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Optimizing location, routing and inventory decisions in an integrated supply chain network under uncertainty

Ehsan Dehghani[1*], NimaBehfar[1], Mohammad Saeed Jabalameli[1]

[1]*School of Industrial Engineering, Iran University of Science and Technology, Tehran, Iran*

ehsandehghan@ind.iust.ac.ir, nimabehfar@yahoo.com, jabal@iust.ac.ir

Abstract

This study extends a mathematical model that integrates the location, allocation, inventory replenishment and routing decisions simultaneously. To cope with inherent uncertainty of parameters, we implement a continuous-time Markov process and derive the performance measures of the system. Using the obtained results, the problem is formulated as a mixed integer nonlinear programing model (MINLP), where the total costs of location, transportation and inventory are minimized. In addition, we develop a simulated annealing (SA) based meta-heuristic algorithm to tackle the computational complexity of the problem. Finally, several computational experiments are provided to assess effectiveness of the proposed algorithm and the model.
***Keywords**:* Markov Chain, Location, Allocation, Routing, Simulated Annealing, Inventory

1- Introduction

For designing a distribution network, three major problems compromising location-allocation, vehicle routing and inventory control have been typically taken into account. Most of the researches have been focused on combining of two of the aforementioned problems; i.e. inventory-routing, location-inventory and location routing (Chew et al., 2007). Since these problems have significant reciprocal impacts on the each other, considerable savings can be realized by integrating them (see,e.g. Miranda and Garrido (2008)). Here, we briefly review the researches that study the joint problems.

The inventory-routing problem (IRP)combines the management of inventory and the distribution of products from facilities to customers over a determined planning horizon.The reader can refer to Qi et al. (2009), Tawarmalani and Sahinidis (2005), Shu et al. (2005) and Javid and Azad (2010) for surveys on the subject.

The decisions of facility location and stocks replenishment are incorporated in the location-inventory problem (LIP). More specifically, the optimal number of DCs, their locations, the customers assigned to each DC and the optimal ordering policy at the DCs are determined in this problem type (Ahmadi-Javid

*Corresponding author
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and Seddighi, 2012). Additionally, Shen (2007), Melo et al. (2009) and Farahani et al. (2015) have provided surveys on the LIP.

Recently, Diabat (2015a) presented a location-inventory problem, in which capacity constraints problem was considered. He reformulated the proposed model using piecewise linearization to accelerate solution time. Diabat and Theodorou (2015) investigated a two-echelon inventory management problem with multiple retailers and presented an equivalent formulation with fewer non-linear terms. Diabat (2015b) considered warehouse capacity in the model and applied a genetic algorithm to solve the problem.

For the first time, Ozsen et al. (2009) and Liu et al. (2010) introduced the concept of the location routing problem (LRP). This type of integration has been surveyed by Liao et al. (2011), Daskin et al. (2002), Qi et al. (2009) and Ross (2014).

Ahmed (2007) was the first authors that considered routing costs in the location-inventory problem. However, he did not determine the routing decisions, and only approximated the routing costs by using location-allocation decisions. He formulated the problem as a nonlinear integer-programming model and solved it by using a Lagrangian relaxation based solution algorithm.Thereafter, Miranda and Garrido (2008) simultaneously determined the location, inventory and routing decisions for the first time and numerically illustrated that by integrating the decisions, considerable saving can be obtained. They casted the problem as a mixed integer nonlinear program and a presented a heuristic method based on a hybridization of Tabu search and simulated annealing for solving it.

Chew et al. (2007) proposed an incorporated LRIP location-routing-inventory problem and presented a mixed-integer programming model for it. Besides, they presented an effective three-phase heuristic in order to solve large-sized instances of the problem.

In connection with integrated supply chain management, researches on complex joint service-inventory systems or production-inventory systems have attracted much attention over the last decades. Typically, by implementing queuing networks and multi-echelon inventory models, interplays between production/service processes and inventory management can be described (Schwarz et al., 2006). As mentioned by Saffari et al. (2013) and Sadjadi et al. (2015), the queuing systems attached inventory are more general and more practical compared with the traditional ones.

An early contribution was provided by Sigman and Simchi-Levi (1992), who proposed a light traffic heuristic for an M/G/1 queue with restricted inventory. A problem of dynamic replenishment was introduced by Berman and Kim (2001), where the problem was formulated using a Markov process.The aim of their study was to determine replenishment policy such that the costumer waiting, order replenishment costs, and inventory holding costs are minimized. Schwarz et al. (2006) examined several single server M/M/1 queuing systems with an attached inventory. They assumed that the arriving demands are lost during stock-out and derived explicit performance measures for the proposed system.Teimoury et al. (2010) devised a production-inventory-queue model and considered that the demands and lead-times were uncertain. Additionally, they implemented the model in a case study and developed a computationally efficient algorithm to solve it. Saffari et al. (2013) analyzed an M/M/1 queuing system with inventory, in which demands and lead-times were uncertain and followed Poisson and exponential distributions, respectively. Using an M/M/1 queue, Baek and Moon (2014) extended a production-inventory problem. They assumed that customers arrive in the system according to a Poisson process and lead-times were exponentially distributed.

In this research, we incorporate the routing problem in the LIP proposed by Sadjadi et al. (2015)**.** Therefore, our model determines the location, allocation, inventory replenishment, and routing decisions simultaneously. In order to deal with the stochastic natures of the parameters, we implement a continuous-time Markov chain and derive the performance measures of the system. Then, we formulate the proposed problem as an MINLP model using the obtained measures. Since the LRPs, as illustrated by Ramezanian and Saidi-Mehrabad (2013), are NP-hard, the LRIPs, as their developments, are definitely NP-hard too. Therefore, we develop a solution approach based on simulated annealing (SA) to solve the model in a reasonable time. The numerical results show that the proposed algorithm is effective and efficient.

The reminder of the paper is organized as follows. In Section 2, the problem is defined and pertaining assumptions are given.The proposed problem is formulated as an MINLP in Section 3. Section 4 provides the solution approach for solving the model. In Section 5, computational experiments are proposed and finally the conclusions are presented in Section 6.

2- Problem definition

The concerned supply chain includes three levels, in which a single supplier sends one type of product via DCs to a set of retailers. Our assumptions and decisions determined by the model are stated as follows.

Assumptions

- The demands of retailers independently arrive according to Poisson processes with intensityλ. In addition, each retailer should be allocated to exactly one DC. Thus, the demand of each DC follows Poisson, where its intensity is obtained by summing up intensities of its allocated retailers. This can be justified as follows. We know that the time between two consensus events that follow Poisson process follows exponential distribution. Now, with respect to constant rates of arriving demands of retailers as well as memory-less feature of the exponential distribution, this assumption can be accepted.
- The replenishment of DCs is done under a continuous review system, namely (S-1,S) inventory policy, in which as the inventory level becomes less than$S$,either due to a demand or a failure, an order is placed to attain the inventory level to S(Kalpakam and Shanthi, 2001). This class of inventory policy can be usedfor low demand, expensive, slow moving, and high inventory holding costs items. More specifically, it can be implemented for computer manufacturing and spare parts inventory systems (Sadjadi et al., 2015; Schultz, 1990).
- The supplier lead-time is uncertain and is exponentially distributed with parameterμ.
- The supplier is not subject to any capacity restrictions, but the storage space of each DC is constrained.
- When there is no on-hand inventory in a DC, the arriving demands are backlogged.
- The intensity of each DC is assumed less than μ in order to avoid perpetual shortages in system.
- The vehicles' capacities are the same, i.e., and fleet type is homogeneous, and each retailer should be placed on exactly one vehicle's route.

The model should simultaneously determine following decisions:

1- The number of the opened DCs,
2- The locations of the opened DCs,
3- The allocation of the retailers to the opened DCs,
4- The optimal inventory policies at the opened DCs and,
5- Vehicles' routes started from an opened DC for serving its allocated retailers.

The model aims at determining the aforementioned decisions such that the total costs of locating DCs, transportation and inventory are minimized. Noteworthy, the inventory costs include the holding, shortage, ordering and purchase costs.

3- Mathematical formulation

In order to obtain the performance measures of the inventory policy, a Markov process is implemented first. As mentioned by Saffari et al. (2013) and Sadjadi et al. (2015), a queuing system with attached inventory yields more practical and general models. The following notation is used in the Markov process:

$M_k(t)$ The level of inventory in opened DC k at time t

Therefore, $M_k(t)$is a continuous time Markov process with state space $E_k = \{S_k, S_k - 1, \ldots, 0, -1. -2, \ldots\}$. Accordingly, the steady-state probabilities of the system can be introduced as following:

$$p_k(i,j,t) = pr[N_k(t) = j | N_k(0) = i] \qquad i,j \in E_k \tag{1}$$

$$p_k(j) = \lim_{t \to \infty} p_k(i,j,t) \tag{2}$$

In this regard, the steady-state equations for an (S-1, S) inventory system will be:

$$\lambda_k p_k(S_k) = \mu p_k(S_k - 1) \tag{3}$$

$$(\lambda_k + \mu) p_k(j) = \lambda_k p_k(j+1) + \mu p_k(j-1) \qquad j \le S_k - 1 \tag{4}$$

Considering that the sum of the all states' probabilities must be equal to 1, i.e., $\sum_{j \in E_k} p_k(j) = 1$, as well as Eqs (3-4), we have:

$$P_K(S_k) = 1 - \frac{\lambda_k}{\mu} \tag{5}$$

$$P_K(j) = \left(1 - \frac{\lambda_k}{\mu}\right) \left(\frac{\lambda_k}{\mu}\right)^{S_k - j} \qquad j \le S_k - 1 \tag{6}$$

Now, we intend to obtain some inventory features by using the achieved results. Hence, lets us define the following notations:

R_K The expected number of annual orders at DC k

P_k The expected number of annual purchases at DC k

Γ_K The expected number of annual backlogged shortages at DC k

MI_k The expected number t of mean inventory level at DC k

In an (S-1,S) inventory policy, when the demands are backlogged, the expected number of annual ordering as well as the expected number of annual purchases would be equal to the expected number of annual arriving demands. In other words:

$$R_K = P_K = \lambda_K, \tag{7}$$

Furthermore, when the level of inventory is less than zero, the arriving demands are backlogged. Thus, expected number of annual backlogged shortages at DC $k$is given by Eq.8

$$\Gamma_K = \lambda_K P_K(x \leq 0) = \lambda_K \left(\frac{\lambda_k}{\mu}\right)^{S_k}, \tag{8}$$

Likewise, the expected mean inventory level is as follows.

$$MI_k = \sum_{j=0}^{S_k} j * p_k\,(j), \tag{9}$$

As a results,

$$MI_k = \frac{\mu\left(\frac{\lambda_k}{\mu}\right)^{S_k}\left(-1+\frac{\lambda_k}{\mu}\right)\left({\lambda_K}^{S_k+1} - \lambda_K\mu^{S_k} - \lambda_K S_k\mu^{S_k} + S_k\mu^{S_k+1}\right)}{{\lambda_K}^{S_k}(\lambda_K-\mu)^2}, \tag{10}$$

In the following, the main LIRP model is formulated according to the obtained results:

Sets:

K	Set of potential DCs,
I	Set of retailers,
V	Set of vehicles,
M	Merged set of potential DCs and retailers, i.e.$(K \cup I)$,

Parameters:

C_k	Unit purchase cost of DC *k* from the supplier*($\forall k \in K$)*,
π_k	Unit shortage cost at DC *k ($\forall k \in K$)*,
F_k	Fixed (per unit time) cost of openingDC *k ($\forall k \in K$)*,
T_{ghv}	Unit transportation cost from node*g* to node*h* by vehicle *v ($\forall k \in K$) ($\forall i \in I$) ($\forall v \in V$)*,
A_k	Unit ordering cost at DC *k($\forall k \in K$)*,
h_k	Unit holding cost at DC *k ($\forall k \in K$)*,
U_k	The storage capacity at DC *k ($\forall k \in K$)*,
$\lambda_i{}'$	Demand rate (Poisson) at retailer *i ($\forall i \in I$)*,
D_v	Maximum capacity of vehicle *v($\forall v \in V$)*,
μ	Exponential parameter of lead-time
ψ	Weight factor associated with transportation cost,
Φ	Weight factor associated with inventory costs.

Decision variables:

Z_k	1 if DC k is opened, 0 otherwise *($\forall k \in K$)*,
y_{ki}	1 if demand at retailer i are assigned to DC k, 0 otherwise *($\forall k \in K$) ($\forall i \in I$)*,
X_{ghv}	1 if *k* precedes l in route of vehicle v, 0 otherwise *($\forall g,h \in M$) ($\forall v \in V$)*,
λ_k	Demand rate (Poisson) atDC k *($\forall k \in K$)*,
S_k	Base stock level maintained at DC *k ($\forall k \in K$)*,
N_{ik}	Auxiliary variable defined for customer *i* for sub-tour elimination in route of vehicle *v($\forall i \in I$)($\forall v \in V$)*.

Based on the results of queuing analysis, the problem is formulated as an MINLP model as follows.

$$Min\ w = \sum_{k \in K} F_k z_k + \psi \sum_{g\,\in M}\sum_{h \in M}\sum_{v\,\in V} T_{ghv}\,X_{ghv} \tag{11}$$

$$+\Phi\left[\sum_{k\in K} h_k z_k \left[\frac{\mu\left(\frac{\lambda_k}{\mu}\right)^{S_k}\left(-1+\frac{\lambda_k}{\mu}\right)\left(\lambda_K{}^{S_k+1}-\lambda_K\mu^{S_k}-\lambda_K S_k\mu^{S_k}+S_k\mu^{S_k+1}\right)}{\lambda_K{}^{S_k}(\lambda_K-\mu)^2}\right] + \pi_k z_k\left[\lambda_K\left(\frac{\lambda_k}{\mu}\right)^{S_k}\right]+(A_k+C_k)z_k\lambda_k\right]$$

Subject to

$$\sum_{g\in M}\sum_{v\in V} X_{giv}=1 \qquad \forall i\in I \qquad (12)$$

$$\sum_{k\in K}\sum_{i\in I} X_{kiv}\leq 1 \qquad \forall v\in V \qquad (13)$$

$$\sum_{g\in M} X_{ghv}-\sum_{g\in M} X_{hgv}=0 \qquad \forall h\in M, \forall v\ \in V \qquad (14)$$

$$\sum_{h\in M} X_{khv}+\sum_{h\in M} X_{ihv}-y_{ki}\leq 1 \qquad \forall k\in K, i\in I, \forall v\in V \qquad (15)$$

$$N_{iv}-N_{gv}+|I|*X_{igv}\leq |I|-1 \qquad i,g\in I, \forall v\in V \qquad (16)$$

$$\sum_{i\in I}\lambda_i{}^{'}\sum_{k\in M} X_{ikv}\leq D_v \qquad \forall v\in V \qquad (17)$$

$$\sum_{i\in I}\lambda_i{}^{'}y_{ki}=\lambda_k \qquad \forall k\in K \qquad (18)$$

$$y_{ki}\leq Z_k \qquad \forall k\in K \qquad (19)$$

$$\lambda_k\ \leq\mu \qquad \forall k\in K \qquad (20)$$

$$S_k\leq U_k \qquad \forall k\in K \qquad (21)$$

$$z_k\in\{0,1\} \qquad \forall k\in K$$

$$y_{ki}\in\{0,1\} \qquad \forall i\in I, \forall k\in K \qquad (22)$$

$$X_{ghv}\in\{0,1\} \qquad h,g\in M, \forall v\in V$$

$$N_{iv}\geq 0 \qquad i\in I, , \forall v\in V \qquad (23)$$

$S_k \geq 0$ *and is integer* $\forall k \in K$

The objective function (11) minimizes the total costs of the location, transportation and the expected inventory costs.

Constraints (12) assure that each retailer is assigned to exactly one vehicle route. Constraints (13) state that multiple DCs cannot serve a rout. Constraints (14) ensure that whenever a vehicle visits a customer's or a distribution center's node, it must leave that node. In addition, they assure that the routes would be circular. Constraints (15) assure that each retailer is assigned to a DC that has a route passing by. Constraints (16) are the sub-tour elimination constraints, which guarantee each tour must contain a distribution center from which it originates. Constraints (17) stipulate that the amount of product transportations for each vehicle is less than its capacity. Constraints (18) show that the demand rate of each opened DC equals the sum of Poisson rate(s) of its assigned retailer(s). Constraints (19) make sure that retailers can only be assigned to DC(s) that is (are) opened. Constraints (20) illustrate stability of inventory system in each opened DC. In other words, if the demand rate of each opened DC is greater thanμ, the system will be in shortage forever. Constraints (21) ensure that the maximum level of inventory is less than storage capacity. Finally, constraints (11) and (12) define the variables' types.

4- Solution approach

LRPs have been proven to be NP-hard (see, Ramezanian and Saidi-Mehrabad (2013)). Accordingly, LRIPs, as their developments, belong to the class of NP-hard problems as well. For this reason, we develop a SA-based algorithm in this section to solve the proposed problem in anefficient way. The SA is a powerful method for solving complex combinatorial optimization problems. This method is a local search heuristic, which uses mechanisms for preventing from being trapped into local optimums. Metropolis et al. (1953) introduced the SA and Kirkpatrick and Vecchi (1983) generalized it. The concept behind the method is obtained from the "annealing" process of metals, which is common in the metallurgical industries. The results of this approach are better-aligned, low energy-state and crystallization than local search methods. A simple form of local search to find optimum values starts with an initial solution. The initial solution is called the current solution.Then, a neighbor solution is generated and the difference in objective function value of the current solution and neighbor solution is calculated. For minimization problems, if the difference was negative, the neighbor solution is substitutedby the current solution. Otherwise, the current solution is retained. These steps are repeated until no improvement in objective function value is attained. The disadvantage of this approach is the possibility of being trapped in local optimums. In order to avoid such issue, the SA may accept a worse neighbor solution. Typically, the Boltzman function is used for this purpose. In this way, the worse neighbor solution is accepted with probability of$exp(-\Delta/kT)$, where Δ is difference of objective function values, k is a outlined constant, and T is the current temperature. This mechanism is the main idea of the SA and helps the algorithm to escape from local optimums.

In the current paper, the proposed LRIP is decomposed into two sub-problems, the location, allocation and routing sub-problem and the inventorysub-problem. Each iteration first generates a solution for the location, allocation and routing sub-problem and then the inventory decisions are made for the opened DCs. A graphical illustration of the proposed algorithm is depicted in Figure 1. In the following, we describe these two sub-problems in more details.

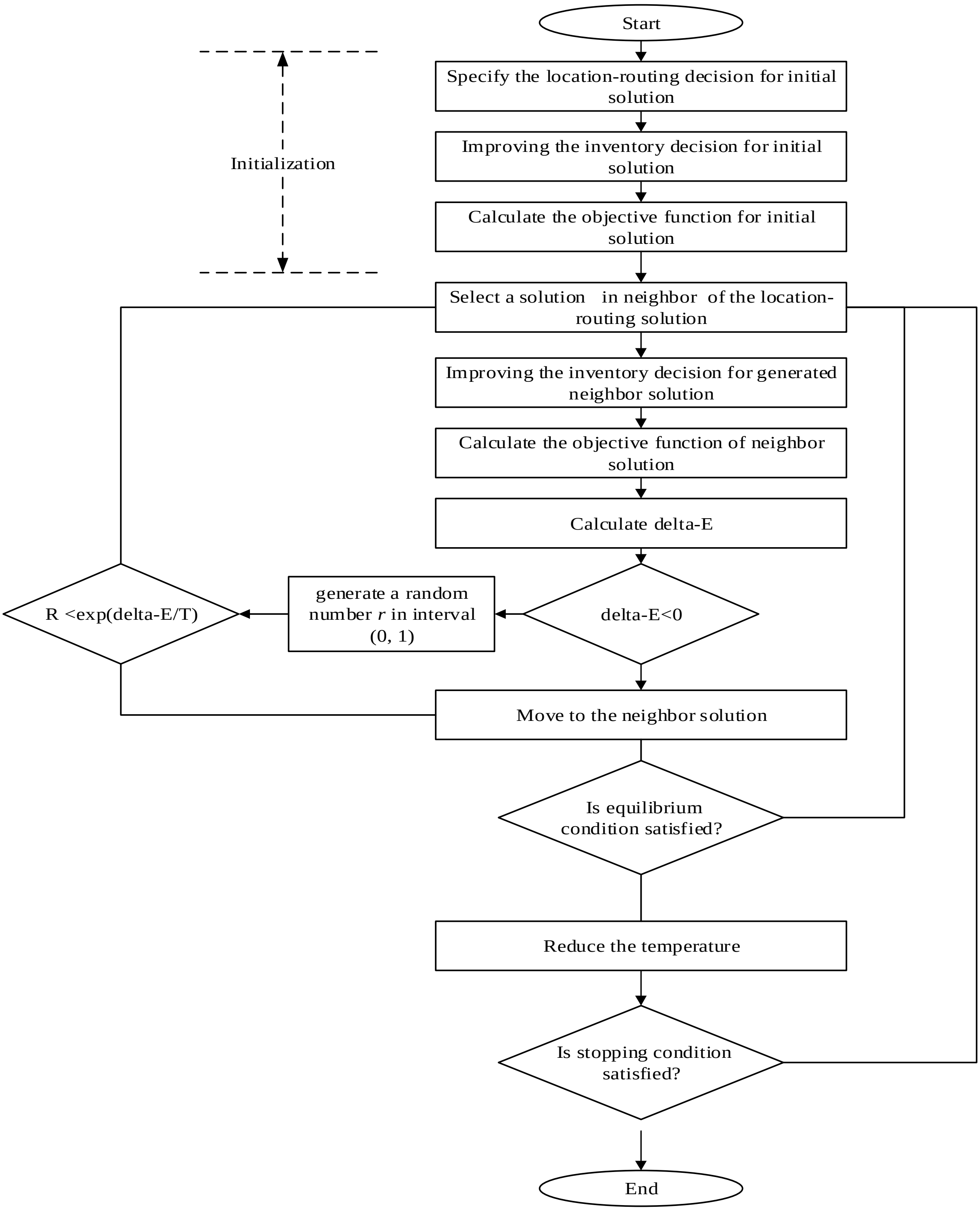

Fig 1. Flowchart of proposed SA

4-1- Location, allocation and routing decisions

At the first stage, the algorithm must determine the location, allocation and routing decisions. For this sake, we consider a string of numbers, including n retailers denoted by set$\{1,2,\ldots,n\}$, m potential DCs denoted by set $\{-1,-2,\ldots,-m\}$ and g dummy zeros. The value of g is obtained from following equation.

$$g = \left\lceil \frac{\sum_i \lambda_i'}{D} \right\rceil$$

In witch, $\lceil \blacksquare \rceil$ yields the smallest integer that is equal to or larger than the enclosed number. Thereupon, the positive and negative numbers denote the pertaining retailers and potential DCs, respectively.

The first number of the aforementioned string belongs to the set of potential DCs (i.e., a negative variable). The positive numbers between two negative numbers represent the retailers that are allocatedto the first negative number. Hereupon, when there is no positive number after a negative number, the pertaining potential DC would not be opened. A route started from an opened DC is terminated, when by adding its allocated retailers to the route; one of above three conditions is occurred:

1- The string arrives to a dummy zero.
2- The vehicle capacity exceeds by total mean demands of added retailers to the route.
3- The string arrives to a negative number.

Nonetheless, we must ensure that the product transferred by each vehicle is less than its capacity.

4-1-1- Illustration of solution representation

Tables 1 and 2 present an example with 20 retailers and 5 potential DCs. The location (x,y) of DCs and the setup cost for opening DCs are rendered in Table 2. The DCs' capacities are 270, the vehicles' capacities are 110, and the fixed cost for setting a route is 1000. A randomly generated sample solution for this example is presented in Figure 2. In the solution, six dummy zeros are given. Figure 3 presents a visual illustration of the sample solution. The first number is 2, followed by (13,15,1,0,3,10). Thus, DC-2 services retailers (13, 15, 1) in the first route. Retailer 1 is followed by a zero, so the first route is terminated. The second route of DC-2 then starts to service retailers 3 and 10. Retailer 10is followed by DC-4, thus second route is terminated. DC-4 is followed by two successive zeros. As there are no actual retailers between DC-4 and DC-3, so DC-4 is closed. Since retailers (2,9,18,16,6,20,17) between DC-3 and DC-1 are actual retailers, DC-3 is opened in order to service these retailers. The first route of DC-3 services retailers (2,9,18,16). As the vehicle capacity exceeds by adding retailers 6 to the route, the first route is terminated. The second route services retailers (6,20,17). As retailer 17 is followed by DC-1, the second route is terminated. DC-1 followed by zero. Because zero is not followed by an actual retailer, it can be relinquished. The first route of DC-1 services retailers (19,11,5,14). As the vehicle capacity exceeds by adding retailers 8 to the route, the first route is terminated. The second route services retailer (8,4). Since retailer 4 is followed by DC-5, the second route is terminated. DC-5 is followed by (7,12,0,0), so the first route services retailers 7 and 12. As retailer 12 is followed by dummy zeros, the route terminates. At this time, all retailers are servicedand no other DC is required to be opened. In this solution, DC-2, DC-3, and DC-1 are opened with two routes and DC-5 is opened with one route. As shown by solution representation, in each route, opened DCs and retailers can be specified from solution representation. It is worthy note that as the total demandsare violated DC capacities, the penalties are added to objective function.

Table1. Retailer location and demand

Retailer No	X	Y	λ
1	26	28	25
2	28	29	27
3	6	26	26
4	30	24	26
5	38	7	28
6	5	5	30
7	9	25	32
8	18	29	30
9	28	30	22
10	37	31	26
11	42	34	28
12	20	38	30
13	25	39	29
14	31	40	27
15	33	2	24
16	36	1	28
17	35	12	29
18	19	15	31
19	12	37	27
20	1	14	30

Table2. DC location and opening cost

DC No	X	Y	DC opening cost
-1	12	25	11000
-2	20	26	12000
-3	30	1	8000
-4	24	25	7500
-5	19	9	7400

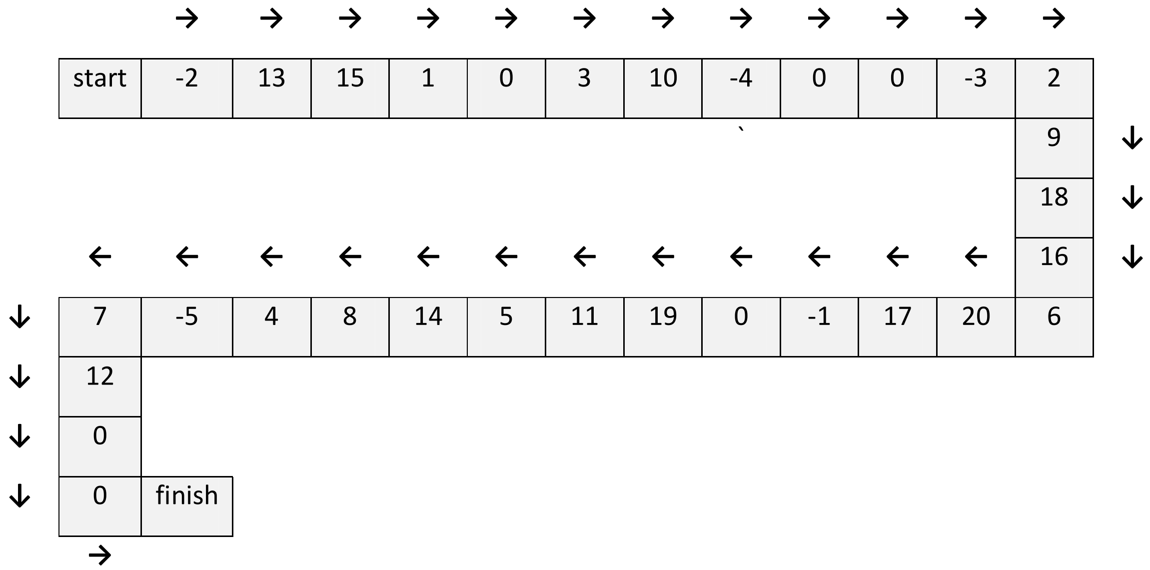

Fig 2. An example of solution representation

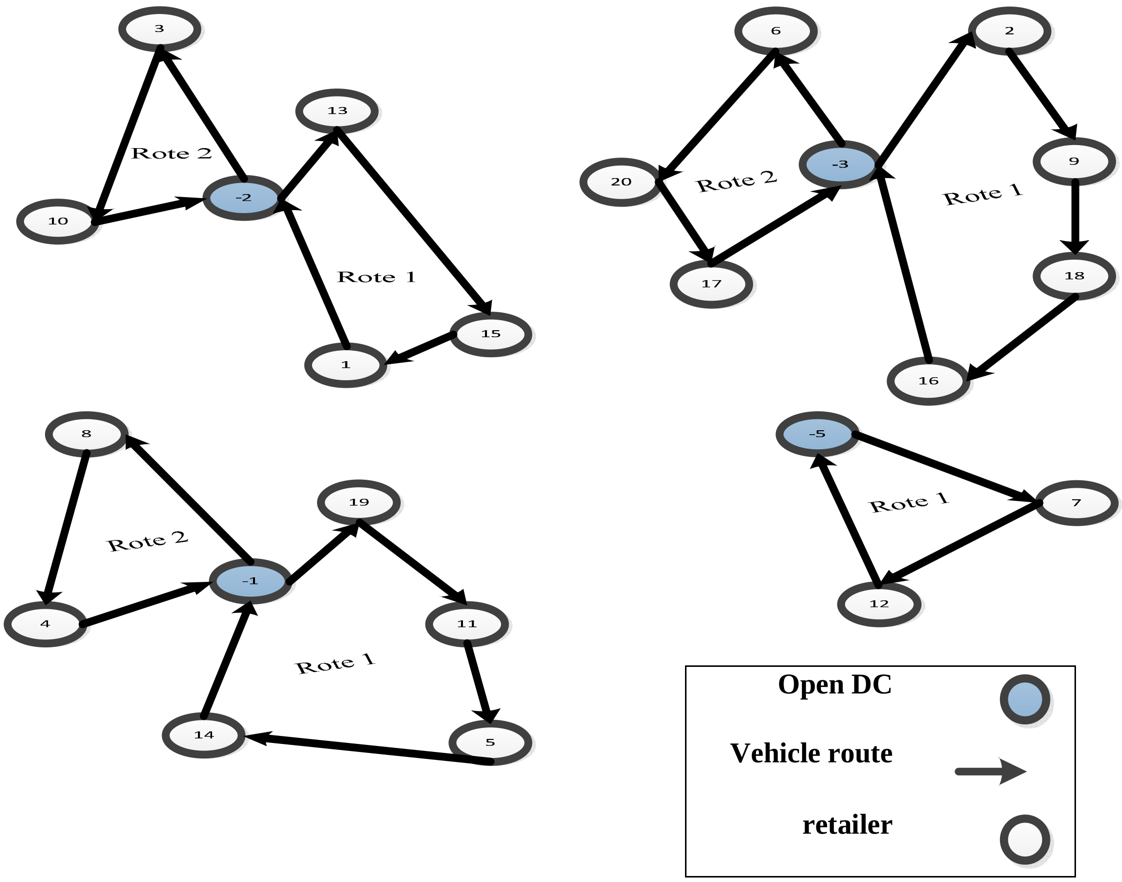

Fig 3. Visual illustration of the example solution given in Figure 2

4-1-2- Neighborhood

We use two kinds of moves, namely insertion move and swap move, for generating random neighbors. Let's assume that *N(X)* is a set of *X* solutions and the neighbor solution *Y* is obtained from *N(X)* using the insertion move or the swap move. Dummy zeros are considered as retailers when carrying out these moves. For performing insertion move, two numbers *i* and *j* are first randomly chosen from *N(X)*. Then, number *i* is inserted exactly before *j*. The insertion move may cause of four effects, as below:

Case1: *i* and*j* are both DCs. Thus, DC *i* will be closed and its retailers will be served by DC *j*.

Case2: *i*and*j* are both retailers. Thus, retailer *i* will be served by the DC that now serves retailer j.

Case3: *i* is a DC and *j* is a retailer. Thus, retailer *j* will be served as the first priority of DC *i*.

Case4: *i*is a retailer and *j* is a DC. Thus,*i*will be served from DC, which was previous priority DC *j* before move.

It seems that Case 3 has more effect on preventing the algorithm from being trapped in a local optimum. For performing the swap move, two numbers *i* and *j* are randomly first chosen. Then, number *i* is replaced with position *j* and vice versa. The swap move may causefour effects, as below:

Case1: *i* and j are both DCs. Thus, retailers of *j* are allocated to DC *i*, and vice versa.

Case2: *i* and *j* are both retailers. Thus, retailer *i* is served by the DC that serves retailer j before the swap move, and vice versa.

Case3: *i* is a retailer and *j* is a DC. Thus, DC *j* will serve the retailers which followed retailer *i* before this move and the retailer *i* will be served by the DC which had *j* previous priority before this move.

Case 4: *i* is a DC and j is a retailer. This case is similar to the case 3.

The new part of solution exploring by algorithm may significantly affected by case 3, 4 and it may change the original solution significantly.

Swap move and insertion move are chosenin the algorithm with the same probability. Note that there three situations while generatingthe neighbor solutions that may cause infeasible solutions.

1) The DC capacity constraints violation, in this situation, a penalty cost is considered in objective function value, as mentioned in section 3.2.
2) The vehicle capacity constraints violation, in this situation, another route will be generated.
3) The first entry may not bechose from {-1,-2,…,-5},so this solution is not considered and another solution will be generated

4-2- Inventory decisions

After the algorithm generates a solution for the location, allocation and routing problem, the inventory decisions for opened DCs are to be determined. In fact, the following model should be solved for each opened DC:

$$TR_k(S_k)\sum_{k\in K} h_k z_k \left[\frac{\mu\left(\frac{\lambda_k}{\mu}\right)^{S_k}\left(-1+\frac{\lambda_k}{\mu}\right)\left({\lambda_K}^{S_k+1}-\lambda_K\mu^{S_k}-\lambda_K S_k\mu^{S_k}+S_k\mu^{S_k+1}\right)}{{\lambda_K}^{S_k}(\lambda_K-\mu)^2}\right] + \pi_k z_k\left[\lambda_K\left(\frac{\lambda_k}{\mu}\right)^{S_k}\right] \tag{24}$$

$$S_k \le U_k \tag{25}$$

For solving the aforementioned model, we suggest following algorithm, that provides optimal inventory policy for each opened DC.

Step 1: Put $S_j = 0$ and let $TR_k^* = inf$ (inf is a big number).

Step 2: obtain TR_k.

Step 3: If $TR_k - TR_k^* \le 0$, then$TR_k^* = TR_k$.

Step 4: Put$S_k = S_k + 1$.

Step 5: If $S_k > U_k$, then stop. Otherwise, follow to step 2.

5- Computational experiments

In this section, we evaluate the performances of the proposed meta-heuristic algorithm using numerical experiments. The experiments are run on a computer with 2.93GHZ CPU and 2GB of RAM. In addition, the meta-heuristic algorithm is implemented in MATLAB R2014a. In all test problems, the parameters are uniformly generatedwith respect to the ranges reported in Table 3.

Table 3. The distributions of model Parameters

C_k	π_k	F_k	T_{ghv}	A_k	h_k
[5 10]	*[70 90]*	*[5000 7000]*	*[5 10]*	*[5 15]*	*[25 40]*
U_k	$\lambda_i{'}$	D_v	μ	ψ	Φ
[15 30]	*[50 150]*	*[250 350]*	*[4500 5500]*	*1*	*1*

5-1- Parameter tuning

Suitable design of parameters has great effect on performance of meta-heuristic algorithms (Ramezanian and Saidi-Mehrabad, 2013). Here, we tune the parameters of the proposed algorithm using the Taguchi method. The Taguchi method is developed based on full factorial experiment (FFE) introduced by Cochran and Cox (1957). Unlike FFE, Taguchi method studies the impacts of different factors on the cost function using fewer experiments. In fact, it investigates complete parameters' space with a particular design of orthogonal arrays. In the Taguchi method, the factors are divided into two main groups, noise and controllable factors. The main objective of Taguchi method is to determine values of controllable factors such that the effects of uncontrollable factors are decreased, while reducing the variability of the cost function.

The results of the Taguchi method can be analyzedthrough two ways. The first way is based on evaluationof variances of tests, where single replicate is done. In the second way, evaluations are performed using the signal to noise (S/N) ratio, which multiple replications are required to investigate the responses. Due to the better performance of multiple replications, the S/N ratio is selected in this paper.
Here, we aim at tuning the values of the temperature (*TI*), final temperature (*TF*), maximum number of no-improvement solutions in inside loop (*NI*), and maximum number of no-improvement solutions in outside loop (*NO*). As shown in Table 4, three levels are considered for each parameter. Therefore, we use the Taguchi L^9 orthogonal array to design the experiment. Table 5 reports various combinations of the parameters' levels in the design and Figure 4 shows mean S/N ratio for each level of the factors. Each level that maximizes the value of S/N ratio is the suitable level of the pertaining factor. Accordingly, the best values of factors are given in Table 6 regarding the Figure 4.

Table4. The levels of factors

factors	Level 1	Level 2	Level 3
TI	1000	1200	1300
CL	0.9	0.95	0.99
NI	20	40	60
NO	10	20	30

Table5. Taguchi experiment design

	TI	TF	NI	NO
1	1	1	1	1
2	1	2	2	2
3	1	3	3	3
4	2	1	2	3
5	2	2	3	1
6	2	3	1	2
7	3	1	3	2
8	3	2	1	3
9	3	3	2	1

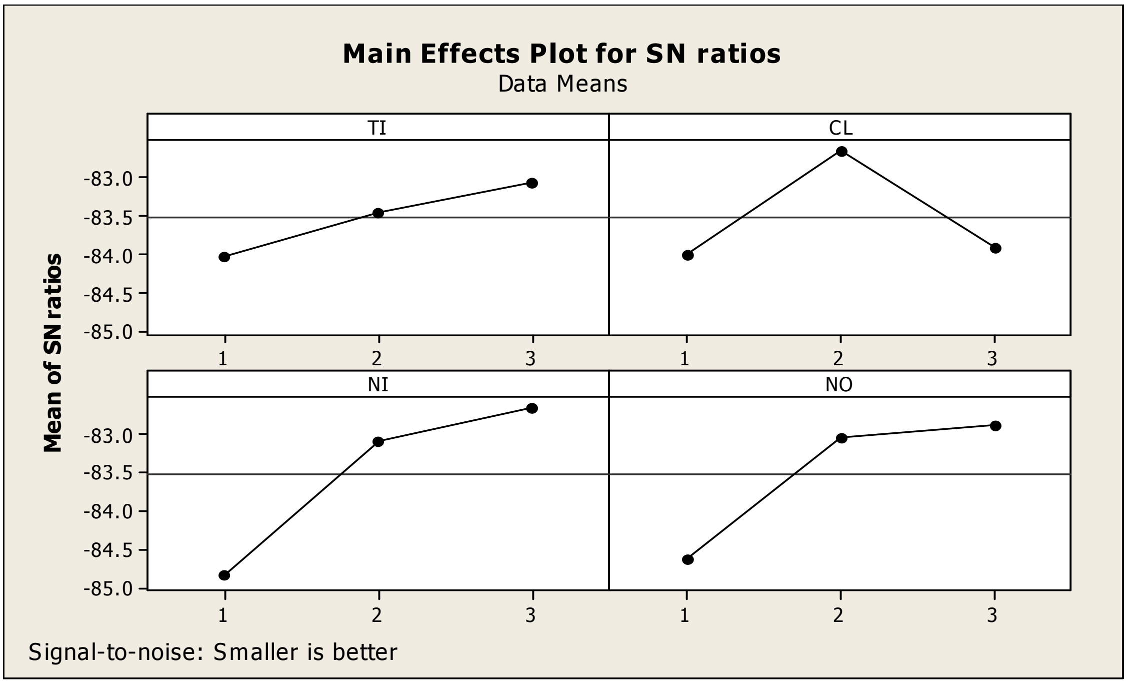

Fig 4. Main effects plots for SN ratios

Table6. The best values of SA factors

Factors	Best value
TI	1300
TF	.95
NI	60
NO	30

5-2- Validation of solution approach

To validate the proposed algorithm, we apply general algebraic modeling system (GAMS), where BARON solver has been applied. BARON employs deterministic global optimization algorithms of the branch-and-bound type. It ensures to obtain global optimal under general assumptions of the global solution of nonlinear (NLP) and mixed-integer nonlinear programs (MINLP).The comparative results are reported in Table 7. From left to right; the columns of the Table show the instance no.,the number of vehicles, the potential DCs, and the retailers. Next columns illustrate thetotal costs and the runtimes of the proposed algorithm and the GAMS, respectively. The last column shows the percentage difference of the objective function of proposed algorithm and the GAMS, which is obtained by$\frac{Total\ cost_{SA}-Total\ cost_{GAMS}}{Total\ cost_{SA}} \times 100$. What is observed from this table is that in instances 1-7, the maximum runtimes of the proposed algorithm and the GAMS are 31.14 and 5366.87 seconds, respectively. With respect to quality of the solution, the average gap is only 2.088%. In other words, the proposed algorithm has approximately attained the same quality achieved by GAMS, while its runtime is significantly less than the GAMS. For instances 8-13, the best solutions of the GAMS in 24-hours runtime are reported. It can be seen that with respect to both quality of solutions and runtime, the proposed algorithm outperforms the GAMS. Therefore, it can be concluded that the proposed algorithm can achieve good-quality solutions within reasonable amount of times.

Table 7. Evaluation the performance of proposed algorithm

NO.	# Vehicles	# Potential DCs	# retailers	meta-heuristic algorithm		GAMS		GAP (%)
				Total cost ($)	Runtime (Sec)	Total cost($)	Runtime (Sec)	
1	2	2	4	11622.75	2	11622.75	21.436	0
2	3	4	6	11951.23	7.95	11951.23	150.385	0
3	4	5	16	12051.36	11.37	11987.24	670.359	0.532
4	5	5	18	12103.45	15.97	11993.84	1265.74	0.905
5	6	6	20	12171.66	21.7	12011.86	2783.5	1.312
6	6	7	23	12283.75	27.39	12098.21	3452.44	1.51
7	6	7	25	12389.13	31.14	12130.36	5366.87	2.088
8	8	9	31	12508.2	43.21	12977.49	24 h limit	-3.751
9	12	12	40	12848.42	63.85	13138.86	24 h limit	-2.260
10	13	14	43	12945.95	69.77	13545.85	24 h limit	-4.633
11	15	15	50	13213.44	77.56	14386.3	24 h limit	-8.876
12	17	17	63	14828.8	82.96	15733.41	24 h limit	-6.100
13	19	19	80	15514.89	88.42	16585.1	24 h limit	-6.89

5-3- Sensitivity analysis

In this sub-section, we aim at analyzing numerically the effects of the parameters on the location decisions. Table 8 shows the impact of the importance weighs on the number of opened DCs. We see that by increasing the importance of the transportation costs, i.e., ψ, the number of opened DCs increases too. On the other hand, as the importance of the inventory costs, i.e., Φ, increases, the number of opened DCs decreases. Table 9 displays the sensitivity analysis of the storage capacities at DCs, i.e., U_k, on the objective function and number of opened DCs. We observe from this Table that at higher values of storage capacities, the total costs decrease. In addition, we take from this Table that by increasing the storage capacities, the number of opened DCs decreases. The reason is that by increasing the storage capacities, more demands can be served by DCs. Therefore, the number of opened DCs would be decreased in the optimal solution in order to save the location costs.

Table 8. The effect of the weight factors on the number of DCs

ψ	Φ	# Vehicles	# Potential DCs	# Potential retailers	Total cost ($)	CPU time	# opened DCs
0.5	1	12	12	40	12778.83	60.97	7
1	1	12	12	40	12848.42	63.85	7
1.5	1	12	12	40	12894.67	61.86	8
2	1	12	12	40	12961.39	66.47	9
1	0.5	12	12	40	12748.59	68.48	8
1	1.5	12	12	40	12923.49	58.694	6
1	2	12	12	40	12982.68	60.78	6

Table 9. The effect of the storage capacitieson the objective function and number of DCs

Changes (%)	# Vehicles	# Potential DCs	# Potential retailers	Total cost ($)	CPU time	# opened DCs
-20	12	12	40	14019.93	42.35	8
-10	12	12	40	13271.582	55.35	7
0	12	12	40	12848.42	63.85	7
10	12	12	40	12565.94	71.96	6
20	12	12	40	12342.519	83.87	6

6- Conclusion

In this paper, we developed a mathematical model for designing a three-level supply chain network. Unlike the most of published papers, the proposed model is capable to tackle simultaneously three major issues of the supply chain network design, i.e., location-allocation, vehicle routing and inventory planning. In fact, the proposed model aims at finding (1) the number of opened DCs, (2) their locations, (3) the allocation of the retailers to the opened DCs, (4) the optimal inventory policies at opened DCs, and (5) vehicles' routes starting from opened DCs for serving their allocated retailers. Additionally, we implemented a continuous-time Markov process to deal with the stochastic nature of the parameters and then derived the performance measures of the system. Using the obtained measures, the proposed problem was formulated as an MINLP model. Synthesizing different NP-hard problems together (i.e., routing problem, facility location problem etc.), yielded more complicated NP-hard problem that exact methods could not persuasively solve it. Accordingly, we proposed a simulated annealing (SA) based meta-heuristic algorithm in order to tackle the complexity of the MINLP model. The computational experiments demonstrated that the proposed algorithm could achieve good-quality solutions within reasonable times.

The current research can be developedin several ways. First, assume that multiple products are distributed in the supply chain. Second, there are lateral transitions between the opened DCs or the retailers in reality. In other words, the transitions between the DCs or the retailers can be incorporated into the model. Finally, it is interesting to consider inventory policy in multi levels. For example, it can be assumed that retailers have inventory policies and their impacts on the inventory policy of opened DCs are investigated.

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