

A new fuzzy multi-objective model for selecting capital projects in the public sector

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Abstract

In evaluating projects, there are many qualitative criteria, weighting, and quantifying, which have no definitive nature and are associated with various ambiguities. Also, because of the relationship between these conflicting criteria (goals), no single and multip optimal solutions (non-dominant set) should be sought. Because of the relationship between these inconsistent criteria (goals), no single and multiple optimal solutions (non-dominant set) should be sought. Accordingly, this study aims to provide an appropriate approach to develop a model for selecting construction projects in the public sector based on a mathematical multi-objective fuzzy model, which can cover the multi-objective nature of the problem and consider inherent inaccuracies and problem uncertainties. This paper first converts the model to a non-linear model by fractional planning concepts, defuzzification according to Jimenez and Yang approaches, then solves by a non-dominated sorting genetic algorithm (NSGA-II) to provide a more comprehensive model for governmental project selection public when allocating budget. This paper is attempted to develop a new model for selecting construction projects while considering the uncertainty of parameters using fuzzy theory in the public sector to show the performance of the developed model. The fuzzy model solution is compared with the deterministic model to analyze the results. The results show the improvements reflect the success rate of accomplishment for the corresponding goals in the fuzzy model compared to the exact one.

Keywords: Capital project selection, fuzzy goal programming, fractional linear programming, NSGA-II algorithm

1- Introduction

In real-world markets, choosing a project is a crucial strategic decision any economic organization can confront (Tofighian & Naderi, 2015). In fact, before starting a project, organizations should identify projects they are willing to invest in, and managers should utilize tools to select and prioritize projects, which leads to an informed accord (Wirick, 2011). The latter induces issues of selecting and allocating funds to candidate projects in analyzing public policies and making strategic, political, and ideological principles affect the executive decisions (Fernandez, et al., 2013). Project selection is not associated with the financial revenue of the project regarding portfolio management in the public sector.

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Still, it undoubtedly involves public interest and some factors, such as the norms, the shareholders, the media, and all citizens, to regulate its procedure. Thus, evaluating the success of a project via indicators such as cost-benefit or return on investment is arduous. Subsequently, government executives assess the effectiveness of projects utilizing quantitative indicators (Nassif, et al., 2013).

Most available data about the projects are scarce, uncertain, and vague (Perez & Gomez, 2016). For instance, uncertain parameters affecting project management are as follows: interest rate, opportunity cost, income, inflation, and cash flow (Medaglia, et al., 2007). Although outsourcing IT projects are profitable to IT Practitioners, Decision-Makers, and leaders of businesses, the fact continues that choosing the right IT Provider is still a challenge that is destined to be circumvented as they proceed to make the mistake of choosing the wrong IT Provider through outsourcing provider choice (Orlu, 2021). Fuzzy logic, a substitute for classical logic, can react to particular real-world situations that are marked by vagueness and uncertainty (Mohagheghi, et al., 2015). The utilization of fuzzy numbers as a possible solution to fuzzy theory not only overcomes the uncertainty and vagueness issue remarkably; but also makes it easier for to estimate parameters for experts (Ebrahimnejad, et al., 2013). The project selection procedure is executed by considering various contradictory goals and objectives. Multiple methods, such as mathematical modeling, have been proposed for this end. All similar issues of real-world project selection can be evaluated by considering different criteria (Ghorbani & Rabbani, 2009). Whereas single-objective optimization searches for the optimal solution, a multi-objective problem has no single optimal solution due to the exchange of opposite objectives and therefore searches for a set of non-dominated solutions (Fonseca & Fleming, 1993). Overall the multi-objective optimization nature of project selection is unquestionable, and its application has attracted much attention (Saborido, et al., 2016). The meta-innovative methods are reputable methods for solving various types of multi-dimensional problems that form real-world issues (Ghorbani & Rabbani, 2009), and in the last two decades, multi-objective evolutionary algorithms have been successfully deployed in solving multi-objective problems since they are capable of achieving approximation for Pareto efficiency per execution (Martinez-Vega, et al., 2017).

This study aims to provide a more comprehensive model for governmental project selection public when allocating budget. In selecting the project's problem for investment in the country's public budget, there are many qualitative criteria that any quantification and weighting of them lacks a definitive nature and is associated with various ambiguities, such as the parameters obtained through interviews with experts or the value a province defined for ideals. Also, the lack of clarity in many technical specifications at the beginning of the project, such as the credit required for each project, is almost normal. In many cases, it is impossible to expect the information to be accurate. Therefore, the necessity of fuzzy modeling of the decision-making problem of this research with imprecise data was emphasized. Also, in the project selection problem in the public sector, unlike single-objective optimization in which a single optimal solution is sought, due to the exchange between conflicting objectives, there is no single optimal solution, and the non-defeated set must be searched for. In this way, the necessity of multi-objective modeling of the problem in this research and pursuing the problem in a multi-objective manner was emphasized. In general, in this research, a fuzzy multi-objective model has been developed to select construction projects in the country's public sector. Therefore, by studying the corresponding literature, we could exploit a suitable model for selecting construction projects in the public sector and develop it through appropriate modification, public acceptability, and real-world adaption. Consequently, the proper solution is proposed, and results are presented ultimately. Also, this paper is followed by literature, problem definition and mathematical model, solving model, computational results, and discussion and conclusion.

2- literature review

A little research has been conducted on project selection in the public sector. In this regard, Joiner and Drake (1983) discussed governmental planning and budgeting and presented multiple objective models (Joiner & Drake, 1983). Accordingly, Leinbach and Cromley (1983) proposed a goal-programming approach to public investment decisions; their case study consisted of rural road projects in Indonesia. Therefore it is correlated with the current study (Leinbach & Cromley, 1983). A linear goal-programming model for public-sector project selection was suggested by Benjamin (1985); it included eight different goals being considered for economic, social, political, and other purposes

(Benjamin, 1985). Two types of water and energy restrictions formed the resource constraints. This case study includes fourteen general development projects. A model for public sector design-build project selection was introduced by Molenaar and Songer (1998), which addresses the "design-build" issue of project selection in the public sector. It uses multi-attribute analysis to identify influencing factors in 122 case studies (Molenaar & Songer, 1988). This study was considered by the public sector, not decision-makers in public budgets. Vonortas and Hertzfeld (1998) examined research and development project selection in the public sector. Chapman et al. (2006) compared the projects based on the discount rate and net present value (NPV) (Chapman, et al., 2006).

The main contribution of this study is comparing projects regarding nuclear waste disposal since it provides a framework through which the discount rate can be calculated more realistic by considering the risks. Puthamont and Charoenngam (2007) discussed a framework for strategic project selection in the public sector based on the 32 criteria. It aimed to identify influential factors for construction projects for the Ministry of Defence in Thailand. Fazli and Madani (2009) introduced a model for selecting construction projects using multiple criteria decision-making and a goal programming approach (Fazli & Madani, 2009). Their case study consisted of three transport projects, and seven specific measures have been proposed for evaluating these projects. Bellos et al. (2010) also suggested a method for ranking public sector projects grounded on the network analysis process, a case study related to one of the Greek municipalities (Bellos, et al., 2010). Mohaghar et al. (2014) presented a mathematical goal-linear model for selecting construction projects in the public sector, which comprehensively examined the topic of this study (Moheghar, et al., 2014).

Carlsson et al. (2007) aimed to reduce productivity efficiency by applying fuzzy logic to R&D project portfolio selection (Carlsson, et al., 2007). Rebiasz (2007) presented a model for project risk assessment using fuzzy or random variables. Wang and Hwang (2007) practiced a fuzzy set approach for R&D portfolio selection using a real options valuation model. Qin et al. (2009) presented a mathematical model that used portfolio selection based on fuzzy cross-entropy. Tan et al. (2010) used a fuzzy TOPSIS approach to aid construction project selection. They utilized linguistic variables in the ranking of options and weight criteria. Ravanshadnia et al. (2010) presented a Hybrid fuzzy MADM project-selection model for diversified construction companies. A novel two-phase group decision-making approach for construction project selection in a fuzzy environment was suggested by Ebrahimnejad et al. (2012).

Wu and Chen (2021) suggested a structural method for policy choice in the smart city, which consists of the modified Delphi method (Wu & Chen, 2021). Issa et al. (2020) studied a strategy to help contractors assess and select proper construction plans (H.Issa, et al., 2020).

Many studies have been performed in the course on multi-objective models and their use in solving project selection issues. In this regard, Greenberg and Nunamaker (1994) suggested a multi-objective budgeting model for public sector organizations (Greenberg & Nunamaker, 1994). Doerner et al. (2004) presented the Pareto ant colony optimization as a metaheuristic approach to multi-objective portfolio selection problems (Doerner, et al., 2004). Zavadskas et al. (2021) represented a fuzzy extension of the method. Their model covers sustainability standards in the municipality (Zavadskas, et al., 2021). Shybalkina (2022) discussed a multi-objective optimization model for project selection with probabilistic considerations (Shybalkina, 2022).

In addition, the study background assumes that budget-based public capital projects as the primary and limiting issue, as it was a recurrent vagueness through prior studies. It is fundamental to assess the extent of consistency between this assumption and the real-life portfolio selection in public budgeting (Youssef, et al., 2023). Therefore, the necessity of fuzzing the problem of selecting Capital projects in the public sector has been examined and emphasized in this study. Another setback of previous studies employed techniques when confronting non-linear models (Dong, et al., 2023). It treated non-linear models by practicing approximate (inventive and metaheuristic) methods or conversing non-linear objective functions to linear relations through initial and approximate changes. Both approaches face an approximation of the reasonable space, which will be inaccurate. Therefore, the current study affirms the mathematical methods required to deal with such issues (Roozkhosh, 2022). Thus, the present study investigates an appropriate approach for selecting Capital projects in the public sector based on a fuzzy multi-objective mathematical model, which can cover both the multi-dimensional nature of the problem and addresses the inherent inaccuracy of the problem for the given reasons.

3 - Problem definition

3-1- Assumptions and model structure

The following five sections shape the structure of this study:

- a. Designing a model for selecting Capital projects in the public sector based on what Mohaghar et al. proposed
- b. Discussion regarding the model mentioned
- c. The research procedure
- d. Results of model solving
- e. Discussion
 1. Designing a model for selecting capital projects in the public sector based Mohaghar model

The proposed model by Mohaghar et al. (2014) (Moheghar, et al., 2014) has been elected as the base developmental model. It is based on the following criteria:

- a. All the projects are required to obtain an environmental license.
- b. All projects are required to obtain a passive defense license.
- c. The total productivity, the better.
- d. It is better to have standard project numbers of organizations (PNOs).
- e. It is better to have standard project numbers of states (PNS).
- f. It is better to have projects more standard in project number of the budget chapters (PNC).
- g. The closer the project Credit Sum of the Organization (CSO) to its standard, the better.
- h. The closer the project Credit Sum of State (CSS) to its standard, the better.
- i. The closer the project Credit Sum of Budget Chapter (CSC) to its standard, the better.
- j. The Credit Sum of All Projects (SAP) should not exceed the total budget allocated to development projects.
- k. Any scheme obtaining 30% or more Physical Progress of Scheme (PPS) must be selected.
 1. Any project from selected schemes obtaining 30% or more Physical Progress of the Project (PPP) must be selected.
- m. The minimum allocated credit to each selected project is 40 percent.

The mathematical model of this study is as follows: Indicators, parameters, and model variables are shown in table 1.

Table 1. the indexes, parameters, and variables of the model

Indexes			
Designs number	n	Provinces number	K
Devices number	j	Programs number	L
Number of organization	J	Projects number	I
Number of Budget Chapters	L	Number of States	K
		Number of Proposed Projects	I
Input parameters			
Environmental License Status of Scheme i .	$ELSP_i$	Total Allocated Credit to capital projects	TAC
Project Number of Scheme n .	I_n^S	Physical Progress of Scheme	PPS_n
Passive Defense License Status of Scheme m	DLS_i^P	Physical Progress of Project i .	PPP_i
Productivity Index of Total	PROIT	Physical Progress of Scheme Selection	MPPSS
Set of projects for organization j .	Set_j^{OP}	Physical Progress of Scheme Selection.	MAPPPS
The standard for project number of organization j	S_j^{PNO}	Minimum allocated percentage for selecting a project	MABPBC
The standard for project number of state k	S_k^{PNS}	Set of projects for state k .	Set_k^{SP}
The standard for project number of budget chapter I	S_l^{PNC}	Set of projects for budget chapter I	Set_l^{CP}
The standard for project credit sum of the organization j ratio to the total allocated credit	S_j^{CSO}	Average weight of productivity index of total	W_{PROIT}
The standard for project credit is the sum of the state k ratio to the total allocated credit	S_k^{CSS}	Deviation ratio average weight of project number of organization	W_{PNO}
The standard for project credit sum of budget chapter i ratio to total allocated credit	S_l^{CSC}	Deviation ratio average weight of project number of state	W_{PNS}
Set of projects for scheme m .	Set_n^{SMP}	Deviation ratio average weight of project number of budget chapter	W_{PNC}
Credit for a running project I in the current year	PC_i	Deviation ratio average weight of project credit sum of organization	W_{CSO}
Deviation ratio average weight of project credit of budget chapter	W_{csc}	Deviation ratio average weight of project credit sum of state	W_{CSS}
Decision variables			
Statue of selecting n scheme from the zero and one variables	y_n^S	The negative deviation of project number of budget chapter I from the standard.	d_l^{-PNC}
Statue of selecting i scheme from the zero and one variables	y_i^P	The positive deviation of project credit sum of organization j from the standard.	d_j^{+CSO}
The percentage of which the project i is allocated from in the year planned.	x_i^P	The negative deviation of project credit sum of organization j from the standard.	d_j^{-CSO}
The positive deviation of Project Number of Organization j from the standard.	d_j^{+PNO}	The positive deviation of project credit sum of state k from the standard.	d_k^{+CSS}
The negative deviation of Project Number of Organization j from the standard.	d_j^{-PNO}	The negative deviation of project credit sum of state k from the standard	d_k^{-CSS}
The positive deviation of Project Number of State k from the standard.	d_k^{+PNS}	The positive deviation of project credit sum of budget chapter I from the standard	d_l^{+CSC}
The negative deviation of Project Number of State k from the standard.	d_k^{-PNS}	The negative deviation of project credit sum of budget chapter I from the standard	d_l^{-CSC}
The positive deviation of Project Number of Budget Chapter I from the standard.	d_l^{+PNC}		

4- Mathematical model

As mentioned, the mathematical model of the model according to the variables and parameters defined in table 1 is expressed as follows.

$$\begin{aligned}
& \text{Max } Z = w_{PROIT} \cdot PROIT \\
& -w_{PNO} \cdot \frac{\sum_{j=1}^J \frac{(d_j^{+PNO} + d_j^{-PNO})}{S_j^{PNO}}}{J} - w_{PNS} \cdot \frac{\sum_{k=1}^K \frac{(d_k^{+PNS} + d_k^{-PNS})}{S_k^{PNS}}}{K} - w_{PNC} \cdot \frac{\sum_{l=1}^L \frac{(d_l^{+PNC} + d_l^{-PNC})}{S_l^{PNC}}}{\hat{L}} \\
& -w_{CSO} \cdot \frac{\sum_{j=1}^J \frac{(d_j^{+CSO} + d_j^{-CSO})}{S_j^{CSO}}}{J} - w_{CSS} \cdot \frac{\sum_{k=1}^K \frac{(d_k^{+CSS} + d_k^{-CSS})}{S_k^{CSS}}}{K} - w_{CSC} \cdot \frac{\sum_{l=1}^L \frac{(d_l^{+CSC} + d_l^{-CSC})}{S_l^{CSC}}}{L}
\end{aligned} \tag{1}$$

$$\text{for all } n: \left\{ y_n^S \cdot \sum_{i \in \text{Set}_n^{SMP}} ELS_i^P = y_n^S \cdot I_n^S \right. \tag{2}$$

$$\text{for all } n: \left\{ y_n^S \cdot \sum_{i \in \text{Set}_n^{SMP}} DLS_i^P = y_n^S \cdot I_n^S \right. \tag{3}$$

$$\text{for all } j: \left\{ \sum_{i \in \text{Set}_j^{OP}} y_i^P - d_j^{+PNO} + d_j^{-PNO} = S_j^{PNO} \right. \tag{4}$$

$$\text{for all } k: \left\{ \sum_{i \in \text{Set}_k^{SP}} y_i^P - d_k^{+PNS} + d_k^{-PNS} = S_k^{PNS} \right. \tag{5}$$

$$\text{for all } l: \left\{ \sum_{i \in \text{Set}_l^{CP}} y_i^P - d_l^{+PNC} + d_l^{-PNC} = S_l^{PNC} \right. \tag{6}$$

$$\text{for all } j: \left\{ \sum_{i \in \text{Set}_j^{OP}} (x_i^P \cdot PC_i) - d_j^{+CSO} + d_j^{-CSO} = S_j^{CSO} \right. \tag{7}$$

$$\text{or all } k: \left\{ \sum_{i \in \text{Set}_k^{SP}} (x_i^P \cdot PC_i) - d_k^{+CSS} + d_k^{-CSS} = S_k^{CSS} \right. \tag{8}$$

$$\text{for all } l: \left\{ \sum_{i \in \text{Set}_l^{CP}} (x_i^P \cdot PC_i) - d_l^{+CSC} + d_l^{-CSC} = S_l^{CSC} \right. \tag{9}$$

$$\sum_{i=1}^I (x_i^P \cdot PC_i) = TAC \tag{10}$$

$$\text{for all } n: \{ \text{forall } i \in \text{Set}_n^{SP}: \{ y_n^S - y_i^P \geq 0 \} \tag{11}$$

$$\text{for all } n: \left\{ \sum_{i \in \text{Set}_n^{SMP}} (y_i^P) - y_n^S \geq 0 \right. \tag{12}$$

$$\text{for all } n: \{ PPS_n - M y_n^S \leq MPPSS \} \tag{13}$$

$$\text{for all } n: \{ \text{forall } i \in \text{Set}_n^{SP}: \{ PPP_i - M y_i^P \leq MAPPPS + M(1 - y_n^S) \} \tag{14}$$

$$\begin{cases} x_i^P + M(1 - y_i^P) \geq MAPBPC \\ x_i^P \leq M y_i^P \end{cases} \tag{15}$$

$$\text{for all } n: \{ y_n^S = 0.1 \} \tag{16}$$

$$\text{forall } i: \begin{cases} y_i^P = 0.1 \\ 0 \leq x_i^P \leq 1 \end{cases} \tag{17}$$

The objective function of this mathematical model is equation (1), which seeks maximization. It is derived from the reasonable determination of various objectives of the problem, which are combined in

a goal weight in the form. But given that the prerequisite of the combination of objective functions is the coherence of their domain, goals are balanced before combining. This equation involves seven parts (Roozkhosh & Motahari Farimani, 2022). The first part seeks to maximize productivity following the c criterion. Based on criteria d to i, the six following sections represent the ideals of the problem and seek to maximize the deviations from them with a negative coefficient representing minimized deviations from the stated goals. Equations (2) and (3) are limitations covering environmental and passive defense license requirements per criteria a and b, respectively. Equations (4) to (9) are goal limitations based on criteria d to i. Equation(10) confirms the budget limitations based on criterion j.

Equations. (11) and (12) indicate "if no project is selected, then none of the scheme projects can be selected" and "a project can be selected if at least one of the subset projects of that scheme is selected as well," respectively. Likewise, equations (13) and (14), according to the k and l criterion, signify the modeling limitations related to thresholds of physical progress of the scheme and project. By the modeling criterion, inequality (15) represents the minimum credit allocated to each project selection. Finally, both equations (16) and (17) equations determine the variations range of variables.

4-1- Discussion on Mohaghar et al. model

The Delphi method helped identify the criteria in this model. In addition to establishing interviews, governmental documents have also been analyzed. Having multiple numbers of prominences is reassuring that this model has done desirable and precise modeling:

- Compared to prior experiences, it covers the majority of public limitations for selecting Capital projects (Carlsson, et al., 2007), (Deb, 2008).

- In addition to issuing general limitations, it also includes the constraints imposed by the state laws in a country (Iran). Specific constraints that are caused by government laws in a country, such as the percentage of physical progress of a plan or project in such a way that if it exceeds a certain level, investment in that plan or project will be mandatory, or possible standards, such as the total amount of projects in a province, or the chapter of the budget program in such a way that the closer the total amount of the province's projects, and or a chapter of the program is to its standard, the better.

I. Many unrealistic assumptions in mathematical models are considered to cause nonconformity with the real world. To name a few, we can point out the following: the identical accuracy of the level of goals, the proper limitation values, and in general, the modeling parameters. For example, facing a majority of decision-making problems involving a mathematical solution, the decision-maker can not precisely determine the values of the problem's coefficients, leading to ambiguity. Experts generally determine informal mathematical models, the coefficients of the decision-making problems with exact values. Still, in fuzzy environments, this assumption is unrealistic (Hejazi & Roozkhosh, 2019). Therefore, in real decision-making problems dealing with inaccurate data, using the fuzzy modeling scheme deems to be appropriate and logical. In this model, the issue of selecting a portfolio of capital projects according to the public budget is one of the restrictive and basic assumptions, even if all the parameters of the model are accurate. The consistency of assumption with the real-time situation is of fundamental importance. Thus, this study has reviewed the necessity of implementing a fuzzy design in capital project selection. Regarding the different yields of construction projects, experts' opinions have been used in the basic model. For this purpose, experts were asked to give their opinion on the returns of various projects and plans in the form of a spectrum from 1 to 10, where 1 means "very little" and 10 means "very much." Then, the average of experts' opinions was calculated and used as yield values. In addition, the "quantitative objectives of the plan" mentioned in the bill's text for each project were provided to the experts as a guide.

- As mentioned, many parameters are obtained through interviews with experts when encountering project selection, such as the effectiveness of designs, which is a set of several parameters determined through scoring. Given the nature of the parameters, it would be ambiguous to allocate a precise amount; then, it would be better to use verbal expressions (linguistic variables) rather than scoring these parameters. Since there are several linguistic variables in current research, to make more precise decisions based on the actual real-life problem, developing an appropriate approach for implementing this inaccuracy into the decision-making model is of paramount importance.

➤ Various criteria (d to i) are of an ideal type, and a threshold value is appointed to them. For example, it is better to have standard project numbers of states (PNS). The PNS is not an exact amount necessary, so that it can be expressed in terms of verbal (low, medium, and high) periods. The closer the project credit sum of the state (CSS) to its standard, the better; this value is not accurate and needs to be fuzzy properly since the credit sum of one-year projects is not a standard for the credits in a state.

➤ Another discussion targets the weighting of the criteria. For instance, productivity criterion c is an amalgamation of several other parameters that are weighted by experts and can also be considered fuzzy values.

➤ There are some exceptional criteria, such as (k and l). To give you an idea, any scheme obtaining 30% or more Physical Progress of Scheme (PPS) must be selected. Nevertheless, some cases can be overlooked under certain circumstances based on the financial regulations overseeing the budget preparation and adjustment (adjusted annually). Therefore, the threshold value of these criteria is also fuzzy.

II. Total Productivity (PROIT) is acquired from the following equation for the Mohaghar objective equation (1).

$$\text{PROIT} = \frac{\sum_{\text{all } n} y_n^S \cdot \text{PROIS}_n}{\sum_{\text{all } n} y_n^S} \quad (18)$$

"PROIS_n" is the productivity value for n scheme. As can be seen in equation (18), the presence of the variable y_n^S in the fraction denominator made the model non-linear. In this model, the non-linear objective function has transformed the model into a linear one via initial and approximate changes. Through the adoption of such methods, approximations of the justified space emerge that is certainly inaccurate.

III. Despite the comprehensive approach to the practical criteria for Capital projects selection in the public sector, this model's weaknesses are grounded in its modeling technique. It issues several objectives as a whole single issue despite the problem being of a multi-objective nature. Unlike single-objective optimization, in which the optimal solution of a unit is searched, there is no single optimal answer due to the exchange of conflicting goals in multi-objective problems. Thus, it requires searching for a set of non-recursive responses.

4-2- The research process

This research aims to develop an essential, linear, ideal, and complex integer-programming model and then utilize an appropriate problem-solving method. This is a deterministic (Crisp) model, which through a development process, has converted from linear to non-linear, altered to multi-objective, and finally changed from deterministic to fuzzy mode. The diagram figure 1 shows the stages of the research.

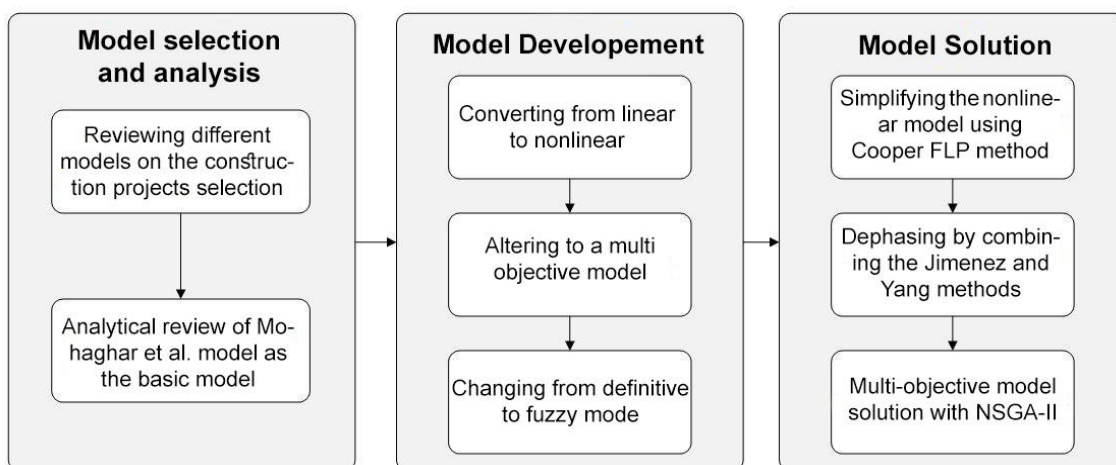


Fig 1. Research process

4-3- Case study

The sample study of this research includes the Iranian public sector. The data provided by Mohaghar et al. (2014) has been used to implement the developed current model and compare it with the identical model of the research history (Moheghar, et al., 2014). These refer to the budget bill of 2020, consisting of 136 projects out of 28 schemes and 16 budget chapters for 56 national and provincial organizations. State projects comprise 20 different provinces across the country. The sum of credit regarding these projects equals 13,733,845 million Rials. It can be inferred that the available budget is 8,952,999 million Rials.

4-4- Model development

4-4-1- Converting the model from linear to non-linear mode

As discussed earlier, the non-linear objective function transforms the model into linear relationships by applying initial and approximate changes to a completely linear model. The objective function is deemed to be non-linear according to equation (19).

$$Max f_1 = \frac{\sum_{all n} y_n^S \cdot PROIS_n}{\sum_{all n} y_n^S} \quad (19)$$

4-4-2- Altering to multi-objective model

Based on equation (20) of the Mohaghar et al. (2014) study model, seven objective functions are deduced.

$$Min f_2 = \sum_{for\ all\ j} (d_j^{+PNO} + d_j^{-PNO}) \quad (20)$$

$$Min f_3 = \sum_{for\ all\ k} (d_k^{+PNS} + d_k^{-PNS}) \quad (21)$$

$$Min f_4 = \sum_{for\ all\ l} (d_l^{+PNC} + d_l^{-PNC}) \quad (22)$$

$$Min f_5 = \sum_{for\ all\ j} (d_j^{+CSO} + d_j^{-CSO}) \quad (23)$$

$$Min f_6 = \sum_{for\ all\ k} (d_k^{+CSS} + d_k^{-CSS}) \quad (24)$$

$$Min f_7 = \sum_{for\ all\ l} (d_l^{+CSC} + d_l^{-CSC}) \quad (25)$$

Equation (19) Maximizing productivity

Equation (20): Minimizing the sum of deviations from the PNO goals.

Equation (21): Minimizing the sum of deviations from the PNS goals.

Equation (22): Minimizing the sum of deviations from the PNC goals.

Equation (23): Minimizing the sum of deviations from the CSO goals.

Equation (24): Minimizing the sum of deviations from the CSS goals.
 Equation (25): Minimizing the sum of deviations from the CSC goals.

The suggested multi-objective model of Mohaghar et al. (2014) has been resolved by illustrating the problem's features. This single-objective model is solved according to one of the objective functions and GOM software. Table 2 demonstrates the resulting data for the optimal solution to the example by considering each target function. A target function in each column of this table addresses the optimal answer. For instance, in the first column, the objective function of maximizing productivity shows the optimal answer.

Table 2. Manifesting the competition between the target functions in the responses

MIN F7	MIN F6	MIN F5	MIN F4	MIN F3	MIN F2	MAX F1	objective function
0.569	0.569	0.569	0.569	0.569	0.569	0.569	F1
95	97	94	94	98	92	95	F2
57	54	57	57	53	57	57	F3
106	110	107	105	111	107	108	F4
123.079	125.079	122.079	122.079	126.079	122	123.079	F5
69.245	66.245	69.245	69.245	66.989	69.245	70.245	F6
113.113	117.113	114.113	117.113	118.113	114.113	115.113	F7

According to table 2, the responses manifest the competition between the target functions. This implies optimizing a target function provokes other objective functions to distance themselves from their optimal value. Thus, this rivalry will result in the development of some non-recursive responses for each issue.

4-4-1- Modifying the model from crisp to fuzzy mode

In cases where no sufficient data is present, gathering them is arduous, or when they are available as phrases and linguistic and subjective variables, implementing fuzzy logic appears appropriate. Based on the nature of the criteria (source and method of gathering information) discussed in the previous section and its mathematical modeling, this section establishes the necessity of applying fuzzy logic to decision-making parameters, and the proposed solution is practical for its assessment. This includes four stages, each containing one question about the parameter. If the response is "negative," the parameter has a fuzzy nature; otherwise, the parameter is precise. Remember that the third stage is a separate process that repeats the process from the beginning. These steps are shown in figure 2.

By practicing this procedure, we found out that there are three types of fuzzy parameters:

1. Parameters that have fuzzy components, such as productivity.
2. Parameters with deterministic components, such as the standard number of state projects.
3. Parameters with data that is unavailable or can not be collected, such as the minimum percentage of allocated credit to each project. Replacing the fuzzy parameters converted the modified model as follows:

$$Max f_1 = \frac{\sum_{all n} y_n^S \cdot \overline{PROIS}_n}{\sum_{all n} y_n^S} \quad (26)$$

$$(20) - (25) \\ for\ all\ j: \begin{cases} \sum_{i \in Set_j^{OP}} y_i^P - d_j^{+PNO} + d_j^{-PNO} \cong S_j^{PNO} \end{cases} \quad (27)$$

$$for\ all\ k: \begin{cases} \sum_{i \in Set_k^{SP}} y_i^P - d_k^{+PNS} + d_k^{-PNS} \cong S_k^{PNS} \end{cases} \quad (28)$$

$$\text{for all } l: \left\{ \sum_{i \in \text{Set}_l^{CP}} y_i^P - d_l^{+PNC} + d_l^{-PNC} \cong S_l^{PNC} \right. \quad (29)$$

$$\text{for all } j: \left\{ \sum_{i \in \text{Set}_j^{OP}} (x_i^P \cdot \widehat{PC}_i) - d_j^{+CSO} + d_j^{-CSO} \cong S_j^{CSO} \right. \quad (30)$$

$$\text{or all } k: \left\{ \sum_{i \in \text{Set}_k^{SP}} (x_i^P \cdot \widehat{PC}_i) - d_k^{+CSS} + d_k^{-CSS} \cong S_k^{CSS} \right. \quad (31)$$

$$\text{for all } l: \left\{ \sum_{i \in \text{Set}_l^{CP}} (x_i^P \cdot \widehat{PC}_i) - d_l^{+CSC} + d_l^{-CSC} \cong S_l^{CSC} \right. \quad (32)$$

$$\sum_{i=1}^I (x_i^P \cdot \widehat{PC}_i) = TAC \quad (33)$$

(11) & (12)

$$\text{for all } n: \{PPS_n - My_n^S \leq \widehat{MPPSS}\} \quad (34)$$

$$\text{for all } n: \{\text{forall } i \in \text{Set}_n^{SP}: \{PPP_i - My_i^P \leq \widehat{MAPPPS} + M(1 - y_n^S)\}\} \quad (35)$$

$$\left\{ \begin{array}{l} x_i^P + M(1 - y_i^P) \geq \widehat{MAPBPC} \\ x_i^P \leq My_i^P \end{array} \right. \quad (36)$$

(16) & (17)

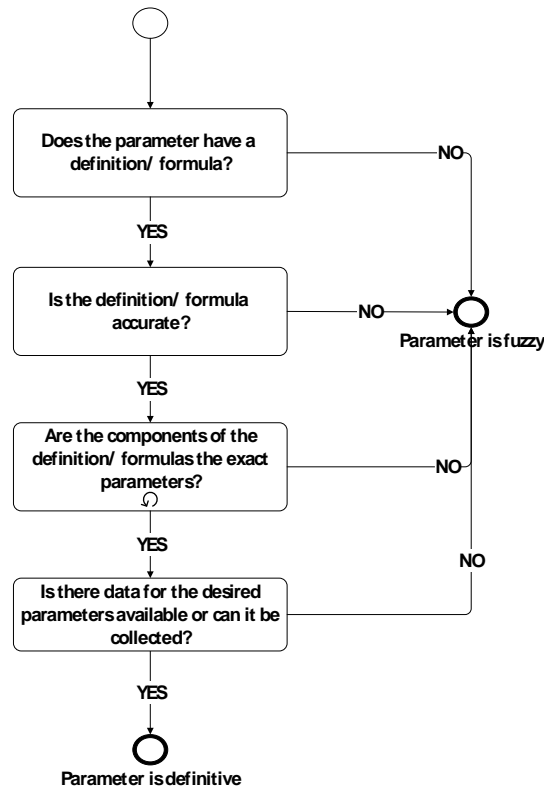


Fig 2. Necessity of applying fuzzy logic on parameters of decision-making

In equation (26), the efficiency of each scheme is acquired from the multiplication of the scheme's effectiveness with its efficiency (inaccurate). The productivity parameter of the "n" scheme is fuzzy. The effectiveness is a blend of the five indicators, such as the Technical Return of Project (TRP),

Economic Return of Scheme (ERS), Social Return of Scheme (SRS), Political Return of Scheme (PRS), and Financial Return of Scheme (FRS). Likewise, efficiency is a blend of the five indicators, such as the Environmental Index of Project (EIP), Passive Defense Index of Scheme (DIS), Performance Score of Organization (PSO), and Performance Score of State (PSS). The experts compile these indicators. In equations (27) to (32), since the definition of these goals is subjective and does not have a single, accurate description, then the sign \cong Represents a fuzzy goal. For instance, in equation (10), the number of selected projects in the j state ($\sum_{i \in \text{Set}_j^{\text{OP}}} y_i^P$) must be roughly equal to the standard number of projects for j organization (S_j^{PNO}). The required credit for implementing the project i in the current year is a fuzzy parameter based on the limitations from equations (30) to (33); this is mainly because of the inaccuracy of project components even though there is a precise definition for the cost of a project. In a similar case, when an organization evaluates the cost of a project, it should notice parameters such as project ratio, risks, human resources, and availability of raw materials (Modares, et al., 2022). Since none of the above parameters are accurate, then the required credit for implementing a project is fuzzy. The minimum physical progress of the project, scheme, and minimum allocated credit for project selection are fuzzy parameters presented in equations (34), (35), and (36), respectively. There is no data for their exact definitions. Thus, the values of these parameters are determined verbally and of a fuzzy nature. Another significant matter includes the input parameters of the problem. Whether all the parameters in the (Moheghar, et al., 2014), study are accurate, in this study, we considered all fuzzy parameters as fuzzy triangular numbers. It is noteworthy to know the reason for this decision is its high computational efficiency and ease of collecting the required information. In order to generate triangular fuzzy parameters, it is required to specify three critical values for each parameter for the most pessimistic, most probable, and most optimistic. For this reason, the exact values of each parameter were considered the "most probable" (c^m) for each of the fuzzy parameters in the study (Nassif, et al., 2013). Moreover, to maintain the model's integrity, a "symmetric triangular distribution" is designed for the fuzzy parameters of the problem. In order to conclude, the "most cynical" (c^p) and "most optimistic" (c^o) values of each of the fuzzy parameters, their most probable value is multiplied by a number proportional to their parameter. The expert's ideas helped indicate these multiplied numbers for each parameter separately.

5- Results

Considering the nature of the model, which is a multiobjective-nonlinear-fuzzy model, the proposed model solution is performed in three stages: linearization, defuzzing, and multi-objective.

5-1- Linearization

The objective function number (26), maximization of productivity, has led to the non-linearization of the model due to the existence of a variable in the fraction denominator. Several approaches are adoptable in such cases; for instance, we can solve the model by non-linear methods with the help of approximate techniques (innovative and meta-innovative) or similar to Mohaghar (2014), change the non-linear objective function model into a linear relationship with the initial and approximate modifications. Both approaches cause inaccurate approximations of reasonable space (Moheghar, et al., 2014). Accordingly, since the current model resembles the linear-deficit model, we have implemented the same acquire approaches used before. Linear-Deficit Programming (FLP¹) is a problem with a target function as a fraction. Until now, several ways to solve this problem have been established. Charles and Cooper introduced an alternate method that transforms this into an equivalent linear program (Odior, 2012). In this method, the transformation can be used, provided that the denominator of the objective function fraction is *positive* at all available space points in equation (38).

$$\text{Max } f(x) = \frac{cx + c_0}{dx + d_0}$$

$$S \cdot t \cdot \quad Ax \leq b \tag{37}$$

$$x \geq 0$$

If we suppose the programming problem is a fraction such ($x = \frac{y}{t}$) then by changing the variable ($t = \frac{1}{dx+d_0}$), it would change into a linear program with associated limitations.

$$\text{Max } Z = cy + c_0t$$

$$S \cdot t \cdot \quad Ay - bt \leq 0$$

$$dy + d_0t = 1 \tag{38}$$

$$y, t \geq 0$$

Considering the first objective function of problem (26), the fraction denominator will always be a positive non-zero value due to the nature of the problem, its limitations on the selected number of schemes, and the percentage of physical progress above 30%. Therefore by altering two variables such as ($t = \frac{1}{\sum_{all\ n} y_n^S}$) and ($Z_n^S = ty_n^S$) and substitute them in the problem, the first function would transform as non-linear. As a result, all the other constraints, including (y_n^S) variable needs revision appropriately. Multiplication of variables (y_i^P) and (t) led to changes in some variables to non-linear ones after revision. Since the range of variables (y_i^P) and (t) are (0, 1), (0-1) respectively, it is plausible to use an auxiliary variable in equation (39).

$$t(y_i^P) = k_i^P \tag{39}$$

Then by applying the following constraints to the original model, the new variable can be defined in the application in equations (40-43).

$$k_i^P \geq 0 \tag{40}$$

$$k_i^P \leq y_i^P \tag{41}$$

$$k_i^P - (M(1 - y_i^P)) \leq t \tag{42}$$

$$k_i^P + (M(1 - y_i^P)) \geq t \tag{43}$$

When the value of the variable (y_i^P) is zero, the constraints such as equations (42) and (43) are eliminated, and the variable (k_i^P) is equal to zero. And if the value of the variable (y_i^P) is equal to one, the value of the variable (k_i^P) will be equal to (t) accordingly. Thus, using a two-step approach which is a variable shift in Cooper's approach for linear-deficit programming, and a linearization approach based on the definition of auxiliary variables and increasing the model constraints, the non-linear model will convert into a linear model ultimately.

5-2- Defuzzification

As mentioned earlier, this is a multi-objective fuzzy model research with two main undefined components. The first is fuzzy coefficients in goals, and the second is fuzzy goals. Therefore, model defuzzification, giving fuzzy sets and corresponding membership accurate values, requires two approaches corresponding to the fuzzy parts of the problem (Modares, et al., 2022). Because of its high-performance attributes, we utilized the Giménez approach in dealing with non-specific coefficients in the objective constraints and functions. This method is grounded in concepts of "expected interval" and "expected value," which was first introduced in Yager's work (Yang, et al., 1991). When confronted with goal-fuzzy orientated models, we employed the Yang method (Zadeh, 1965), which first minimizes the set of degrees for goals, then selects the maximum criteria among the minimized set. At this time, we try to convert the "multi-objective, non-linear fuzzy model" into a "linear, accurate multi-objective model" via the Cooper method (linearization) and the Giménez and Yang methods (Defuzzing).

$$\text{Max } f_1 = \sum_{\text{all } n} Z_n^S \cdot \text{EV}(\text{PROIS}_n) \quad (44)$$

$$\text{Max } f_2 = \lambda \quad (45)$$

$$\text{for all } j: \lambda \leq 1 - \frac{\sum_{i \in \text{Set}_j^{\text{OP}}} y_i^P - S_j^{\text{PNO}}}{d_{j2}^{\text{PNO}}} \quad (46)$$

$$\text{for all } j: \lambda \leq 1 - \frac{\sum_{i \in \text{Set}_j^{\text{OP}}} y_i^P - S_j^{\text{PNO}}}{d_{j1}^{\text{PNO}}} \quad (47)$$

$$\text{for all } k: \lambda \leq 1 - \frac{\sum_{i \in \text{Set}_k^{\text{SP}}} y_i^P - S_K^{\text{PNS}}}{d_{k2}^{\text{PNS}}} \quad (48)$$

$$\text{for all } k: \lambda \leq 1 - \frac{S_K^{\text{PNS}} - \sum_{i \in \text{Set}_k^{\text{SP}}} y_i^P}{d_{k1}^{\text{PNS}}} \quad (49)$$

$$\text{for all } l: \lambda \leq 1 - \frac{\sum_{i \in \text{Set}_l^{\text{CP}}} y_i^P - S_l^{\text{PNC}}}{d_{l2}^{\text{PNC}}} \quad (50)$$

$$\text{for all } l: \lambda \leq 1 - \frac{S_l^{\text{PNC}} - \sum_{i \in \text{Set}_l^{\text{CP}}} y_i^P}{d_{l1}^{\text{PNC}}} \quad (51)$$

for all $j: \lambda$

$$\leq 1 - \frac{\sum_{i \in \text{Set}_j^{\text{OP}}} (x_i^P \cdot [(1 - \alpha)E_2^{\text{PC}_i} + \alpha E_1^{\text{PC}_i}]) - S_j^{\text{CSO}}}{d_{j2}^{\text{CSO}}} \quad (52)$$

for all $j: \lambda$

$$\leq 1 - \frac{S_j^{CSO} - \sum_{i \in \text{Set}_j^{OP}} (x_i^P \cdot [(1 - \alpha)E_2^{PCi} + \alpha E_1^{PCi}])}{d_{j1}^{CSO}} \quad (53)$$

for all $k: \lambda$

$$\leq 1 - \frac{\sum_{i \in \text{Set}_k^{SP}} (x_i^P \cdot [(1 - \alpha)E_2^{PCi} + \alpha E_1^{PCi}]) - S_k^{CSS}}{d_{k2}^{CSS}} \quad (54)$$

for all $k: \lambda$

$$\leq 1 - \frac{S_k^{CSS} - \sum_{i \in \text{Set}_k^{SP}} (x_i^P \cdot [(1 - \alpha)E_2^{PCi} + \alpha E_1^{PCi}])}{d_{k1}^{CSS}} \quad (55)$$

for all $l: \lambda$

$$\leq 1 - \frac{\sum_{i \in \text{Set}_l^{CP}} (x_i^P \cdot [(1 - \alpha)E_2^{PCi} + \alpha E_1^{PCi}]) - S_l^{PNC}}{d_{l2}^{CSC}} \quad (56)$$

for all $l: \lambda$

$$\leq 1 - \frac{S_l^{PNC} - \sum_{i \in \text{Set}_l^{CP}} (x_i^P \cdot [(1 - \alpha)E_2^{PCi} + \alpha E_1^{PCi}])}{d_{l1}^{CSC}} \quad (57)$$

$$\sum_{i=1}^I (x_i^P \cdot [(1 - \frac{\alpha}{2})E_2^{PCi} + \frac{\alpha}{2}E_1^{PCi}]) \geq TAC \quad (58)$$

$$\sum_{i=1}^I (x_i^P \cdot [\frac{\alpha}{2}E_2^{PCi} + (1 - \frac{\alpha}{2})E_1^{PCi}]) \leq TAC \quad (59)$$

$$\text{for all } n: \{tPPS_n - MZ_n^S \leq [(1 - \alpha)E_2^{mppss} + \alpha E_1^{mppss}]t \quad (60)$$

for all $n: \{ \text{for all } i$

$$\begin{aligned} & \in \text{Set}_n^{SP}: \{tPPP_i - Mk_i^P \\ & \leq [(1 - \alpha)E_2^{\text{MAPPPS}} + \alpha E_1^{\text{MAPPPS}}]t \\ & + M(t - Z_n^S) \end{aligned} \quad (61)$$

$$\begin{cases} x_i^P + M(1 - y_i^P) \geq \alpha E_2^{\text{MAPBPS}} + (1 - \alpha)E_1^{\text{MAPBPS}} \\ x_i^P \leq My_i^P \end{cases} \quad (62)$$

$$\text{for all } n: \{\text{forall } i \in \text{Set}_n^{SP}: \{Z_n^S - k_i^P \geq 0\} \quad (63)$$

$$\text{for all } n: \left\{ \sum_{i \in \text{Set}_n^{MP}} k_i^P - Z_n^S \geq 0 \right. \quad (64)$$

$$k_i^P \leq y_i^P \quad (65)$$

$$k_i^P - (M(1 - y_i^P)) \leq t \quad (66)$$

$$k_i^P + (M(1 - y_i^P)) \geq t \quad (67)$$

$$\text{for all } n: \begin{cases} 0 < t \leq 1 \\ 0 < Z_n^S \leq 1 \end{cases} \quad (68)$$

$$\lambda, k_i^P \geq 0 \quad (69)$$

$$\sum_n Z_n^S = 1 \quad (70)$$

$$\text{forall } i: \begin{cases} y_i^P = 0,1 \\ 0 \leq x_i^P \leq 1 \end{cases} \quad (71)$$

According to Yang's approach to fuzzy-goal programming in the above inequalities, the following values are the maximum deviation from goals in a negative and positive direction, respectively (Yang, et al., 1991). ($d_{j2}^{PNO}, d_{j1}^{PNO}, d_{k2}^{PNS}, d_{k1}^{PNS}, d_{l2}^{PNC}, d_{l1}^{PNC}, d_{j2}^{CSO}, d_{j1}^{CSO}, d_{k2}^{CSS}, d_{k1}^{CSS}, d_{l2}^{CSC}, d_{l1}^{CSC}$). The set initially select the degrees of goals in equations (46) to (56) as a minimum, and then in equation (45), select the maximum (λ) from the minimum values. In response to the Giménez fuzzy approach (Yang, et al., 1991), EV , E_1 and E_2 values respectively illustrate the expected mean, the expected lower bound and expected upper bound.

5-3-Model solution with the (NSGA-II) method

Multi-objective evolutionary algorithms are one of the most broadly used methods that mimic natural evolution principles and search for optimal solutions with two different search and selection operations. These algorithms don't abide by certain principles and do not follow a specific structure in problem-solving. When the complexity of the problem precludes the use of precise methods to find the approximation of the Pareto efficiency set, multi-dimensional evolutionary algorithms gain importance. This is a problem with a large data set; hence, practicing the exact approaches does not have a proper function. Among the well-known algorithms is the Non-dominated Sorting Genetic Algorithm (NSGA-II), the most widely used algorithm for multi-objective optimization. The first version of this algorithm was introduced in 1995. Afterward, its creators, such as the renowned Deb, presented a second version in 2002 known as the NSGA-II abbreviation (Deb, 2008). This algorithm is altered to a multi-objective algorithm by combining two essential operators to the ordinary single-element genetic algorithm, which instead of finding the best answer, gives us a set of the best solutions known as Pareto frontier. These two operators are defined as follows: An operator that assigns a ranking criterion (rank) according to

the members of the population and an operator that retains the variation of the answer among the non-dominated responses. (crowding distance). The following steps in table 3 have been taken to solve and implement this algorithm in the model. The word credit means the amount or budget allocated to the project. For example, Total Allocated Credit means the total amount that can be allocated, or Credit Sum of All Projects means the sum of all construction projects.

Table 3. The algorithm implementation steps

Steps	Title
1	<p>Creating a mechanism to generate, store and recall model information.</p> <p>Activity description: All parameters of the model such as the allocated credit to capital projects, the productivity of scheme and etc are recalled.</p>
2	<p>Create a mechanism to generate and display the initial response.</p> <p>Activity description: The percentage of the credit for the projects is genetically modeled in order to display the initial response. In other words, for one percentage of the budget of each project 7 genes are coded as binary code numbers 0 and 1 (numbers 0-100 require a maximum of 7 bits for the binary basis). For instance, the following chart represents a part of a chromosome as an example of the initial response to a problem. These genes are randomly quantified from the left to right, respectively.</p>
3	<p>Implementation of a mechanism to convert raw to primary problem variables.</p> <p>Activity description: The percentage of allocated credit to projects is presented as a binary basis of 0 and 1. Therefore, it is necessary to bring these numbers to radix 10 if we are to convert raw variable to primary. In the previous example, the percentage of allocated credit to the first and second projects are 40% and 36%, correspondingly.</p>
4	<p>Implementing cost function and estimation of limitations.</p> <p>Activity description: According to section 3, two objective functions (44) and (45) are assumed as the main goals. In order to implement the constraints we applied intelligent binding optimization approaches [36] including the violation concept (penalty function) (58) and(59), the modification of responses (60) and space modeling (based on the response coding) for other model constraints (61).</p>
5	<p>Applying the cost function to the optimization algorithm.</p> <p>Activity description:</p> <ol style="list-style-type: none"> 1. Generating 100 chromosomes as primary population (initial response creation). 2. Determine the fitting number or cost value associated with each of the chromosomes regarding two specified target functions. 3. Randomized selection of 20% of the population for the integration of the binary method. 4. Selecting 40% of the integration results for the mutation (to escape the local algorithm). 5. Calculate the fitting value for the results of integration and mutation according to two considered functions. 6. Reviewing the termination process: 50 repetitions are considered from the the algorithm process. If provided, the algorithm ends, otherwise it enters the next cycle. At this point, some of the results should be eliminated to keep the population constant. Therefore, the population are sorted based on the rank and then on base of the crowd distance, now those with a plausible condition are selected and rest are eliminated to retain population constant, finally the algorithm function as before.

We use MATLAB software to implement and solve the model via the NSGA-II method. The computer hardware for running the model featured an Intel Core i7 2.50 GHz CPU and 6GB of RAM.

Parameters setting. Specific important parameters were determined to conduct the NSGA-II algorithm. These parameters include the number of generations, primary population size, the cross-section rate, and the rate and operator of mutation. Since the quality and quantity of effective responses

are vital, the components such as "number generated non-dominated responses" and "the deviation ratio from credit limitations" is regarded as the optimal result of the parameters obtained by comparing the generated answers in all experiments. Therefore, the problem was regarded with randomly generated parameters. For this purpose, the problem was considered with parameters that were generated randomly. After 20 runs, the best combination for the algorithm parameters was obtained according to the following table 4.

Table 4. The best combination for algorithm parameters after 20 runs

Mutation Operator	Cross Section Operator	Mutation Rate	Cross Section Rate	Population Size	Generation Num.
Inversion	Single point	0.08	0.7	100	50

The following tables illustrate the outcome of five initial integration. Also, note the coefficient of α is equal to "0.5" and the coefficient of budget constraint penalty is equal to " 10^5 " through these performances. To calculate the penalty coefficients from the budget constraint, we assumed 10 different coefficients associated with 10 frequencies, the value " 10^5 " was selected regarding credit deviation comparison.

Table 5. Results of using the NSGA-II algorithm

Run	Solution	First objective function	Second objective function	Credit deviation
1	1	0.523	0.046	13.702
	2	0.521	0.075	-14.348
2	1	0.545	0.078	5.889
3	1	0.564	0.0505	-11.193
	2	0.471	0.085	325.662
	3	0.5603	0.0505	-3.569
4	1	0.519	0.074	151.8558
	2	0.373	0.077	827.2424
	3	0.599	0.068	32.4857
	4	0.566	0.060	-8.5708
5	1	0.556	0.087	-35.1007

As it is evident, credit deviations caused by a budget limitation in terms of the penalty function are presented in the fifth column. Negative and positive values signify surplus and credit deficit, respectively. The "Third" implementation yielded the response "three," and this is the best answer since

only "3.569" credit units are not allocated to projects, which is a tiny proportion when compared to the available budget of "8,952,999" million rials. According to the results of the first, third, and fourth performances in table 5, it is evident that the NSGA-II algorithm has searched the non-dominated set for a multi-objective model. Figure 3 shows the three basic Pareto fronts for the fourth run.

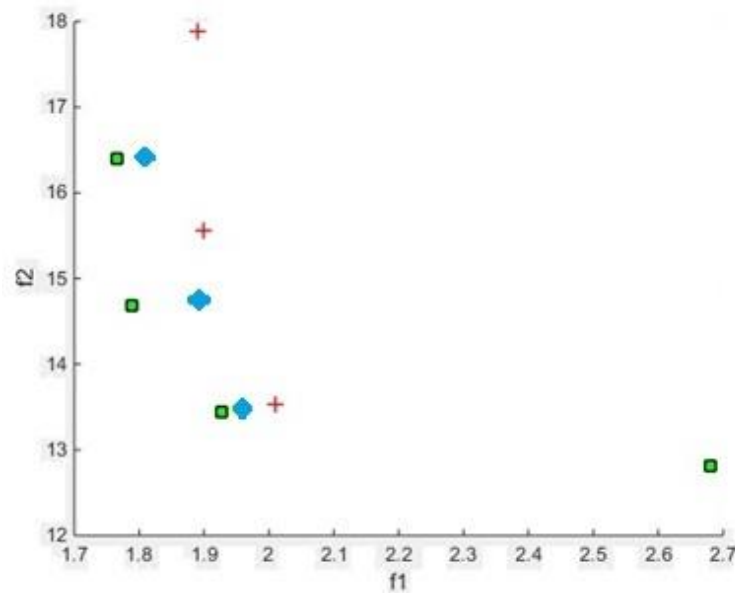


Fig 3. First three Pareto Fronts for the fourth run

The "square," "asteroid," and "cross" points are subsequently the first to third Pareto Fronts. It is worth to mention the goal function values are shown inversely illustrated in the above figures (inverse minimization of goal functions). Therefore, the proximity to the coordinates' origin indicates the responses' superiority. The figure 3 depicts four square-shaped points representing four responses on the first Pareto Front consistent with the results of the fourth run. The status of the project and project selection for an answer "3" in the implementation of "third" is given in tables 6 and 7.

Table 6. Status of design selection for answer number "3" in "Third" implementation using NSGA-II

Number of Selected Projects	The status of the Schemes selection	Schemes Number	Number of Selected Projects	The status of the Schemes selection	Schemes Number	Number of Selected Projects	The status of the Schemes selection	Schemes Number
3	1	21	4	1	11	4	1	1
1	1	22	2	1	12	4	1	2
4	1	23	0	1	13	2	1	3
4	1	24	6	1	14	2	1	4
5	1	25	8	1	15	3	1	5
4	1	26	6	1	16	5	1	6
1	1	27	4	1	17	2	1	7
3	1	28	0	1	18	4	1	8
			7	1	19	1	1	9
			1	1	20	9	1	10

Table 7. The status of the project selection for the answer "three" in the "Third" implementation of NSGA-II

percentage of credit	The status of the project	Schemes Number	Project Number	percentage of credit	The status of the project	Schemes Number	Project Number	percentage of credit	The status of the project	Schemes Number	Project Number	percentage of credit	The status of the project	Schemes Number	Project Number	percentage of credit	The status of the project	Schemes Number	Project Number
1.00	1	25	121	0.45	1	19	91	0.63	1	15	61	0.98	1	8	31	1.00	1	1	1
0.63	1	25	122	0.87	1	19	92	0.64	1	15	62	0.79	1	8	32	0.00	0	1	2
0.00	0	25	123	0.00	0	19	93	0.00	0	15	63	0.00	0	9	33	0.35	1	1	3
0.67	1	25	124	1.00	1	19	94	0.50	1	15	64	0.56	1	9	34	0.92	1	1	4
1.00	1	25	125	1.00	1	19	95	1.00	1	15	65	0.57	1	10	35	1.00	1	1	5
0.83	1	26	126	0.00	0	20	96	0.00	0	15	66	1.00	1	10	36	0.75	1	2	6
0.92	1	26	127	0.39	1	20	97	0.63	1	15	67	0.43	1	10	37	0.00	0	2	7
0.87	1	26	128	0.00	0	20	98	0.60	1	15	68	0.61	1	10	38	0.67	1	2	8
0.00	0	26	129	0.00	0	21	99	0.70	1	15	69	0.93	1	10	39	0.65	1	2	9
0.45	1	26	130	0.00	0	21	100	0.74	1	16	70	1.00	1	10	40	0.00	0	2	10
0.76	1	27	131	0.00	0	21	101	0.51	1	16	71	0.41	1	10	41	1.00	1	2	11
0.94	1	27	132	0.49	1	21	102	0.48	1	16	72	0.00	0	10	42	0.63	1	3	12
1.00	1	28	133	0.00	0	21	103	0.95	1	16	73	0.93	1	10	43	0.00	0	3	13
0.91	1	28	134	0.98	1	21	104	0.85	1	16	74	0.94	1	10	44	0.35	1	3	14
0.00	0	28	135	0.68	1	21	105	0.80	1	16	75	0.86	1	11	45	0.62	1	4	15
1.00	1	28	136	0.00	0	22	106	0.45	0	16	76	1.00	1	11	46	0.60	1	4	16
				0.75	1	22	107	0.00	0	17	77	1.00	1	11	47	0.59	1	5	17
				0.00	0	23	108	1.00	1	17	78	0.57	1	11	48	0.43	1	5	18
				0.34	1	23	109	0.0	0	17	79	0.94	1	12	49	1.00	1	5	19
				0.00	0	23	110	0.00	0	17	80	0.00	0	12	50	1.00	1	6	20
				1.00	1	23	111	0.62	1	17	81	0.47	1	12	51	1.00	1	6	21
				0.64	1	23	112	0.56	1	17	82	0.00	0	13	52	0.98	1	6	22
				0.00	0	23	113	0.85	1	17	83	0.70	1	14	53	1.00	1	6	23
				1.00	1	23	114	0.00	0	18	84	1.00	1	14	54	0.80	1	6	24
				0.00	0	24	115	0.00	0	18	85	0.00	0	14	55	0.00	0	7	25
				0.85	1	24	116	0.75	1	19	86	0.82	1	14	56	1.00	1	7	26
				1.00	1	24	117	0.00	0	19	87	0.67	1	14	57	0.88	1	7	27
				1.00	1	24	118	0.37	1	19	88	1.00	1	14	58	1.00	1	8	28
				0.88	1	24	119	0.89	1	19	89	0.78	1	14	59	1.00	1	8	29
				0.88	1	25	120	0.00	0	19	90	0.41	1	15	60	0.00	0	8	30

The following indices were employed to correlate the results of the developed model with the exact model of the literature review:

- a. Total productivity, according to the first objective function, is better (maximizing productivity).
- b. Deviation Ratio Average for the number of projects (organization, state, and budget chapter) and the project credit sum (organization, state, and budget chapter). These indices correlate with the six objectives of the problem. The lower is better.
- c. Ratio of selected schemes (projects) to the total number of schemes (projects) suggests the percentage of candidate schemes, any value closer to one is better.
- d. Distribution coefficient of projects for each scheme. It expresses the ratio of the SD for each project to the average number of projects per scheme; the lower, the better.
- e. The ratio of the given credit to the total allocated credit (TAC) represents a percentage of resources not utilized by projects; any value closer to one is better. Table 8 shows the obtained results obtained from the sample problem-solving by the NSGA-II algorithm in terms of indices

Table 8. The obtained results obtained from the sample problem-solving by the NSGA-II algorithm

Evaluation criteria	Crisp model (Moheghar, Mehregan, Azar, & Motahari, 2014)	Fuzzy model
	Weighted	NSGA-II
Total productivity	0.569	0.5603
Deviation Ratio Average for the number of projects organization	0.478	0.516
Deviation Ratio Average for the number of projects state	0.483	0.593
Deviation Ratio Average for the number of projects budget chapter	1.156	1.141
The project credit sum organization	0.969	0.8
The project credit sum state	0.998	0.744
The project credit sum budget chapter	0.986	2.011
Ratio of selected projects to the total number of projects	1	0.92
Ratio of selected schemes to the total number of schemes	0.44	0.73
Distribution coefficient of projects for each scheme	0.77	0.63
The ratio of the given credit to total allocated credit	1	0.99

In order to analyze the results fuzzy model solution, it is required to compare them with the deterministic model. To this end, table 8 represents the results obtained from the sample problem-solving by the NSGA-II algorithm in terms of indices. Subsequently, they are compared with the exact model literature review. According to the third column, only one response from the Pareto border has been identified, following indicators have improved, such as: "standard deviation ratio of CSS," "standard deviation ratio of CSO," and "standard deviation ratio of CSC." These improvements reflect the success rate of accomplishment for the corresponding goals in the fuzzy model compared to the exact model of the research. However, some components, such as total productivity, have faced no alteration. Some other indicators, including the "standard deviation ratio of CSS," have declined. The third column of this table maintains only one non-recessive response that signifies an improvement over the measurement indicators. Thus, analyzing the whole set of non-recessive solutions in which some indices have improved significantly is mandatory. The multi-objective-fuzzy model gains some advantages, such as a) the improved rate of goal realization regarding the inherent ambiguities of the

problem and b) identifying the set of non-recursive responses via a multi-objective solution method rather than finding the optimal answer.

6- Discussion and conclusion

Project selection in the public sector requires great decision-makers' goals. Several qualitative standards are associated with project investment credit. Any quantification and weighting of them, such as "parameters obtained through interviews with experts or threshold value defined for goals," is indefinite and ambiguous. Moreover, the lack of clarity in many technical specifications at the beginning of the project, such as credibility, is almost normal. Therefore, the necessity of fuzzy modeling in decision-making has been emphasized via inaccurate data. Likewise, unlike the single-objective that searches for the optimal unit, project selection in the public sector seeks a non-recursive set due to the exchange of conflicting goals and no single optimal answer. Thus, this research is by the necessity of multi-dimensional modeling and against the combination of objectives in the form of a single. We developed an integrated fuzzy multi-objective model to help Capital project selection for the public sector. It sought model selection for projects and credit allocation to them by emphasizing the multi-dimensional nature of the problem and its intrinsic negligence. The study criteria are based on the model extraction from the literature review, which includes technical, economic, political, social, and financial objectives in conjunction with the observance of standards. Therefore, a three-stage process was utilized for its solution. The initial stage converted it to a non-linear model by fractional planning concepts, defuzzification according to Jimenez and Yang approaches, and finally solved by NSGA-II algorithm. The results yielded improvement in the realization of the goals of the problem regarding the inherent ambiguities of the problem, in addition to the identification of non-recursive solutions set instead of finding the optimal solution by a multi-objective method. Formulating the project's required limitations and minimizing risk is advised to boost and develop the subject presented in this study. By incorporating other evolutionary multi-objective techniques, such as MOPSO and MOEA/D, we could evaluate the obtained computational records of the study algorithm. Also, decision-makers in the field of capital projects in the country's general budget, such as the parliament to review and approve investment projects during the preparation of the country's available budget and the President's Strategic Management and Supervision Organization to prepare and adjust the list of proposed projects for allocation, can use the proposed model.

Reference

- Bellos, E., Voulgridou, D., Kirytopoulos, K., & Panopoulos, D. (2010). An MCDA approach for project selection in public sector. *Centre for Construction Innovation*.
- Benjamin, C. O. (1985). A linear goal-programming model for public-sector project selection. *Journal of the Operational Research Society*, 36, 13-23.
- Carlsson, C., Fuller, R., Helkkila, M., & Majlender, P. (2007). A fuzzy approach to R&D project portfolio selection. *International Journal of Approximate Reasoning*, 44, 93-105.
- Chapman, C., Ward, S., & Klein, J. (2006). An optimised multiple test framework for project selection in the public sector, with a nuclear waste disposal case-based example. *International journal of project management*, 24, 373-384.
- Deb, K. (2008). *Genetic algorithm with Multi-objective optimization approach*. Tehran, Pelk.
- Doerner, K., Gutjahr, W., Hartl, R. F., Strauss, C., & Stummer, C. (2004). Pareto ant colony optimization: A metaheuristic approach to multiobjective portfolio selection. *Annals of operations research*, 131, 79-99.
- Dong, T., Yin, S., & Zhang, N. (2023). New Energy-Driven Construction Industry: Digital Green Innovation Investment Project Selection of Photovoltaic Building Materials Enterprises Using an Integrated Fuzzy Decision Approach. *Systems*.

- EBRAHIMNEJAD, S. M., Tavakkoli-Moghaddam, R., Hashemi, H., & Vahdani, B. (2012). A novel two-phase group decision making approach for construction project selection in a fuzzy environment. *Applied Mathematical Modelling*, 36, 4197-4217.
- Ebrahimnejad, S., Hosseinpour, M. H., & Nasrabadi, A. (2013). A fuzzy bi-objective mathematical model for optimum portfolio selection by considering inflation rate effects. *The International Journal of Advanced Manufacturing Technology*, 69, 595-616.
- Fazli, S., & Madani, S. (2009). introducing a model for selecting construction projects using Multiple criteria decision making and Goal programming approach. *International Project Management Conference*.
- Fernandez, E., Lopez, E., Mazcorro, G., Olmedo, R., & Coello, C. A. (2013). Information Sciences. *Application of the non-outranked sorting genetic algorithm to public project portfolio selection*, 228, 131-149.
- Fonseca, C., & Fleming, P. (1993). Genetic Algorithms for Multiobjective Optimization: Formulation Discussion and Generalization. *ICGA*, 416-423.
- Ghorbani, S., & Rabbani, M. (2009). A new multi-objective algorithm for a project selection problem. *Advances in Engineering Software*, 40, 9-14.
- Greenberg, R., & Nunamaker, T. (1994). Integrating the analytic hierarchy process (AHP) into the multiobjective budgeting models of public sector organizations. *Socio-Economic Planning Sciences*, 28, 197-206.
- H.Issa, U., A.A.Mosaad, S., M., & Hassan, S. (2020). Evaluation and selection of construction projects based on risk analysis. *Structures*, 27, 361-370.
- Hejazi, T. H., & Roozkhosh, P. (2019). Partial inspection problem with double sampling designs in multi-stage systems considering cost uncertainty. *Journal of Industrial Engineering and Management Studies*, 1-17.
- Joiner, C., & Drake, A. E. (1983). Governmental planning and budgeting with multiple objective models. *Omega*, 11, 57-66.
- Leinbach, T. R., & Cromley, R. G. (1983). A goal programming approach to public investment decisions: a case study of rural roads in Indonesia. *Socio-Economic Planning Sciences*, 17, 1-10.
- Martinez-Vega, D., Sanchez, P., Castilla, G., Fernandez, E., & Cruz-Reyes, L. (2017). Evaluation of the Evolutionary Algorithms Performance in Many-Objective Optimization Problems Using Quality Indicators. *Nature-Inspired Design of Hybrid Intelligent Systems*. Springer.
- Medaglia, A. L., Graves, S. B., & Ringuest, J. L. (2007). A multiobjective evolutionary approach for linearly constrained project selection under uncertainty. *European journal of operational research*, 179, 869-894.
- Modares, A., Farimani, N. M., & Emroozi, V. B. (2022). A new model to design the suppliers portfolio in newsvendor problem based on product reliability. *Journal of Industrial and Management Optimization*.
- Modares, A., Farimani, N. M., & Emroozi, V. B. (2022). Developing a Newsvendor Model based on the Relative Competence of Suppliers and Probable Group Decision-making. *Industrial Management Journal*, 115-142.

- Mohagheghi, V., Mousavi, S. M., & Vahdani, B. (2015). A new optimization model for project portfolio selection under interval-valued fuzzy environment. *Arabian Journal for Science and Engineering*, 40, 3351-3361.
- Moheghar, A., Mehregan, M., Azar, A., & Motahari, N. (2014). Designing a model for selecting construction projects in public sector. *Journal of Industrial Management*, 6, 831-847.
- Molenaar, K. R., & Songer, A. D. (1988). Model for public sector design-build project selection. *Journal of Construction Engineering and Management*, 124, 467-479.
- Nassif, L. N., Santiago Filho, J. C., & Noguera, J. M. (2013). Procedia-Social and Behavioral Sciences. *Project portfolio selection in public administration using fuzzy logic*, 74, 41-50.
- Odiar, A. (2012). An approach for solving linear fractional programming problems. *International Journal of Engineering & Technology*, 1, 298-304.
- Orlu, G. U. (2021). Outsourcing Provider Selection Model in Public Sector . *Turkish Journal of Computer and Mathematics Education (TURCOMAT)*, 12(3).
- Perez, F., & Gomez, T. (2016). Multiobjective project portfolio selection with fuzzy constraints. *Annals of Operations Research* , 245, 7-29.
- Roozkhosh, P. P. (2022). Blockchain acceptance rate prediction in the resilient supply chain with hybrid system dynamics and machine learning approach. *Operations Management Research*, 1-21.
- Roozkhosh, P., & Motahari Farimani, N. (2022). Designing a new model for the hub location-allocation problem with considering tardiness time and cost uncertainty. *International Journal of Management Science and Engineering Management*, 1-15.
- Saborido, R., Ruiz, A. B., Bermudez, J. D., Vercher, E., & Luque, M. (2016). Evolutionary multi-objective optimization algorithms for fuzzy portfolio selection. *Applied Soft Computing* , 39, 48-63.
- Shybalkina, I. (2022). Toward a positive theory of public participation in government: Variations in New York City's participatory budgeting. *Public Administration*, 841-858.
- Tofighian, A., & Naderi, B. (2015). Modeling and solving the project selection and scheduling. *Computers & Industrial Engineering*, 83, 30-38.
- Wirick, D. (2011). Public-sector project management: Meeting the challenges and achieving results. *John Wiley & Sons*.
- Wu, Y. J., & Chen, J.-C. (2021). A structured method for smart city project selection. *International Journal of Information Management*, 56.
- Yang, T., Ignizio, J. P., & Kim, H. (1991). Fuzzy programming with nonlinear membership functions: piecewise linear approximation. *Fuzzy sets and systems*, 41, 39-53.
- Youssef, H., Janzer-Araji, A., & Mazahreh, J. R. (2023). Public Investment-Maximizing the Development Impact. *Jordan Economic Monitor*.
- Zadeh, L. A. (1965). Fuzzy sets. *Information and control*, 8, 338-353.
- Zavadskas, E. K., Turskis, Z., Šliogerienė, J., & Vilutienė, T. (2021). An integrated assessment of the municipal buildings' use including sustainability criteria. *Sustainable Cities and Society*, 67.