Journal of Industrial and Systems Engineering
Vol. 10, special issue on Quality Control and Reliability, pp 109-124
Winter (February) 2017

Reliability based maintenance and human resources work-rest scheduling in manufacturing system

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Abstract

In today's competitive market, all manufacturers attempt to improve their maintenance policy in order to decrease the cost of failure and increase the quality of products, but most of these attempts do not consider the role of humans involved in a manufacturing system. Human resources are the main factor in manufacturing that has an undeniable effect on products quality, machines reliability, safety and maintenance policy. In this paper we propose a nonlinear mathematical model that optimizes the maintenance policy considering the humans fatigue to investigate its effects on reliability and associated Costs in manufacturing system. That is to say, the model is a reliability based maintenance optimization that aims to maintain the reliability of machines and their human resources in a proper predetermined interval. The performance of the proposed model was examined by some instances and the obtained results indicated this model can provide effectiveness maintenance policy for manufacturing systems.

Keywords: Maintenance, human resource, fatigue, optimization, reliability, production, scheduling

1- Introduction

Many researchers believed human resource is the main factor in all manufacturing system. Both human and manufacturing system influence each other, according to this fact, researchers aim to investigate the human performance in manufacturing systems and the effects of manufacturing system on human.

There are useful papers that investigated on considering human effects in manufacturing system (Godwin and Aniekan 1999; Baines, Asch, Hadfield, Mason, Fletcher and kay 2005; Mason, Baines, Kay, Ladbrook 2004; Dawson 2011) but few of them studied the interaction between human and manufacturing system. MacCarthy and Wilson (2001) have edited a book considering human factors in scheduling and planning, but did not really consider the interaction between human and manufacturing factors and interpretation of factors affecting human performance.

The role of human resource in manufacturing system has been investigated in some aspects such as scheduling, maintenance and reliability.

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Lodree et al. (2009) proposed that the interface between scheduling and human resources can be observed in three primary areas of research: (i) job rotation scheduling, (ii) work–rest (or rest break) scheduling, and (iii) shift work scheduling. Job rotation is a management technique that assigns employees to various jobs and departments over a period of time (Hsieh and Chao 2004). The aim of work–rest scheduling is to optimize the number, placement and duration of rest times during manufacturing period (Bechtold 1984; Bechtold, Janaro and Sumners 1984; Bechtold and Sumners 1988). Shift scheduling involves the assignment of human resources within a given period of time based on work volume and work force in each shift period.

Since in all manufacturing systems humans and machines work together, some papers studied the human resources scheduling and machines scheduling problem with consideration of three categories proposed by Lodree et al.(2009). Mahdavi et al. (2010) proposed a mathematical model for a dynamic cellular manufacturing systems considering production planning and worker assignment, in this paper human resources constraints change the optimal assignment of works to cells. Cappadonna et al.(2013)addressed the unrelated parallel machine scheduling problem with limited human resources. They proposed how both the number of human resources and the number of machines employed within the manufacturing system, play a key role in minimizing makespan value.

Researchers also investigated on maintenance and its interaction with human resources. Martorell et al. (2010) proposed that appropriate development of each maintenance strategy depends on the resources scheduling such as human and material. Taylor (2000) studied the human as an important resource in maintenance actions. He believed that maintenance resources scheduling is a part of maintenance human factors which addresses the issues of management. The interface between human scheduling and maintenance is mostly considered in maintenance resource management (MRM) (Siddiqui, Iqbal and Manarvi 2012).

Human scheduling affects on manufacturing system reliability. Many stress factors such as fatigue can reduce the reliability of human, if there is no proper work-rest schedule for all involved human resources in manufacturing system. Although some papers proposed the best work-rest schedule for human resources, few of them considered machines scheduling or system reliability. Nader Azizi et al. (2013) focused on effect of fatigue on human performance and proposed the best work-rest schedule for each human resource. Jamshidi and Seyyed Esfahani (2013) proposed a mathematical model that obtains the optimal work and rest schedule based on reliability of each worker.

Although there are researches that studied on human scheduling, production scheduling and maintenance scheduling but there are no outstanding researches in which scheduling of human resources, production and maintenance have been studied simultaneously.

In this paper we focused on scheduling the human resources, production and maintenance based on reliability of manufacturing system. Work and rest and maintenance for human resources and machines are scheduled based on humans' fatigue and machines reliability. We proposed a mathematical model to provide the optimal production schedule, optimal scheduling of work and rest for each worker and optimal maintenance policy. In the proposed model we assumed that machines reliability and humans fatigue effect on total reliability of manufacturing system. If the total reliability is not in a proper interval, cost of low product quality is imposed to the manufacturing system. We used some examples to show that the model can obtain the optimal results effectively.

The rest of the paper is organized as follows: Section 2 presents the models of fatigue and recovery. Section 3 describes the maintenance and reliability function. Section 4 proposes the problem statement and its assumptions. Section 5proposes the linearized model and section 6presents some instances with computational results to validate and verify the proposed model and finally section 7 concludes the paper.

2- Fatigue and recovery models

In real world we face many forms of fatigue such as tardiness, physical discomfort, lack of motivation and sleepiness. The effect of fatigue on human performance has been investigated in many manufacturing systems and the obtained results indicates that the loss of throughput (Wang and Hu 2010), human fault increasing (Kopardekar and Mital 1994), job dissatisfaction (Jiang, Baker and Frazier 2009), performance decrement (Koo, Kim and Kim 2014; Griffith and Mahadevan 2011) and injuries (Guadalupe 2003) are the harmful effects of human fatigue.

There a many models for fatigue and recovery but in this paper we use the fatigue and recovery model proposed by Konz (1998), this model assumes that fatigue and recovery can be formulated as follows.

$$F(t) = 1 - e^{-\lambda p.t} \tag{1}$$
$$R(\tau_i) = F(t)e^{-\mu\tau_i} \tag{2}$$

Where *F(t)* is the fatigue accumulated by time *t*, $R(\tau_i)$is the residual fatigue after a rest break of lengthτ_i. And $\lambda p, \mu$ are fatigue rate and recovery rate, respectively.Figure1 shows the trend of fatigue for a worker based on equation 1 and 2.

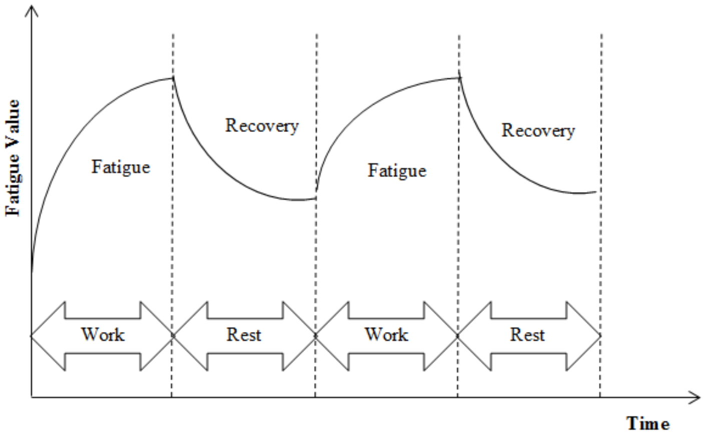

Figure 1. The trend of the fatigue and recovery over time position

As proposed by Jaber et al. (2013), in practice, rest breaks during work cycles are usually short and cannot retrieve the fatigue perfectly, thus human faces with residual fatigue after improper rest time. The residual fatigue, carried forward into cycle*i*+1 and equation I can be re-written as follows.

$$F_{i+1}(t) = R(\tau_i) + \left(1 - R(\tau_i)\right).(1 - e^{-\lambda p(t_u - t_i)}) \tag{3}$$

Where $t_u$is the production time of the cycle *i* and t_i is determined by projecting the value of $R(\tau_i)$on the fatigue curve as follows.

$$t_i = -\ln\,(1 - R(\tau_i))/\lambda p \tag{4}$$

Since in the proposed model, the production period is divided into equal time positions, the value of (t_u-t_i) is equal to one unit of time position, thus equation 3 can be re-written as follows.

$$F_{i+1}(t) = R(\tau_i) + \left(1 - R(\tau_i)\right).(1 - e^{-\lambda p}) \tag{5}$$

Also the equation 2 should be developed to obtain the residual fatigue according to workers status in each time position. This relation is described in next section.

3- Reliability and maintenance of machine

According to Smith (1976) the reliability function for an element with failure rate λ(t) is calculated as follows:

$$R(t) = e^{-\int_0^t \lambda(t)dt} \qquad (6)$$

Relation 7 can be derived from relation 6, if the failure rate is constant.

$$R(t) = e^{-\lambda t} \qquad (7)$$

The reliability decreases during useful life of an element. Figure 2 shows the reliability trend of an element with constant failure rate 0.03.

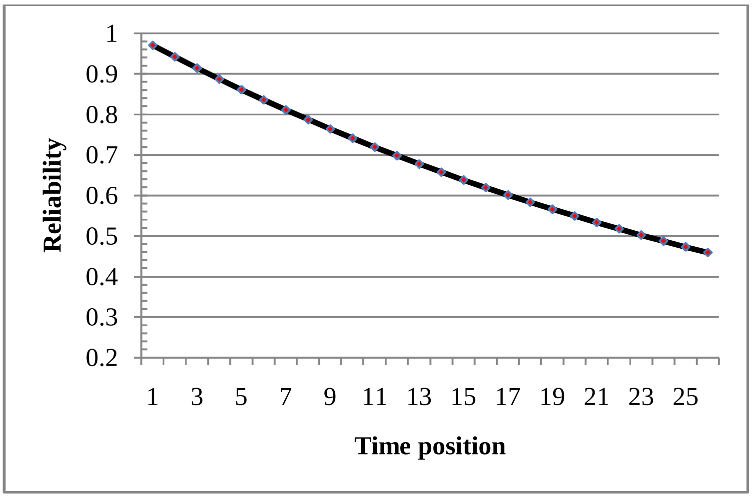

Figure 2. The reliability trend for an element with constant failure rate

In this paper the machine reliability decreases, if it works without stopping and maintenance action. Maintenance can retrieve the reliability of a machine by decreasing the failure rate. Fig.3 shows the reliability trend with maintenance for an element.

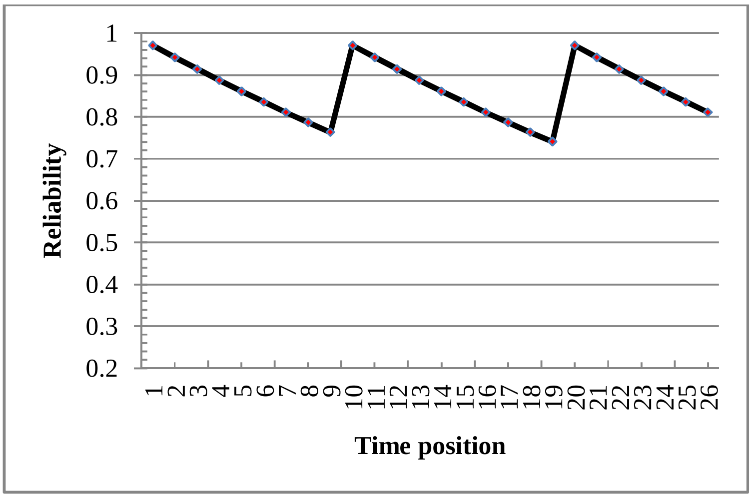

Figure 3. Recovery of reliability by each repair action

As can be seen in figure3 since the element is repaired in time positions 10 and 20 the reliability increases and returns to its primary value. That is to say, maintenance keeps the reliability of an element in a proper interval.

4- Problem statement

In this section, we formally describe the considered problem and its assumption. The aim of proposed model is to determine the optimal policy for human resources work-rest schedule, production schedule and maintenance action schedule based on six cost components.

- Idleness cost of machines
- Idleness cost of human resources
- Cost of unpredictable failure
- Cost of corrective maintenance
- Cost of poor quality product
- Unavailability cost of machines

The main advantage of this model is to use the reliability as a main decision variable for maintenance of machines and rest of human resources. In fact the proposed model is a reliability based model for scheduling of human, production and maintenance. In this model each machine and human resource has a special impact on total reliability of manufacturing system. If the maintenance of machine and rest of human resources are postponed, the total reliability decreases and as a result manufacturing system cannot produce quality products and cost of poor quality products is imposed on the manufacturer. Since the proposed model is a nonlinear one, we then linearize such model in next subsection. Other assumptions made in the model are proposed in next subsection.

4-1-Assumptions

- The demand of each product is known
- Operation time to produce each product is known
- Each machine can produce only one type of products
- Each human has a special impact on total reliability of manufacturing system
- Each machine has a special impact on total reliability of manufacturing system
- The reliability of each machine must be greater than a predetermined value in each time position
- Every machine has one worker and the worker won’t work on other machines.
- Each human has a specific maximum fatigue value and cannot work with a fatigue value greater than this fatigue limit.
- Humans' reducing rate of fatigue in idle status and rest status are not equal.
- The fatigue of human resources decreases in proportion with rest duration.
- The maintenance actions decrease machine failure rate.
- The planning horizon is equal to the required time to produce the entire products.

Notations

4-1-1- Subscripts

i	Index for machine and its worker (i=1, 2,…I)
k	Index for time position(k=1, 2,…K)

4-1-2- Input parameters

L_i	Process time of machine *i*
CH_i	The unitary cost of idleness for the worker of machine *i*
CM_i	The unitary cost of machine *i* idleness
CF_i	The unpredictable failure cost for machine *i*
CR_i	The unitary cost maintenance for machine *i*
CQ_i	The unitary cost of poor quality for machine *i*
CA_i	The unavailability cost for machine *i*
DEM_i	The demand must be supplied by machine *i*
PR_i	The primary reliability for machine *i*
$HEXUS_{\mathrm{i}}$	Fatigue upper limit for the worker of machine *i*
$MEXUS_i$	Fatigue lower limit for the worker of machine *i*
$MRNE_i$	Minimum reliability required for machine *i*
$PEXUS_i$	Primary fatigue for the worker of machine *i*
λ_i	The failure rate of machine *i*
λp_i	The fatigue rate for worker of machine *i*
σ_i	The impact of machine *i* on total reliability of manufacturing system
σp_i	The impact of worker *i* on total reliability of manufacturing system
μ_i	Fatigue decreasing rate for worker *i* per unit of rest time
μp_{i}	Fatigue decreasing rate for worker *i* per unit of idle time

4-1-3- Decision variables

$x_{i,k}$	1 if the machine *i* is available in position *k*; =0 otherwise
$xp_{i,k}$	1 if the worker of machine *i* is available in position *k*; =0 otherwise
$z_{i,k}$	1 if the machine *i* works in position *k*; =0 otherwise
nr_i	The number of time positions that machine *i* is unavailable
$r_{i,k}$	The reliability of machine *i* in position *k*
$tr_{i,k}$	The total reliability value for worker-machine *i* in position *k*
$exus_{i,k}$	The fatigue of worker *i* in position *k*
$rexus_{i,k}$	The residual fatigue for worker *i* in position *k*
ava_i	The availability of machine *i*

With respect to the above notations, the mathematical model can be formulated as mentioned in next subsection.

4-2- The mathematical model

$$\begin{aligned}\min Z:&\sum_{i=1}^{I}\sum_{k=1}^{K} CH_i.xp_{i,k}.(1-z_{i,k})+\sum_{i=1}^{I}\sum_{k=1}^{K} CM_i.x_{i,k}.(1-z_{i,k})+\sum_{i=1}^{I}\sum_{k=1}^{K} CF_i\,.\big(1-r_{i,k}\big)\\&+\sum_{i=1}^{I} CR_i.nr_i+\sum_{i=1}^{I} CA_i.ava_i+\sum_{i=1}^{I}\left[CQ_i.(\sum_{k=1}^{K}\big(1-tr_{i,k}\big).z_{i,k})/(L_i.DEM_i)\right]\end{aligned}\tag{8}$$

s.t.

$$\sum_{k=1}^{K} z_{i,k}=L_i.DEM_i \qquad \forall m; \tag{9}$$

$$z_{i,k}\le x_{i,k} \qquad \forall i,k; \tag{10}$$

$$z_{i,k}\le xp_{i,k} \qquad \forall i,k; \tag{11}$$

$$r_{i,1}=PR_i \qquad \forall i \tag{12}$$

$$\begin{aligned}r_{i,k}=e^{-\lambda_i}.z_{i,k-1}.r_{i,k-1}+e^{\lambda_i}.(1-x_{i,k-1}).r_{i,k-1}\\+(1-z_{i,k-1}).x_{i,k-1}.r_{i,k-1}\end{aligned} \qquad \forall i,k\ge 2; \tag{13}$$

$$ava_i=1-(nr_i/(L_i.DEM_i)) \qquad \forall i \tag{14}$$

$$tr_{i,k}=\sigma_i.r_{i,k}+\sigma p_i.(1-exus_{i,k}) \qquad \forall i,k; \tag{15}$$

$$exus_{i,k}\le HEXUS_i \qquad \forall i,k; \tag{16}$$

$$MRNE_i\le r_{i,k} \qquad \forall i,k; \tag{17}$$

$$r_{i,k}\le 1 \qquad \forall i,k; \tag{18}$$

$$MRNE_i-r_{i,k}\le 1-x_{i,k} \qquad \forall i \tag{19}$$

$$rexus_{i,1}=PEXUS_i \qquad \forall i \tag{20}$$

$$\begin{aligned}rexus_{i,k}=e^{-\mu_i}.exus_{i,k-1}.\big(1-xp_{i,k-1}\big)\\+e^{-\mu p_i}.exus_{i,k-1}.\big(1-z_{i,k-1}\big).xp_{i,k-1}\\+exus_{i,k-1}.z_{i,k-1}\end{aligned} \qquad \forall i,k\ge 2; \tag{21}$$

$$exus_{i,k}=rexus_{i,k}+\big(1-rexus_{i,k}\big).(1-e^{-\lambda p_i}).z_{i,k} \qquad \forall i,k; \tag{22}$$

$$nr_i = \sum_{k=1}^{K} (1 - x_{i,k}) \qquad \forall i \qquad (23)$$

$$z_{i,k}, x_{i,k}, xp_{i,k} \in \{0,1\} \qquad \forall i, k; \qquad (24)$$

$$nr_i, tr_{i,k}, exus_{i,k}, rexus_{i,k}, ava_i \geq 0 \qquad \forall i, k; \qquad (25)$$

Objective function (8) consists of six cost components. First component calculates the workers idleness. Similarly, the second component shows that if machine is available but does not work, manufacturing system faces with machine idleness cost. The third component considers the cost of unpredictable maintenance action. The cost of unpredictable maintenance increases in proportion with the difference between reliability value and ideal reliability (100%).In fact, this component tries to keep the total reliability in a proper interval. The fourth component computes the maintenance cost based on the number of time positions that machines are maintained. The unavailability cost of machines is calculated in fifth component. The last component calculates the quality cost. This component shows that quality cost is calculated based on proportion of work time that total reliability of worker-machine system is not ideal. Relation (9) assures that the total working time of a worker-machine system is equal to required time to produce all demand. Relations (10-11) show that worker-machine system can work if both worker and machine are available. Relation (12) shows that the reliability of each machine in first time position is equal to machine primary reliability. Relation (13) calculates the reliability of each machine in each time position based on its status in prior time position. Relation (14) calculates the availability of each machine. Relation (15) computes the total reliability of worker-machine system. Relation (16) sets a maximum limit on workers fatigue. Relation (17) sets a lower limit for machines reliability. Relation (18) indicates that reliability cannot be greater than 100%. Relation (19) shows that if the reliability of a machine is less than minimum required reliability, the machine should be maintained and cannot be available to work. Relation (20) shows that the residual fatigue of each worker in first time position is equal to worker primary fatigue value. Relation (21) calculates the residual fatigue for each worker in each time position also relation (22) computes the fatigue value, based on residual fatigue of worker in each time position. Relation (23) obtains the number of time positions that machine is unavailable and relations (24-25) determine the type of variable.

As mentioned in the previous paragraph, relation (18) calculates the reliability value for each machine in time positions. Each machine has three statuses in each time position, work, idleness and unavailability. If a machine works in previous time position, its reliability decreases. If machine is available and idle, its reliability does not alter and is the same as reliability of previous time position, also if machine is unavailable because of maintenance action, its reliability increases in comparison with previous time position.

5- Linearization of proposed model

Since the objective function and relations (13),(21),(22) are nonlinear; in this section an attempt is made to linearize the proposed model.

The nonlinear terms of the relation (13) could be linearized via three transformations. We have extended this relation as follows:

$$r_{i,k} = e^{-\lambda_i}. z_{i,k-1}. r_{i,k-1} + e^{\lambda_i}. (r_{i,k-1} - x_{i,k-1}. r_{i,k-1}) + (x_{i,k-1} - z_{i,k-1}. x_{i,k-1}). r_{i,k-1} \qquad (26)$$

To linearize the relation (26) we should define four auxiliary variables and their related constraints. The first variable is $xr_{i,k} = x_{i,k}. r_{i,k}$,the second is$zr_{i,k} = z_{i,k}. r_{i,k}$and the third is $zx_{i,k} = z_{i,k}. x_{i,k}$.These variables used with constraints (17-19), (20-22), (23-25) respectively.

$$xr_{i,k} \leq r_{i,k} + A(1 - x_{i,k}) \qquad \forall i,k; \tag{27}$$

$$xr_{i,k} \geq r_{i,k} - A(1 - x_{i,k}) \qquad \forall i,k; \tag{28}$$

$$xr_{i,k} \leq x_{i,k} \qquad \forall i,k \tag{29}$$

$$zr_{i,k-1} \leq r_{i,k} + A(1 - z_{i,k}) \qquad \forall i,k; \tag{30}$$

$$zr_{i,k} \geq r_{i,k} - A(1 - z_{i,k}) \qquad \forall i,k; \tag{31}$$

$$zr_{i,k} \leq z_{i,k} \qquad \forall i,k \tag{32}$$

$$zx_{i,k-1} \leq z_{i,k} + A(1 - x_{i,k}) \qquad \forall i,k; \tag{33}$$

$$zx_{i,k} \geq z_{i,k} - A(1 - x_{i,k}) \qquad \forall i,k; \tag{34}$$

$$zx_{i,k} \leq z_{i,k} \qquad \forall i,k \tag{35}$$

The fourth auxiliary variable is $zxr_{ik} = zx_{i,k}.r_{i,k}$ that added to model with the following constraints.

$$zxr_{i,k} \leq r_{i,k} + A(1 - zx_{i,k}) \qquad \forall i,k; \tag{36}$$

$$zxr_{i,k} \geq r_{i,k} - A(1 - zx_{i,k}) \qquad \forall i,k; \tag{37}$$

$$zxr_{i,k} \leq zx_{i,k} \qquad \forall i,k \tag{38}$$

Considering these variables with their related constraints we have

$$r_{i,k} = e^{-\lambda_i}.zr_{i,k-1} + e^{\lambda_i}.(r_{i,k-1} - xr_{i,k-1}) + (xr_{i,k-1} - zxr_{i,k-1}) \tag{39}$$

By considering relation (10),it can be proved that $zxr_{ik} = zr_{i,k}$ and we can eliminate zxr and use zr instead of it. Consequently relations (36-38) should be removed from linear model. Based on this fact, the relation (6) can be replaced by following relation.

$$r_{i,k} = e^{-\lambda_i}.zr_{i,k-1} + e^{\lambda_i}.(r_{i,k-1} - xr_{i,k-1}) + (xr_{i,k-1} - zr_{i,k-1}) \tag{40}$$

To linearize the relation (14) three auxiliary variables should be defined. The first variable is $exx_{ik} = exus_{i,k}.xp_{i,k}$ by using this variable; relation (21) turns into relation (41)

$$\begin{aligned} rexus_{i,k} = {} & e^{-\mu_i}.\left(exus_{i,k-1} - exx_{i,k-1}\right) + e^{-\mu p_i}.\left(exx_{i,k-1} - exx_{i,k-1}.z_{i,k-1}\right) \\ & + exus_{i,k-1}.z_{i,k-1} \end{aligned} \tag{41}$$

The second auxiliary variable is $exxz_{i,k} = exx_{i,k}.z_{i,k}$. This variable changes relation (41) to relation (42)

$$\begin{aligned} rexus_{i,k} = {} & e^{-\mu_i}.\left(exus_{i,k-1} - exx_{i,k-1}\right) + e^{-\mu p_i}.\left(exx_{i,k-1} - exxz_{i,k-1}\right) \\ & + exus_{i,k-1}.z_{i,k-1} \end{aligned} \tag{42}$$

The last variable utilized to linearize relation (31) is $exz_{ik} = exus_{i,k}.z_{i,k}$. By using this variable relation (42) can be replaced by relation (43).

$$rexus_{i,k} = e^{-\mu_i}.\left(exus_{i,k-1} - exx_{i,k-1}\right) + e^{-\mu p_i}.\left(exx_{i,k-1} - exxz_{i,k-1}\right) + exz_{i,k-1} \tag{43}$$

Considering relation (11) it can be proved that *exz* is always equal to *exxz*, we can replace variable *exxz* by *exz* and eliminate the related constraints to *exxz*. Based on this fact the linear relation for residual fatigue and its constraints are as follows.

$$rexus_{i,k} = e^{-\mu_i}.\left(exus_{i,k-1} - exx_{i,k-1}\right) + e^{-\mu p_i}.\left(exx_{i,k-1} - exz_{i,k-1}\right) + exz_{i,k-1} \qquad (44)$$

$$exx_{i,k} \leq exus_{i,k} + A(1 - xp_{i,k}) \qquad \forall i,k; \qquad (45)$$

$$exx_{i,k} \geq exus_{i,k} - A(1 - xp_{i,k}) \qquad \forall i,k; \qquad (46)$$

$$exx_{i,k} \leq xp_{i,k} \qquad \forall i,k \qquad (47)$$

$$exz_{i,k} \leq exus_{i,k} + A(1 - z_{i,k}) \qquad \forall i,k; \qquad (48)$$

$$exz_{i,k} \geq exus_{i,k} - A(1 - z_{i,k}) \qquad \forall i,k; \qquad (49)$$

$$exz_{i,k} \leq z_{i,k} \qquad \forall i,k \qquad (50)$$

After linearizing the residual fatigue we have to eliminate the nonlinearity of relation (22). In this case we substitute *rexz* for *rexus.z*in relation (22) and add constraints (51-54) to linear model, we have.

$$exus_{i,k} = rexus_{i,k} + (1 - e^{-\lambda p_i}).\left(z_{i,k} - rexz_{i,k}\right) \qquad (51)$$

$$rexz_{i,k} \leq rexus_{i,k} + A(1 - z_{i,k}) \qquad \forall i,k; \qquad (52)$$

$$rexz_{i,k} \geq rexus_{i,k} - A(1 - z_{i,k}) \qquad \forall i,k; \qquad (53)$$

$$rexz_{i,k} \leq z_{i,k} \qquad \forall i,k \qquad (54)$$

After linearizing all constraint, we introduce two auxiliary variables to linearize the objective function and propose the linear mathematical model. In objective function we have two nonlinear cases, thus two auxiliary variables *zxx=xp.z and ztr=z.tr* are introduced to make them linear, also constraints (55-60) will be added to linear model.

$$zxx_{i,k} \leq xp_{i,k} + A(1 - z_{i,k}) \qquad \forall i,k; \qquad (55)$$

$$zxx_{i,k} \geq xp_{i,k} - A(1 - z_{i,k}) \qquad \forall i,k; \qquad (56)$$

$$zxx_{i,k} \leq z_{i,k} \qquad \forall i,k \qquad (57)$$

$$ztr_{i,k} \leq tr_{i,k} + A(1 - z_{i,k}) \qquad \forall i,k; \qquad (58)$$

$$ztr_{i,k} \geq tr_{i,k} - A(1 - z_{i,k}) \qquad \forall i,k; \qquad (59)$$

$$ztr_{i,k} \leq z_{i,k} \qquad \forall i,k \qquad (60)$$

According to all the introduced auxiliary variables and their related constraints, the linear mathematical model is as follows:

$$
\begin{aligned}
\min Z: \sum_{i=1}^{I}\sum_{k=1}^{K} CH_i.\left(xp_{i,k} - zxx_{i,k}\right) + \sum_{i=1}^{I}\sum_{k=1}^{K} CM_i.\left(x_{i,k} - zx_{i,k}\right) \\
+ \sum_{i=1}^{I}\sum_{k=1}^{K} CF_i\,.\left(1 - r_{i,k}\right) \\
+ \sum_{i=1}^{I} CR_i.nr_i \\
+ \sum_{i=1}^{I} CA_i.ava_i + \sum_{i=1}^{I}\left[CQ_i.\left(\sum_{k=1}^{K}\left(z_{i,k} - ztr_{i,k}\right).\right)/\left(L_i.DEM_i\right)\right]
\end{aligned}
\tag{61}
$$

Subject to constraints (9)−(12), (14)−(20), (23)−(25), (27)−(35), (40), (43), (44)−(50), (51-54), (55-60).

6- Numerical illustration

In this section we use the proposed mathematical model to determine the optimal work-rest schedule for human resources, maintenance schedule for machines and optimal production schedule for manufacturing system in some instances and report computational results to evaluate the effectiveness of the proposed model. Five instances are solved by branch-and-bound (B&B) method using Gams software on a PC including Intel®Core i5 and 2 GB of RAM. The instances designs are detailed in Table 1.

Table 1. Instances design

	Instance 1		Instance 2		Instance 3			Instance 4				Instance 5				
Machine no.	*M1*	*M2*	*M1*	*M2*	*M1*	*M2*	*M3*	*M1*	*M2*	*M3*	*M4*	*M1*	*M2*	*M3*	*M4*	*M5*
L_i	2	3	4	5	4	5	3	3	5	4	6	2	3	5	3	4
CH_i	20	30	25	20	30	32	26	27	35	30	32	22	26	27	23	21
CM_i	40	60	55	45	30	42	50	60	33	47	33	60	58	38	49	32
CF_i	35	45	40	40	36	35	30	32	32	40	41	38	37	33	38	39
CR_i	75	64	70	76	70	76	63	66	69	72	76	72	63	64	63	71
CQ_i	34	26	32	35	40	30	25	30	39	38	39	35	39	34	33	25
CA_i	28	30	30	37	40	35	36	28	38	36	32	30	29	40	35	33
DEM_i	9	10	12	16	15	10	12	13	16	15	10	13	11	12	13	10
PR_i	0.8	0.8	0.85	0.85	0.75	0.8	0.7	0.72	0.74	0.82	0.83	0.72	0.83	0.84	0.75	0.75
$HEXUS_i$	0.6	0.7	0.6	0.6	0.65	0.7	0.6	0.62	0.65	0.63	0.64	0.65	0.68	0.68	0.70	0.69
$MEXUS_i$	0.2	0.2	0.3	0.3	0.25	0.25	0.25	0.2	0.2	0.2	0.2	0.21	0.28	0.29	0.25	0.27
$MRNE_i$	0.4	0.4	0.5	0.5	0.5	0.55	0.5	0.45	0.47	0.48	0.50	0.47	0.55	0.41	0.50	0.49
$PEXUS_i$	0.3	0.35	0.32	0.32	0.3	0.3	0.3	0.27	0.38	0.27	0.27	0.35	0.29	0.32	0.34	0.32
λ_i	0.03	0.04	0.035	0.021	0.02	0.01	0.02	0.01	0.01	0.02	0.03	0.02	0.01	0.02	0.03	0.02
λp_i	0.035	0.025	0.03	0.03	0.015	0.02	0.03	0.02	0.02	0.03	0.02	0.03	0.02	0.02	0.03	0.03
σ_i	0.6	0.55	0.5	0.5	0.5	0.5	0.5	0.5	0.6	0.5	0.6	0.52	0.55	0.57	0.55	0.50
σp_i	0.4	0.45	0.5	0.5	0.5	0.5	0.5	0.5	0.4	0.5	0.4	0.48	0.45	0.43	0.45	0.5
μ_i	0.03	0.03	0.03	0.03	0.04	0.03	0.035	0.03	0.04	0.04	0.04	0.03	0.03	0.03	0.03	0.03
μp_i	0.025	0.025	0.02	0.02	0.03	0.02	0.03	0.03	0.02	0.02	0.03	0.03	0.02	0.03	0.03	0.03

Table 2 shows the objective function values and the minimum and maximum reliability of machines for each instance. The values for human resources indicate that fatigue of workers does not exceed the maximum fatigue limit and the proposed model maintains the total reliability of manufacturing system in a proper interval.

Table 2. The results of proposed model on instances

Instance No.	5	4	3	2	1
Objective value	18294.570	18857.57	11352.56	8867.52	2994.933
Min rel M1	0.720	0.720	0.500	0.820	0.480
Max rel M1	0.990	0.990	0.850	0.980	0.800
Ave	0.946	0.910	0.748	0.951	0.700
Min rel M2	0.820	0.470	0.550	0.500	0.440
Max rel M2	0.990	0.730	0.850	0.940	0.980
Ave	0.921	0.591	0.740	0.797	0.790
Min rel M3	0.420	0.530	0.690	---	---
Max rel M3	0.890	0.980	0.980	---	---
Ave	0.673	0.860	0.876	---	---
Min rel M4	0.730	0.510	---	---	---
Max rel M4	0.980	0.990	---	---	---
Ave	0.855	0.837	---	---	---
Min rel M5	0.490	---	---	---	---
Max rel M5	0.830	---	---	---	---
Ave	0.698	---	---	---	---
Min exus W1	0.220	0.200	0.250	0.300	0.230
Max exus W1	0.350	0.350	0.470	0.360	0.510
Ave	0.244	0.224	0.307	0.329	0.300
Min exus W2	0.280	0.250	0.250	0.320	0.320
Max exus W2	0.320	0.540	0.540	0.590	0.610
Ave	0.298	0.427	0.315	0.480	0.421
Min exus W3	0.310	0.200	0.250	---	---
Max exus W3	0.610	0.610	0.460	---	---
Ave	0.471	0.362	0.290	---	---
Min exus W4	0.330	0.230	---	---	---
Max exus W4	0.480	0.400	---	---	---
Ave	0.408	0.307	---	---	---
Min exus W5	0.280	---	---	---	---
Max exus W5	0.520	---	---	---	---
Ave	0.388	---	---	---	---

Using the solutions obtained by proposed model, we have the optimal work-rest schedule and maintenance schedule. Table 3 shows the optimal scheduling for machine 2 and its worker in instance1.

Table 3. Optimal solution for machine 2 in instance 1

Time position	Machine Status	Worker status	Worker-Machine status	Machine reliability	Worker fatigue	Time position	Machine Status	Worker status	Worker-Machine status	Machine reliability	Worker fatigue
1	0	0	0	0.8	0.35	24	0	0	0	0.87	0.38
2	0	0	0	0.83	0.34	25	0	0	0	0.9	0.37
3	0	0	0	0.87	0.33	26	1	1	1	0.94	0.37
4	1	1	1	0.9	0.34	27	1	1	1	0.9	0.39
5	0	0	0	0.87	0.34	28	1	1	1	0.87	0.4
6	0	0	0	0.9	0.33	29	1	1	1	0.83	0.42
7	0	0	0	0.94	0.32	30	1	1	1	0.8	0.43
8	1	1	1	0.98	0.32	31	1	1	1	0.77	0.45
9	1	1	1	0.94	0.34	32	1	1	1	0.74	0.46
10	0	0	0	0.9	0.34	33	1	1	1	0.71	0.47
11	1	1	1	0.94	0.35	34	1	1	1	0.68	0.49
12	0	0	0	0.9	0.35	35	1	1	1	0.65	0.5
13	1	1	1	0.94	0.35	36	1	1	1	0.63	0.51
14	0	0	0	0.9	0.35	37	1	1	1	0.6	0.52
15	0	0	0	0.94	0.34	38	1	1	1	0.58	0.54
16	1	1	1	0.98	0.35	39	1	1	1	0.56	0.55
17	1	1	1	0.94	0.37	40	1	1	1	0.54	0.56
18	1	1	1	0.9	0.38	41	1	1	1	0.52	0.57
19	1	1	1	0.87	0.4	42	1	1	1	0.5	0.58
20	0	0	0	0.83	0.4	43	1	1	1	0.48	0.59
21	1	1	1	0.87	0.4	44	1	1	1	0.46	0.6
22	0	0	0	0.83	0.4	45	1	1	1	0.44	0.61
23	1	1	0	0.87	0.39						

Number 0 in machine status shows that machine is under maintenance action and for worker status shows that worker rests to retrieve his fatigue level, by contrast, number 1 shows that machine and its worker are available to work. The model aims to synchronize the time of machine maintenance and rest time of its worker to minimize the idleness of both machine and its worker.

The figure 4 and figure 5 also show the trend of machine 2 reliability and fatigue of its worker in instance 2.It should be noted that the minimum reliability in entire positions time is larger thanminimum required reliability, this fact also holds for worker fatigue.

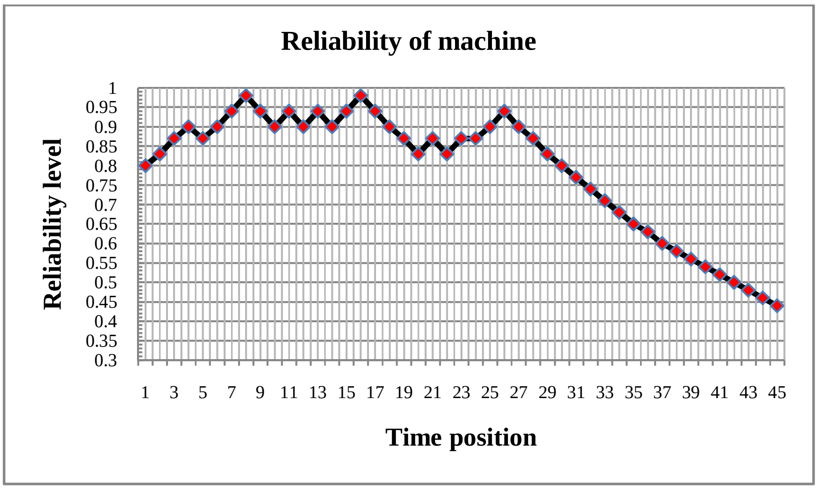

Figure 4. Reliability trend for machine 2 in instance 1

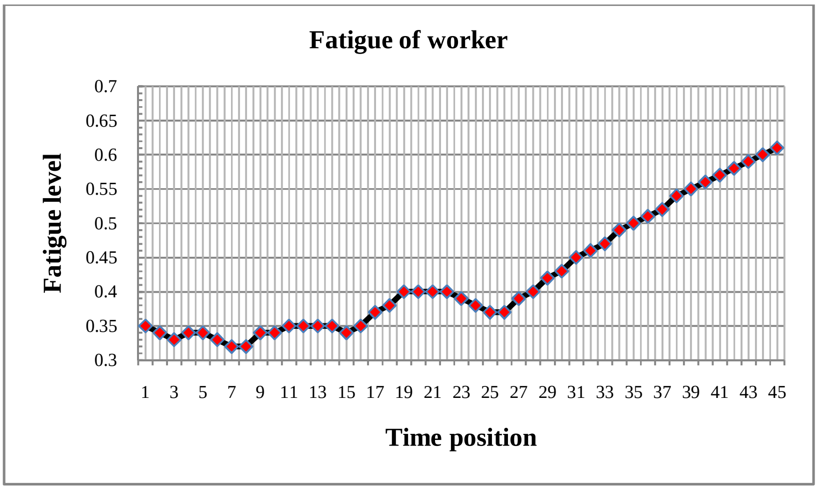

Figure 5. Fatigue trend for worker of machine 2 in instance 1

7- Conclusions

This paper presented a novel integer nonlinear model to optimize the human resource scheduling, production scheduling and maintenance policy considering machines reliability and fatigue of workers, as two main factors in manufacturing system. The proposed model aims to minimize the machines idleness, workers idleness, unpredictable failure cost, maintenance cost, unavailability cost and quality cost. The machines ‘reliability and workers' fatigue have an effect on the total reliability of manufacturing system. If the total reliability is not in a proper interval, manufacturing system produces poor quality products, considering this fact the proposed model obtains the optimal work-rest schedule for each worker and best maintenance schedule for each machine to maintain the total reliability in a proper interval. Given the complexity of proposed model, we used the linearization technique to convert it to linear form and decrease the solving time. The performance of proposed model was examined by five instances and the provided results indicated the model can obtain efficient and effective work- rest schedule and maintenance schedule.

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