method is different based on the kind of the strategy that each distributor chooses. In the Hawk-Dove game, if both of distributors choose the Dove procedure, they will get equal but less revenue. In the Nash Bargaining Game, the equilibrium strategy will obtained when distance of revenues of both distributors form its breakdown payoff is maximum.
Keywords: Vehicle routing problem, Two-Person Nonzero Sum game, Hawk-Dove game, Nash Bargaining Game, Equilibrium solution.

1- Introduction

Transportation problems have the most important role in various fields, such as transit of goods, public transportation, materials handling in factories and so on. Based on the importance of the subject, researchers are interested in shortening and improving the routes, to omit the unnecessary travels and costs and create the best routes. Hence, many mathematical problems such as traveling salesman problem (TSP), vehicle routing problem (VRP) (Dantzig et al., 1959) and so on, are developed to address the real transportation problems. The Vehicle Routing Problem is a problem that a set of vehicles must deliver goods to customers regarding to different situations and constraints. Generally minimizing the total routing cost or traveling distance, is the main objective for the VRP. This objective is sometimes insufficient to provide a good practical solution. In this research, we attempt to redefine the problem in competitive environment by using two-person nonzero sum games approaches namely Hawk-Dove game and Bargaining game model. In our paper the main objective of each company is to maximize the total routing benefit. In the real world, there are a lot of problems within the framework competitive VRP, such as distribution of dairy products to retailers by two rival distributors. Where, compared with rival, choosing a suitable route can increase the level of customer service and profits and reduce costs. The customers' visit time by distributors is the key factor in competitive VRP. Depending on the nature of the goods, the quality of services, and the characteristics of customers and vehicles, there are several variants of the VRP. Some typical complications are heterogeneous vehicles located at different depots, customers incompatible with certain vehicle types, customers accepting delivery within specified time windows, multiple-day planning horizons and vehicles performing multiple routes. In all cases, the objective is to supply the customers at minimum cost (Baldacci et al., 2012).

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The different types of VRP are presented in the Table 1. Each type changes one or more main assumptions of basic VRP.

Table 1. Types of VRP

Name	Description	Abbreviation
Vehicle Routing Problem with Pickup and Delivery	A number of goods must be moved from specific pickup points to some delivery points. The aim of VRPPD is to find the best routes for a set of vehicles to visit the pickup and delivery points.	VRPPD
Vehicle Routing Problem with LIFO	Is similar to the VRPPD but there is an extra limitation on the vehicles for loading goods.	VRP+LIFO
Vehicle Routing Problem with Time Windows	The customers have interval time (time windows) that distributors must visit them.	VRPTW
Capacitated Vehicle Routing Problem	The fleet of vehicles has limited capacity for goods that must be carried.	CVRP
Capacitated Vehicle Routing Problem with Time Windows	The fleet of vehicles has limited capacity for goods that must be carried + time windows	CVRPTW
Vehicle Routing Problem with Multiple Trips	The vehicles can do more than one route.	VRPMT
Open Vehicle Routing Problem	The vehicles are not required to come back to the depot.	OVRP
Vehicle Routing Problem with Stochastic Demand	A number of goods must move from the depot to customers that their demands are stochastic.	VRPSD
Competitive Vehicle Routing Problem	Are defined in the following paragraph.	COVRP
Competitive Vehicle Routing Problem with Stochastic Demand	Are defined in the following paragraph.	COVRPSD

Capacitated VRP (CVRP) is the most simple and studied member of the VRP family. In the CVRP, a fleet of identical vehicles located at a central depot has to be optimally routed to supply a set of customers with known demands. Each vehicle can perform at most one route and the total demand of the customers visited by a route cannot exceed the vehicle capacity. Another important variant of the VRP is the VRP with time windows (VRPTW) that generalizes the CVRP by imposing each customer to be visited within a specified time interval, called time window (Baldacci et al., 2012). VRPTW is defined in the following section. The objective of this paper is to present mathematical formulations and solution algorithms for the VRPTW in competitive environment by using game theory models.

Specifically, this research uses the mathematical game theory model to answer the following questions:

(1) What are the best strategies for the simultaneouse optimization of the distributors' profit?

(2) Which customers are assigned to each distributor?

(3) What is the sequence of customer servicing by distributors?

This paper is organized as follows. In Section 2, literature of past works is reviewed. Then, problem definition and the proposed model are presented in Section 3. A numerical example is provided to demonstrate how the theoretical results, in this paper can be applied in practice in Section 4. Finally, Section 5 presents the conclusion remarks and future research directions.

2- Literature of past works

In this section first, the literature of previouse works is briefly reviewed. Then, the location of the current study is determined among the existing works.

2-1- Vehicle routing problem with time windows

The vehicle routing problem with time windows (VRPTW) is a very popular version of the VRP that delivery of goods to customer *i* must be done in the interval time [$t1_i$, $t2_i$], such that $t1_i$ is the earliest allowable time and $t2_i$ are the latest allowable times that the distributor must serve the customer. Table 2 describes literature review of VRPTW:

Table 2. Literature review of VRPTW

Authors	Title	Method
Solomon MM , (1986)	On the worst-case performance of some heuristics for the vehicle routing problem with time window	Variety of heuristics
Solomon MM (1987)	Algorithms for the vehicle routing and scheduling problem with time windows constraints.	Insertion-Type Heuristic
Golden BL and Assad AA (1988)	Vehicle routing: methods and studies	Modeling and Implementation
Solomon MM, Desrosiers J (1988)	Time window constrained routing and scheduling problems	Dial-a-Ride Problem
Taillard E et al. (1997)	A tabu search heuristic for the vehicle routing problem with soft time windows.	Tabu Search
Cordeau JF et al. (2002)	The VRP with time windows	Discrete Mathematics and Applications
Geiger MJ (2003)	A computational study of genetic crossover operators for multi-objective vehicle routing problem with soft time windows.	Genetic Algorithm
Tavakkoli-Moghaddam R et al. (2005)	A multi-criteria vehicle routing problem with soft time windows by simulated annealing.	Simulated Annealing
Braysy O and Gendreau (2005)	Vehicle routing problem with time windows, part I: route construction and local search algorithms.	Local Search Algorithms
Ombuki B, Ross BJ and Hanshar F. (2006)	Multi-objective genetic algorithm for vehicle routing problem with time windows	Genetic Algorithm
Qureshi AG et.al. (2009)	An exact solution approach for vehicle routing and scheduling problems with soft time windows	Dantzig-Wolf decomposition
Li X. et al. (2010)	Vehicle routing problems with time windows and stochastic travel and service times: Models and algorithm	Tabu Search
Tavakkoli-Moghaddam R et al. (2011)	A new mathematical model for a competitive vehicle routing problem with time windows solved by simulated annealing	Simulated Annealing
Baldacci R et al. (2012)	Recent exact algorithm for solving the vehicle routing problem under capacity and time window constraints	State-of-the-art exact Algorithm
Errico F. et al. (2013)	The Vehicle Routing Problem with Hard Time Windows and Stochastic Service Times	Branch-price-and-cut Algorithm
Fakhrzad and Sadri Esfahani (2014)	Modeling the Time Windows Vehicle Routing Problem in Cross-docking Strategy Using Two Meta-heuristic Algorithms	Meta-heuristic Algorithms
Batista BM et al. (2014)	A bi-objective vehicle routing problem with time windows: A real case in Tenerife	Mixed integer linear Model
Wang, Z. et al. (2015)	A heuristic approach and a tabu search for the heterogeneous multi-type fleet vehicle routing problem with time windows and an incompatible loading constraint	A heuristic approach and a tabu search
Hernandez, f. et al. (2016)	Branch-and-price algorithms for the solution of the multi-trip vehicle routing problem with time windows	Branch-and-price algorithms and dynamic programing
Schneider, M. (2016)	The vehicle-routing problem with time windows and driver-specific times	Tabu search

In the real world VRP problems, distributor companies specify the routes of vehicles based on situation of other competitor distributors to gain the maximum sale. This situation would highlight the importance of service time of each rival company to the customers.

To the best of the authors' knowledge, no research was found in the context of VRPTW, which surveys the competition between distributors by game theory. Due to this gap in the literature, the main contribution of this research is considering the VRPTW problem as a game theory problem. The distributors are regarded as players which should choose the equilibrium routes and the possible routes are the strategies for players. Bargaining game model as a nonzero cooperative model is used to find the nash equilibrium.

2-2-Competitive vehicle routing problem

Competitive Vehicle Routing Problem presents a new version of VRP, that the cost of routes is minimized in the competitive environment. In this method considering the situation of competitors is needed.

Competitive Vehicle Routing Problem is a different version of vehicle routing problem with time windows and some competitors for servicing the customers. In this situation, it is very important to be aware that if the service time of each customer will be later than other rival, it will miss a part of its sale. So the revenue of distributor's sale is depended on the time of distributor visits the customers (Tavakkoli-Moghaddam et al., 2011).

2-3- TWO-PERSON nonzero sum games

In Two-Person nonzero sum games, each person has a revenue matrix. The revenue matrices are as follows:

$$\mathrm{A} = \begin{pmatrix} a_{11} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{nm} \end{pmatrix} \quad B = \begin{pmatrix} b_{11} & \cdots & b_{1m} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nm} \end{pmatrix}$$

In these games, if player A chooses strategies in a row for example *row n* and the other hand player B chooses strategies in a column for example *column 1*, the revenue of players A and B will be a_{n1} and b_{n1}. In zero sum games there is always A+B=0 rule. But in nonzero games there is not. Instead, the revenue of game is (a_{ij}, b_{ij}) when player A chooses *row i* and player B chooses *column j*. Since valuable goods and services can be created, destroyed, or badly allocated in a number of ways, and any of these will create a net gain or loss of utility for numerous stakeholders, many economic situations are not zero-sum. Specifically, all trade is by definition positive sum, because when two parties agree to an exchange, each party must consider receiving goods more valuable than delivering goods. In fact, all economic exchanges must pay both parties to the point that each party can overcome its transaction costs, or the transaction would simply not take place.

According to the many use of the Two-Person nonzero sum games, Table 3 describes literature review of the Two-Person nonzero sum games in extended fields:

Table 3. Literature review of Two-Person nonzero sum games

Authors	Title	Fields
Crowley P.H. (2000)	Hawks, Doves, and Mixed-symmetry Games	Social interactions
Yann Bramoullé (2001)	Complementarily and Social Networks	Social Networks
Ahmed et al. (2002)	ON SPATIAL ASYMMETRIC GAMES	Telegraph reaction
Pedersen (2003)	Moral Hazard in Traffic Games	Traffic safety behavior
Cantrell and Cosner (2004)	Deriving reaction–diffusion models in ecology from interacting particle systems	Reaction-diffusion models
Sarmiento and Wilson (2005)	Spatial Modeling in Technology Adoption Decisions: The Case of Shuttle Train Elevators	Shuttle train grain elevators
Sandholm et al (2006)	The Projection Dynamic, the Replicator Dynamic, and the Geometry of Population Games	Strategy distribution
Liu and Wang (2007)	Study on evolutionary games based on PSO-neural networks	learning and strategy-choosing
Altman et al. (2008)	An evolutionary game approach for the design of congestion control protocols in wireless networks	Biological sciences
Helbing (2009)	Pattern formation, social forces, and diffusion instability in games with success-driven motion	Agglomeration of cooperators
Xin Miao et al. (2010)	Modeling of bi-level games and incentives for sustainable critical infrastructure system	Critical infrastructure management
Tembine et al. (2011)	Bio-inspired delayed evolutionary game dynamics with networking applications	Competition
Asher et al. (2012)	Reciprocity and Retaliation in Social Games With Adaptive Agents	Risk-taking and cooperative behavior
Yunrui and Rui (2013)	Evolutionary game of motorized and non-motorized transport in city	Motorized and non-motorized transport
Jones and Briffa (2014)	Boldness and asymmetric contests: role- and outcome-dependent effects of fighting in hermit crabs	Differences in behavior
Liao and Chen (2015)	Use of Advanced Traveler Information Systems for Route Choice: Interpretation Based on a Bayesian Model	Route Choice
Flisberg et al. (2015)	Potential savings and cost allocations for forest fuel transportation in Sweden: A country-wide study	Cost allocation
Ko (2016)	An airline's management strategies in a competitive air transport market	Airlines management
Kellner (2016)	Allocating greenhouse gas emissions to shipments in road freight transportation: Suggestions for a global carbon accounting standard	Green issues in transportation

According to literature review, Table 2 and Table 3, there is not any research in the field of VRP using Two-Person nonzero sum games. To the best of the authors' knowledge, no research was found in the context of the competitive vehicle routing problem, which modeled the competition using Bargaining game or Hawk-Dove game models. Due to this gap in the literature, there are two main contributions in this research. First, the goal of the paper is choosing the best route to gain the highest revenue in competitive environment. Second, Two-Person nonzero sum games have been used to achieve the goal.

3- Model formulation

3- 1- Problem definition

The problem is related to ***two*** rival distributors in a city. Also there are ***n*** customers that are scattered within the city. Thus, the distributors want to choose the best routes to service more customers and consequently gain more revenue. In this paper, two-person nonzero sum games is chosen to evaluate the problem.

Each distributor can serve all ***n*** customers. So each distributor tries to choose a short path to visit more customers to sell its goods. Figure1 illustrates schema of the problem where there are ***two*** distributors and ***n*** customers in a city.

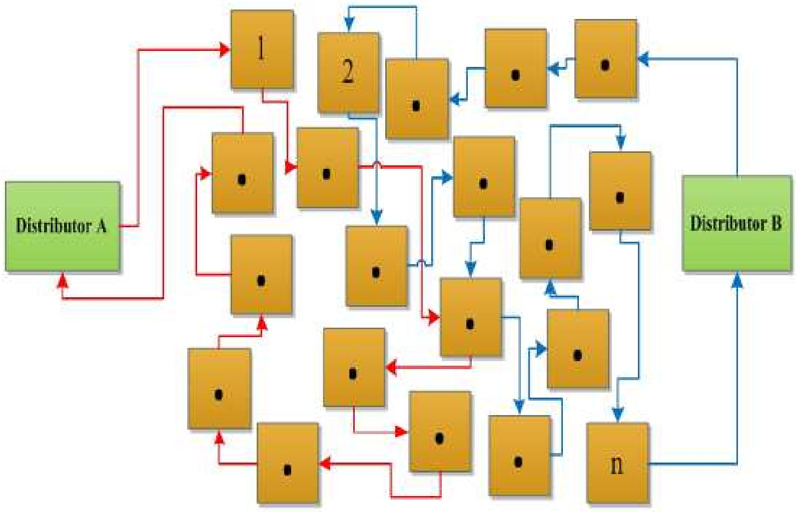


Figure 1. A conceptual framework of game model

The proposed model is established upon the following assumptions:

Assumption 1. the number of selectable routes for distributors is finite and the customers have been considered as potential stations in routes**.**

Assumption 2. The cost of each distributor is proportional by distance between depot of each distributor and customers.

Assumption 3. If two distributors arrive to a customer simultaneously, the sale of distributors becomes half.

Assumption 4. In the context of the Bargaining game model, the power of distributors is considered equal, and equal to 1.

The indexes, parameters and variables used in the model formulae are as follows

Indexes:

i the index of the route of the distributer A

j the index of the route of the distributer B

Parameters:

n the number of customers

e the number of possible routes for the distributer A

f the number of possible routes for the distributer B

X_m the customer m

w_m the cost per unit of displacement in distances

R_A the reservation revenues for the distributer A

R_B the reservation revenues for the distributer B

Variables:

$Revenue\ (A_i, B_j)$ the profit of route i for the distributor A, when the distributor B chooses the route j

Suppose that each distributor has some routes that these routes are depended on to other distributor's routes. The number of routes for distributor A is ***e***, and the number of routes for distributor B is ***f***. The matrix of routes for distributors and their revenue is as follows:

Table 4. Revenue-matrix of routes for distributors

		B			
		Route 1	**Route 2**	**…**	**Route *f***
A	**Route 1**	(a_{11}, b_{11})	(a_{12}, b_{12})	$\cdots$	(a_{1f}, b_{1f})
	Route 2	(a_{21}, b_{21})	(a_{22}, b_{22})	$\cdots$	(a_{2f}, b_{2f})
	...	**…**	**…**	**…**	**…**
	Route *e*	(a_{e1}, b_{e1})	(a_{e2}, b_{e2})	$\cdots$	(a_{ef}, b_{ef})

Where (a_{ij}, b_{ij}) represent revenue gained by distributers A and B from routs *i* and *j*, respectively.

3-2- Revenue function of distributor

The cost of each distributor is proportional by distance between depot of each distributor and customers. For example if $C(A, X_m)$ and $d(A, X_m)$ show the cost of distributor A and distance between distributor A and costumer *m*, respectively, and w_m shows the cost per unit of displacement in distances between distributor A and costumer *m*, the cost function of costumer *m* for distributor A is:

$$C(A, X_m) = w_m d(A, X_m) \tag{1}$$

According to equation 1 the cost function of one route, for example route *i*, for distributor A is:

$$Total\ cost\ of\ route\ i\ for\ distributor\ A = TC(A,i) = \sum_{m \in I} C(A, X_m) = \sum_{m \in I} w_m d(A, X_m) \tag{2}$$

where $I = \{m \mid X_m \in route\ i\}$.

The revenue function of route *i* for the distributor A when the distributor B chose route *j* , $Revenue(A_i, B_j)$, is:

$$\begin{aligned} &Revenue(A_i, B_j) = Total\ Demand(A_i, B_j) \times price - cost \\ &= \sum_{m \in I} (D_{X_m}(A_i, B_j) \times p) - TC(A,i), \qquad I = \{m \mid X_m \in route\ i\} \end{aligned} \tag{3}$$

Notice that if two distributors arrive to a customer simultaneously, the sale of distributors becomes half. $TC(B, j)$ and $Revenue(B_j, A_i)$ have the same definition.

3-3- Equilibrium strategy

3-3-1- Hawk-Dove game

Hawk-Dove game is one of the simple examples of Two-Person nonzero sum games. In this game there are two strategies: 1- aggressive strategy (hawk) and 2- passive strategy (Dove). The players choose one of the strategies simultaneous (Barron, 2013). This game is a kind of discrete non-zero sum game that is stationary with full information. For more explanation, this game is solved by an example.

Suppose that there are two pedestrians (A, B) find a 100$ bill. If each one chooses one of the aggressive strategy (hawk) or passive strategy (Dove), the revenue of the game is in table 5.

Table 5. Revenue of Hawk-Dove game

		B	
		Hawk (B1)	**Dove (B2)**
A	**Hawk (A1)**	0, 0	90, 10
	Dove (A2)	10, 90	50, 50

As it is seen aggressive strategy for two persons causes to destroy money and loss of both of them, where Dove procedure for two persons cause to gain equal revenue for them.

3-3-2- Bargaining game model

The goal of the Nash bargaining game, as a cooperative game, is dividing the benefits or utility between two players based on their competition in the market place. The Nash bargaining game has recently been used in a lot of different fields like as energy (Mahmoudi et al., 2014), justice (Alexander and Skyrms, 1999), supply chain management (Hafezalkotob and et al., 2014, Alaei and Setak, 1888) and etc.

The Nash bargaining game model (Nash, 1950) requires the feasible set to be compact and convex. It contains some payoff vectors, so that each individual payoff is greater than the individual breakdown payoff. Breakdown Payoffs are the starting point for bargaining which represent the possible payoff pairs obtained if one player decides not to bargain with the other player. In this study, the Nash bargaining game is used for achieving the equilibrium point of the game. It is believed that a distributor dose not stay in the business unless it can meet his minimum needs; therefore, the breakdown point of the game for each distributor depends on his individual policy.

If $Revenue(A_i, B_j)$ is the profit of *route i* for the distributor A and $Revenue(B_j, A_i)$ is the profit of *route j* for the distributor B, they will maximize $(Revenue(A_i, B_j) - R_A)(Revenue(B_j, A_i) - R_B)$, where R_A and R_B , are reservation revenues (breakdown points) for the distributor A and B, respectively. They would withdraw from the competitive market, if they obtain optimal revenues lower than the reservation revenues (i.e. $Revenue(A_i, B_j) \le R_A$ and $Revenue(B_j, A_i) \le R_B$). Therefore, according to the Nash bargaining model, best rout model for the distributors is given by

$$\begin{aligned} \max \quad & (Profit(A,i) - R_A)(Profit(B,j) - R_B) \\ s.t \quad & \\ & Profit(A,i) \ge R_A \\ & Profit(B,j) \ge R_B \end{aligned} \tag{4}$$

As mentioned in Binmore et al. (1986) the choice of the breakdown point is an issue of modeling judgment. Let $\theta_{\min}^A$ and $\theta_{\min}^B$ the worst achievable revenue of possible routes for distributor A and B, respectively. It is believed that a player does not stay in the business unless it can meet his minimum revenue; therefore, $\theta_{\min}^A$ and $\theta_{\min}^B$ has been used as the breakdown points.

The algorithm of the game is as follows: by solving problem (1-3) first, the total revenues of each possible rout for each optimal distributor are achieved and the revenue matrix is determined. Afterwards, the distributors must determine the optimal strategies from the Nash bargaining problem (4).

3-4-The algorithm and flowchart

According to above discussions, the algorithm for choosing the equilibrium routes in competitive environment using Two-Person Nonzero Sum game is expressed as:

Algorithm:
Step 0. Start
Step 1. Identify the network structure and the customers
Step 2. Collect data
Step 3. Determine the distances, costs and revenues
Step 4. Form the revenue-matrix of routes for distributors
Step 5. Identify the break down point of distributors
Step 6. Determine the equilibrium routes by models 4
Step 7. Finish

The flowchart of algorithm has been shown in figure 2.

4-Numerical example

In this section, two numerical examples are provided to demonstrate how the theoretical results, in this paper, can be applied in practice.

4-1- Example A

Suppose that there are 9 customers in a competitive market in a region that two distributors can support them. In figure 3 scattering of customers and a route of each distributor is determined.

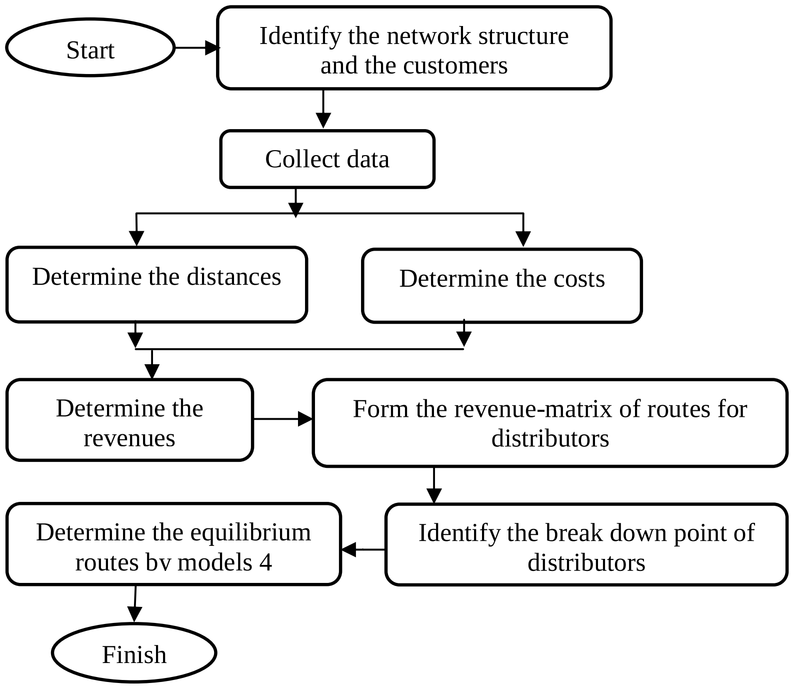


Figure 2. The flowchart

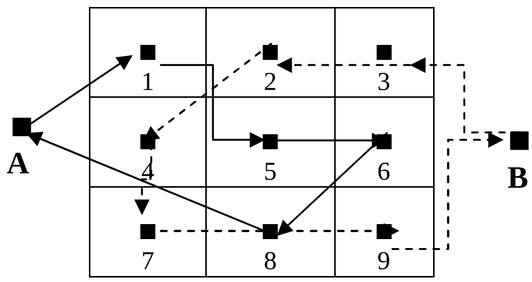


Figure 3. Scattering of customers and routes

In this example w_i is considered 10 for all costumers. Therefore costs of each costumer for each distributor are given in the Table 6 due to figure 3 :

Table 6. Cost of each distributors

Costumers	Distributor A	Distributor B
C. 1	20	40
C. 2	30	30
C. 3	40	20
C. 4	10	30
C. 5	20	20
C. 6	30	10
C. 7	20	40
C. 8	30	30
C. 9	40	20

choosing the route of each distributor rule is that if each distributor chooses a customer, another distributor does not choose it.

It is supposed that each customer demand is equal to 50 and each good price is 10. If two distributors arrive to a customer simultaneously, the sale of distributors becomes half. So the routes of distributors and their profits are shown in the next table.

Table 7. Routes, Cost and Profit

Distributor	Route 1	Cost 1	Revenue 1
A	0,1,5,6,8,0	100	1900
B	0,3,2,4,7,9,0	140	2400
Distributor	**Route 2**	**Cost 2**	**Revenue 2**
A	0,4,2,6,8,7,0	140	2400
B	0,9,5,1,3,0	100	1900
Distributor	**Route 3**	**Cost 3**	**Revenue 3**
A	0,7,5,3,2,1,0	130	2400
B	0,6,4,8,9,0	90	1900

So the Revenue-matrix of routes for distributors can be shown in table 8:

Table 8. Revenue-matrix of routes for distributors

		B		
		Route 1	**Route 2**	**Route 3**
	Route 1	(1900,2360)	(1400,1400)	(1400,1410)
A	**Route 2**	(1610,1610)	(2360,1900)	(1610,1260)
	Route 3	(1620,1610)	(1870,1400)	(2370,1910)

The table 8 is calculated for when two distributors arrive to a customer simultaneously, the sale of distributors becomes half.

4-2- Analyzing the distributor's behaviors in example A

According to the Table 8 there are three strategies for each distributor. Due to Hawk-Dove game in the route 1, distributor A is dove and another is hawk. In the route 3, distributor A is hawk and B is dove.

Considering Hawk-Dove game, if each distributor chooses one of the routes 1, 2 and 3 simultaneously, it causes to one distributor gains more revenue than other. But if A chooses route 2 and B chooses route 1 or A chooses route 2 and B chooses route 2, the distributors will have the same revenue (Dove-Dove performance).

Table 9. Result of Problem

		B		
		Route 1	**Route 2**	**Route 3**
A	**Route 1**	(1900,2360)	(1400,1400)	(1400,1410)
	Route 2	(1610,1610)	(2360,1900)	(1610,1260)
	Route 3	(1620,1610)	(1870,1400)	(2370,1910)

Dove-Dove performance: more revenue Hawk-Dove performance Dove-Dove performance: Less profit

According to table 9 the Dove-Dove performance by more revenue has 1610 unit of revenue for each distributor, that means if A chooses Route 2 and B chooses Route 1 they gain equal revenue. But in Haw-dove performance for example if A chooses Route 3 and B chooses Route 3, A gains 2370 units of revenue and B gains 1910 units of revenue. 2370 and 1910 both are more than 1610. In the following section by using Nash bargaining model the equilibrium point of this game will be calculated.

4-3- Calculating the equilibrium point in example A

According to table 8, the worst achievable benefit of the first and second distributors for different strategies is 1400 and 1260 respectively, therefore $\theta^{A}_{\min} = 1400$ and $\theta^{B}_{\min} = 1260$. Regarding these break down values, the results of the bargaining game model can be shown in Table 10.

Table 10. Numerical results for the Nash bargaining game model

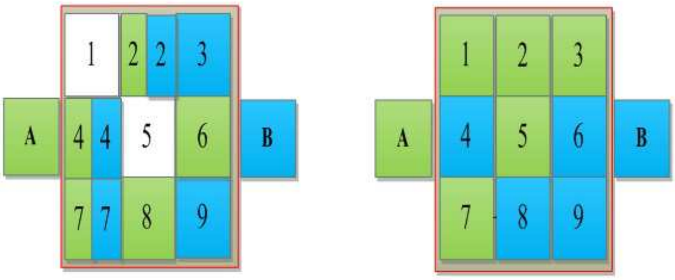
		B		
		Route 1	**Route 2**	**Route 3**
A	**Route 1**	550000	0	0
	Route 2	73500	614400	0
	Route 3	77000	0	630500

Less profit he distributors would withdraw from the competitive market Equilibrium point

Therefore, the equilibrium strategy is (A_3, B_3). In other words, if distributor 1 chooses route 3 in its possible routes and distributor 2 chooses route 3 in its possible routes, the distributor would obtain maximum revenue.

4-4- Numerical example analysis for example A

Figure 4 presented a conceptual comparison of customers choosing layout for the Dove-Dove performance by more revenue ((A_2, B_1)) and the Hawk-Dove strategy that is the equilibrium strategy ((A_3, B_3)).



Dove-Dove performance

Hawk-Dove performance
(the equilibrium strategy)

Figure 4. A conceptual comparison between Equilibrium and Dove-Dove strategies

Survey figure 3 shows, three factors are the most impressive ones for gaining more revenue: 1) the distance between distributor and customers; 2) the number of served customers; 3) sharing the customers. Less distance, more served customers and not sharing in customers gain more revenue for distributors. As shown in figure 3, in the Dove-Dove strategy, the consumers 1 and 5 are not served by any of the distributors. While the customers 2, 4 and 7 are served by both of the distributors. But in the Hawk-Dove strategy or in other word in the equilibrium strategy all customers have been served and there is no customers covered by both of the distributors. Therefore distributors' revenues in equilibrium strategy of Nash bargaining game are more than Dove-Dove performance. A comparison between results of two strategies shows that if distributors decide to cooperate they will gain more profit but not equal.

4-5- Example B

Consider a region by 16 customers in a competitive market that two distributors can support them. In figure 4 Scattering of customers has been shown. In figure 5. Different routs are available for each distributors.

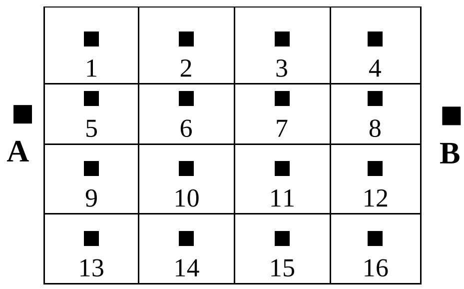


Figure 5. Scattering of customers and routes

For second example, w_i is considered 20 for all costumers. Therefore the costs of each costumer for each distributor are given in the Table 11 due to figure 5:

Table 11. Cost of each distributors

Costumers	Distributor A	Distributor B
C. 1	40	100
C. 2	60	80
C. 3	80	60
C. 4	100	40
C. 5	20	80
C. 6	40	60
C. 7	60	40
C. 8	80	20
C. 9	40	100
C. 10	60	80
C. 11	80	60
C. 12	100	40
C. 13	60	120
C. 14	80	100
C. 15	100	80
C. 16	120	60

The rules are similar to example A. It is supposed that the demand of each customer is equal to 20 and price of each good is 5. The routes of distributors and their profits are given in the next table.

Table 12. Routes, Cost and Profit

Distributor	Route 1	Cost 1	Revenue 1
A	0,1,2,3,7,11,15, 14,10,5,0	580	320
B	0,4,6,9,13,16,12,8,0	400	300
Distributor	**Route 2**	**Cost 2**	**Revenue 2**
A	0,1,2,6,10,14, 13,9,5,0	400	400
B	0,4,3,7,11,15,16, 12,8,0	400	400
Distributor	**Route 3**	**Cost 3**	**Revenue 3**
A	0,1,2,7,11,14,9, 5,0	380	320
B	0,4,7,10,11,15,12,8,0	360	340

The Revenue-matrix of routes for distributors can be shown in Table 13:

Table 13. Revenue-matrix of routes for distributors

		B		
		Route 1	**Route 2**	**Route 3**
A	**Route 1**	(320,300)	(120,200)	(120,140)
	Route 2	(250,150)	(400,400)	(350,290)
	Route 3	(270,250)	(220,300)	(220,240)

According to Table 8, $\theta^{A}_{\min} = 120$ and $\theta^{B}_{\min} = 140$. Regarding these break down values, table 13 shows the equilibrium point of this game using Nash bargaining:

Table 13. Numerical results for the Nash bargaining game model

		B		
		Route 1	**Route 2**	**Route 3**
A	**Route 1**	32000	0	0
	Route 2	1300	72800	34500
	Route 3	16500	16000	10000

Equilibrium

Therefore, the equilibrium strategy is (A_2, B_2). In other words, if distributor 1 chooses route 2 in its possible routes and distributor 2 chooses route 2 in its possible routes, the distributor would obtain maximum revenue by cooperation.

Figure 6 presented a conceptual comparison of customers choosing layout for the strategy (A_3, B_1) and the the equilibrium strategy (A_2, B_2).

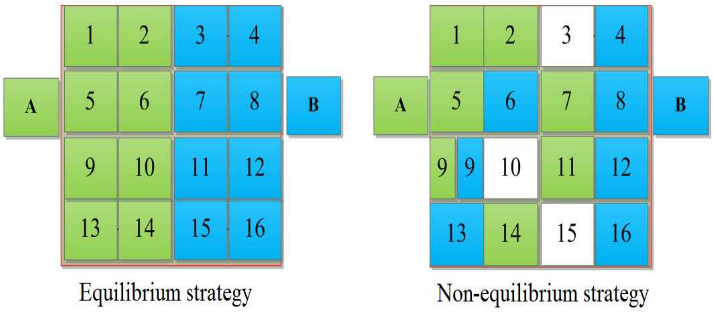


Figure 6. A conceptual comparison between equilibrium and an non- equilibrium strategies

Figure 6 shows the same results. As can be seen in figure 6, in the non-equilibrium strategy, the consumers 3, 10 and 15 are not served by any of the distributors, while the customer 9 is shared by both of the distributors. But in the equilibrium strategy all customers have been served and there is no customers covered by both of the distributors. For this reasons revenue of distributors in equilibrium strategy of Nash bargaining game is more than other strategies.

5- Conclusion

According to this paper, the idea of competitive VRP using Two-Person Nonzero Sum game was explained. By this method the equilibrium routes can be chosen in competitive environment without any problem. The result shows that these methods are proper for real problems. In this model, some kinds of two-person nonzero sum game has been used, for example Hawk-Dove game, Prisoners Dilemma and so on to solve the problem. To find the equilibrium strategy the Nash bargaining game has been used. For large-sized problems, if the number of players are large, the bargaining game model can be used yet, also the population game models are useful for this problem. If the number of customers is large, the suggested model must be coded by heutistic and meta heuristic methods.

For the future research, there are many fields and subject to do. Although our model is restricted to two distributors, one can easily generalize it to competition of more than two distributors. In this prospective condition, the model would be transform into a three-person or multiple-person or n-person nonzero sum game between the distributors. There are also other directions and suggestions for the future research. Firstly, VRP can be modeled using other cooperative games. secondly, the presented model in this paper may be extended to cover other categories of routing and scheduling problems for freight distribution.

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