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A Model for Runway Landing Flow and Capacity with Risk and Cost Benefit Factors

Babak Ghalebsaz Jeddi[1*], John F. Shortle[2]

[1]Dept. of Industrial Engineering, Sharif University of Technology, Tehran, Zip 14588,Iran
bgjeddi@sharif.edu, babak.jeddi@gmail.com

[2]Dept. of Systems Engineering and Operations Research, George Mason University, Fairfax, VA22030, USA
jshortle@gmu.edu

ABSTRACT

As the demand for the civil aviation has been growing for decades and the system becoming increasingly complex, the use of systems engineering and operations research tools have shown to be of further use in managing this system. In this study, we apply such tools in managing landing operations on runways (as the bottleneck and highly valuable resources of air transportation networks) to handle its optimal and safe usage. We consider a uniform aircraft fleet mix landing on a runway with two major landing risks of *wake-vortex encounter* and *simultaneous runway occupancy*. Here, we empirically estimate minimum safe wake-vortex separation thresholds, extend *go-around* procedure to avoid wake-vortex encounter, and enforce the *go-arounds* assumed to be risk free. We introduce cost-benefit factors to study implications of enforced go-arounds, and develop models to adjust the average separation to maximize the net economic outcome. This also estimates the runway's true landing capacity, and provides a ground for quantifying effect of separation variance on optimal throughput. An estimation of the economic effect of wake-vortex phenomenon is also presented. Illustrations are provided through real world data.

Keywords: Aircraft separation; Landing safety; Safety; Wake-vortex; Landing capacity; Go-around procedure; Variance reduction; Cost and benefit analysis.

1. INTRODUCTION

According to the Airports Console International (2010) over 1.62 billion passenger traffic (total passengers enplaned and deplaned, passengers in transit counted once) and 1.4 million traffic movement (total movements: landing + take off of an aircraft, including both passenger and cargo aircraft) were handled by 30 busiest airports around the globe in 2010. These facts underline the high impact of the studies concerning runway efficiency, runway safety, and correct capacity estimation.

Furthermore, the increasingly high demand for the runway arrival and departure slots in congested airports around the world and the expense of runways makes them very limiting and highly valuable

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*Corresponding Author

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resources of air transportation networks. This capacity restriction limits the service rate and results in unsatisfied demand of passengers and aircraft. On the other hand, scheduling a facility beyond its capacity would cause increased delays and system instability. Thus, it is our desire in this study to obtain the maximum possible outcome (e.g., the flow) of the runway operations without over scheduling. Aircraft flow in landing, takeoff, or en route can be increased by reduced aircraft separation, nevertheless, the consequences of any reduced separation can be loss of safety (so can be stated that risk is the other side of the throughput coin).

This study introduces a procedure to mitigate the increased risk due to separation reduction, and empirically estimate minimum safe wake-vortex separation threshold, introduce some cost-benefit factors into the separation adjustment procedure and formulate a model to balance them, and consequently quantify economic effect of wake-vortex phenomenon and effect of separation variance on system capacity. In this section, after focusing on a capacity definition, we briefly look at safety risks in the approach process and the background in capacity analysis.

1.1. Capacity Definition

Through some discussion, Jeddi (2012) provides a revised version of existing runway capacity definitions as follows: Runway's true capacity is the maximum (landing or takeoff) throughput on average in a given period of time which could be *safely sustained* (for an infinitely long time if we have a large pool of aircraft continuously coming to land or takeoff).In this manner, safety is explicitly taken as a no compromise factor as we also follow that path in this study. Thus, in this definition, probability of some kind of hazard is meant to be zero while calculating the capacity.

1.2. Safety risks

Some models are developed to address the collision risk and safety (e.g. Reich, 1964; Hockaday and Chatziioanou, 1986). Here we are not to calculate collision risks but are concerned with two major safety risks in aircraft approach process (i.e., the risk of a wake-vortex encounter and the risk of simultaneous runway occupancy (SRO), which is a precursor for a runway collision). A wake-vortex hazard occurs when a following aircraft encounters the wake-vortex from its leading aircraft. When the wake is strong enough relative to the weight of trailing aircraft, the encounter may cause a loss of control, which may result in passenger injuries or aircraft crash and fatalities; see Gerz*et al.* (2002) and Robins and Delisi (2002) for a review of wake-vortex dynamics and theorems. SRO risk is the probability that a following aircraft reaches the runway threshold before its leading aircraft clears of the runway. These two risks are to be avoided to assure a safe landing. Separation requirements to mitigate these two risks are the major constraints on the landing capacity of runways. The in-trail separation minima are given in Table 1, but some exceptions may apply to reduce the 3 nm minimum to 2.5 nm (FAA, 1993).

Table 1 Approach in-trail separation minima under ILS

Following Aircraft	***Leading Aircraft***			
	Small	**Large**	**B757**	**Heavy**
Small	3	4	5	6
Large	3	3	4	5
B757	3	3	4	5
Heavy	3	3	4	4

Existing probabilistic approaches to this problem study the relation between these risks or safety and the throughput rate (e.g. Lee *et al*., 1997; Jeddi*et al*., 2006). In these approaches, the maximum throughput occurs at the point of maximum allowable risk. While the previous studies aim to decrease the chance of SRO solely by increasing the separation, in this study we attempt to eliminate the risk of SRO by enforcing the known *go-around* procedure; for a briefing on this procedure see Nolan (2011, p 96 and pp 268-269). Here, we extend and adopt that concept to eliminate the risk of a hazardous wake-vortex encounter. We assume that such procedures are strictly enforced and respected whenever the separation distance is not sufficient to assure the safety. Note that the assumption of risk free *go-around* may not be true in some runways.

1.3. Capacity analysis background

Early studies of landing capacity go back to the 1940's when runways became congested and delays became an important concern. Thus the early works typically were concerned with the aircraft delay analysis, some of which cited by Hockaday and Kanafani (1974); this reference also presented a probabilistic model for calculating runway capacity in which the aircraft separation is considered to be normally distributed. Newell (1979) provided an early survey and critique of the literature on airport capacity including few important technical reports. He presented a probabilistic analysis for capacity evaluation also (like the majority of such studies) considering a normal probability distribution for the aircraft separation.

Some studies (e.g. Newel, 1979;Gilbo, 1993) also optimize runway utilization by considering arrival and departure operations as interdependent processes and by suitably allocating runway time between these two operations. Our study is focused on landing operations but not on takeoff although our separation concepts (in this paper) can be applied for takeoff and en route processes as well. Some other works utilize simulation to evaluate the capacity and delay benefits of different operating scenarios (e.g. Boesel and Bodoh, 2004, use simulation to analyze Air Route Traffic Control Centers (ARTCC) redesign for arrival traffic). These simulation based studies consider a normal probability distribution for the capacity. Logistics Management Institute (LMI) provided a method to evaluate airport capacity and delays as discussed in Lee *et al.* (1997). This method based on probabilistic calculations also assumes a normal probability distribution behavior for in-trail separation (like Newell, 1979;Hockaday and Kanafani, 1974) and considers 5% allowance of violating the minimum separation.

In this study, by enforcing *go-around* procedures to obtain a desired zero risk, as we put aircraft closer together a separation is reached where throughput actually goes down because aircraft must break out due to avoid the safety violation. In other words, the output (e.g. throughput) function is not monotonic but has a maximum. This paper is concerned with determining the optimal separation of landing aircraft (to maximize the system output, e.g. number of successful landings, the economic benefit, *etc*) on runways operating almost independently of other runways (typically called single runways). We consider a given pair of follow-lead aircraft types (e.g. large-large). We adopt the U.S. Federal Aviation Administration's (FAA) aircraft classification as the following three weight ranges in addition to Boeing 757 (indicated by B_{757}): Small (≤ 41,000 lbs), Large (41,000 lbs < - ≤ 255,000 lbs), and Heavy (≥ 255,000 lbs) (note: lbs = 0.454 Kg).

The rest of the paper is organized as follows. Major notations are introduced following this paragraph. In Section 2, we discuss a *go-around* procedure for wake-vortex avoidance in addition to enforced *go-around* for SRO avoidance, and calculate *go-around* probabilities as a function of the attempt-to-land rate (or equally the average separation). Section 3 provides a model to maximize the net economic benefit of the landing operation when some cost and benefit factors are introduced. A special case for maximizing sustainable and safe runway throughput and estimating runway landing capacity is provided. A sensitivity analysis of separation variance on optimal solutions is also provided. A case study and illustrations are provided in Section 4. A conclusion is provided in the last section.

1.4. Notations

The following notations are used throughout the paper.

LTI	landing time interval between an aircraft pair measured at the runway threshold, in seconds
L	the lowest observed value for *LTI* in the large volume of separation data
ROT	runway occupancy time of aircraft *k*, in seconds
B	expected dollar benefit of one successful landing for all the system beneficiaries
C	expected average cost of a *go-around* or unsuccessful landing to all the system beneficiaries
ENB	expected net benefit
t_0	minimum safe wake-vortex *go-around* separation of successive aircraft, in seconds
p	probability of a *go-around*

The decision variables are:

ω	landing attempt per hour (i.e., flow rate through the glide path); ω = 3600 /mean(*LTI*)
λ	arrival rate to airport area, or equally the runway throughput, landing/hour
ELA	economic landing attempts (the optimal ω)
ELS	economic landing separation
ELT	economic landing throughput (the optimal λ)

The abbreviations are:

ILS	Instrument Landing System
SRO	Simultaneous Runway Occupancy
WV	wake-vortex
GSRO	*go-around* to avoid SRO; the position to execute GSRO is called missed approach point (MAP), also known as the decision height under the precision approach. It is in a distance from threshold where pilot/controller decides whether to execute GSRO.
GWV	*go-around* to avoid wake-vortex encounter

2. PROBABILITY OF GO-AROUND

It is desired that the chance of a simultaneous runway occupancy or a hazardous wake-vortex encounter be nearly or exactly zero. In conventional models in the literature (e.g. MLI model in Lee *et al.* 1997) increasing the target separation between successive aircraft decreases these two risks, and a target level of chance for unwanted incidents, such as 10^{-5}, indicates what the average separation should be. The risk can also be reduced by implementing *go-around* procedures. In current reality, the *go-around* procedure to avoid an SRO is not always taken. In this paper, we assume that:

- an aircraft always executes a *go-around* procedure whenever a safe separation minimum (either due to SRO or wake-vortex encounter risk) is not or will not be met,

- the *go-around* procedure is risk free, and
- complete, accurate, and timely information to make *go-around* decisions exist.

With these assumptions, the risk of an SRO or a wake-vortex encounter can be reduced to zero when *go-around* is enforced; however, there is possibly an increase in the number of *go-around*s. Assuming the system to be risk free due to perfect execution of the *go-around* procedure creates a different dynamic and may change the optimal level of operations. As shown later in Section 3, the optimal level of separation and the rate of landing attempt depend on the *go-around* probability, and some cost-benefit factors. This section calculates this probability for two cases of with and (theoretically) without wake-vortex effect.

When the net benefit is maximized, we interpret the optimal solution as economic landing attempts (*ELA*) per time unit and the corresponding separation as the economic landing separation (*ELS*). The economic landing throughput (*ELT*) per time unit will be obtained by deducting the rate of *go-around* aircraft from *ELA*. Numerical examples in this paper are based on data analysis and probability distributions of landing time interval (*LTI*) at runway threshold and runway occupancy time (*ROT*) for a single runway estimated in Jeddi et al. (2009). These distributions are for the pairs with 3 nautical miles (nm) separation minima in the FAA separation standard rules (FAA, 1993). In contrast with the common assumption of normal probability distribution for LTI in the capacity literature, we use lognormal distribution as estimated in Jeddi et al. (2009).

In the aircraft approach/landing process, two different aircraft flows are observed: the flow through the glide slope (the common aircraft path) denoted here by ω measured in landing attempts per time unit, and the flow through the runway (i.e., the throughput λ measured in successful landings per hour). Assuming a continuous separation monitoring system (by controller, pilot, *etc*) at any point through the glide path, the trailing aircraft executes a *go-around* when the separation is less than a specific value t_0. We may name this operation as the wake-vortex *go-around* or wake-vortex missed approach procedure in contrast with the well-known *go-around* procedure executed to avoid an SRO (so the other reason for a *go-around* is to avoid an SRO). Figure 1 demonstrates this dynamic in the aircraft approach process with enforced *go-around* procedures.

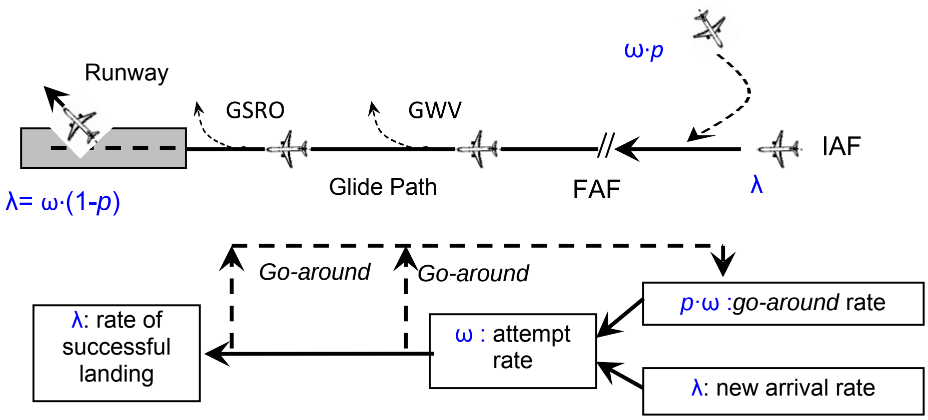

Figure 1 *Go-around* procedures on the glide slope to avoid wake-vortex encounter and SRO

We believe that such a minimum wake-vortex safe separation threshold exists and can be estimated theoretically. The minimum might be different under different meteorological conditions. We are not to theoretically calculate this minimum in this study, but we provide an intuitive empirical method to estimate the minimum wake-vortex safe separation in Section 2.2.1 based on historical minimum observed separation. For a review of available wake-vortex strength, position and decay theories see Gerz, *et al.* (2002), for example.

If a safe separation is achieved through the glide path, the aircraft continues the approach. Then, at GSRO (well known as missed approach point, also called decision height under the precision approach) the follower decides whether or not to execute a *go-around* to avoid simultaneous runway occupancy with its leading aircraft, Nolan (2011). Before computing probability of *go-around*, we note some additional assumptions as follows:

- *LTI* and *ROT* are independent random variables.
- Shifting *LTI* probability distribution function to the right or left does not change its shape.
- Zero risk assumed for execution of *go-around* procedures.
- *Go-around* is completely respected and enforced.
- The number of *go-around* in a given period is not restricted.
- Wake-vortex *go-around* and SRO *go-around* conditions do not occur simultaneously for an aircraft pair.

Go-around probability p is a function of the average separation and attempt rate ω. We calculate $p(\omega)$ in two parts: firstly, we theoretically assume that there is no wake-vortex effect and derive the necessary formula. Secondly, we complete the calculation by taking the wake-vortex phenomenon into consideration.

2.1. Probability of *go-around* assuming no wake-vortex effect

In this section, we theoretically ignore the possible wake-vortex encounter, and only consider the risk of an SRO and not the risk of a hazardous wake-vortex encounter. The probability of an SRO is:

$$\begin{aligned} P\{\text{SRO}\} &= P\{LTI<ROT \text{ and Follow aircraft lands}\} \\ &= P\{\text{Follow aircraft lands} \mid LTI<ROT\} \cdot P\{LTI<ROT\}. \end{aligned}$$

LTI is the landing time interval that would be observed if the following aircraft lands. $P\{\text{SRO}\}$ can be reduced to zero by enforcing the *go-around* procedure. In this case,

$$P\{\text{Follow aircraft lands} \mid LTI<ROT\} = 0 \text{ and } p(\omega) = P\{\textit{go-around}\} = P\{LTI<ROT\}.$$

The latter probability can be calculated by conditioning on specific *ROT* or *LTI*. It results

$$P\{LTI < ROT\} = \int F_{LTI} \cdot dF_{ROT} \tag{1}$$

where F_{LTI} and F_{ROT} are cumulative distribution functions of the landing time interval and runway occupancy time, respectively. *LTI* and *ROT* are assumed to be independent. Further, we assume that changes in average arrival rates (that is, changes in λ or ω) can be achieved by shifting the location parameter of the *LTI* distribution to the left or right. Shifting the *LTI* distribution also changes the probability that *LTI*<*ROT*, or equivalently, the probability of a *go-around*. As illustrated in Section 5, *p* exponentially increases as separation decreases.

2.2. Probability of *go-around* with wake-vortex effect

Now consider the case where an aircraft may possibly execute a *go-around* through the glide path for two different reasons: to avoid wake-vortex encounter and to avoid simultaneous runway occupancy. For this situation, let t_0 to be the minimum safe separation (of successive aircraft given in seconds) to execute *go-around* to avoid a wake-vortex encounter, and *L* to be the lower limit of *LTI* distribution; e.g., *L* = 40 second.

The basic assumption here is that the safety associated with potential wake encounters can be determined relative to a single threshold in time. Separations above the threshold are considered safe. Separations below this level can be hazardous. Note that this threshold is with respect to meteorological conditions for the given aircraft pair type. Any historical wake-vortex encounter information at the runway of study must be taken into consideration. For example, if an encounter has occurred but it is controlled without a hazard, the corresponding observed minimum separation cannot be considered to be safe.

2.2.1. Safe wake-vortex threshold

Samples of *LTI* are representative of the process performance, which we use to estimate the safe wake-vortex threshold. The proposed procedure is as follows for a given aircraft pair type, meteorological conditions, and set of technologies involved in the separation control of a specific runway.

Step 1. Collect historical information on aircraft separation for a long enough time using a precise aircraft tracking technology (e.g. Multi lateration surveillance system).
Step 2. Using the separation information, indicate the minimum separation which has occurred without a wake-vortex encounter.
Step 3. Add a reaction-time to the observed minimum in *Step 2* to obtain the minimum safe wake-vortex *go-around* separation t_0. The reaction time is the delay to start executing *go-around* procedure from the time that such a decision is made.

It is obvious that total $p(\omega)$ depends on threshold t_0 and increases as t_0 increases for any fixed ω. To calculate $P\{go\text{-}around\}$, two cases for lower limit of *LTI* distribution shall be considered as follows:

Case 1: Lower limit of *LTI* distribution $L<t_0$.
In this case, $p = \mathrm{P}\{LTI<t_0\} + \mathrm{P}\{t_0 \le LTI<ROT\}$, and by conditioning on *ROT*, the second term can be written as $\int_{t_0}^{\infty} F_{LTI}(r)dF_{ROT}(r) - F_{LTI}(t_0)[1 - F_{ROT}(t_0)]$. Thus we have

$$p = \int_{t_0}^{\infty} F_{LTI}(r)dF_{ROT}(r) + F_{LTI}(t_0)F_{ROT}(t_0). \tag{2}$$

Case 2: $L \ge t_0$. This case means that *LTI* shifted to the right as much that its lower point *L* is above the wake-vortex *go-around* threshold t_0. No wake-vortex *go-around* would occur in this case, and $P\{go\text{-}around\}$ is obtained from equation (1).

In Sections 3 and 4, having $P\{go\text{-}around\}$ as a function of average landing attempt rate, we formulate some models to analyze the tradeoff between the average cost of a *go-around* and benefit gained from a successful landing when wake-vortex hazard and SRO risks are reduced to a practical zero by the enforced *go-around*.

3. MAXIMIZING THE OUTPUT

The objective is to maximize the overall system output by adjusting the average aircraft separation (which determines the landing attempt rate ω), while $P\{\text{SRO}\}$ + $P\{\text{WV encounter hazard}\}$ is maintained at zero by the enforced *go-around* procedures. Since this is an economic system, the overall output may be well represented by the overall net economic benefit to all the system beneficiaries.

System beneficiaries include airlines, passengers, airports, pilots, crew, *etc*. It should be noted that maximizing the number of (successful) landings does not necessarily guarantee the overall economic optimality of the landing operations. This is because the costs and benefits influence the optimality and should be taken into account, the task we are committed to do in this research. However, we will show that maximizing the successful landing rate is a special case of maximizing the overall net economic gain.

As mentioned, the *go-around* probability p is a function of the attempt rate ω. The average *go-around* rate (number of *go-around* per hour) is $\omega \cdot p(\omega)$ and the average successful landing rate is

$$\lambda(\omega) = \omega \cdot [1\text{-}p(\omega)]. \tag{3}$$

Clearly, the rate of aircraft attempting to land is the arrival rate of aircraft $\lambda(\omega)$ plus the *go-around* rate $\omega \cdot p(\omega)$, that is, $\omega = \omega \cdot [1- p(\omega)] + \omega \cdot p(\omega)$. A graphical illustration was given in Figure 1 in Section 2.

3.1. Maximizing expected net benefit (ENB)

For the landing operations when safety is guaranteed by the enforced *go-around* procedure, two major benefit and cost factors are observable:

a) The landing benefit is the total benefit (or revenue) to all the system beneficiaries minus the operational cost, except the cost due to performing a *go-around* procedure. Because of uncertainties involved in the benefit factors, this overall benefit is a random variable. Let B represent the expected value of this overall benefit.

b) The *go-around* cost due to the loss of one landing attempt, for any given type of aircraft, is the summation of cost components such as the costs of: fuel, passenger delay, crew delay, disrupted schedule of downstream flights, airport operations, and so forth. These cost components depend on parameters like aircraft load factors and the arrival rate at a given time, which involve some level of uncertainty. In other words, the *go-around* cost is a random variable, and we consider its expected value C in this study.

Estimations of B and C are not subjects of this study; rather, their effect on the optimal level of operation is the concern.

We seek to maximize the net economic benefit (that is, the total benefit minus the total cost of operations in a peak period) with respect to the average attempt rate ω (or, equally, with respect to the average aircraft separation). This net benefit is a random variable and we consider maximizing its expected value (*ENB*).

For every landing attempt, a successful landing occurs with probability 1- $p(\omega)$ and a *go-around* occurs with probability $p(\omega)$. Thus the optimization problem of interest can be formulated as

$$\max_{\omega} ENB(\omega; B, C) = \omega \cdot \left[\left(1 - p(\omega)\right) \cdot B - p(\omega) \cdot C\right]. \tag{4}$$

To obtain a more generalized formulation, we write *ENB* in terms of cost to benefit ratio *C/B*. Factoring out *B* in equation (4) gives

$$\begin{aligned} ENB(\omega; B, C) &= B \cdot \omega\left[\left(1 - p(\omega)\right) - p(\omega)\, C/B\right] \\ &= B \cdot \omega[1 - (1 + C\ /\ B)p(\omega)\ ]. \end{aligned} \tag{5}$$

The optimal solution of this problem, ω^*, is the Economic Landing Attempt rate (*ELA*) as a variable, so we may name (4) as the Economic Landing Attempt (ELA) model. Letting $g(\omega; C\ /\ B) = \omega \cdot [1 - (1 + C/B)p(\omega)]$, results in $ENB(\omega;B,C) = B \cdot g(\omega;C/B)$.

Then (4) is equivalent to

$$\max_{\omega} g(\omega; C/B) = \omega \cdot [1 - (1 + C/B)p(\omega)]. \tag{6}$$

The problem can be solved numerically as explained later in Section 3.1.1, or analytically if one has a formulation for $p(\omega)$.

The optimal solution for this problem $ELA = \omega^*$ also indicates the optimal values of ELS = 3600/ELA and $ELT = \omega^* \cdot [1\text{-}\ p(\omega^*)]$, where the landing stream is sustainable and safe. Note that *ELA*, *ELS*, and *ELT* all depend on the *C*/*B* ratio.

The derivative of $g(\omega;C/B)$ and $ENB(\omega;B,C)$ with respect to ω are zero at ω^*. We shall discuss some general properties of $g(\omega;C/B)$, or equally $ENB(\omega;B,C)$, as the following. These properties generalize behavior of *g* and *ENB*, and help to understand the nature of interactions among the parameters, the decision variable and their influence on the output. The properties are independent of the type of follow-lead aircraft pair. Illustrations are given in Section 4.

Property 1: $g(\omega;C/B) \to \omega$ *when* $\omega \to 0$*, i.e.,* $g(\cdot) \approx \omega$ for small ω.

This is clear as $dp/d\omega > 0$. Then it is immediate from equations (5) and (6) that as ω reduces and $p(\omega) \to 0$, the term inside brackets goes to 1. □

Property 2: $g(\omega;C/B)$ is concave for a given *C/B* and $\omega \le \text{Argmax}\{dp/d\omega\}$. That is, it has a unique maximum in this range.

Proof: We have $dg/d\omega = 1 - (1 + C/B)[p(\omega) + \omega \cdot p'(\omega)]$. So $dg/d\omega = 1$ at $\omega = 0$, and it is decreasing in ω because ω, $p(\omega)$, and $p(\omega)$ are increasing in ω for $\omega \le \text{Argmax}\{dp/d\omega\}$. This means that the second derivative of $g(\cdot)$ is negative and this completes the proof. □

Property 3: $g(\omega;C/B)$ decreases as C/B increases for any given ω.

Proof: This property directly follows from equation (6). The partial derivative of g with respect to C/B is clearly negative. □

Property 4: $ELA = \omega^*(C/B) = \text{Argmax}\{g(\omega;C/B)\}$ is decreasing in C/B for $\omega \leq \text{Argmax}\{dp/d\omega\}$.

Proof: The proof is by contradiction. By definition, $dg/d\omega = 0$ at ω^*. Then, it is straight forward to observe $p(\omega^*) + \omega^* \cdot p'(\omega^*) = 1/(1 + C/B)$. Increasing C/B, decreases the right hand side, and consequently the left hand side of this equation. On the other hand, ω, $p(\omega)$, and $p'(\omega)$ are all increasing in ω for $\omega \leq \text{Argmax}\{dp/d\omega\}$. Thus, if ω^* does not decrease, the left hand side will not decrease. This means that $ELA = \omega^*$ shall decrease by contradiction. □

Property 5: Economic landing through put $ELT = \omega^*[1- p(\omega^*)]$ is decreasing in C/B for $\omega \leq \text{Argmax}\{dp/d\omega\}$.

Proof: This property clearly follows from Properties 3 and 4, based on which the peak of g moves down and left as C/B increases. □

Note that the limitation $\omega \leq \text{Argmax}\{dp/d\omega\}$ for Properties 2, 4 and 5 is not binding in practice since $\text{Argmax}\{dp/d\omega\}$ is beyond practical attempt rates, e.g. 40 landing per hour. On the other hand, these properties are still valid for some ω beyond $\text{Argmax}\{dp/d\omega\}$ as far as $p'' \geq -2\, p'/\omega$, where p'' is the second derivative of p with respect to ω.

3.1.1. The Algorithm Summary

Here, we summarize the proposed steps to determine optimal level of operations, separation and landing throughput. (For illustration and figures, see Section 4.)

Step 0. Choose an aircraft pair type for separation optimization. Indicate meteorological conditions and set of separation control/adjustment technologies in effect under which the optimized control is desired. Indicate the ratio of average cost of a *go-around* to the average benefit from a successful landing for the given follow-lead aircraft, C/B.

Step 1. Collect *ROT* and *LTI* data as precise as possible, e.g. by processing Muti lateration system data, in peak arrival periods for the condition specified in *Step 0*.

Step 2. Estimate the probability distributions of *ROT* and *LTI*, using Maximum Likelihood Method, for example.

Step 3. Find the safe wake-vortex *go-around* threshold using the collected data and procedure in Section 2.2.

Step 4. Identify a practical range for attempt rate ω. Starting from the lower bound of the range, calculate $p(\omega)$ from equations (1) and (2).

Step 5. Increase ω by a small increment Δ, and repeat Step 4 until $p(\omega)$ is computed for the end of the selected range.

Step 6. Calculate $g(\omega;C/B)$ for a given C/B and for all $p(\omega)$ computed in *Steps 4* and *5*.

Step 7. Indicate the maximum value of $g(\omega;C/B)$ among all values computed in *Step 6*.

Step 8. Using the result of *Step 7*, indicate the optimal solution $ELA = \omega^* = \text{Argmax}\{g(\omega;C/B)\}$. Calculate the economic landing throughput $ELT = \lambda^*$ as a function of *ELA* and the economic landing separation through the glide path as $ELS = 3600/ELA$.

Steps 6, 7, and 8 solve problem (5). Desired precision can be achieved by suitably reducing the increment value Δ in Step 5.

3.2. Maximizing runway throughput

We show that maximizing the runway throughput is a special case of maximizing the expected net benefit. This special case provides an estimate for the runway landing capacity as the maximum sustainable safe landing throughput. From equation (3), $\lambda(\omega) = \omega\cdot[1\text{-}p(\omega)]$. Then, (5) can be written as

$$\max_{\omega} g(\omega; C/B) = \lambda(\omega) - \frac{C}{B} \cdot \omega \cdot p(\omega). \tag{7}$$

Property 6: $g(\omega; C/B) \rightarrow \lambda(\omega)$when $C/B \rightarrow 0$. The validity of this property is immediate from equation (7).□

Properties 3 and 5 together, indicate that $g(\omega; C/B)$ is bounded by $\lambda(\omega)$ for any ω(i.e. $g(\omega; C/B)$ lies inside and under $\lambda(\omega)$). Based on this property, when C is negligible relative to B, the problem reduces to maximizing the runway throughput $\lambda(\omega) = \omega\cdot[1\text{- } p(\omega)]$. Properties 1 and 2 are valid for $\lambda(\omega)$ (i.e. $\lambda(\omega) \rightarrow \omega$ when $\omega \rightarrow 0$, and $\lambda(\omega)$ is concave with a unique maximum).

One can intuitively think about the concavity of this function as follows: by increasing the attempt rate, the *go-around* percentage increases but the percentage of successful landings decreases. So that, after a point, the decrease in the rate of successful landings dominates the increase in the attempt rate. In other words, throughput $\lambda(\omega)$ has a unique maximum as an optimal point. This can also be explained in mathematical terms. $p(\omega) > 0$ is increasing, and 1- $p(\omega)$ is decreasing in ω. Thus, after a point, the decrease of 1- $p(\omega)$ dominates the increase of ω so that $\lambda(\omega) = \omega\cdot[1\text{-}p(\omega)]$ has a maximum.

Note that the economic landing throughput equals λ at economic landing attempt when C/B is zero, i.e. $ELT(C/B = 0) = \lambda^*(\omega^*)$.

3.3. Runway landing capacity

As discussed in introduction, capacity can be well defined as the maximum sustainable and safe throughput in average. Sustainability also implies that the maximization is over the average throughput. While one may observe some higher or lower throughput rates under an adjusted throughput (or separation) at a certain level, we are making sure that the process remains safe and sustainable under constantly sufficient landing demand. The maximum sustainable safe throughput rate $\lambda^*(\omega^*)$ is the runway sustainable landing capacity calculated for any specific follow-lead aircraft pair types, e.g. large-large.

Finally, one interesting conclusion of the properties regarding cost and benefit factors is that the best economic use of the system does not necessarily achieved by using its capacity to the fullest extent.

3.4. Variance reduction effect

Variance may cause a loss of productivity and is not a desirable factor. Generally speaking, operations with lower variability are easier to manage. In the case of separation variance, intuitively, risks of SRO and hazardous wake-vortex encounter are lower for a smaller separation variance. It is important to quantify effect of reducing separation variance on the risk/safety and on the optimal level of operations. Variance reduction can occur due to systemic or systematic reduction of errors and increased precision. These can be realized by utilizing new technologies (e.g. communication, navigation, surveillance, etc.) or new procedures in separation control and adjustment.

To evaluate sensitivity of optimal solutions to separation variance, we use the following steps for a given aircraft pair type and meteorological condition. Illustration is provided in Section 4.3.

Step1: Manipulate parameters of *LTI* distribution so that the mean remains constant and the standard deviation reduces by a desired factor.

Step2: Follow Steps 4 to7 of algorithm 3.1.1 to obtain *ELA*, *ELS*, and *ELT* for the new case.

Formula for Step 1 when *LTI* has a log-normal distribution is given in Section 4. In these calculations, an additional assumption is that reduced separation variance does not change the form of separation probability distribution; however, this does not change the concepts.

4. A CASE STUDY AND ILLUSTRATION

As a practical study, we apply methodologies of this paper to landings on runway 21L at Detroit Metropolitan Wayne County airport. We follow the steps of algorithm 3.1.1 for this runway. In Step 0, we chose the follow-lead aircraft pairs with 3 nm separation minima which include S-S, L-S, B_{757}-S, H-S, L-L, B_{757}-L, and H-L indicated in FAA (1993) where S, L, B_{757} and H respectively stand for small, large, Boeing 757 and heavy. The landing system in use is ILS.

As Steps 1 and 2, we use the results reported in Jeddi *et al.* (2009). In that study, the aircraft track data from the Multi lateration surveillance system is analyzed and probability distributions of the approach process are estimated. The distributions in peak periods (when there are at least 7 aircraft landings per quarter hour), under *ILS*, for 3 nm pairs (indicated by LTI_3) along with their corresponding *ROT* probability distributions are as follows:

LTI_3 ~ Log-normal (shift = 40, scale = 4.06, shape = 0.45),

$ROT_{S,L}$~ 0.62 Beta ([20,90], 11.23, 26.33) + 0.38 Beta ([30,110], 13.60, 27.39),

where in Beta distributions, the values inside brackets indicate the range, and the next two values are the distribution parameters. For completeness we present basic forms of log-normal and Beta distributions here.

Log-normal distribution: Probability distribution function of log-normally distributed random variable X with the shift value of zero is

$$f(x;\mu,\sigma) = \frac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right), \text{ for } x > 0,$$

where μ and σ are scale and shape parameters, respectively. Expected value and variance of X are

$$E(X) = \exp(\mu + \sigma^2/2), \text{ and } V(X) = \exp(2\mu + \sigma^2)\cdot(\exp(\sigma^2) - 1).$$

It is straight forward to show that to change the variance by a factor of $c > 0$, for a fixed $E(X)$, it is enough to obtain the new scale parameter μ' and shape parameter σ' as

$$\mu' = \mu + \frac{\sigma^2}{2} - \frac{1}{2}\ln\left[1 - c + c\exp\left(\sigma^2\right)\right], \text{ and } \sigma' = \left(\ln\left[1 - c + c\exp\left(\sigma^2\right)\right]\right)^{1/2}.$$

Beta distribution: Probability distribution function of a Beta distributed random variable Y with parameters α and β in the range (0, 1) is:

$$f(y;\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} y^{\alpha-1}(1-y)^{\beta-1}$$

where $\Gamma(n) = (n - 1)!$. Expected value and variance of Y respectively are:

$$E(X) = \frac{\alpha}{\alpha+\beta} \text{ and } V(X) = \frac{\alpha\beta}{(\alpha+\beta)(\alpha+\beta+1)}.$$

Figure 2 shows distributions of LTI_3 and $ROT_{S,L}$. For simplicity, we assume that these are related to large-large aircraft pairs since 63.3% of data was from this pair type.

For 3 nm pairs, at runway 21L in Detroit airport, $p\{LTI < ROT\}$ is estimated to be 0.0034 with a 95% confidence interval of [0.0021, 0.0051], and the LTI mean is 104 s, Jeddi *et al.* (2009). So the average number of attempts is $\omega = 3600/104 = 34.6$per hour. In this case, with no *go-around*, $\omega = \lambda$ and theoretically $P\{\text{SRO}\} = 0.0034$ but not zero.

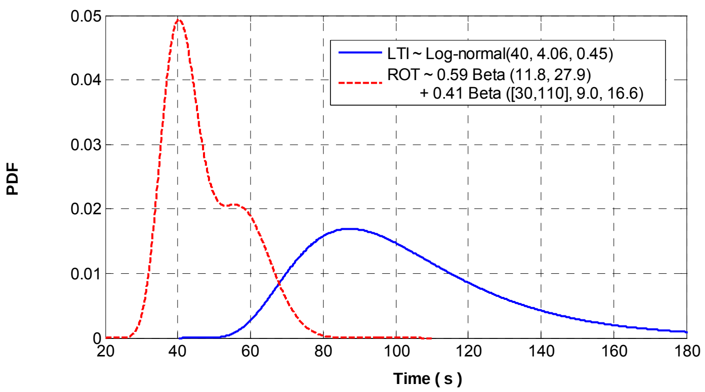

Figure 2 ROT and LTI density functions for 3 nm pairs under Instrument Landing System

Figure 3 shows $p(\omega)$ in solid line and its derivative $dp/d\omega$ in dashed line, but 10 times larger to provide a better visibility.

As Step 3, we estimate the minimum safe separation to execute wake-vortex *go-around* using procedure 2.2.1. Table 6 in Jeddi, *et al.* (2009) indicates that, for 3 nm pairs, aircraft was as close as 48 s without a wake-vortex encounter. (To the best of our knowledge, no severe wake-vortex encounter was reported during the time the data was collected.) Thus, this can indicate a lower limit for safe separation. There needs to be more time than this minimum to assure a safe *go-around*. We add 7seconds as the reaction-time in order to execute the *go-around* procedure to avoid wake-vortex hazard, which results in $t_0 = 55$ s and $P\{LTI_3 < t_0 = 55\} = 0.0013$.

4.1. Optimal solution without wake-vortex effect

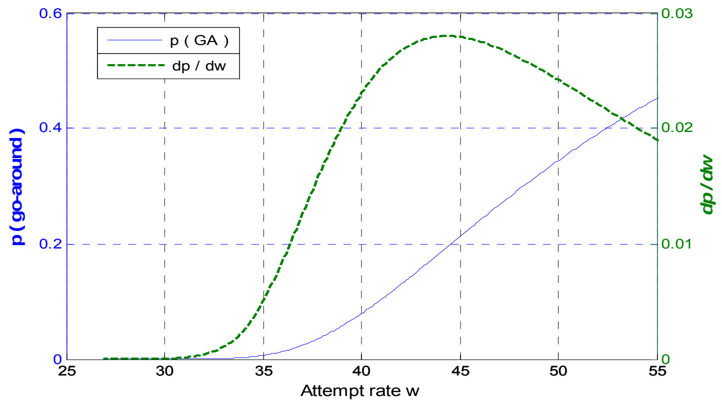

Figure 3 Probability of *go-around* $p(\omega)$ (left axis) and its derivative in ω (right axis)

First consider the case where wake-vortex effect is theoretically ignored. In this case, in Steps 4 and 5, $p(\omega)$ is calculated from (1), for ω in range [25,55] with increments of $\Delta = 0.001$. As Step 6, $p(\omega)$ is plugged into the function g for given C/B values in (7) to maximize the expected net economic benefit.

$g(\omega, C/B)$ is concave and has a unique maximum (Property 2). The ones with larger C/B ratio lie under the other ones (Property 3). The solid curve at the top corresponds to $C/B = 0$ and represents the throughput $\lambda(\omega)$ (Property 4). Also, as in Properties 4 and 5, peak of $g(\omega;C/B)$ moves down and left by increasing C/B (i.e., the maximal ω^* decreases as C/B increases). Argmax$\{dp/d\omega\}$ is calculated 49.7 attempt/h for the LTI_3 and $ROT_{S,L}$ at hand, needed for Properties 2, 4, and 5. One can plot $ENB = B{\cdot}g$, for a given B.

As Steps 7 and 8, for $C/B = 0$, the optimal solution ($ELA{=}\omega^*$, $ELT{=}\lambda^*$, p^*) is (46.5, 40.2, 0.137). To have a stable approach system, the arrival rate to airport area λ, is adjusted so that ω is maintained in the optimal level of 46.5 attempt/*h*. At most, 40.2 landing/*h* can successfully go through the runway; that is, capacity is 40.2 landing/h for large-large aircraft pair when there is theoretically no wake-vortex effect present. In terms of optimal separation, $\omega^* = 46.5$ results in economic landing separation $ELS \approx 77$ s.

4.2. Optimal solution with wake-vortex effect

Now we maximize the net economic benefit and runway throughput under the natural situation where wake-vortex encounter risk is also taken into consideration. In this case, the problem is maximizing (4) or (7). In the case of 3 nm pairs, as for *Steps 4* and *5* of Algorithm 3.1.1, $p(\omega)$ is calculated from equation (1) and equation (2), for $t_0 = 55$ s and for ω in [25,55] with increments of $\Delta = 0.001$. As *Step 6*, $p(\omega)$ is plugged in function g for given C/B values, in (7), to maximize the expected net economic benefit. Optimal values are calculated using *Steps7* and *8*. Figure 4 shows $g(\omega;C/B)$ and optimal solutions for the calculated $p(\omega)$. In this figure, to obtain $ELT(C/B)$, one should locate $\omega^*(C/B)$ on the throughput curve, i.e., $ELT = \lambda(\omega^*) = g(\omega^*;C/B)$.

In Figure 4, C/B is 4, 2, 1, 0, from the bottom to the top curves, respectively. All five properties of $g(\omega, C/B)$ can be verified from this plot: $g(\omega) \approx \omega$ for smaller ω (Property 1), it is concave(Property

2), the ones with larger *C/B* ratio lie under the other ones (Property 3), and $\omega^*(C/B)$ and *ELT* decrease as *C/B* increases (Properties 4 and 5). In this case, Argmax$\{dp/d\omega\}$ is about 43 attempt/h, see Figure 3, the condition for Properties 2, 4, and 5.

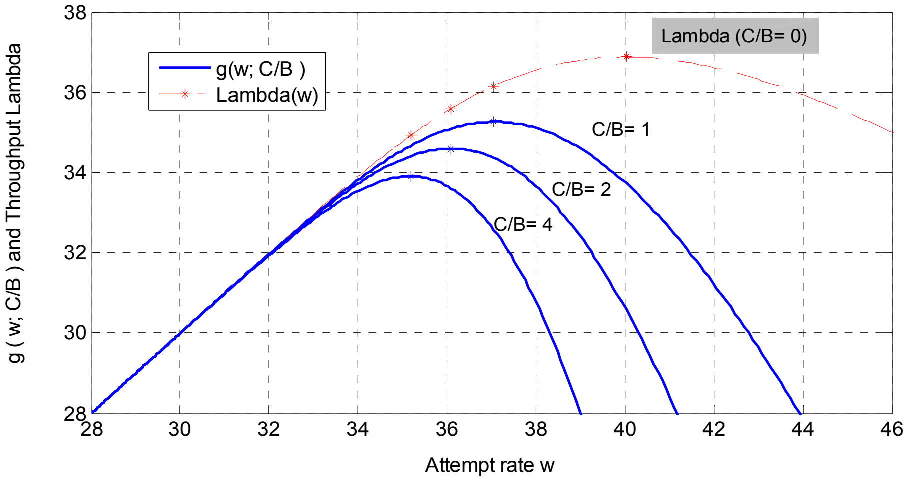

Figure 4 g(ω;*C/B*) of 3 nm pairs for *C/B* = 0, 1, 2, 4 and safe threshold t_0 = 55 s

Optimal values are summarized in Table 2 for wake-vortex *go-around* threshold of t_0 = 55 sec. The optimal solution is *ELA*= 40 attempt/*h*, *ELS* = 90 s, *ELT* = 36.9 landing/*h*, and p^* = 0.079 for *C/B*= 0. Thus, the *landing capacity* of the system is *ELT*(*C/B*=0) = 36.9 landing/*h* independent of the market parameters. As another example, from Table 2, the optimal throughput is about 35.6 landing/*h* if the cost of *go-around* is two times of the benefit gained from a successful landing.

Table 2 *ELT,* λ^*, for 3 nm pairs for given *C/B* and a safe wake-vortex threshold of t_0= 55s

C/B	ω^*	**g**	λ^*	***P****
0	40.0	36.9	36.9	0.0785
1	37.1	35.3	36.2	0.0242
2	36.1	34.6	35.6	0.0138
4	35.2	33.9	34.9	0.0072

Comparing the system capacity with and (theoretically) without wake-vortex effect can indicate how much this phenomenon is affecting the landing capacity. In other words, it estimates the cost of wake-vortex in the system. Wake-vortex is costing the system about 40.2 – 36.9 = 3.3 landings per peak hour in average. This means that wake-vortex phenomenon is causing an approximate loss of 12,000 landings per year, assuming 10 hours of peak periods every day on this runway. This is for the case of 3 nm pairs, the majority of which are large aircraft. Now, one can multiply this value by *B* and estimate the cost of wake-vortex to the system in monetary terms. Then multiply the result by the number of all runways utilized in busy airports in the U.S., for example, to obtain an estimated annual wake-vortex cost (i.e. the loss due to wake-vortex) in the U.S.

As a closing discussion of this section, it is worth obtaining a capacity estimation using a common method, the reciprocal of the minimum safe separation. We take the 2.5 nm or equally 75 s (considering the average aircraft speed through the glide path of 120 knots) as the minimum safe separation (note that 55 s is considered as the wake-vortex *go-around* threshold in this study for large-large pairs). Then we obtain the capacity of 48 large aircraft landings per hour. Intuitively this number is not practical since the realized landing capacity is generally between 30 and 40 landings per hour per runway for a fleet of large aircraft.

The concern with this method is that it ignores the probabilistic nature of the process. Achieving this level of throughput requires that the mean of *LTI* to be adjusted at 75 s. This implies $P\{go\text{-}around\} \approx 0.33$. For risk-free and safe landing with the enforced *go-around* procedure, there would be 33% loss of attempted landings, (leave it alone that this go-around rate can cause other safety risks by itself). This would lead to the successful throughput level of 32 landing/*h*, and the system should tolerate the high cost of 16 *go-arounds* per hour if even possible. Surely this level of *go-arounds* is not safe, and 32 successful landings is less than 36.9 which can be obtained as shown in Table 2 and Figure 4. So the reciprocal method for capacity estimation is not practically useful.

Our methodology estimates the average capacity of 36.9 landings per hour for 55 s safe wake-vortex threshold and risk free landings. Economic considerations (cost to benefit ratio of 4) may reduce the optimal throughput to 34.9 landing per hour (corresponding to an optimal separation of 98s in average) with a maximized net economic gain, 1% *go-around* rate and sustainable safe operations, Table 2.

4.3. Variance reduction effect

This section illustrates the two step procedure explained in Section 3.4. Jeddi, *et al.* (2009) provided two most suitable distribution fittings for LTI_3 (i.e., log-logistic and log-normal distributions). In Step 1, we are to manipulate parameters of *LTI* distribution so that the mean remains constant and the standard deviation is reduced by a desired factor. Performing this operation on log-normal probability distribution function is more convenient than log-logistic distribution, and we chose the log-normal distribution here to discuss the variance reduction effect. Formula for Step 1 is given earlier in this section.

Figure 5 shows that the overlap of *LTI* and *ROT* reduces sharply by reducing variance of LTI_3 while maintaining the mean at 104s. Accordingly, $P\{\text{SRO}\}$ reduces from 0.0034 to 0.00062 and 0.00002 when we reduce *LTI* standard deviation by 25% and 50% from its original value, respectively.

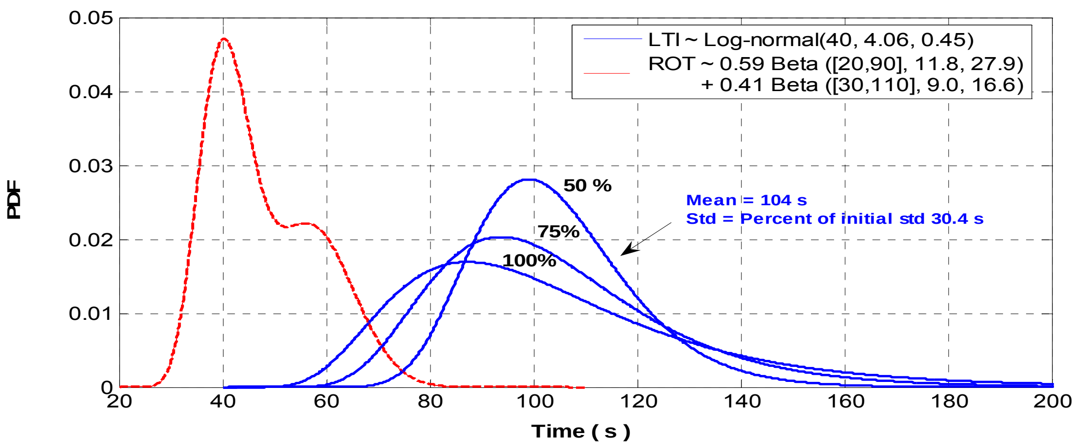

Figure 5 $P\{LTI<ROT\}$ decreases as variability of *LTI* decreases for a fixed mean

Here we show the effect of reducing *LTI* variance on optimal throughput. For brevity, we limit examples to 3 nm pairs and *C/B*= 0 (i.e., the throughput curves). Figure 6 shows the effect of reducing standard deviation of LTI_3 up to 50% of its original value 30.4 s, while keeping the mean unchanged. For example, 30% decrease in standard deviation (labeled 70% in Fig 6) leads to an extra throughput of 3.5 landing/*h* in average. Other values are seen on the plots. Overall, 30% reduction of *LTI* standard deviation yields 12,755 extra landings per year for a single runway, assuming 10 hours of daily peak periods.

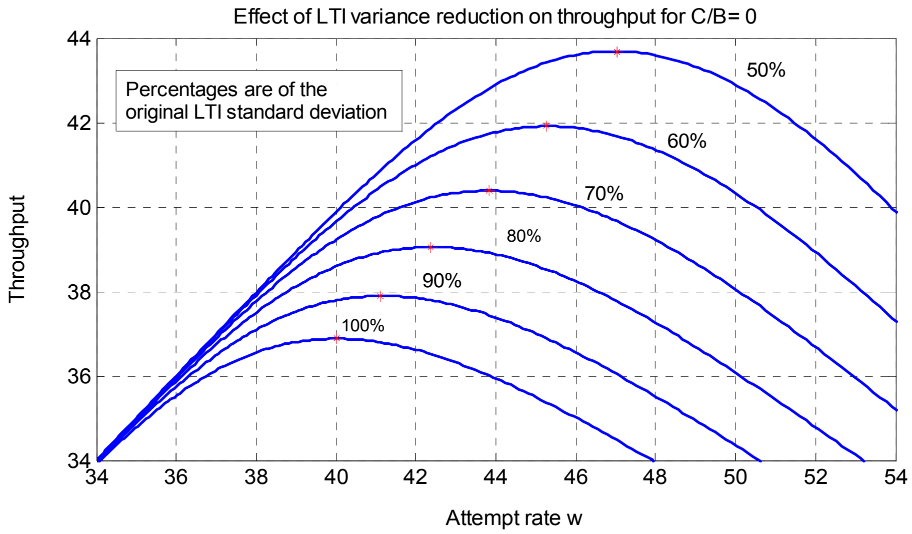

Figure 6 Decreasing standard deviation of LTI_3 increases optimal landing throughput

5. CONCLUSION

We studied the aircraft approach process on a runway, operating independently of other runways in the airport under ILS. We use an augmented capacity definition as the maximum sustainable and safe landing throughput in a given period of time. The goal is to properly estimate the runway capacity and to take the most advantage of this scarce air transportation resource. Increasing utilization is made possible by reducing aircraft separation but consequences of this reduction can be risking the safety and human lives. We studied and formulated this trade off to maximize the utilization with regard to the cost and benefit considerations. The main concepts and results are:

1) A *go-around* procedure can be utilized to avoid wake-vortex incidents and to assure the landing safety.

2) For a given aircraft pair type, we are to adjust the average landing attempt rate (or equally the average aircraft separation) to maximize the overall net economic benefit while the probabilities of SRO and wake-vortex encounter hazard are maintained at zero. Total net economic benefit is the result of a successful landing and the overall cost of possible *go-around* procedures. (We note that executing the *go-around* procedures might be limited for some runways or airports.) We considered maximizing expected value of the net economic benefit (*ENB*) with respect to the average landing attempt rate, and solved the model numerically.

3) We showed that *ENB* is a concave function with a unique maximum, and its optimal solution (economic landing attempt(*ELA*) and corresponding separation) depend on the cost to

benefit ratio *C/B*. Also, the economic landing throughput (*ELT*) is the throughput level at *ELA* and is decreasing in *C/B*.

4) When the cost of *go-around* is ignored or is negligible relative to the landing benefit, *ELT* gives the maximum runway throughput. This value of *ELT* is the runway landing capacity (i.e., the maximum landing throughput that can be safely sustained) for a given aircraft pair. A novel aspect of our approach (contrary to the existing methodologies) is that we do not need to set a target safety level to calculate the capacity.

5) By estimating the capacity for two cases of with and without wake-vortex effect, we quantitatively estimated the economic effect of wake-vortex phenomenon. In our illustration, we roughly estimated this cost to be about 12,000 landings of large aircraft per year for a typical landing runway with 10 hours of daily peak ILS periods.

6) We discussed the effect of reducing variance of aircraft separation on optimal attempt rate, separation, and runway landing capacity. This provides a ground for an objective evaluation of alternative technologies and procedures involved in separation control. For example, for the case of all large aircraft in the fleet, we showed that 30% reduction in *LTI* standard deviation, for example, may result in over 12,750 extra landings per year for a single runway assuming 10 hours of daily peak periods (at Detroit Metropolitan Wayne County airport).

We illustrated the methodologies and concepts for 3 nm aircraft pairs (of the minimum separation standard, FAA 1993) for the probability distributions presented in a 2009 study (obtained using some precise aircraft track data). The extension of the models and methodologies for any given fleet mix can be practically very useful. Real world estimations of the cost (of a *go-around*) to benefit (of a successful landing) ratio for different aircraft types are necessary as a future research. A study on aircraft reaction time to execute a *go-around* procedure is valuable.

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REFERENCES

[1] Airports Console International (2010), Annual Traffic data;http://www.aci.aero/Data-Centre/Annual-Traffic-Data/Movements/2010-final

[2] Boesel J., Bodoh D. (2004), Simulating Airspace Redesign for Arrivals to Detroit Wayne County Airport;*Proceedings of Winter Simulation Conference*; 1318-1325.

[3] Federal Aviation Administration (1993),*FAA Order* 7110.65, *Para* 2,1,19 *and Para* 3,9,6 (September).

[4] Gerz T., Holzäpfel F., Darracq D. (2002), Commercial aircraft wake vortices;*Progress in Aerospace Sciences* 38; 181-208.

[5] Gilbo E.P. Sept. (1993), Airport Capacity: representation, estimation, optimization;*IEEE Transactions on Control Systems Technology* 1(3); 144-154.

[6] Hockaday S.L.M., Chatziioanou A. (1986), An analytical method for aircraft collision risk estimation;*Transportation Research Part B* 20B(5); 415-428.

[7] Hockaday S.L.M., Kanafani A.K. (1974), Developments in airport capacity analysis;*Transportation Research* 8; 171-180

[8] Jeddi B.G., Shortle J.F., Sherry L. (2006), Statistical separation standards for the aircraft–approach process;*Proceedings of 25th Digital Avionics System Conference, Portland, Oregon, USA*; 2A11-13.

[9] Jeddi, B.G., Shortle J.F. (2007), Throughput, Risk and Economic Optimality of Runway Landing Operations;*7th Air Traffic Management R&D Seminar, Barcelona, Spain.*

[10] Jeddi, B.G., Donohue G.L., Shortle J.F. (2009), A statistical analysis of aircraft landing process;*Journal of Industrial and Systems Engineering* 3(3); 152-169.

[11] Jeddi, B.G., (2012), A Note on Runway Capacity Definition, *Journal of Industrial and Systems Engineering*, 5(4); pp

[12] Lee D., Kostiuk, P.F., Hemm, R.V., Wingrove III, E.R., Shapiro, G. (1997), Estimating the effects of the terminal area productivity program;*Logistics Management Institute, McLean, Virginia, NASA Contractor Report* 201682.

[13] Newell G.F. (1979), Airport capacity and delays;*Transportation Science* 13(3); 201-241.

[14] Nolan M.S. (2011), Fundamentals of Air Traffic Control 5th Ed.; *Delmar, Cengage Learning*.

[15] Reich P. (1964), An Analysis of Planned Aircraft Proximity and Its Relation to Collision Risk, with Special Reference to the North Atlantic Region 1965-1971*; Royal Aircraft Establishment Technical Report 64042, Famborough. England*.

[16] Robins R.E., Delisi D.P. (2002), NWRA AVOSS wake-vortex prediction algorithm version 3.1.1;*NASDA/CR*-2002-211746.