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Proposing a mathematical model of parallel machine scheduling with the aim of minimizing the task completion time and energy cost using a meta-heuristic algorithm

Samira Sameye[1], Javad Rezaeian Zaidi[2*], Mohammad Reza Lotfi[3]

[1]Department of Industrial Management, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran
[2]Department of Industrial Engineering, School of Management and Industrial Engineering, Mazandaran University of Science and Technology, Babol, Iran
[1]Department of Industrial Engineering, Firoozkooh Branch, Islamic Azad University, Firoozkooh, Iran

samiee.samira@yahoo.com, m.reza_lotfi@yahoo.com, javadjf@gmail.com

Abstract

This research proposes and solves a mathematical problem of parallel machine scheduling to minimize the total completion time and energy cost. This research aims to design and optimize a multi-objective mathematical model by minimizing energy consumption and total completion time for the parallel machine scheduling problem in Semnan Polyethylene Factory. First, the mathematical model of the problem is provided, and then the solution method is investigated using the epsilon constraint method in the GAMS optimization software and the meta-heuristic imperialist competitive algorithm (ICA). The mathematical model is validated using GAMS software and the constraint epsilon method and a real problem is implemented in large dimensions regarding the case study of the polyethylene factory in Semnan province using the meta-heuristic ICA. Finally, the performance of the ICA is measured in terms of the RPI index for small dimensions and the MID index for examples with large dimensions. Numerical results show that the value of the index for distance from the ideal point in the ICA is lower than that of the index obtained from solving the problem in GAMS. With these interpretations, it can be concluded that the ICA has a better performance than GAMS for optimizing the parallel machine scheduling problem in this research. According to the obtained answers, it can be concluded that with the increase in the time to do a task, the time to complete all tasks also increases and the cost of energy remains constant. While the cost of doing the task and the price of the electricity signal increase, energy costs increase and the time to complete tasks remains constant.

Keywords: Scheduling problem, parallel machines, energy cost, meta-heuristic algorithm

1- Introduction

Manufacturers make great efforts to invest in sustainability in order to decrease the environmental effect of manufacturing centers and thus increase social responsibility, which is one of the important ways for manufacturers for the accountability of managing their operations. These efforts may prepare the manufacturing environment for thinking differently about how to manage operations, identify opportunities, and better execute plans. In an environment where profitable companies use dynamic pricing of electricity energy to more effectively manage generation capacity by increasing demand during high-price periods to avoid additional costs and investments, a manufacturing environment can use intelligent scheduling and scheduling operations to simultaneously consider energy pricing and scheduling objectives.

*Corresponding author
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For a manufacturing center, changes in capacity utilization in different periods or machines create differences in the opportunity cost of operations during the planning horizon (Robaei et al., 2019; Salahi et al. 2023). Energy acts as the blood stream for the economy of manufacturing centers and modern society. Energy efficiency, advanced technologies, Internet of Things (IoT), and products and services related to them are among the projects with the fastest growth rate for investment in recent years. This is done by many manufacturers who are considered investor owners rather than followers of technological advances. Energy suppliers with stable prices are at the center of the financial reform and improvement of many manufacturing sectors (Robaei et al., 2017; Babaeinesami et al. 2022). In today's competitive environment, the survival of economic firms depends on the continuous improvement of performance in order to maintain and increase the competitive power and gain more benefits. In fact, manufacturing planning and scheduling is a kind of decision-making process for optimization that plays a fundamental role in improving performance and improving productivity in manufacturing and service industries and is very essential for the survival of companies (Abolghasemian et al, 2021; Sohrabi et al., 2021).

With the passage of time and the complexity of problems in the real world, the human's need for methods that are efficient in terms of time and cost and have good accuracy has increased (Chobar et al., 2022). One of the most important discussions that is given special attention in companies and factories is the issue of correct scheduling and optimal use of resources or optimization. In scheduling problems, as the size of the problem increases, the time required to find the optimal solution increases exponentially. Therefore, the scheduling problem is considered NP-hard (Pourghader Chobar, 2022; Maadanpour Safari et al. 2021).

On the other hand, the increase in energy costs is one of the important factors related to the increase in manufacturing costs, encouraging manufacturing managers and decision makers to face this problem in different ways. An important step in this process is to decrease energy consumption costs in manufacturing systems and to consider different energy prices during a day. By decreasing the energy cost, the manufacturing cost and the finished price of products will decrease, and as a result, the competitiveness and profit of the company will increase.

In fact, manufacturing planning and scheduling is a kind of decision-making process for optimization that plays an essential role in improving productivity in manufacturing and service industries. In today's competitive world where the speed of changes is one of its undeniable features, it is very necessary to have proper planning and scheduling of activities for the survival of companies (Daneshvar et al., 2023).

On the other hand, the increase in energy costs is one of the important factors related to the increase in manufacturing costs, which encourages manufacturing managers and decision makers to face this problem in different ways. An important step in this process is to decrease energy consumption costs in manufacturing systems and consider different energy prices during a day, i.e. decreasing energy costs can be based on not allocating in periods with high energy costs. This minimization process also brings positive environmental effects. In Semnan Polyethylene Company, the optimal use of the company's resources, including machinery, is one of the important goals of the managers, they always try to make the most of the available resources so that they can achieve more market share and profit by reducing the cost and the finished price. Also, due to the use of electric machines in the production line and the significant figure of the company's electricity costs, managers are looking for a solution to reduce these costs. Furthermore, in line with the company's social responsibilities, they define the decrease of energy consumption a step towards reducing environmental effects and adhere to it. The scheduling plan created in this research makes the polyethylene company's managers capable of minimizing the total task completion time and also the cost of energy consumed in the manufacturing lines. In this research, continuous changes in energy prices are identified as an important factor in the scheduling problem, simultaneous attention to these two objectives creates a problem with two objective functions, which is considered as an important gap in past studies, and for the first time, this problem in parallel machines is investigated in this research.

In the following, the research background is discussed in the second section. In the third section, the mathematical model and solution method are provided. In the fourth section, the numerical results of the research and sensitivity analysis are presented, and finally, the conclusions and suggestions are stated in the fifth section.

2- Literature review

In this section, studies conducted on the optimization of parallel machine scheduling are reviewed and the objectives and methods used in these studies are discussed. Finally, a comparison is made between previous studies and the present research, and research gaps are discovered.

Ding et al. (2015) provided the parallel machine scheduling problem under time-of-use electricity prices. They used new models and new optimization approaches in their research. Their main objective was to minimize the total electricity cost by appropriately scheduling the jobs such that the overall completion time did not exceed a predetermined production deadline. To solve this problem, two solution approaches were presented. The first approach modeled the problem with a new time-interval-based mixed integer linear programming formulation. In the second approach, they reformulated the problem using Dantzig-Wolfe decomposition and proposed a column generation heuristic to solve it. Computational experiments were conducted under different TOU settings and the results confirmed the effectiveness of the proposed methods. Based on the numerical results, they provided some practical suggestions for decision makers to help them in achieving a good balance between the productivity objective and the energy cost objective. Abikarram et al. (2019) considered the minimization of energy cost for the parallel machine scheduling in a research in which they examined demand and real-time pricing. For this purpose, a mathematical optimization model was proposed to schedule jobs in which both demand charges and consumption charges were considered for a facility with parallel machines. Results showed that total electricity costs could be reduced significantly when demand charges were included in scheduling decisions versus considering only consumption charges. They further indicated a significant reduction in peak demands for the facility, which provided a benefit for both the facility and the utility. Taking into account consumption and demand charges did not increase the total energy consumption. Li et al. (2016) studied an unrelated parallel machine scheduling considering the cost of tardiness and energy. In their research, it was assumed that the energy consumption on each machine was also unrelated. First, they presented a mathematical model of the problem. Then, 10 algorithms were, respectively, proposed based on the priority rules, the energy consumption, and the combinational rules due to the complexity of this problem. Finally, in order to test the performance of these 10 algorithms, computational experiments were designed. In these experiments, lots of instances were generated, and the computational results indicated that the algorithms based on the combinational rules outperformed the ones based on the priority rules and energy consumption, with respect to the unrelated parallel scheduling problem.

Kianpour et al. (2021) addressed the optimization of unrelated parallel machine in job shops where the maximum allowed tardiness limit was considered. They analyzed unrelated parallel machine scheduling models. The researchers proposed a new mixed integer linear programming model considering effects of maximum limit on the allowable tardiness. An efficient solution technique was developed in MATLAB to minimize computational time. The model was validated using real-life data from local aircraft industry. It was shown that the proposed model leads to lower cost compared to the current models in the literature. In addition, reduction in computational time enables corporate scheduling staff to use the model efficiently.

Guo et al. (2022) used decomposition approaches for parallel machine scheduling of step-deteriorating jobs to minimize total tardiness and energy consumption. The actual processing time of a job was assumed to be a step function of its starting time and its deteriorating threshold. In this paper, a mixed integer linear programming model was proposed to minimize the total tardiness cost and the extra energy cost. Decomposition approaches based on logic-based Benders decomposition (LBBD) were developed by reformulating the studied problem into a master problem and some independent sub-problems. Moreover, MILP and heuristic based on Tabu search were used to solve the sub-problems. The results demonstrated that the proposed decomposition approaches could compute competitive schedules for medium- and large-size problems in terms of solution quality. In particular, the LBBD with Tabu search performed the best among the suggested four methods.

Rego et al. (2022) proposed a mathematical modeling and an NSGA-II algorithm for time and energy cost minimization under time-of-use in an unrelated parallel machine scheduler. We also developed a new bi-objective unrelated parallel machine scheduling problem with sequence-dependent setup times, in which the objectives were to minimize the time and the total energy cost. They proposed a mixed-integer linear programming formulation based on the weighted sum method to obtain the Pareto front. They also developed an NSGA-II method to address large instances of the problem. The results showed that the proposed NSGA-II is able to find a good approximation for the Pareto front when compared with the weighted sum method in small instances. Besides, in large instances, NSGA-II outperforms, with 95% confidence level, the MOGA and NSGA-I that are multi-objective techniques. Thus, the proposed algorithm finds non-dominated solutions with good convergence, diversity, and uniformity. Moon et al. (2013) proposed the optimization of production scheduling with time-dependent and machine-dependent electricity cost for industrial energy efficiency. This paper deals with the production and energy efficiency of the unrelated parallel machine scheduling problem. This method allows the decision maker to seek a compromise solution using the weighted sum objective of production scheduling and electricity usage. Reliability models are used to consider the energy cost aspect of the problem. This paper aims to optimize the weighted sum of two criteria: the minimization of the makespan of production and the minimization of time-dependent electricity costs. They suggested a hybrid genetic algorithm with blank job insertion algorithm and demonstrated its performance in simulation experiments. Cataldo et al. (2016) presented the problem of optimizing on-line the production scheduling of a multiple-line production plant composed of parallel equivalent machines which can operate at different speeds corresponding to different energy demands. The transportation lines may differ in length and the energy required to move the part to be processed along them is suitably considered in the computation of the overall energy consumption. The optimal control actions are recursively computed with Model Predictive Control aiming to limit the total energy consumption and maximize the overall production. Finally, simulation results were reported to witness the potentialities of the approach in different scenarios. Meng et al. (2018) investigated an energy-aware hybrid flow shop scheduling problem with unconnected parallel machines (HFSP-UPM) with a power-on-off strategy. First, the energy consumption of HFSP-UPM was analyzed and five mixed integer linear programming (MILP) models were formulated based on two different modeling ideas, i.e. idle time and idle energy. The results show that MILP models based on different modeling ideas differ dramatically in size and computational complexity. The results show that IGA performs more effectively than Genetic Algorithm (GA), Simulation Cooling Algorithm (SA) and Migratory Bird Optimization (MBO). Compared with the best MILP model, IGA can obtain a solution close to the optimal solution with a gap of more than 2.17% for small-scale samples. For large samples, IGA can provide a better solution than the best MILP model in up to 10% of the running time of the best MILP model.

Pan et al. (2019) considered low carbon parallel machines scheduling problem (PMSP), in which total tardiness is regarded as key objective and total energy consumption is a non-key one. A lexicographical method is used to compare solutions and a novel imperialist competitive algorithm (ICA) was presented, in which a new strategy for initial empires was adopted. Some new improvements were also added in ICA to obtain high quality solutions, which are adaptive assimilation, adaptive revolution, imperialist innovation, and alliance and the novel way of imperialist competition. Extensive experiments were conducted to test the search performance of ICA by comparing it with methods from literature. Computational results showed the promising advantages of ICA on low carbon PMSP. Shao et al. (2021) studied a distributed hybrid flow shop scheduling problem (DHFSP) with makespan criterion, which combines the characteristic of distributed flow shop scheduling and parallel machine scheduling. For solving the DHFSP, this paper proposes two algorithms: DNEH with smallest-medium rule and multi-neighborhood iterated greedy algorithm. The DNEH with smallest-medium rule constructive heuristic first generates a seed sequence by decomposition and smallest-medium rule, and then uses a greedy iteration to assign jobs to factories. The proposed algorithms are evaluated by a comprehensive comparison, and the experimental results demonstrate that the proposed algorithms are very competitive for solving the DHFSP.

Environmental and economic pressures caused by energy consumption raise the awareness of energy saving in the manufacturing industry. To this end, Zhou et al. (2021) integrated energy awareness into the unrelated parallel machine scheduling problem with multiple auxiliary sources common in the photolithography process of wafer fabrication. By comprehensively considering jobs that require different processing requirements, startup times, different ready times, resource constraints, and energy consumption, a multi-objective scheduling model was developed to minimize the total weighted completion time and total system energy consumption. Based on this model, a modified multi-objective artificial safety algorithm with non-dominated sorting strategy was presented for solving. In addition, in order to improve the performance of the proposed algorithm, clone operators, neighborhood search operators, and elitist preservation operators were applied to the algorithm. Finally, the experimental results and analysis confirmed that the proposed algorithm is efficient and effective. Al-harkan et al. (2022) focused on the problem of scheduling a set of jobs on unrelated parallel machines subject to release dates, sequence-dependent setup times, and additional renewable resource constraints. The objective was to minimize the maximum completion time (makespan). To optimize the problem, a modified harmony search (MHS) algorithm was proposed. The parameters of MHS were regulated using full factorial analysis. The MHS algorithm was examined, evaluated, and compared to the best methods known in the literature. Four algorithms were represented from similar works in the literature. A benchmark instance was established to test the sensitivity and behavior of the problem parameters of the different algorithms. The computational results of the MHS algorithm were compared with those of other meta-heuristics. The competitive performance of the developed algorithm was verified, and it was shown to provide a 42% better solution than the others.

In table 1, a summary of related research on parallel machine scheduling is discussed, and accordingly, research gaps are identified and innovations that can be presented are expressed.

Table 1. Summary of research review

Considerations	**Case example**		**Solution method**	**Objective functions**	**Uncertainty**	**Modeling**			**Authors**	**Year**
	Taken from literature	**Real example**				**MILP**	**MINLP**	**MIP**		
Unrelated parallel machines under time-of-use energy pricing		*	Dantzig-Wolfe decomposition, column generation method	Total cost of electrical energy with task scheduling, efficiency, and energy cost objective		*			Ding	2015
Unrelated parallel machine scheduling		*	Algorithms based on evolutionary rules, algorithms based on priority rules	Energy cost, tardiness cost			*		Li	2016
Investigating demand and real-time pricing	*		Meta-heuristic algorithm	Total energy costs				*	Abikarram	2019
	*		Meta-heuristic algorithm	Total energy cost			*		Aghel inejad	2019
Distributed hybrid flow shop scheduling (DHFSP) with makespan criterion	*		Multi-neighborhood iterated greedy algorithm	Total processing time			*		Shao	2021
Considering the maximum allowable tardiness limit, the effects of the maximum limit on the allowable tardiness	*		Meta-heuristic algorithm	Total energy cost				*	Kianpour	2021

Table 1. Continued

Considerations	Case example		Solution method	Objective functions	Uncertainty	Modeling			Authors	Year
	Taken from literature	Real example				MILP	MINLP	MIP		
	*		Decomposition-based approaches based on logic-based boundary decomposition (LBBD), tabu search meta-heuristic	Total tardiness and energy consumption		*			Guo	2022
Sequence-oriented times	*		NSGA-II algorithm, weighted sum method, MOGA and NSGA-I	Minimizing time and energy cost under electricity pricing		*			Rego	2022
Parallel machine scheduling in polyethylene pipe factory in Semnan province		*	Multi-objective genetic algorithm (NSGA-II), epsilon constraint	Total energy consumption cost, total task completion time		*			Present research	

Based on the review of the above literature, as you know, most of the models have considered a specific objective. There are few studies that have investigated and optimized more than one objective. Accordingly, in this research, optimization and modeling of two objective functions are discussed simultaneously. On the other hand, since according to the research literature, the problem of energy consumption is of great importance in the parallel machine scheduling problem, in this research, the main objective is to minimize the energy consumption costs, which consequently reduces the energy consumption in manufacturing plants. Furthermore, since the present problem is the parallel machine scheduling, an inherent objective of scheduling problems is to minimize the task completion time, which is investigated in this research, and based on this, the mixed integer linear programming mathematical model for the parallel machine scheduling problem is presented to simultaneously minimize two objective functions.

3- Methodology

In this problem, the issue of planning, energy consumption, and parallel machines in the pipe and polyethylene manufacturing industry is investigated and analyzed in order to minimize the total task completion time, as well as the total energy consumption cost for parallel machine scheduling. Since the problem has two objectives: minimizing the completion time and minimizing the energy consumption, and it is NP-hard, multi-objective meta-heuristic algorithms need to be used to solve it, and since the efficiency of meta-heuristic algorithms depends on the selection of parameters and solutions with different quality can be achieved with different combinations of parameters involved in the implementation of an algorithm, it is tried to identify and determine the parameters affecting the performance of the algorithm using a systematic method such as the Taguchi method. To check the accuracy of the model's performance and the efficiency of the algorithm in solving the problem, first, some examples of simple, medium, and complex problems are designed and solved. After ensuring the accuracy of the proposed model, it can be implemented in the real world. Finally, after implementing the proposed model, we should be able to minimize the total task completion time and the energy consumption cost.

3-1- Mathematical modeling

In this section, we provide a mathematical model for the parallel machine scheduling modeling problem in order to minimize the total completion time and energy cost, so that the objectives of minimizing energy consumption and completion time are realized.

3-1-1- Optimization model

In this section, a network of parallel machines is designed including specific constraints and objectives of minimizing energy consumption and task completion time, which is determined by proposing a mathematical model. The assumptions of the problem are given below.

- Assumptions

- Machines are parallel and unrelated, which means the speed of different machines is different.
- The cost of electricity varies at different times of the day. During the peak hours of electricity consumption, the price of electricity is more expensive, and a faster machine consumes more electricity, and these types of machines cost more. A slow machine is cheaper, a fast machine is more expensive, and a normal speed machine has an average price.
- A learning model is defined for the worker. The role of the operator in the device and the processing time are considered. For example, a worker does a task with a fast machine in two minutes and another worker does the same task with the same machine in four minutes.
- Access to energy is within a certain limit.
- All jobs are available simultaneously.
- Each machine is only able to process one job at a time.
- Each job is processed on only one machine during its processing time.
- Electricity pricing varies hourly.
- Preventive: It means that a device can process only one task, but by starting a new task process, the previous task can be interrupted and restarted.

In the following, the sets, parameters, and variables of the mathematical model of the problem are presented. Then, based on this, the mathematical model of the problem is provided.
The sets, indicators, parameters, and variables are as follows.

- Sets

N	Number of tasks n $\in\{1,...,N\}$
T	Planning horizon t $\in\{1,...,T\}$
S	The indicator related to the speed of machine k, where s_k is the number of different speeds of the *k-th* machine ($1 \leq s \leq s_k$).
r	Set of machine-related sequences
T'	Subset of the planning horizon $T' \subseteq \mathrm{T}$
j‘ i	Tasks $j, i \in N$
K	Set of machines needed for scheduling and doing tasks
m	Set of workers
t‘ t'	Time interval $t \epsilon T$ *and* $t' \epsilon T'$
c	Set of cells

- Parameters

Et	Price of the electricity signal at time t
Vk	Speed of machine k
p_j	Processing time of task j
t_{ik}	Time required to do task j by machine k
C_{ik}	Cost of doing task j by machine k

QM_{kt}	Number of machines of type k available in time period t after calculating for machines that have been processed.
N_{kct}	Number of machines of type k at the current time in cell c at the beginning of time period t
NP_{kt}	Number of machines of type k processed at time t
α	Electricity production rate per unit of time in kilowatts
En_j	Energy required to do task j per unit of time

- Variables

Et_j	Time to complete task *j*
pjk	Processing time of task *j* by machine *k*
x_{jtk}	Equal to 1 if task *j* is assigned to machine *k* at time t, otherwise 0.
x_{jtkw}	Equal to 1 if task *j* is done at time t by machine *k* and operator *w*, otherwise 0.
x_{jkrs}	Equal to one If task *j* is assigned to the *r-th* sequence of machine k under speed S_k s, (1), otherwise (0)
b_{kr}	Starting moment of a task in sequence *r* and machine *k*
$yadd_{kct}$	Ratio of the number of added machines of type *k* to cell *c* at time *t*
$yrem_{kct}$	Ratio of the number of machines of type *k* removed from cell *c* at time *t*

Thus, a mixed integer linear programming model for the parallel machine scheduling problem is proposed as follows, which is a two-objective optimization model with the minimization of total completion time and total energy. One of the important outputs of the problem is the programming of a device with certain processing time (pjk).

In this section, the parallel machine problem is defined. In this problem, there is a set of n tasks $j = \{j_1, j_2, ..., j_n\}$ so that all tasks are available for processing at zero time and zero setup time. The machine can handle one task at a time and no prerequisites are allowed for it. Each task has a normal processing time p_j, and the normal processing time of a task is in a sequence $p_{[r]}$, provided that it is scheduled at the r-th position. On the other hand, in order to consider the learning rate in the parallel machine scheduling problem model, it is necessary to assume that each operator gradually increases their skill over time and by doing different tasks, and in this way the operators have skills in the given problem. To consider the learning rate of operators, the following expression is formulated (r is the position number in each sequence and β is the learning rate of the operator).

The proposed model is as follows:

$$Min \quad \sum_{j,k} Et_{jk} \tag{1}$$

$$Min \quad \sum_{j,k,t} E_t En_j x_{jkt} \tag{2}$$

$$\sum_{t=1}^{|T|} x_{jt} = p_j \quad \forall j \in J \tag{3}$$

$$\sum_{w,j} x_{jtkw} \le 1 \quad \forall t,k \tag{4}$$

$$\sum_{s,r,k} x_{jkrs} \le 1 \quad \forall j \tag{5}$$

$$\sum_{s,j} x_{jkrs} \le 1 \quad \forall k,r \tag{6}$$

$$\sum_{j} x_{jkt} \le 1 \quad \forall k,t \tag{7}$$

$$\sum_{t} x_{jkt} = p_j \quad \forall j \tag{8}$$

$$\sum_{t} x_{jkt} \le c_{jk} \quad \forall j,k \tag{9}$$

$$b_{k,r+1} - b_{kr} = \sum_{s,j} x_{jktl} t_{jks} \tag{10}$$

$$p_{jk} = (\frac{p_j}{v_k}) s_j \quad \forall j,k \tag{11}$$

$$c_{jk} = \sum_{k,j} p_{j,k} \tag{12}$$

$$C_j \ge x_{jt} + 1 \quad \forall j \in N; t \in T \tag{13}$$

$$N_{kct} = N_{kc(t-1)} + yadd_{kct} - yrem_{kct} \quad \forall k,c,t \tag{14}$$

$$P_m / P_{jr} = p_j (1 + p_{[1]} + p_{[2]} + ... + p_{[r-1]})(r)^{\beta}, \quad 0 < \beta < 1 \tag{15}$$

$$\sum_{j} x_{jt} En_j \le \alpha \qquad \forall t \tag{16}$$

$$C_j \ge 0 \quad \forall j \in N \tag{17}$$

$$x_{jt} \in \{0,1\} \quad \forall j \in N; t \in T \tag{18}$$

According to the proposed mathematical model, it is clear that the first objective function (1) minimizes the total completion time. The second objective function (2) minimizes the energy cost. Constraint (3) states that the total processing time of any task is acceptable. Capacity constraint (4) ensures that the device can perform at most one task per time interval. Constraint (5) states that each task must be assigned to a sequence of one machine and processed under a certain speed. Constraint (6) indicates that each sequence of each machine can be assigned to one task at most. Constraint (7) guarantees that at most one job is assigned to each machine and worker in each time interval. Constraint 8 guarantees that the time allocated for each task is equal to its processing time. Constraint (9) provides the completion time of each job. Constraint (10) specifies the start time of each sequence of machines. Constraint (11) and (12) calculates the completion time of each task. Constraint (13) provides the completion time for each job. Constraint (14) guarantees that the number of machines of type k in the current period is equal to the number of machines in the previous period plus the number of added machines minus the number of removed machines from cell c. According to the given explanation represents, Constraint (15) is the constraint of learning rate of operators. Constraint (16) also guarantees that the energy required to do task j is less than the energy manufacturing rate. It should be noted that the value of energy manufacturing rate per unit of time is a specific and constant value and the constraint of energy availability can also be inferred from this expression. Constraint (17) shows the non-negativity of the variables. Constraint (18) is the constraint of variables being zero and one.

3-2- Imperialist Competitive Algorithm (ICA)

Imperialist competitive algorithm (ICA) is one of the evolutionary algorithms inspired by the social phenomenon called colonialism. This algorithm was first proposed by Atashpaz Gargari and Lucas (2007). Like other meta-heuristic algorithms, it is a population-based algorithm in which the solution space is searched by points called country. Some of these countries are called colonizers and some are colonies under the control of colonizers, and the changing of the position of the colonies during this competition are two important principles of this algorithm. In this research, this algorithm is used to solve the proposed problem. The steps of the multi-objective ICA are as follows:

- Step 1 (formation of the initial empire)

Since ICA has a population-based approach, as the first step of this algorithm, a population with the size of N_{pop} of the country is generated, among which N_{imp} of the country is the colonizer and the rest are the colonies of these colonizers are considered. The number of countries controlled by each colonizer is proportional to its power. For this purpose, first the rank of each country is calculated according to non-dominated sorting indices, and then the power of each colonizer is calculated using the Sigma method, according to what is mentioned in the following relation.

$$Power_n = \frac{1}{\sum_{j=1}^{D}\left[\frac{f_i(n)}{\sum_{i=1}^{N_{rank}(C)} f_j(i)}\right](Rank(C)-1)\times D} \tag{19}$$

Where $Power_n$ is the power of the *n*-th user, *Rank(C)* is the rank of the *n*-th country, *D* is the number of objective functions of the problem, $f_j(i)$ is the *j*-th value of the objective function of the *i*-th country and $N_{rank}(C)$ is the number of countries with rank *C*.

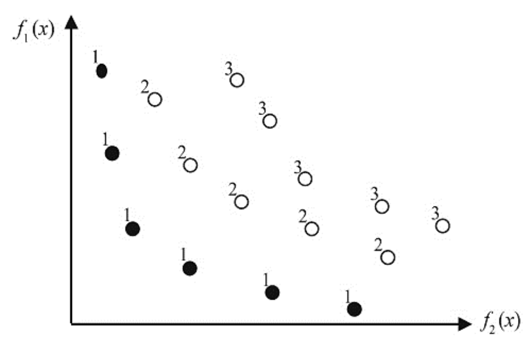

Fig 1. Pareto front of minimization problem

Step 2 (movement of colonies towards their colonizers or assimilation)

Assimilation is the process of moving colonies towards colonizers within an empire. This process is considered one of the most important steps in ICA because it is related to the improvement of colonies in a certain empire. In this step, the dominated colonies take some characteristics of their colonizers and move towards their colonizers with a slight deviation. These movements are calculated by the following relations.

$$x \sim U(0.\beta \times d) \tag{20}$$

$$\theta \sim U(-\gamma . \gamma) \quad (21)$$

Where x and θ indicate the values of movement and deviation in movement, respectively, d is the distance of each colony to its colonizer and β and γ are two parameters.

Figure 2 shows the assimilation process used in this algorithm. In this figure, black circles and red triangles represent the non-dominated solutions (set of global non-dominated solutions) that colonize the entire colonies. The red triangle is the chosen colonizer that is chosen randomly and other colonies move towards it. For simplification, only one moving colony is shown in this figure.

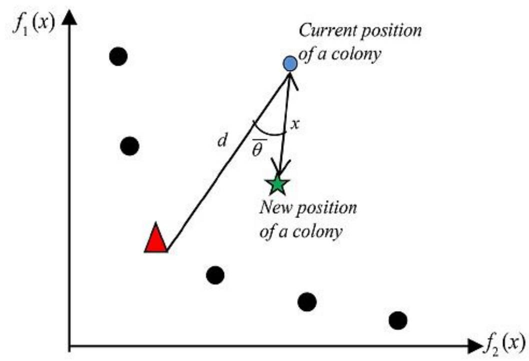

Fig 2. Assimilation process in the multi-objective ICA

- Step 3 (changing the position of colonizer and colony)

In this step, a colony may gain a better position in terms of power than the corresponding colonizer due to the improvement of its position. Therefore, after the second step, the power of each country in each empire is calculated again and the strongest country acts as the colonizer of the empire.

- Step 4 (calculating the total cost of empires)

In order to determine the strongest and weakest empire, the total cost of each empire is calculated, which is proportional to the cost of the corresponding colonizer and a ratio of the average cost of the colonies under domination. In order to calculate the total cost, all the objective functions need to be considered according to the two criteria of non-dominated sorting and ranking according to the following relation.

$$TC_n = Cost(imperialist_n) + \varsigma mean\{cost(colonies)\} \quad (22)$$

Where ς is a value less than one.

- Step 5 (competition of colonists)

In this step, the colonists compete to take over each other's colonies. For this purpose, the most powerful empire subjugates the weakest colony of the weakest empire. The power of each empire is normalized according to their total cost by the following relation.

$$NTC_n = TC_n - max\{TC_n\} \quad (23)$$

- Step 6 (eliminating the powerless empire)

If an empire loses all its colonies, it is eliminated and the competition between other empires continues. The revolution operation in the proposed algorithm is completely different from the classical colonial competition because no new generation is generated randomly. The new revolution operation has two parts which are based on a possible number. The first part is creation of a new colony by randomly selecting elements from two randomly selected colonists in the set of local non-dominated solutions of an empire. If there is only one colonizer in the set of local non-dominated solutions, another solution is randomly generated as a colonizer. In the second part, a number of colonists are randomly selected and replaced by a number of colonies that are also randomly selected.

- Stopping condition

If the stopping condition (only one empire remaining in the population) is not met, we return to step 2 of the algorithm and continue the algorithm with new empires. The figure 3 shows the flowchart related to this algorithm.

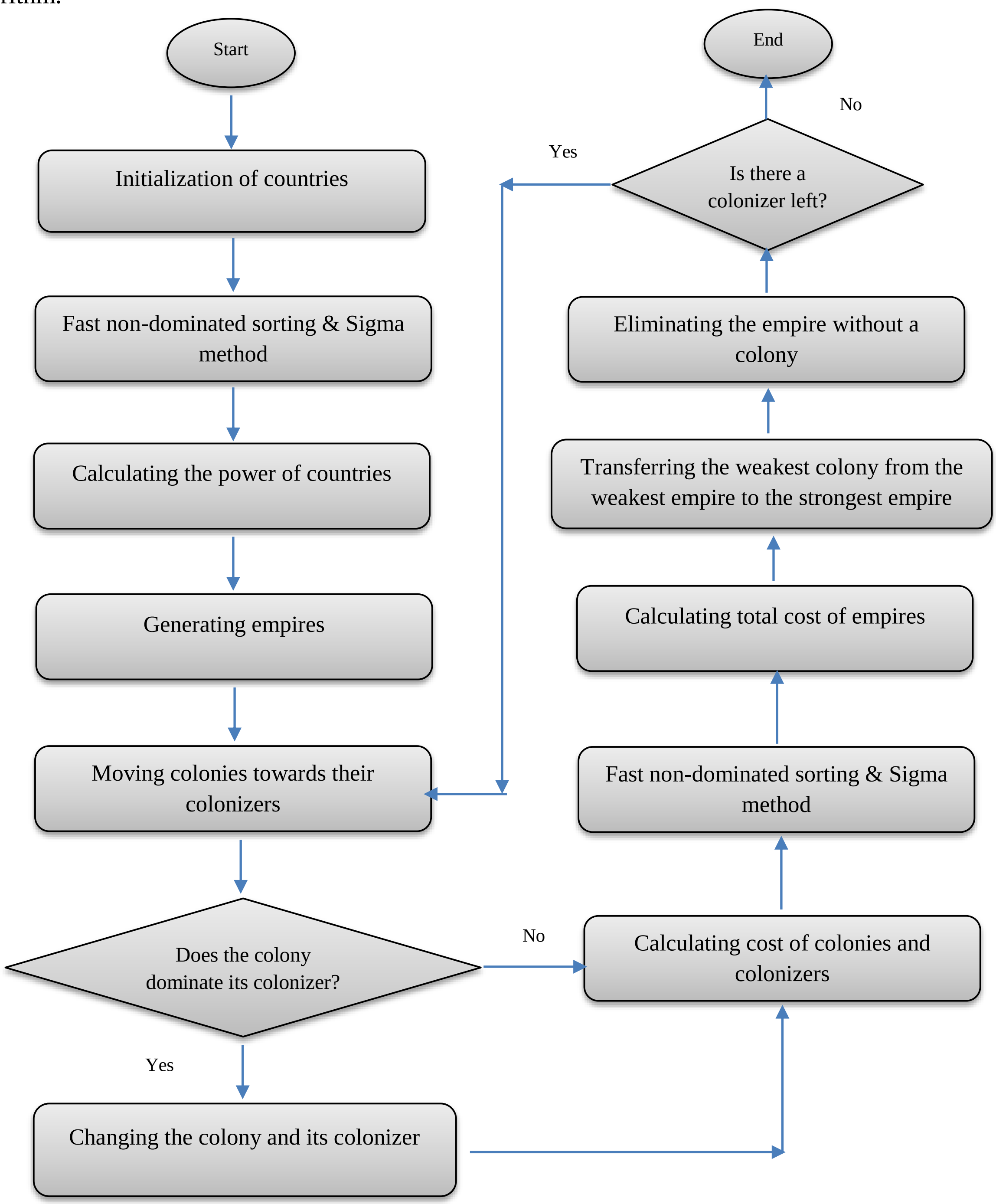

Fig 3. Flowchart of the multi-objective ICA

4- Numerical results and data sensitivity analysis

To solve the mathematical model proposed in this research, in this section, the numerical problem in different sizes is implemented and run using commercial GAMS software and the powerful CPLEX optimization solver, and the results are analyzed. Thus, the characteristics of 10 numerical problems are displayed, whose dimensions are collected according to the literature of the subject. Also, we will use the ε-constraint method to convert the multi-objective problem into a single-objective problem in the GAMS software. On the other hand, to compare the results of solving the problem and validating the model proposed in the third section, the data of a case study in Semnan polyethylene factory is used, and to solve the real problem, we use the meta-heuristic ICA, and compare and analyze the results of solving both methods to solve problems with different dimensions. For the meta-heuristic ICA, we use the Taguchi method and signal-to-noise value analysis to achieve the optimal values of the parameters in the algorithm. Finally, we measure the performance of the ICA in terms of the RPI index for small dimensions and the MID index for examples with large dimensions and present the results in the form of graphic diagrams.

4-1- Solving the problem with the ε-constraint method

As it can be seen, with the increase in the dimensions of the problem, the number of tasks required for execution, the number of machines required for scheduling, and the number of planning periods gradually increase, and the complexity and difficulty of solving the problem increases.

Table 2. Dimensions of numerical problems

Number of numerical problems	Number of tasks	Number of machines	Number of time periods
1	2	1	1
2	3	2	2
3	5	2	3
4	8	3	3
5	10	3	4
6	12	4	4
7	14	4	5
8	15	5	6
9	17	5	6
10	20	6	7

Other parameters of experimental problems are randomly generated from uniform distributions. It should be noted that the cost of doing each task by each machine and the price of the electric signal are reported in the unit of thousand Tomans, and time parameters such as the time interval and the time required to do a task by each machine are in minutes. After reporting the initial data of the problems and the obtained results, the validation of the mathematical model and sensitivity analysis are provided in the next section.

Table 3. Values of parameters used in experimental problems

Parameters	Uniform range
Et	Uniform(0.2,0.3)
pj	Uniform(10,20)
tjk	Uniform(7,10)
Cjk	Uniform(0.6,0.9)
QM_{kt}	Uniform(10,15)
N_{kct}	Uniform(3,4)
NP_{kt}	Uniform(2,3)
En_j	Uniform(0.3,0.5)
p_{jk}	Uniform(0.3,0.5)
v_k	Uniform(0.5,0.75)
pr_r	Uniform(0.1,0.18)

The results of solving all 10 test problems using the ε-constraint method in GAMS software are reported in table 4. In this table, the second column shows the optimal values of the total completion time of tasks. The third column also represents the cost of energy. The objective of the problem, which is to minimize the total completion time of and the energy cost, is shown in table 4 for different levels of numerical problems. In Problem 1, there were two tasks needed to be done, while in Problem 2, three tasks were needed. Hence, with the increase of a task for execution, the cost of the total completion time of tasks and the energy cost of the problem increases, because with the increase of the required tasks, the amount of time required to implement all the tasks increases, and as a result, the objective function of the total completion time increases. Likewise, as the number of tasks required to be done increases, the number of machines required to be employed increases as well, leading to an increase in energy consumption.

Table 4. Results of optimal values of objective functions for different levels of experimental examples

Number of numerical problem	Total task completion time	Energy cost
1	45.236.114	12.366.125
2	47.652.156	13.654.457
3	48.346.771	15.478.399
4	51.156.358	17.456.743
5	53.645.661	21.368.943
6	55.146.459	22.145.789
7	56.789.119	24.651.115
8	57.446.478	28.412.365
9	59.759.478	31.693.145
10	61.456.781	34.145.478

Figures 4 and 5 show the results of the above table, from which it can be concluded that with the gradual increase in the dimensions of the problem, i.e. the increase in the number of tasks required for execution, the number of planning periods and the number of machines, the amount of total completion time of tasks, and also the amount of energy consumption increase gradually and with a small difference compared to the previous stages.

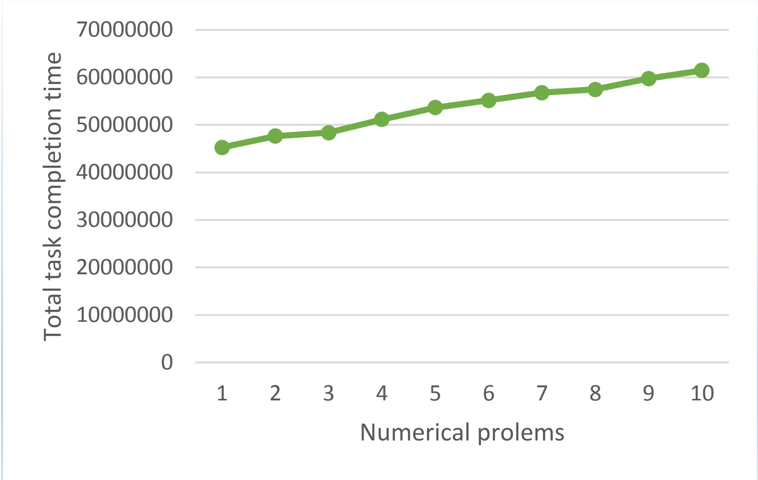

Fig 4. Comparison of the objective function value of the optimal total completion time of tasks in different numerical problems

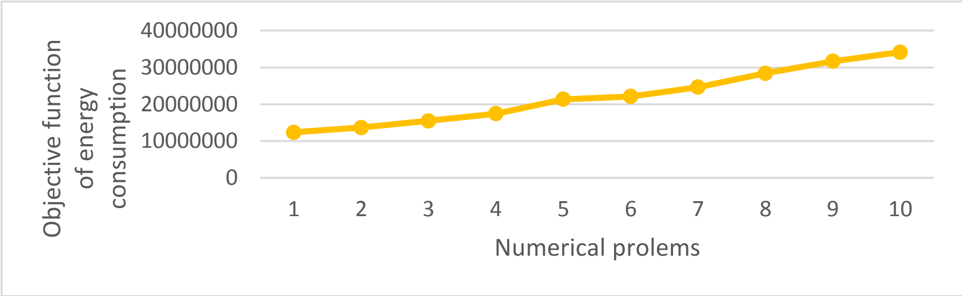

Fig 5. Comparison of the objective function value of energy consumption in numerical examples

In table 5, the variable value of the assignment of each task to each machine can be seen for each of the experimental examples. According to this table, in Problem 3, the variable value of task assignment *i* to machine k is shown. By looking at the results of the table below, it is clear that task 1 is assigned only to machine 2. On the other hand, task 2 can be assigned to machines numbers 3, 4, 5, 6, 7, and 8. Task 3 is only assigned to machine 5. Task 4 is assigned to machines 6 and 7. Task 5 can only be done by machine 8, and task 6 can be done with machines 7 and 8.

Table 5. How to assign task *i* to station j

Assignment of task 1 to Machine 2	X12=1
Assignment of task 2 to Machine 3	X23=1
Assignment of task 2 to Machine 4	X24=1
Assignment of task 3 to Machine 5	X35=1
Assignment of task 4 to Machine 6	X46=1
Assignment of task 4 to Machine 7	X47=1
Assignment of task 5 to Machine 8	X58=1
Assignment of task 6 to Machine 7	X67=1
Assignment of task 6 to Machine 8	X68=1

In the following, the variable value of allocating each task to the time unit t can be seen for each of the experimental examples. According to this table, in problem 3, the variable value of task assignment *i* to time unit t is shown. By observing the results of the table below, it is clear that task 1 is assigned only to time unit 1. On the other hand, task 2 can be assigned to time units 1 and 2. Task 3 is assigned to time units 1 and 3. Task 4 is assigned to time units 2 and 3. Task 5 can be done in time units 1 and 2.

Table 6. How to assign task *i* to time unit *t*

Assignment of task 1 to time unit 1	X11=1
Assignment of task 2 to time unit 1	X21=1
Assignment of task 2 to time unit 2	X22=1
Assignment of task 3 to time unit 1	X31=1
Assignment of task 3 to time unit 3	X33=1
Assignment of task 4 to time unit 2	X42=1
Assignment of task 4 to time unit 3	X43=1
Assignment of task 5 to time unit 1	X51=1
Assignment of task 5 to time unit 2	X52=1

4-2- Solving the problem with the meta-heuristic ICA

When working with meta-heuristic algorithms, the parameter adjustment operation is very important, because with incorrect parameter choices, the efficiency of the algorithm may decrease, and as a result, the obtained solutions are far from the optimal solutions. These parameters are performed through numerical tests and there are different methods for designing numerical tests. One of the simplest methods is to perform the test through complete factors, which can have errors because with a large number of factors, it becomes complicated to perform calculations and there would be a possibility of errors. According to Taguchi's presentations, the factors affecting parameter adjustment are generally divided into two groups of controllable factors and uncontrollable factors. In this method, optimal levels of controllable factors and reducing the effects of uncontrollable factors are desired.

Since in the proposed model, two objectives of the total task completion time and the amount of energy consumption are considered, the complexity of the model increases. Thus, to solve the model, it is better to use meta-heuristic methods in order to obtain approximately optimal solutions. In the following, the results of solving the meta-heuristic ICA are presented. To evaluate the proposed model, we use the mentioned algorithm and adjust the parameters of the algorithm so that we can solve the model using a meta-heuristic algorithm. Also, by changing the parameters of the model, we can examine the behavior of the objective functions of the model, i.e. the total task completion time and the amount of energy consumed.

4-2-1- Parameter adjustment in the meta-heuristic ICA

To adjust parameters in ICA, it is necessary to specify the parameters of this method. In ICA, there are six parameters, the maximum number of iterations (*ni*), initial population size (*np*), number of countries (*nc*), assimilation coefficient (*ac*), rotation probability (*rp*), and mean cost coefficient (*cmcc*). We use a three-level Taguchi design for the mentioned parameters, that is, three different values are considered for each parameter, as shown in the table below.

Table 7. The levels of the ICA parameters according to Taguchi method

Parameter	*ni*	*Np*	*Nc*	*ac*	*rp*	*cmcc*
Three-level values	50, 75, 100	20, 30, 40	40, 60, 80	1,2,3	0.1, 0.2, 0.3	0.1, 0.2, 0.3

To perform numerical tests in order to adjust the above parameters, Taguchi tests are run from the Stat tab and the DOE section through the MINITAB software. In this way, each test is run five times in the software to reduce the effect of their randomness. Finally, the value obtained from the mean execution of tests is considered equivalent to the value of the response level in the algorithm. Thus, the test is analyzed in the MINITAB software.

The following figures are the results of the data analysis obtained from the MINITAB software, in which the values of the parameters are given.

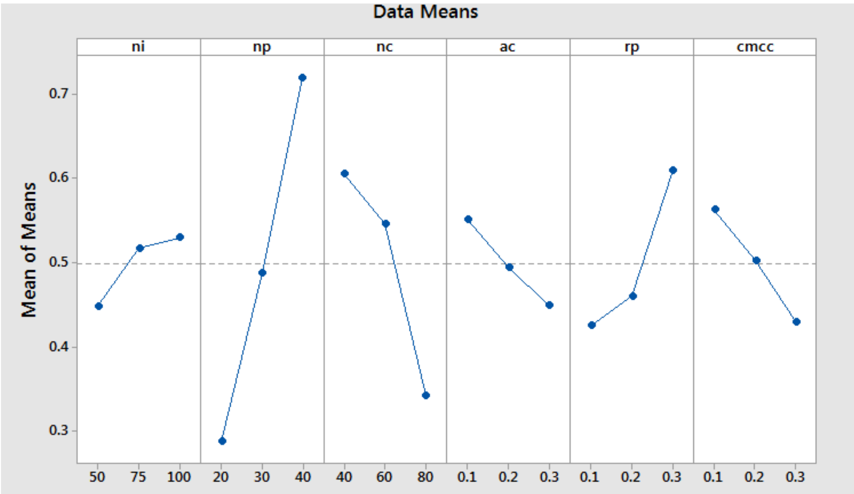

Fig 6. Analysis charts of adjusting ICA parameters in Taguchi method

Table 8. Adjusted values of ICA parameters

Algorithm (index)	*Ni*	*Np*	*nc*	*ac*	*rp*	*cmcc*
ICA	100	40	40	0.1	0.3	0.1

4-2-2- Methods for comparing the performance of algorithms in large dimensions

To measure the performance of meta-heuristic algorithms in optimization problems, there are different methods that can be described as follows:

1. **Number of Pareto solutions (NPS):** The multiplicity of solutions obtained on the Pareto front is one of the important and useful criteria for comparing meta-heuristic algorithms. It can be said that the higher the number of these solutions, the better the algorithm works.

2. **Spacing metric (SM) index:** Using this index, the uniformity of distribution of Pareto solutions in the solution space can be displayed and analyzed. This criterion is calculated as follows:

$$SM = \sqrt{\frac{1}{N-1} \times \sum_{i=1}^{n} (d_i - \bar{d})^2} \tag{24}$$

- *di* is the value of the Euclidean distance between answer *i* and the answer solution to it in the Pareto space.
- $\bar{d}$ is the mean of distances and n is equal to the number of Pareto solutions. The lower this index is, the better the performance of the algorithm.

3. **Mean ideal distance (MID):** It means the distance of the Pareto points from the ideal point, which is calculated as follows.

$$MID = \frac{\sum_{i=1}^{n} \sum_{j=1}^{3} \sqrt{(\frac{f_{ij} - f_{j}^{best}}{f_{ij}^{max} - f_{ij}^{min}})^2}}{n} \tag{25}$$

n is the number of Pareto solutions, f max and f min represent the maximum and minimum objective functions between the investigated algorithms. f best indicates the coordinate value of the ideal point for each objective function. The lower this criterion, the better the performance of the algorithm.

4. **Diversification metric (DM):** Using this criterion, the breadth of Pareto's responses can be understood and is calculated by the following equation. The higher value of this index indicates the optimal performance of the algorithm.

$$\mathrm{DM}= \sqrt{\sum_{j=1}^{3} (\frac{\max f_{ji} - \min f_{ji}}{f_{j,total}^{max} - f_{j,total}^{min}})^2} \tag{26}$$

4-3- Comparison of the performance of algorithms in large dimensions

In this section, the efficiency of the algorithm proposed in the previous section, i.e. the colonial competition algorithm, is compared and the problem is solved in the GAMS software in large-scale problems. To compare the performance of algorithms in large dimensions according to section 4.3.2, the MID criterion is used. For this purpose, to solve the problem using ICA, we used the real data of the case study of the polyethylene factory in Semnan province, so that in addition to solving the problem with large-scale data, we validated the proposed model using a real problem. Since in this research the objective is to reach the best value of the objective function and as a result the shortest distance to the ideal point, the MID index is used to compare the performance of the algorithms. In the following, the specifications of 10 large-scale numerical problems are presented, according to which they are solved and the performance of the algorithms is compared. These data were collected using a case study in a polyethylene factory in Semnan province.

Table 9. Dimensions of numerical problems in large scales

Number of numerical problem	Number of tasks required	Number of machines	Number of time periods
1	10	8	3
2	10	11	3
3	10	15	4
4	10	17	4
5	15	20	5
6	15	23	5
7	15	24	6
8	15	26	6
9	20	28	7
10	20	30	8

By solving problems in large dimensions according to the proposed methods, the resulting solutions for the objective functions of the problem are calculated and displayed.

Table 10. Results of the objective function of completion time in 10 numerical tests

Numerical problem	GAMS	ICA
1	65.453.118	62.366.321
2	69.145.366	69.0253.146
3	73.145.412	72.365.003
4	76.842.954	76.802.954
5	-	79.142.117
6	-	81.475.145
7	-	83.142.146
8	-	85.652.632
9	-	88.036.144
10	-	89.656.702

Table 11. Results of the objective function of energy cost in 10 numerical tests

Numerical problem	GAMS	GA
1	92.336.145	86.113.201
2	98.366.144	89.475.588
3	101.223.541	91.214.986
4	114.125.147	92.563.258
5	_	94.175.144
6	_	97.144.310
7	_	99.110.338
8	_	102.378.715
9	_	105.025.756
10	_	108.963.127

In table 10, 10 numerical tests are generated and for each of them, the value of the objective functions, completion time, and cost are reported in tables10 and 11. In the following, the two methods proposed in the research are analyzed and compared using the MID index.

Table 12. MID obtained from the most optimal solutions of 10 numerical tests

Numerical problem	GAMS(Cost)	GAMS(Time)	ICA(Cost)	ICA(Time)
1	1.52	1.78	1.25	1.15
2	1.69	1.92	1.65	1.53
3	1.82	2.26	1.84	1.95
4	1.75	2.45	1.61	2.16
5	1.51	2.47	1.42	2.17
6	1.45	2.11	1.42	2.94
7	1.69	1.36	1.55	1.06
8	1.85	1.47	1.89	1.32
9	1.9	1.15	1.66	1.08
10	1.19	1.46	1.26	1.52

According to table 13, it can be concluded that the value of the MID index in ICA is lower than the value of the index resulting from problem solving in the GAMS software, except for some numerical problems which it is slightly higher. With these interpretations, it can be concluded that ICA has a better performance than the GAMS software for optimizing the parallel machine scheduling problem in this research. The results of table 13 can be seen in the figure 7, which shows the superiority of ICA over the GAMS method to minimize the energy cost and task completion time.

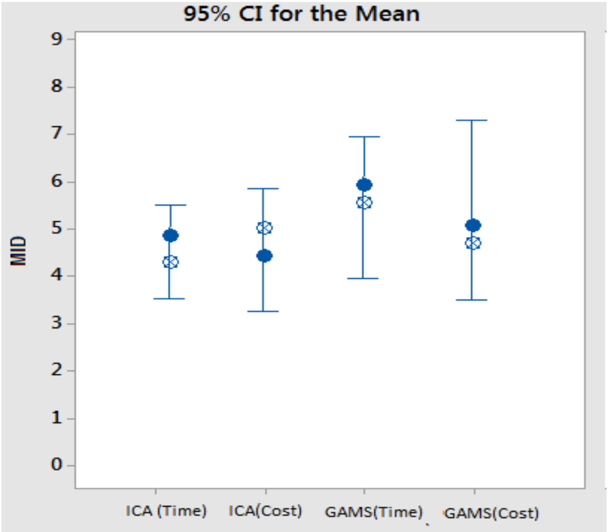

Fig 7. Interval charts of ICA and GAMS method for MID index

4-4- Numerical sensitivity analysis

In this section, in order to validate the presented model and examine the behavior of the model in more detail, by changing the parameters of the model, such as the electricity price, task processing time, task completion cost, number of processed machines, we examine the behavior of the first and second objective functions of the model, i.e. task completion time and energy cost. The chart below shows the result of the sensitivity analysis of the cost objective function relative to the changes in the parameter of the electricity signal price, and it can be concluded that as the electricity price in the problem increases, the value of the objective function of the energy costs of the problem increases.

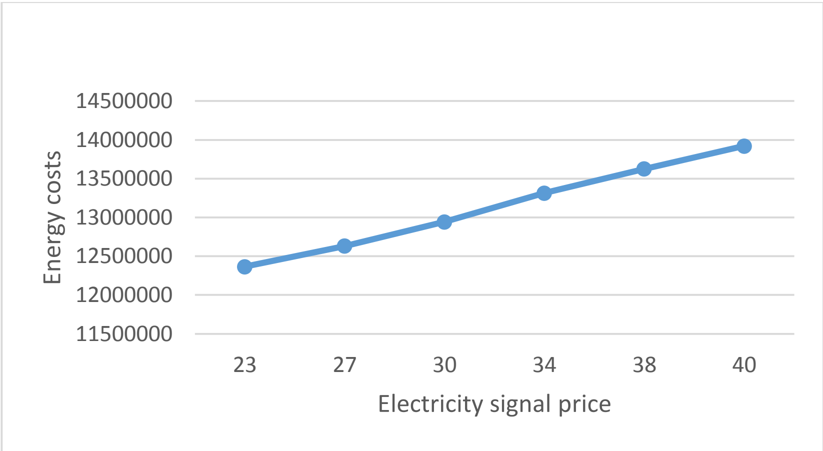

Fig 8. Sensitivity analysis chart of the objective function of the energy cost in relation to the electricity signal price

In the following, the result of the sensitivity analysis of the objective function for the task completion time in relation to the changes in the parameter of the electricity signal price is shown, and it can be concluded that with the increase in the price of electricity in the problem, the value of the objective function of the task completion time remains constant.

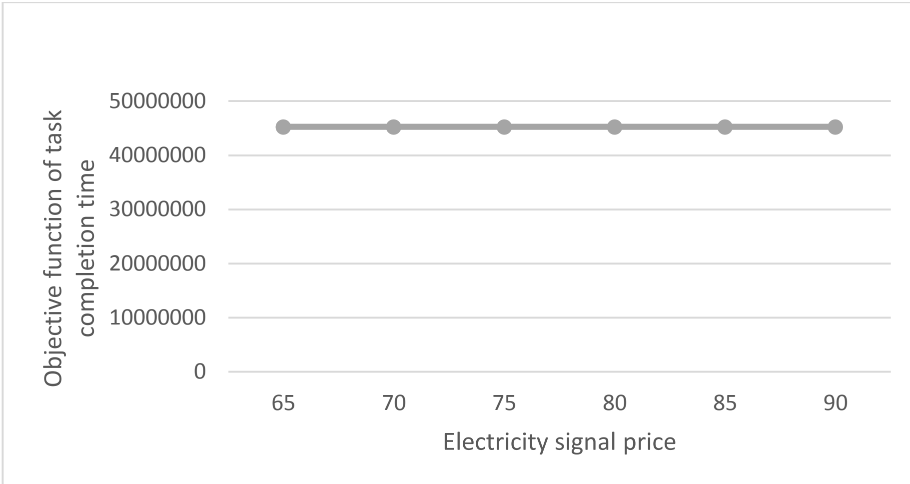

Fig 9. The sensitivity analysis chart of the objective function of task completion time in relation to the changes in the electricity price

In the following, the behavior of the objective functions in relation to the processing time of each task is examined. Figure 10 shows the value of the objective function of the task completion time in the problem for different values of the task processing time. From the chart in figure 10, it can be concluded that as the processing time of each task increases, the value of the objective function of the task completion time also increases.

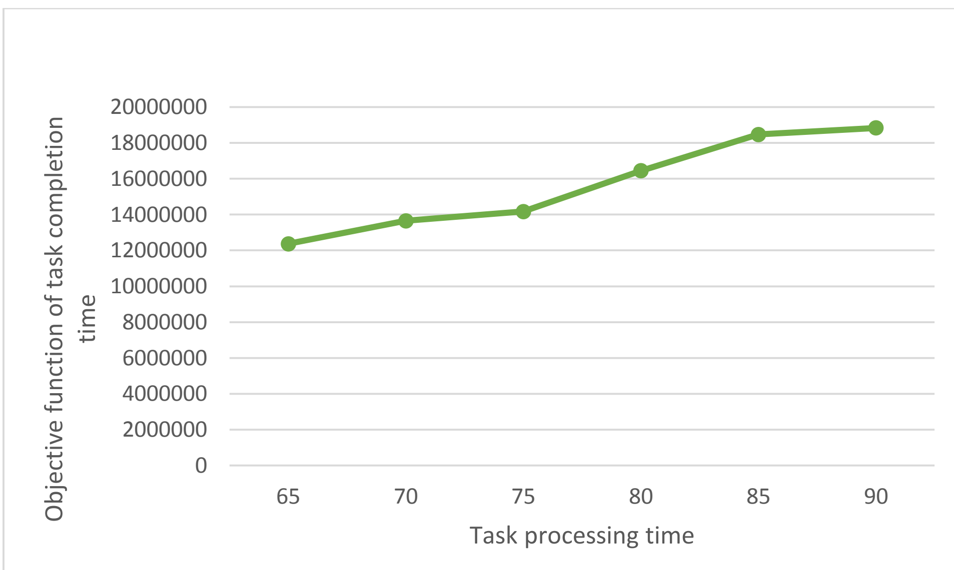

Fig 10. The sensitivity analysis chart of the objective function of the task completion time in relation to the changes in task processing time

Figure 11 shows the value of the energy cost objective function in exchange for different values of task processing time. From the chart in figure 11, it can be inferred that the value of the energy cost objective function increases as the task processing time increases.

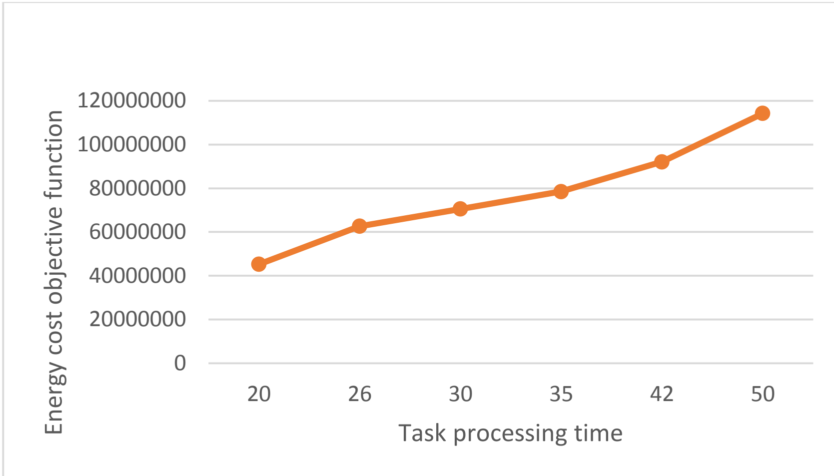

Fig 11. The sensitivity analysis chart of the energy cost objective function to the changes in the task processing time

5- Conclusion

In this research, a mathematical problem of parallel machine scheduling was provided and solved in order to minimize the total completion time and energy cost. First, the mathematical model of the problem was presented and the solution method was investigated using the ε-constraint method in the GAMS optimization software and the meta-heuristic imperialist competitive algorithm (ICA). Then, the mathematical model was implemented using the GAMS software and the solution results were provided. Also, using the meta-heuristic ICA, a real problem was implemented in large dimensions related to the case study of the polyethylene factory in Semnan province. Then, the results of both methods were examined, compared, and analyzed. For the meta-heuristic ICA, the Taguchi method and signal-to-noise value analysis were used to achieve the optimal values of the parameters in the algorithm. Finally, the performance of ICA was measured in terms of the RPI index for small dimensions and the MID index for examples with large dimensions and the results were presented in the form of tables and charts. After examining the obtained solutions, it can be concluded that the value of the mean ideal distance in ICA is lower than that of the index obtained from solving the problem in the GAMS software, except for some numerical examples, which is slightly higher. With these interpretations, it can be concluded that ICA has a better performance than the GAMS software for optimizing the parallel machine scheduling problem in this research.

Also, in order to validate the proposed model and to check the behavior of the model more closely by changing the model parameters such as task completion time, task completion cost, and electricity signal price, we investigated the behavior of the objective function of the energy cost and the task completion time in the model. According to the obtained solutions, it can be concluded that with the increase in the time to perform a task, the time to complete all tasks also increases and the cost of energy remains constant. While the cost of doing a task and the electricity signal price increase, the amount of energy costs increases and the time to complete the work remains constant. On the other hand, by increasing the parameter of the number of processed machines, the objective function of task completion time in the problem decreases, while by increasing this parameter, the energy cost increases due to the use of more machines and more energy consumption in the problem.

5-1- Suggestions for future research

For future research on the design of parallel machine scheduling problem, uncertainty in the problem parameters such as task completion time and the electric signal price can be used. This uncertainty can be of the type of robust uncertainty or fuzzy planning to bring the conditions of the designed problem closer to the conditions of the real world. It is also possible to use more constraints in the problem and consider objectives of customer satisfaction, social issues, and costs other than energy such as the cost of producing the final product. In this way, the number of assumptions and constraints of the problem increases and it can be used in real problems.

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