

Location of health care facilities in competitive and user choice environment

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Abstract

The location of facilities anywhere in an area in which several competing facilities already exist and serving the demand, has been brought to light in this work. Creation and maintenance of competitive advantage in health care systems requires optimizing the location decision and understanding customers' behaviors. Customers' behavior is considered and explicitly modeled in this work. Each facility attracts customers within a "sphere of influence" defined by a "gravity-like spatial interaction model". Customers have full control over which system they choose to patronize and they do so by applying the attractive elements with each center. The attractive factors that affect the user choice behavior are: the lower travelling time, the quality of the services or the reputation of centers. We also investigate how various parameters will affect the market shares of ours and competitors' facilities in the user choice environment. The hospitals have several low level sections to offer low level services (such as primary services) and several high level sections to offer high level services (such as professional services) and the patients will refer to different sections of the hospitals according to their requirements and their health status. Two metaheuristic algorithms including ant colony optimization and tabu search are developed to solve the model and be applied to some numerical examples. TOPSIS method and statistical t- test are performed to evaluate the results of the proposed algorithms.

Keywords: Discrete network location, Competitive environment, Health care system, Hierarchical structure, Queuing theory, Consumer choice behavior

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1- Introduction

Being single player in the part of the market is distant from the reality in facility location problems, particularly, urgent systems in the health and welfare of people, and in practice, most situations do not fit in such models; so, the need arises to incorporate competition with other players. In competitive models, a firm operates v facilities in a market and a competing firm affects the performance of this market by locating v facilities and its purpose is to maximize the market share. So, the location of health care facilities (hospitals) in the competitive environment has been studied in this work. Each customer feels some attraction towards each of the competing facilities, usually referred as “patronizing behavior”. There are two quite different types of customer behavior models to choose the facilities to patronize:

1. The customer chooses the competing facilities in a deterministic manner so that each customer’s total demand is served by the most attractive facility.
2. The customer chooses the competing facilities in a probabilistic manner so that the whole demands of each customer are satisfied in various hospitals according to a probabilistic function.

The attractiveness function describes how the customers choose the facilities to patronize. We need a function that specifies a tradeoff between attractiveness and travel time. Some of these functions are gravity models according to Reilly’s models. According to these models “the probability that a consumer patronizes a shop (or the proportion of demand capture from a node by one shop) is proportional to its attractiveness and inversely proportional to a power of distance to it” (Reilly 1929). In this work, the HUFF model has been used. The Huff probability function applies the travel time or distance from customer’s demand nodes to the service centers. Furthermore, the size of retail centers is as the attractiveness measure. HUFF was also the first one that represented the Luce axiom of discrete choice in the gravity model. According to this axiom, customers may choose more than one center to go and the probability of going to a specific facility is equal to the utility ratio of that facility to the sum of utilities of all the facilities visited by customers (Huff 1964).

The competitive facility location models can be categorized in three spatial representations:

1. Continuous space, where the potential location of the facilities can be anywhere in the space.
2. A network, where facilities are allowed to locate anywhere along the edges of the network.
3. discrete space, where facilities are allowed to locate at a finite set of possible locations on the network.

Therefore, discrete facility location can be represented as a particular case of network facility location problem. In this work, the space is discrete and is defined by a connected graph. Arcs are the possible paths between nodes, and the demand nodes are allowed to locate at specific points such as the vertices of the graph. The vertices represent potential locations for new facilities and the model does not allow location at the same site for both firms. The idea of hierarchical structure is used in this work which is being introduced in detail in the next sections. By using the queuing theory, we describe how to use this structure. Hierarchical service systems can be classified according to their structures such as referral and non referral systems. In a referral system, the users can go to a higher level server only when they are referred by a low level server.

The structure of the remainder of this paper is organized as follows: In the next section, relevant current studies found in the literature are reviewed. Section 3 contains the problem definition and mathematical model of the problem. Section 4 proposes the solution methods. Section 5 validates the model. Section 6 contains numerical illustration to demonstrate the application of the proposed algorithms and to evaluate its performance. And finally, section 7 contains the conclusions and future study.

2- Literature Review

Hotelling (1929) introduced the competitive facility location problem. His work on two firms competing in a linear market with consumers distributed uniformly along the line and set the foundation of what is today the burgeoning field of competitive location, and his assumption was that, each demand point is attracted to the closest facility. A comprehensive review of the competition on a line can be found in Eiselt and Laporte (1989). Several studies using the same spatial representation as Hotelling but

modifying the economic assumptions were carried out in the economic sector (Hoover, 1936; Lerner and Singer, 1937; Smithies, 1941; He et al. 2016; Ebina, Matsushima and Shimizu, 2015; Buettner and Schwerin, 2016). Reville (1986) defined the competitive facility location problem as a maximum capture problem. The model finds the optimal location on a network considering that each demand point will patronize the closest facility. Reville's formulation has been expanded by Serra et al. in some works. Serra, Marianov and Reville (1992) and Serra and Reville (1993) introduced models with facilities that are hierarchical in nature and where there is competition at each level of the hierarchy, a second extension took the possible reaction from competitors to taking that firm into account (Serra and Reville 1994). Finally, Serra, Ratick and Reville (1996) offer a modification of the maximum capture problem in which they consider uncertainty. The authors consider different future scenarios with respect to the demand and/or the location of competitors. A good review of these models can be found in Serra and Reville (1996), considering that a future competitor will locate one or more competing facilities in the area. Freire, Moreno, and Yushimito (2016) discussed the linear and nonlinear integer reformulation of the maximum capture problem with random utilities and introduced a new branch-and-bound algorithm based on a greedy approach for solving a relaxation of the original problem. Drezner, Drezner and Kalczynski (2015) investigated a leader-follower (stackelberg equilibrium) competitive location model and solved the follower's problem by a branch and bound algorithm and designed an effective Tabu search algorithm for the solution of the leader's problem. Blanquero et al. (2016) studied the p -facility Huff location problem on networks, though the p -facility Huff location problem has been deeply studied in the field of continuous location. Studying the consumer's behavior to choose the facilities indicated that the customer not only cares about patronizing the closest facility but also considers other variables to make the decision. These models are based on Newton's Law of Gravitation, Reilly (1929) and Converse (1949) and then the Luce axiom of discrete choice that was introduced by Huff (1964). Facility location-allocation problems arise in many practical settings from various industries including health care and emergency services to manufacturing networks (Teresinha Arns Steiner et al. 2015; Tate et al. 2014; Tohidi 2015; Sainathuni et al. 2014). The first facility location model for health care systems was introduced by Hakimi (1964). Also, it was followed by many innovative efforts of the other researchers. Guerriero, Miglionico and Olivito (2016) studied the problem of location and reorganizing the Calabrian health care network. Ghaderi and Jabalameli (2013) presented a model that is concerned with the determination of the optimal locations of incapacitated health care systems. Vatsa and Jayaswal (2016) studied a multi-period problem of allocating doctors to primary health centers. Mohammadi, Dehbari and Vahdani (2014) proposed a bi-objective reliable location model for health care management and under limited capacity and a patient queue system with two patient groups is created. Cooper (1963) categorized the location-allocation (LA) problems into two different classes: One of them is called uncapacitated LA problem (Damgacioglu et al. 2015; Kratica, Dugošija and Savić, 2014). And, the other category of LA problems is also considered by many researchers, including Alizadeh et al. (2015); Zhou and Liu (2007); and Marinakis (2015). Church and Eaton (1987) and Gerrard and Church (1994) provided the reviews of early hierarchical models, and a comprehensive review of newly-developed hierarchical location models can be found in Sahin and Sural (2007) and Zanjirani Farahani et al. (2014). The first fuzzy model for location-allocation in the hierarchical systems was developed by Shavandi and Mahlooji et al. (2006). They introduced a fuzzy hierarchical queuing location-allocation model for maximal covering location problem (MCLP) in coherent systems. In another work, Shavandi and Mahlooji (2007) developed fuzzy hierarchical queuing models for MCLP, in both nested and referral systems. Furthermore, a successively inclusive hierarchical model for the location of health centers in term of patients' transference from a lower level to a higher level of health centers has been studied by Alinaghian, Hejazi and Bajoul (2014).

Zarrinpoor, Shavandi and Bagherinejad (2012) developed a covering location-allocation model for congested systems in the competitive environment. They used the HUFF model to specify a tradeoff between attractiveness and travel time. In their model, the quality of the services or reputation of the centers and less travelling time is considered as the attractiveness measure to influence the user choice behavior. We would explain the quality of the services or reputation of the centers in our work. One important topic that is not mentioned in health care's literature review is the particular consideration for

the attractiveness measure to influence the user choice behavior. Our goal is to create a model that is more practical in the real world. Therefore, the concept of “user choice environment” has been used in this research. Customers have full control over the facility they choose to patronize and they do so by associating the attractive factors with each facility. In this model, the quality of the services or reputation of the centers and the lower travelling time are considered as the attractiveness measure to influence the user choice behavior. The quality of the services is determined by the convenience amenities of the centers. To explain this measure, we consider the following issues: the idea of hierarchical structure has been used in many efforts. According to the literature of hierarchical structure, there exist many hierarchical structures at service networks, such as health care systems. In these systems, general centers provide low level services, such as primary health care and specialized hospitals provide high level services. In this research, we use this structure inside any systems, because there are some low and high levels and sections at different levels of hospitals that provide different types of services and have different resources and personnel. So, one purpose of the model is to determine the optimal capacity of resources and personnel at different sections of the two levels of the hospitals according to the requirements of patients. Therefore, we represent this measure according to the three categories of characteristics; first: number of sections of the low and high levels of the hospitals and the capacity of personnel and resources of these sections will affect the quality of the services and reputation of the service centers. Second: staff experience in various sections of the hospitals will affect the quality of the services. And finally, patient requirements to the various sections of the hospitals are brought into light such as a measure that shows whether the opening of each section inside the hospitals is reasonable or not.

3- Problem Definition

In order to model the problem, the paper considers the following indices, parameters and variables and assumptions:

3-1- Sets and Indices

- i Index for customer nodes ($i = 1, \dots, N$, where N is the number of customer nodes)
- B' Set of locations occupied by the existing competing hospitals (Number of these locations is equal to v')
- B Set of potential locations for the new facilities $B = N - B'$
- k_j Index for low level sections inside the hospital j , where K' is the maximum number of low level sections inside the hospitals
- z_j Index for high level sections inside the hospital j , where Z' is the maximum number of high level sections inside the hospitals
- l Index for types of patients in terms of physical health status and their requirements to the various sections of the hospitals $l = 1, \dots, L$
- r_L Index for resource types inside the low level sections of the hospitals $r_L = 1, \dots, R_L$
- r_H Index for resource types inside the high level sections of the hospitals $r_H = 1, \dots, R_H$

c_L Index for personnel types inside the low level sections of the hospitals $c_L = 1, \dots, C_L$

c_H Index for personnel types inside the high level sections of the hospitals $c_H = 1, \dots, C_H$

3-2- Parameters

v Number of new facilities to be located

Q Upper bound for resource capacity in low levels

Q' Upper bound for resource capacity in high levels

O Upper bound for personnel capacity in low levels

O' Upper bound for personnel capacity in high levels

H_j The attractiveness of existing and new facilities

β, α, δ Are the numbers between (0, 1) represent the importance of different factors

$a_{i,l}$ The population of type l in demand node i

f_j Staff experience of the hospitals

$t_{i,l,j}$ The travelling time of patient type l from demand node i to the facility j

$y'_{k,j}$ Location parameter shows that a low level section k at the hospital j is open

$y''_{z,j}$ Location parameter shows that a high level section z at the hospital j is open

cap_j System capacity of the existing facility j in the competitive environment ($j \in B'$)

$w_{l,k,j}$ Indicates whether or not the patient type l needs a low level section k of the hospital j , (binary parameter)

$w'_{l,z,j}$ Indicates whether or not the patient type l from low level sections needs a high level section z of the hospital j , (binary parameter), (for patients who have been allocated to the low level sections)

$w''_{l,z,j}$ Indicates whether or not arrived patients of type l need a high level section z of the hospital j , (binary parameter), (for patients who are allocated to the high level sections directly, without going to the low level sections)

$p'_{l,k,j}$ Refers to the fraction of the arrived patients type l , that need the low level section k from hospital j , (a number between (0, 1))

$p''_{l,z,j}$ Refers to the fraction of the arrived patients type l , to the low level section k , also need the high level section z , (a number between (0,1))

$p'''_{l,z,j}$ Refers to the fraction of the arrived patients type l to the hospital j , that need the high level section z , (a number between (0, 1)), (for patients that directly visit the high level sections without referring the low level sections)

$\lambda_{l,j}$ Patient type l arrival rate at the open facility j

$\lambda'_{l,k,j}$ Patient type l arrival rate at the low level section k of the hospital j

$\lambda''_{l,z,j}$ Patient type l arrival rate at the high level section z of the hospital j

3-3- Variables

$p_{i,l,j}$ The probability that customer type l from demand node i will refer to the facility j , based on HUFF model.

y_j Location variable that takes value 1, if facility is located at node j , and zero otherwise

$x_{l,k_j,z_j,j}$ Allocation variable that takes value 1, if patients type l at the hospital j are allocated to the low level section k and then are referred to the high level section z , otherwise it is zero.

$Rcap_{r_L,k_j,j}$ Resource capacity r_L at the low level section k of the hospital j

$Rcap'_{r_H,z_j,j}$ Resource capacity r_H at the high level section z of the hospital j

$Pcap_{c_L,k_j,j}$ Personnel capacity c_L at the low level section k of the hospital j

$Pcap'_{c_H,z_j,j}$ Personnel capacity c_H at the high level section z of the hospital j

3-4- Assumptions

1. The system under study is represented as a network.
2. A model for locating health care facilities (hospitals) in the competitive environment is proposed.
3. In our competitive model a firm operates v' facilities in a market and a competing firm affects the performance of this market by locating v facilities. Furthermore, its purpose is to maximize its market share.

4. The idea of hierarchical structure (referral systems) is used inside the hospitals.
5. Each facility is acting as an M/M/C/K queuing system, there are C servers in each hospital and K is the total capacity of the hospitals.

We consider a discrete location space and on each node of the network, there is a given number of customers that generates a demand for hospitals, and either candidate locations for hospitals. It is assumed that each hospital consists of both low and high levels. Low level sections of the hospitals offer low level services and high level sections of the hospitals offer high level services. Each of these two levels has different sections. The low level sections of the hospitals consist of sections such as hospital emergency departments, general practitioners departments, and high level sections of the hospitals consist of sections such as ICU, CCU, and specialist physicians and surgeons departments. "The attraction function" describes how a customer's attraction towards a facility is obtained and then the probability that a customer patronizes a facility is obtained. In this paper we use the HUFF model (Huff 1964). That is proportional to a power of facility's attractiveness and inversely proportional to a power of travel time to it. Patients based on their physical health status and their requirements can be allocated to each of these centers and choose the best of them according to their criteria. We assume that each low level section of each hospital has C_L personnel and its capacity is finite and equal to

$$R_1 = \sum_{r_L=1}^{R_L} \sum_{k_j=1}^{K_j} R_{cap_{r_L, k_j, j}} + \sum_{c_L=1}^{C_L} \sum_{k_j=1}^{K_j} P_{cap_{c_L, k_j, j}} \text{ and each high level section of the hospitals has } C_H \text{ personnel and}$$

$$\text{its capacity is finite and equal to } R_2 = \sum_{r_H=1}^{R_H} \sum_{z_j=1}^{Z_j} R_{cap'_{r_H, z_j, j}} + \sum_{c_H=1}^{C_H} \sum_{z_j=1}^{Z_j} P_{cap'_{c_H, z_j, j}} \text{ then each hospital behaves as an}$$

M/M/C/K queuing system, $C=C_L+C_H$ is the total number of servers in each hospital and $K=R_1+R_2$ is the total capacity of the hospitals. The service distribution in each section is assumed to be exponential, and arrivals to each section follows a Poisson process.

3-5- Mathematical model

$$\text{Max } Z = \sum_{j \in B} \sum_{i \in N} \sum_{l \in L} a_{i,l} p_{i,l,j} \quad (1)$$

$$\sum_{j \in B} y_j = v \quad (1)$$

$$H_j^B = \sum_{r=1}^{R_1} \sum_{k(j)=1}^{K(j)} R_{cap_{r, k(j), j}} + \sum_{r=2}^{R_2} \sum_{z(j)=1}^{Z(j)} R_{cap'_{r, z(j), j}} + \sum_{c=1}^{C_1} \sum_{k(j)=1}^{K(j)} P_{cap_{c, k(j), j}} + \sum_{c=2}^{C_2} \sum_{z(j)=1}^{Z(j)} P_{cap'_{c, z(j), j}} \quad (2a)$$

$$+ f_j^\beta + \sum_{l=1}^L \sum_{k(j)=1}^{K(j)} D_l w_{l, k(j), j} + \sum_{l=1}^L \sum_{z(j)=1}^{Z(j)} D_l w'_{l, z(j), j} + \sum_{l=1}^L \sum_{z(j)=1}^{Z(j)} D_l w''_{l, z(j), j} \quad j \in B$$

$$H_j^{B'} = cap_j + f_j^\beta + \sum_{l=1}^L \sum_{k(j)=1}^{K(j)} D_l w_{l, k(j), j} + \sum_{l=1}^L \sum_{z(j)=1}^{Z(j)} D_l w'_{l, z(j), j} + \sum_{l=1}^L \sum_{z(j)=1}^{Z(j)} D_l w''_{l, z(j), j} \quad j \in B' \quad (2b)$$

$$q_{i,l,j} = H_j^\alpha / t_{i,l,j}^\delta \quad j \in B \cup B' \quad (3)$$

$$p_{i,l,j} = q_{i,l,j} y_j / \sum_{m \in B} q_{i,l,m} y_m + \sum_{m \in B'} q_{i,l,m} \quad j \in B \cup B' \quad (4)$$

$$\lambda_{l,j} = \sum_{i=1}^N a_{i,l} p_{i,l,j} \quad j \in B \cup B' \quad (5)$$

$$\lambda'_{l, k(j), j} = \lambda_{l,j} w_{l, k(j), j} p'_{l, k(j), j} \quad j \in B \cup B' \quad (6)$$

$$\lambda''_{l,z(j),j} = \sum_{k(j)=1}^{K(j)} [\lambda'_{l,k(j),j} x_{l,k(j),z(j),j} w'_{l,z(j),j} p''_{l,z(j),j}] + [(\lambda_{l,j} - \sum_{k(j)=1}^{K(j)} \lambda'_{l,k(j),j}) \cdot (w''_{l,z(j),j} p''_{l,z(j),j})] \quad j \in B \cup B' \quad (7)$$

$$Rcap_{r1,k(j),j} \leq Q y'_{k(j),j} \quad (8)$$

$$Rcap'_{r2,z(j),j} \leq Q' y''_{z(j),j} \quad (9)$$

$$Pcap_{c1,k(j),j} \leq O y'_{k(j),j} \quad (10)$$

$$Pcap'_{c2,z(j),j} \leq O' y''_{z(j),j} \quad (11)$$

$$\lambda'_{l,k(j),j} \leq Rcap_{r1,k(j),j} \quad j \in B \quad (12)$$

$$\lambda''_{l,z(j),j} \leq Rcap'_{r2,z(j),j} \quad j \in B \quad (13)$$

$$\lambda'_{l,k(j),j} \leq Pcap_{c1,k(j),j} \quad j \in B \quad (14)$$

$$\lambda''_{l,z(j),j} \leq Pcap'_{c2,z(j),j} \quad j \in B \quad (15)$$

$$\sum_{l=1}^L \lambda_{l,j} \leq cap_j \quad j \in B' \quad (16)$$

$$y_j \in \{0,1\} \quad j \in B \quad (17)$$

$$x_{l,k(j),z(j),j} \in \{0,1\} \quad \forall l \in L, k(j) \subseteq K', z(j) \subseteq Z', j \in B \cup B' \quad (18)$$

$$P_{i,l,j} \geq 0, \quad \forall j \in B \cup B', l \in L \quad (19)$$

$$Pcap_{c1,k(j),j} \geq 0, Pcap'_{c2,z(j),j} \geq 0, Rcap_{r1,k(j),j} \geq 0, Rcap'_{r2,z(j),j} \geq 0 \quad (20)$$

$$\forall c1 \in C1, c2 \in C2, r1 \in R1, r2 \in R2, k(j) \subseteq K', z(j) \subseteq Z', j \in B$$

The objective function of the model maximizes the total demand captured by the new hospitals and expression 1 shows the number of new facilities that can be opened.

In order to discuss equation 2, the following assumptions have been considered:

Our goal is to create a model that is more practical in the real world. So the concept of “user choice environment” has been used in this research. Customers have full control over the facility they choose to patronize and they do so by associating the attraction factors with each facility. In this model, the quality of the services or reputation of the centers is considered as the attractiveness measure to influence the user choice behavior. The quality of the services or reputation of the centers is considered as the attractiveness measure and the quality of the services is determined by the convenience amenities of the centers. To explain the attractiveness measure, we consider the following issues:

1. It is assumed that each hospital consists of both low and high levels and patients based on their physical health status and their requirements can be allocated to each of these sectors. The number of these sections is different inside the various hospitals. In fact, in all hospitals, K' is the maximum number of low level sections and Z' is the maximum number of high level sections that can be opened. Center managers based on their own criteria decide which sections can be opened inside the hospitals, and what equipment and human resources can be assigned to them. Number of hospitals' sections, and the capacity of personnel and resources of these sections will affect the quality of the services and reputation of the service centers.

2. Staff experience in various sections of the hospitals.

3. Patient requirements to the various sections of the hospitals are considered such as a measure,

$D_l = \sum_{i=1}^N a_{i,l}$, that is the whole demand of patients type l from all of the demand zones, when multiplied by

the $w_{l,k,j}$, $w'_{l,z,j}$, $w''_{l,z,j}$, that shows the requirements of patients to the various sections of the hospitals, so a measure is obtained that shows whether the opening of each section inside the hospitals is reasonable or not, and the attractiveness measure is expressed by the following expressions:

$$H_j^B = \sum_{r_l=1}^{R_l} \sum_{k_j=1}^{K_j} R_{cap_{r_l,k_j,j}} + \sum_{r_H=1}^{R_H} \sum_{z_j=1}^{Z_j} R_{cap'_{r_H,z_j,j}} + \sum_{c_L=1}^{C_L} \sum_{k_j=1}^{K_j} P_{cap_{c_L,k_j,j}} + \sum_{c_H=1}^{C_H} \sum_{z_j=1}^{Z_j} P_{cap'_{c_H,z_j,j}} + f_j^\beta + \sum_{l=1}^L \sum_{k_j=1}^{K_j} D_l w_{l,k_j,j} + \sum_{l=1}^L \sum_{z_j=1}^{Z_j} D_l w'_{l,z_j,j} + \sum_{l=1}^L \sum_{z_j=1}^{Z_j} D_l w''_{l,z_j,j} \quad j \in B \quad (21)$$

$$H_j^{B'} = cap_j + f_j^\beta + \sum_{l=1}^L \sum_{k_j=1}^{K_j} D_l w_{l,k_j,j} + \sum_{l=1}^L \sum_{z_j=1}^{Z_j} D_l w'_{l,z_j,j} + \sum_{l=1}^L \sum_{z_j=1}^{Z_j} D_l w''_{l,z_j,j} \quad j \in B' \quad (22)$$

Capacity of the existing competing hospitals is already predetermined and we are going to determine the capacity of new facilities that can be opened.

Attractiveness function; the HUFF model (Huff 1964), (equation 3) is defined by the expression:

$$q_{i,l,j} = H_j^\alpha / t_{i,l,j}^\delta \quad j \in B \cup B' \quad (23)$$

The probability that customer type l from demand node i choosing to go to the facility j (equation 4) is as below:

$$p_{i,l,j} = q_{i,l,j} y_j / \sum_{m \in B} q_{i,l,m} y_m + \sum_{m \in B'} q_{i,l,m} \quad j \in B \cup B' \quad (24)$$

It is obvious that the demand originating at the demand nodes can be served by more than one facility according to the probabilistic function. Figure 1 shows the network of our model.

Patients decide to choose the best hospital according to the lower travel time or the quality of the services or reputation of the centers (Number of the low and high level sections of the hospitals and the capacity of personnel and resources of these sections, staff experience in various sections and patient requirements to the various sections of the hospitals will affect the quality of the services and reputation of the service centers.

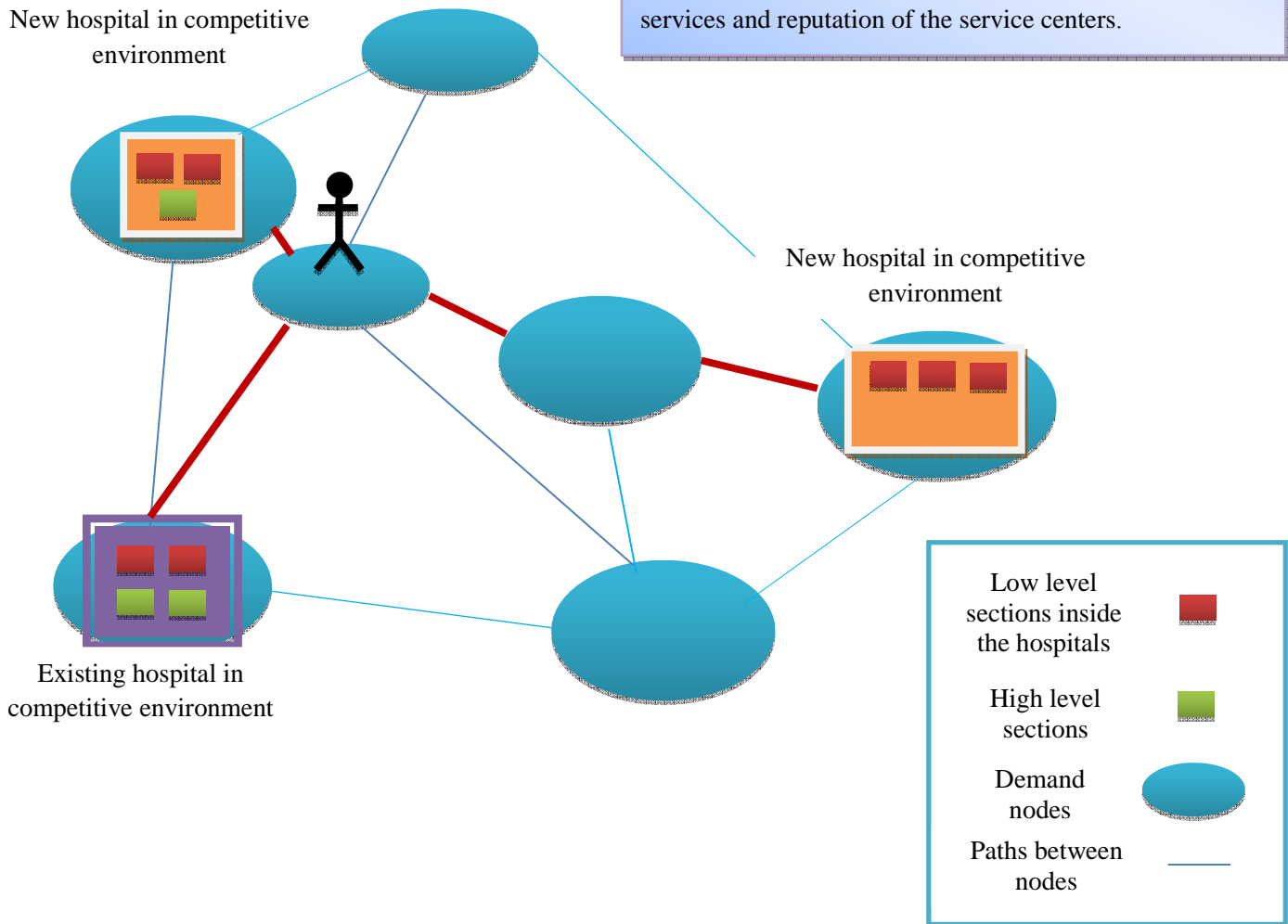


Fig.1.The graphical representation of the network of our problem ($N=7$, $K'=3$, $Z'=3$, $v=2$, $v'=1$)

In order to discuss equations 5 the following explanations have been considered. We assume that the request for service at each demand node appears according to a Poisson process with average demand rate $a_{i,j}$, since $\lambda_{l,j}$ is equal to the sum of the average demand rates of the processes at the demand nodes that is served by the facility, then $\lambda_{l,j}$ is also Poisson process. $\lambda'_{1,k_j,j}$, $\lambda''_{1,z_j,j}$ are the linear combinations of some Poisson processes as well. So they are also Poisson processes.

$$\lambda_{l,j} = \sum_{i=1}^N a_{i,l} P_{i,l,j} \quad (25)$$

To calculate $\lambda'_{1,k_j,j}$ (equations 6); the rate of entering patients type l into the low level section k , at the hospital j , patients who have been allocated to the hospital j based on their health status and their requirements are allocated to the low level sections if the condition $\sum_{k_j=1}^{K_j} p'_{1,k_j,j} = 1$ is observed, the arrival rate to the low level parts is calculated below:

$$\lambda'_{1,k_j,j} = \lambda_{l,j} w_{1,k_j,j} p'_{1,k_j,j} \quad (26)$$

To calculate $\lambda''_{1,z_j,j}$ (equations 7); the amount of entering patients type l to the high level segments of the hospital, we assume that the patients who have been assigned to the low level sections, with probability $p''_{1,z_j,j}$ will need the high level sections, if condition $\sum_{z_j=1}^{Z_j} p'_{1,z_j,j} = 1$ is observed; therefore, these patients are referred from low levels to high levels. Allocation variable $x_{l,k_j,z_j,j}$ expresses referring patients from low levels to high levels. (the idea of hierarchical structure has been used inside the hospitals).

$$\lambda''_{1,z_j,j} = \sum_{k_j=1}^{K_j} [\lambda'_{1,k_j,j} x_{1,k_j,z_j,j} w'_{1,z_j,j} p''_{1,z_j,j}] \quad (27)$$

However, patients with serious situations should be directly allocated to the high level sections, without being allocated to the low level sections, if the condition $\sum_{z_j=1}^{Z_j} p''_{1,z_j,j} = 1$ is observed. In this case, the patients who have been assigned to the low levels will be deducted from the patients entered the hospital, the remaining patients, are the patients who will need only the high level sections.

$$[(\lambda_{l,j} - \sum_{k_j=1}^{K_j} \lambda'_{1,k_j,j}) \cdot (w''_{1,z_j,j} p''_{1,z_j,j})] \quad (28)$$

Then $\lambda''_{1,z_j,j}$ (equations 7) is as follows:

$$\lambda''_{1,z_j,j} = \sum_{k_j=1}^{K_j} [\lambda'_{1,k_j,j} x_{1,k_j,z_j,j} w'_{1,z_j,j} p''_{1,z_j,j}] + [(\lambda_{l,j} - \sum_{k_j=1}^{K_j} \lambda'_{1,k_j,j}) \cdot (w''_{1,z_j,j} p''_{1,z_j,j})] \quad (29)$$

Expressions 8-11 mean that resource and personnel capacity inside low and high level sections of the new facilities are bounded and low levels and high levels inside each hospital are different from others and are already predetermined, because these levels are predetermined and when the hospitals open, the

various sections of the hospitals will open; so $y'_{k_j,j}$, $y''_{z_j,j}$ are location parameters. Expressions 12-15 assure that the arrival rate for each low and high level section inside the new facilities must be less than their resource and personnel capacity (because the shortage is not allowable) and expression 16 assures that all types of arrival patients to each existing facility must be less than its total capacity (total capacity of existing centers consists of both resource and personnel capacity in various sections of the hospitals). Finally, expressions 17, 18 show the binary variables and expressions 19-20 show the nonnegative variables.

4- Solution Methods

4-1- Ant Colony Optimization

Ant colony optimization (ACO) is population based meta-heuristic and can be used to find approximate solutions to difficult optimization problems. This algorithm was initially proposed by Dorigo (1992) in his PhD thesis. In ACO, a set of software agents called artificial ants search for good solutions and take inspiration from the behavior of real ant colonies. They use chemical cues called pheromones to provide a sophisticated communication system. An isolated ant moves essentially at random but an ant encountering a previously laid pheromone will detect it and decide to follow it with high probability and thereby reinforce it with a further quantity of the pheromone. When they arrive at a decision point, they make a probabilistic choice, biased by the intensity of pheromone they smell. In ACO algorithm, the optimization problem is formulated as a graph $G=(S, C)$, where, S is the set of components of the problem (instantiated decision variables), and C is the set of possible connections or transitions among the elements of S . The solution is expressed in terms of feasible paths on the graph G , with respect to a set of given constraints. The population of agents (ants) collectively solves the problem under consideration using the graph representation. A pheromone trail value $\tau_{(i,j)}$ is associated with each component S , pheromone values and the attractiveness $\eta_{(i,j)}$ of the move, as computed by some heuristic indicating the a priori desirability of that move. This attractiveness, which remains constant during the run of the program, is determined by $\eta_{(i,j)}=1/l_{(i,j)}$, where $l_{(i,j)}$ is the cost of move from vertex i to the vertex j and allows the probability distribution of different components of the solution to be modeled to compute the transition probabilities. In this work $l_{(i,j)}$ is the travelling time from demand nodes to the service centres. Starting from the initial vertex i , an explorer ant m chooses probabilistically vertex j to observe next, using the following transition rule:

$$P_m(i,j) = \begin{cases} \frac{[\tau_{(i,j)}]^\alpha \cdot [\eta_{(i,j)}]^\beta}{\sum_{k \in S_m(i)} [\tau_{(i,k)}]^\alpha \cdot [\eta_{(i,k)}]^\beta} & \text{if } j \in S_m(i) \\ 0 & \text{otherwise} \end{cases} \quad (30)$$

α and β are two parameters that control the relative weight of pheromone trail and heuristic value and $S_m(i)$ is a set of vertices that remain to be observed by ant m positioned at vertex i . Equation 31 shows that the quality of the path (i, j) is proportional to its shortness and to the highest amount of pheromone deposited on it. The ants move from vertex to vertex along the edges of the construction graph exploiting information provided by the pheromone values and incrementally building a solution.

Additionally, the ants deposit a certain amount of pheromone on the components, that is, either on the vertices or on the edges that they traverse. The amount $\Delta\tau_{m(i,j)}$ of pheromone deposited may depend on the quality of the solution found and is the mechanism by which ants communicate to share information about good paths.

$$\Delta\tau_{m(i,j)} = \begin{cases} Q/l_{m(i,j)}, & \text{if ant } m \text{ uses curve } ij \text{ in its tour} \\ 0 & \text{otherwise} \end{cases} \quad (31)$$

Q is constant.

Ants change the pheromone level on the paths between vertices using the following updating rule:

$$\tau_{(i,j)} \leftarrow \rho \cdot \tau_{(i,j)} + \sum_m \Delta\tau_{m(i,j)} \quad (32)$$

ρ is the trail evaporation parameter.

4-1-1- Stopping Criteria

The maximum number of iterations must be met to stop the algorithm (Maxit).

4-2- Tabu Search

Tabu search was introduced by Glover (1989, 1990). Tabu search is a meta-heuristic that guides a local heuristic search procedure to explore the solution space beyond local optimality. Figure 2 shows Tabu search process. The steps of Tabu search are as below:

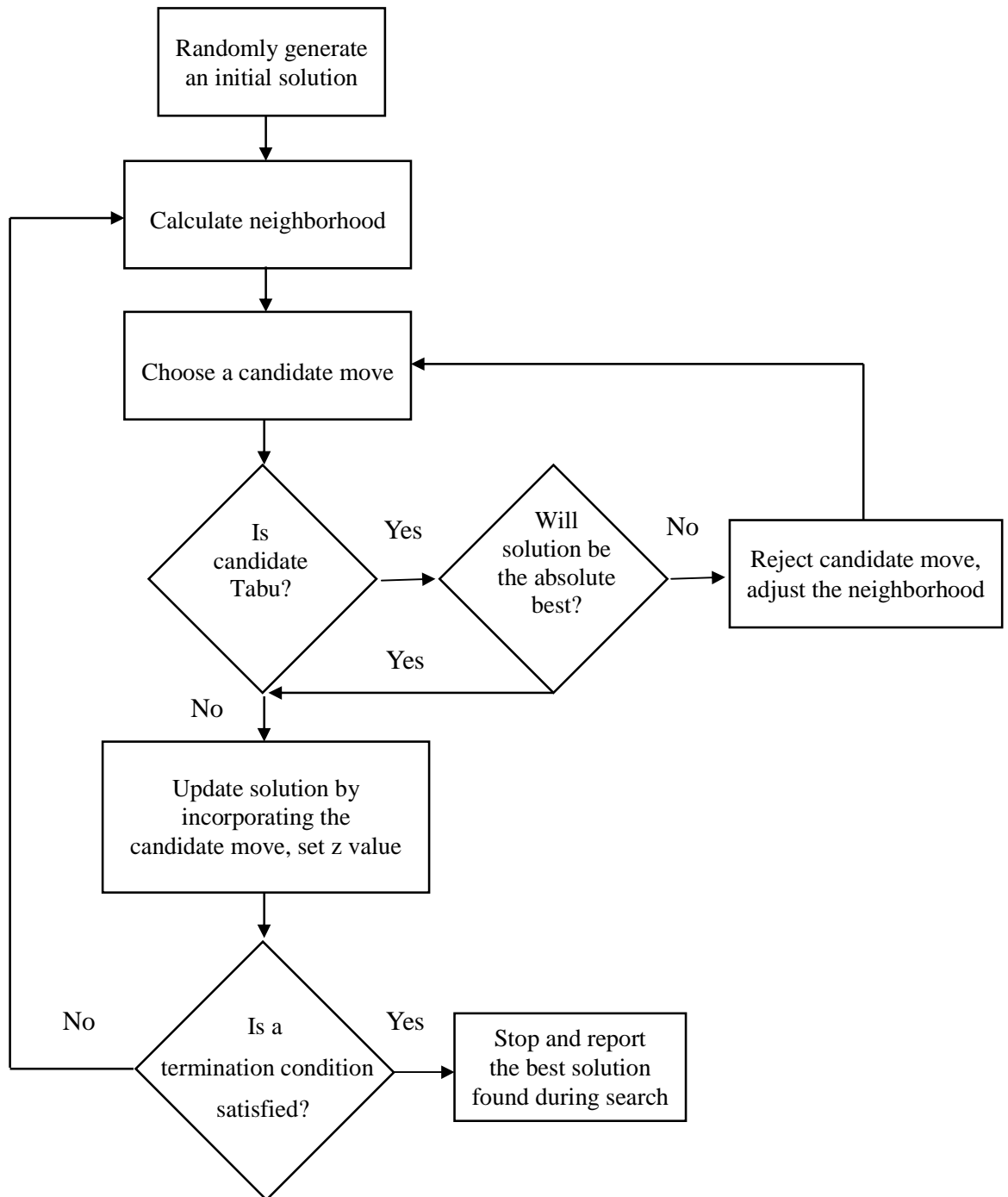


Fig.2.Tabu search process

4-2-1- Initial Solution

To create the initial solution, it must be considered that the existing facilities are not regarded in the new solution, because the model did not allow the location at the same site for both firms. The chromosome consists of a string of 0 and 1s with the length of B, and the ones of a chromosome represent

the location of new hospitals and their sum is equal to v , (the first constraint has been considered). With respect to the expressions 8-11, and 18-22, a corrective process has been done to make the initial solution feasible.

4-2-2- Neighborhood structure

To create a neighborhood of the current solution, swap mutation is used.

4-2-3- Aspiration Conditions

The aspiration function is simply a matter of whether or not the next solution is better than any move we have seen so far. If it is better, and it is Tabu, we still accept it. Then, the best solution criterion is as follow: If a tabu solution encountered at the current iteration, it is better than the best solution found so far, then its tabu status is overridden.

4-2-4- Termination Condition

The maximum number of iterations must be met to stop the algorithm.

4-3- Parameter Adjustment

Since the results of all meta-heuristic techniques are sensitive to their parameter setting, it is required to do extensive simulations to find suitable values for various parameters.

4-3-1- Factors affecting the performance of the algorithms

The parameters of ACO are α (weight of pheromone trail), β (weight of heuristic value), ρ (trail evaporation weight), Q (a constant), τ_0 (Initial Pheromone), nAnt (Number of ants), Maxit (Maximum Number of Iterations) and the parameters of TS are maxit (maximum number of iterations), TL0 (the length of Tabu list), and Nmove (the number of neighbors). The number of neighbors in TS is not constant during the algorithm running, thereby, it is variable and a function of length of the chromosome, therefore if the length of B (LB) <20 ; Nmove=3LB, if LB >20 and LB <100 ; Nmove=8LB, otherwise Nmove=LB. Some of the combination values of these parameters are given in tables 1 and 2.

Table 1. Description of the parameter levels for the experiment of TS

Parameters	Low	Medium	High
maxit	70	100	200
TL0	20	40	50

Table 2. Description of the parameter levels for the experiment of ACO

Parameter	Low	Medium	High
MaxIT	100	150	200
nAnt	30	50	70
Q	0.1	10	100
α	1	1	1
β	0	0.1	0.2
ρ	0.02	0.05	0.1
τ_0	1	1	1

The Taguchi method involves reducing the variation in a process through robust design of experiments. The Taguchi method was developed by Taguchi (1986). The experimental design proposed by Taguchi involves using orthogonal arrays to organize the parameters affecting the process and the levels at which they should vary. The Taguchi L_{27} orthogonal array is used for five factors of ACO, at three levels with a total of twenty seven observations on the response. Three examples of different sizes were generated and used four times for different twenty seven combinations of the parameter levels, where the stopping criterion is met. Taguchi L_9 orthogonal array is used for two factors of TS, at three levels with a total of nine observations on the response. Three examples of different sizes were generated and used four times for different nine combinations of the factor levels, where the stopping criterion is met. Figures 3 and 4 show the mean S/N ratio plots for each parameter level in problems, respectively. Tables 3 and 4 contain all of the best parameter level combinations for all of the problems.

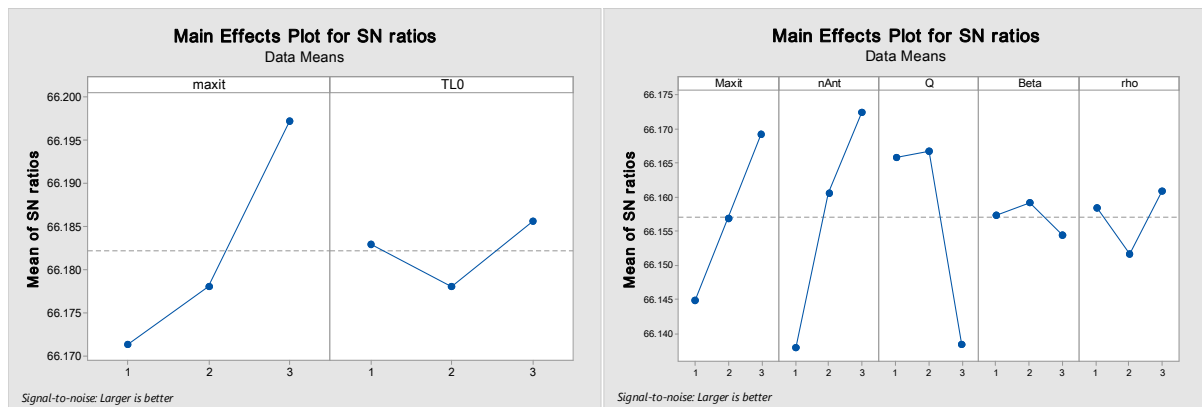


Fig.3. The mean S/N ratio plot for the parameters of TS **Fig.4.** The mean S/N ratio plot for the parameters of ACO

Table 3.TS parameter results

Parameters	maxit	TLO
Parameter Values	50	200

Table 4.ACO parameter results

Parameters	MaxITnAnt	Q	ρ	β
Parameter Values	200	70	10	0.1

5- Validation of the Model

For validation of the model first, 16 test problems in small and average sizes were generated because the GAMS Software is incapable of solving large size problems and needs long time runs for large size problems. Then, these problems were solved in optimization software. The meta-heuristic algorithms are coded and compiled in mathematical software. Every example was run three times in each meta-heuristic algorithm and the average of them has been compared with the results of the optimization software in table 6. In this section, the performance of parameter tuned TS and ACO and optimization software are compared using a statistical t- test. Randomly generated parameters for solving the model are shown in table 5.

Table 5. Randomly generated parameters for solving the model

Factors	Levels
Number of demand nodes	[5 50]
Number of existing facilities	[1 6]
Number of new facilities	[1 6]
Types of patients	[1 2]
Type of resources in low level sections of the hospitals	[1 2]
Type of resources in high level sections of the hospitals	[1 2]
Available personnel in low level sections of the hospitals	[1 2]
Available personnel in high level sections of the hospitals	[1 2]
The population in demand nodes	[1 100]
The traveling time	[1 100]
Personnel experience in service centers	[1 100]
Number of low level sections inside the hospitals	[1 5]
Number of High level sections inside the hospitals	[1 5]
Upper limit to the capacity of resources inside the low level sections in every new hospital	400
Upper limit to the resource capacity of the high level segments inside each new hospital	400
Upper bound for the personnel capacity in low level sections inside new hospitals	400
Upper bound for the personnel capacity in high level sections inside new hospitals	400
Upper bound for the hospital capacity in the existing hospitals	1100

Table6. Comparison of the results of TS and ACO and optimization software (λ is our objective function value, λ' is the competitor's objective function value, A is our market share and B is the competitor's market share and T is the CPU time)

Optimization software							TS					ACO					
N	v	λ	λ'	A	B	T	λ	A	B	T	λ	A	B	T			
5	1	1	1.6	9.4	0.53	0.47	34	1.6	9.4	0.53	0.47	7	1.5	9.5	0.52	0.48	38
	2	1	13.8	6.2	0.69	0.31	40	13.8	6.2	0.69	0.31	7	13.8	6.2	0.69	0.31	47
	1	2	6.98	13.01	0.35	0.65	7	6.98	13.01	0.35	0.65	7	6.95	13.05	0.35	0.65	45
	2	2	1.5	9.5	0.52	0.48	14	1.5	9.5	0.52	0.48	7	1.4	9.6	0.52	0.48	57
10	2	2	2.3	19.7	0.51	0.49	448	2.3	19.7	0.51	0.49	15	2.4	19.6	0.51	0.49	66
	2	3	16.07	23.9	0.4	0.6	388	16.07	23.93	0.4	0.6	14	16.06	23.94	0.40	0.6	14
	3	2	24.5	15.5	0.61	0.39	501	24.57	15.43	0.61	0.39	17	24.59	15.41	0.62	0.38	81
	3	3	2.54	19.46	0.51	0.49	580	2.59	19.41	0.52	0.48	18	2.61	19.39	0.52	0.48	81
12	2	3	14.94	21.06	0.42	0.58	939	14.71	21.29	0.41	0.59	20	14.65	21.35	0.407	0.593	76
	3	2	14.47	9.53	0.6	0.4	766	14.54	9.46	0.61	0.39	24	14.55	9.45	0.61	0.39	76
15	3	3	773.6	430.55	0.65	0.35	1611	773.5	430.51	0.64	0.36	50	773.58	430.42	0.64	0.36	109
	4	3	1021.3	603.7	0.63	0.37	1883	1021.3	603.7	0.63	0.37	59	1021.3	603.7	0.63	0.37	125
18	3	4	910.5	1001.5	0.46	0.54	2297	911.35	1000.67	0.48	0.52	72	911.36	1000.63	0.48	0.52	131
	4	4	1341.1	502.8	0.7	0.3	3248	1343.2	500.81	0.73	0.27	85	1343.41	500.58	0.73	0.27	152
20	4	5	980.56	825.58	0.52	0.48	3610	986.35	819.65	0.55	0.45	88	986.45	819.56	0.55	0.45	169
	5	5	1175.5	785.1	0.65	0.35	4026	1173.1	787.88	0.6	0.4	99	1171.9	789.1	0.6	0.4	186

5-1- Statistical Comparison

Sixteen test problems were generated and solved with TS, ACO algorithms and optimization software. In this regard, paired t- test is performed at 95% significant level for the comparison of each algorithm's results with the results of optimization software. While for the mean of fitness value comparison, the hypotheses are as follows:

$$H_0: \mu_{TS} = \mu_{GAMS}$$

H_1 : Otherwise

(33)

And

$$H_0: \mu_{ACO} = \mu_{GAMS}$$

(34)

H_1 : Otherwise

Table 7. Table of paired sample t- test, for the equality of means for Objective Function Values (OFVs) of TS and ACO and optimization software

	Paired Differences					t	Df	Sig. (2-tailed)
	Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
				Lower	Upper			
Optimization software-TS	-0.38835	1.67499	0.41875	-1.28089	0.50419	-0.927	15	0.368
Optimization software-ACO	-0.32904	1.86411	0.46603	-1.32235	0.66428	-0.706	15	0.491

The results of tests for the equality of means are presented in table 7. As the results show, significance for the equality of means for both algorithms is greater than 0.05, therefore, the assumption of the equality of means will be accepted and it can be concluded that, the proposed algorithms in the significance level of 0.95 are similar to the results of optimization software.

6- Numerical Experiment

In this section, several test problems with different sizes (small, medium, large) are solved to evaluate the performance of two presented meta-heuristic algorithms. Totally, ten test problems were randomly generated and these problems have been run for three times and their average has been compared with the results of the other algorithm. Therefore, thirty runs were done with every algorithm and the results are shown in table 8.

Table 8. Comparison of the OFVs and CPU Times of the proposed algorithms (λ is our objective function value, λ' is the competitor's objective function value, A is our market share and B is the competitor's market share)

Order	TS								ACO				
	N	v	v'	λ	λ'	A	B	CPU	λ	λ'	A	B	CPU
1-3	10	2	3	633	273	0.698707	0.301293	22	633	273	0.698367	0.301633	77
4-6	10	3	2	992	271	0.785147	0.214853	23	991	271	0.784933	0.215067	93
7-9	20	2	3	1073	906	0.541977	0.458023	46	1073	906	0.54211	0.45789	101
10-12	20	3	2	1443	528	0.732073	0.267927	49	1443	528	0.73206	0.26794	106
13-15	30	3	4	2082	1022	0.67068	0.32932	159	2082	1022	0.670857	0.329143	159
16-18	30	4	3	2247	730	0.75464	0.24536	166	2247	731	0.75456	0.24544	162
19-21	40	4	5	2463	1477	0.625147	0.374853	343	2458	1482	0.62383	0.37617	232
22-24	40	5	4	2740	1048	0.723373	0.276627	347	2726	1062	0.71951	0.28049	234
25-27	50	5	6	2794	2070	0.574513	0.425487	584	2823	2041	0.58033	0.41967	315
28-30	50	6	5	2883	1800	0.615667	0.3843333	619	2874	1809	0.613727	0.386273	328

6-1- Statistical Comparison

To evaluate the performance of the mentioned parameter tuned algorithms paired t- test at 95% significant level is performed while for the mean fitness value and run time comparison, the hypotheses are as below:

$$\mathbf{H}_0: \mu_{TS} = \mu_{ACO} \quad (35)$$

\mathbf{H}_1 : Otherwise

The results of tests for the equality of means for Objective Function Values (OFV) and CPU times are presented in table 9.

Table 9. Table of paired sample t- test for the equality of means for OFVs and CPU times of ACO and TS

		Paired Differences					t	Df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
OFV	TS – ACO	0.06057	11.18259	3.53625	-7.93898	8.06011	0.017	9	0.987
CPU	TS-ACO	54.96467	135.64485	42.89467	-42.06981	151.99915	1.281	9	0.232

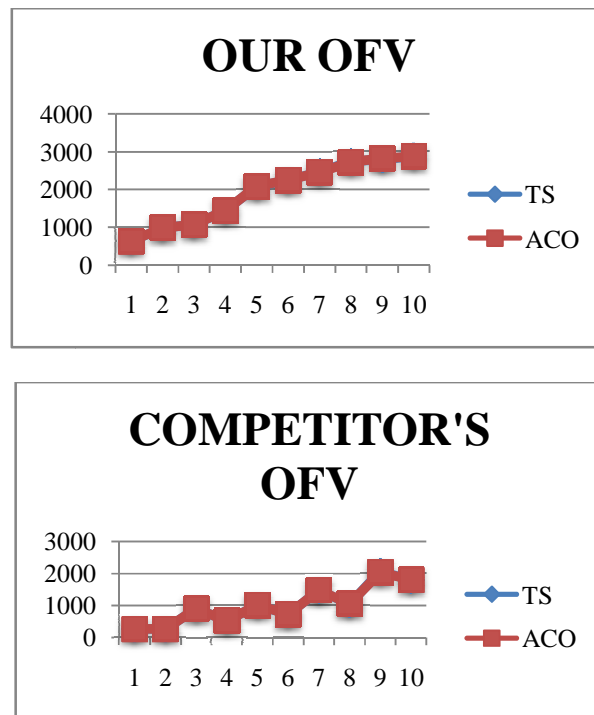


Fig.5. Comparison of the competitor's (existing facilities) and our (new facilities) OFVs of the proposed algorithms

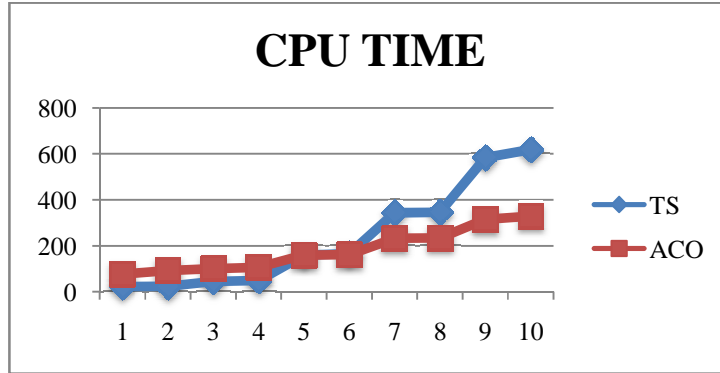


Fig.6. Comparison of the CPU Times of the proposed algorithms

As the results show, significance for the equality of fitness function means and CPU times are greater than 0.05, the assumption of the equality of means will be accepted. Therefore, it can be concluded that, the proposed algorithms in the significance level of 0.95 are similar. The comparison of the OFVs and CPU times are presented in figures 5 and 6.

6-2- TOPSIS Method

Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) was originally developed by Hwang and Yoon (1981) with further developments by Yoon (1987), and Hwang, Lai and Liu (1993). TOPSIS is the Multi Attribute Decision Maker (MADM) method that was developed under the concept that the selected alternative is the nearest from the ideal solution and the farthest from the negative ideal solution. Decision matrix and weights of the alternatives are the inputs of this method. To estimate the weights of the alternative vector, the entropy method has been used.

6-2-1- Entropy Method

Entropy was developed by Shannon (1984). We use Entropy method to evaluate the weights of the alternatives, when x_{ij} (score of option i with respect to criterion j) is known, and decision maker has no idea about the weights of the alternatives. We have an $X = (x_{ij}) m \times n$ matrix as the input of Entropy method, Therefore we create an evaluation matrix consisting of m alternatives (2 algorithms) and n criteria (OFV, CPU). Inputs of the decision matrix are the means of OFVs and CPU times in each algorithm. And the steps of this method are as follows:

$$X = \begin{matrix} & \text{OFV (+)} & \text{CPU (-)} \\ \text{TS} & \left[\begin{matrix} 1935.0077567 & 235.73 \end{matrix} \right] \\ \text{ACO} & \left[\begin{matrix} 1934.9471897 & 180.76 \end{matrix} \right] \end{matrix} \quad (36)$$

Step 1 Construct normalized decision matrix.

$$n_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}} \quad \text{for } i = 1, \dots, m; j = 1, \dots, n \quad n_{ij} = \begin{bmatrix} 0.5 & 0.566 \\ 0.5 & 0.434 \end{bmatrix} \quad (37)$$

Step 2 Calculate the below expression.

$$E_j = 1 + \frac{1}{\ln m} \sum_{i=1}^m (n_{ij} \ln n_{ij}) \quad (E_1 = 0, E_2 = 0.0126) \quad (38)$$

Step 3 Normalize the weight vector.

$$w_j = \frac{E_j}{\sum_{j=1}^n E_j} \quad \sum_{j=1}^n w_j = 1 \quad (39)$$

And the weight vector is obtained.

$$W = \begin{bmatrix} \text{OFV} & \text{CPU} \\ 0 & 1 \end{bmatrix} \quad (40)$$

The TOPSIS process is carried out as follows: we have a decision matrix $(x_{ij})_{m \times n}$.

Step 1 Construct normalized decision matrix. This step transforms various attribute dimensions into non dimensional attributes, which allows the comparisons across criteria.

$$r_{ij} = \frac{x_{ij}}{\sum_{i=1}^m x_{ij}^2} \quad \text{for } i = 1, \dots, m; j = 1, \dots, n \quad r_{ij} = \begin{bmatrix} 0.0002584 & 0.0027 \\ 0.0002584 & 0.002 \end{bmatrix} \quad (41)$$

Step 2 Calculate the weighted normalized decision matrix

$$v_{ij} = w_j r_{ij} \quad v_{ij} = \begin{bmatrix} 0 & 0.0027 \\ 0 & 0.002 \end{bmatrix} \quad (42)$$

Step 3 Determine the ideal and negative ideal solutions

Ideal solution:

$$A^* = \{ v_1^*, \dots, v_n^* \}, \text{ where } v_j^* = \{ \max_i (v_{ij}) \quad \text{if } j \in J; \min_i (v_{ij}) \quad \text{if } j \in J' \} \quad (43)$$

Negative ideal solution:

$$A' = \{ v_1', \dots, v_n' \}, \text{ where } v_j' = \{ \min_i (v_{ij}) \quad \text{if } j \in J; \max_i (v_{ij}) \quad \text{if } j \in J' \} \quad (44)$$

where J is associated with benefit criteria (more is better), and J' is associated with cost criteria (less is better)

Step 4 Calculate the separation measures for each alternative.

The separation from the ideal alternative is:

$$d_i^* = \left[\sum_{j=1}^n (v_j^* - v_{ij})^2 \right]^{\frac{1}{2}} \quad i = 1, \dots, m \quad (d_1^* = 0.0007, d_2^* = 0) \quad (45)$$

Similarly, the separation from the negative ideal alternative is:

$$d_i' = \left[\sum_{j=1}^n (v_j' - v_{ij})^2 \right]^{\frac{1}{2}} \quad i = 1, \dots, m \quad (d_1' = 0, d_2' = 0.0007) \quad (46)$$

Step 5 And finally calculate the relative closeness to the ideal solution C_i^*

$$C_i^* = d_i' / (d_i^* + d_i') , \quad 0 < C_i^* < 1 \quad (47)$$

Step 6 Then rank the preference order. For ranking alternatives using this index, rank the alternatives in decreasing order.

$$C_i^* = \begin{bmatrix} \text{TS} & \text{ACO} \\ 0 & 1 \end{bmatrix} \quad (48)$$

The value of C_i^* for ACO 1, and this shows in terms of the objective function value and CPU time, its performance is better than TS's performance.

6-3- Sensitivity Analysis

To evaluate the impacts of various parameters in our (new hospitals) and competitor's (existing hospitals) market shares in the user choice environment, the 25- node network is used, and the number of existing and new hospitals in the competitive environment is the same and equal to 3.

6-3-1- Evaluation of the Changes of δ

δ represents the importance of travel time at the customer's choice. To evaluate the effects of δ_1 (the impacts of new hospital's travel time), our and competitor's market shares are calculated for different values of δ_1 . The results are shown in figure 7. According to the figure we can say by increasing δ_1 , our market share decreases, and the competitor's market share increases. In fact, the attractiveness of the new hospitals decreases.

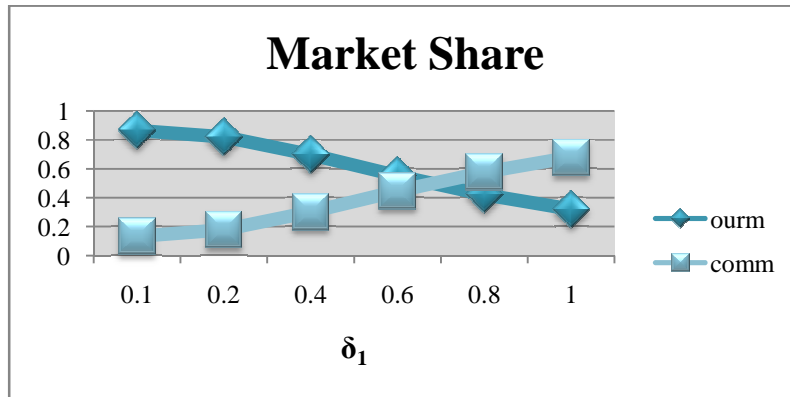


Fig.7.Evaluation of the changes of δ_1 , $\delta_2=0.5$ (competitor's travel time importance factor is constant), $\alpha = 0.5$, $\beta=0.5$, (L, R1, R1, C1, C2) = 2

6-3-2- Evaluation of the Changes of α

To evaluate the effects of α_1 (the importance of service quality of the new hospitals at the customer's choice), by increasing α_1 , the market share of new hospitals increases and the competitor's market share decreases. This means that the attractiveness of our centers increases. The results are shown in figure 8.

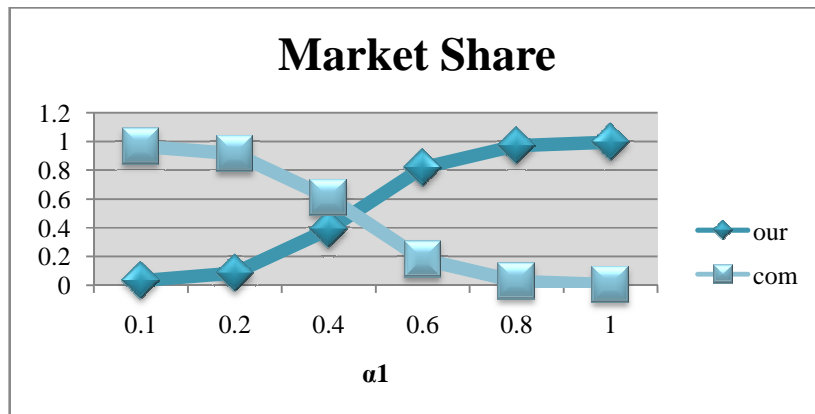


Fig.8.Evaluation of the changes of $\alpha_1, \alpha_2=0.5$ (competitor's convenience services importance factor is constant), $\beta=0.5$, $\delta=0.5$, (L, R1, R1, C1, C2) = 2

6-3-3- Evaluation of the Changes of β

B is the importance factor of personal experience. To evaluate the effects of β_1 (the effects of personal experience at the new centers), our and competitor's market shares are calculated for different values of β_1 , by increasing β_1 , our market share increases and the competitor's market share decreases. The results are shown in Figure 9.

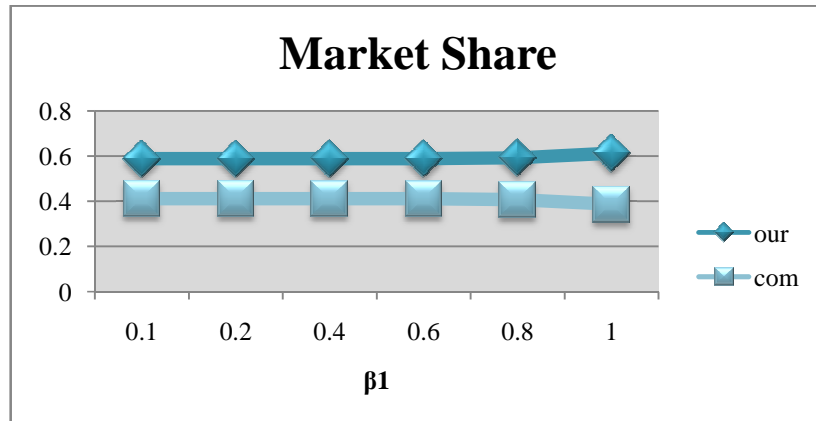


Fig.9.Evaluation of the changes of β_1 , $\beta_2=0.5$ (competitor's personnel experience importance factor is constant)
 $\alpha=0.5$, $\delta=0.5$, $(L, R1, R1, C1, C2) = 2$

7- Conclusions and Future Study

This paper has tried to create a model for locating health care facilities (hospitals) in the competitive location environment, which incorporates the theories of customer choice behavior to patronize the facilities. Customer's attraction, towards a facility is obtained by the attractiveness factors such as less travelling time, the quality of services or reputation of the centers. It is assumed that, each hospital consists of both high and low levels, and patients can be allocated to high levels when they are referred from low levels, and directly visit the high level sections without referring to the low level sections in emergency situations. Two meta-heuristic algorithms including ACO and TS were executed for the produced test problems. Their performances were compared in terms of CPU run times and fitness function values. For the comparison purposes, paired t- test and TOPSIS method were employed. The results of several numerical examples showed that, there is no significant difference in the objective function means and run time means of ACO and TS. Furthermore, TOPSIS results showed that, ACO is a better procedure than TS.

The following approaches can be proposed for future research:

- 1) Considering other measures rather than the quality of service in centers and travelling time, such as the service cost of servers.
- 2) Considering the multi objective function problem and solving it by suitable meta-heuristic algorithms (such as NSGA-II and NPGA).
- 3) Making the model closer to the reality, considering some of the parameters, fuzzy or random. (Such as demand rate)
- 4) Employing other meta-heuristic algorithms or heuristic algorithms or hybrid of heuristic and meta-heuristic algorithms to solve the model and investigate their efficiencies.
- 5) Developing other queuing system rather than M/M/C/K
- 6) Developing heuristic approach instead of generating random solution in the initial segment.

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