Journal of Industrial and Systems Engineering
Vol. 14, No. 4, pp. 67-80
Autumn (November) 2022

A new method for monitoring multivariate simple linear profile

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Abstract
In some statistical process monitoring applications, the quality of a product or process can be determined by a linear or nonlinear regression relationship between a response variable and one or more explanatory variables called "profile". Sometimes, it is possible to describe the quality of a process or product by several simultaneous profiles in which the response variables are interdependent and are modeled as a set of linear functions of one explanatory variable, that is referred to as multivariate simple linear profile structure. In this paper, we propose a new method for monitoring phase II of multivariate simple linear profile. In this method, namely MHWMA/EWMA, a MHWMA control chart and a EWMA control chart based on generalized variance are used for monitoring the differences between the reference profile and the sample profile. Using the Mont-Carlo simulation, the statistical performance of the proposed method is evaluated in terms of the average run length criterion. The obtained results showed that the method effectively detects shifts in the profile parameters. The proposed MHWMA/EWMA method is compared with an existing method, and the results showed that this method has a good performance in detecting different types of shifts in profiles' parameters. In addition, the applicability of the proposed methods is illustrated using a real case of calibration application.
Keywords: Profile monitoring, multivariate simple linear profile, average run length, control chart

1-Introduction
Nowadays quality plays a very important role in success of production and service rendering companies i.e. it can be considered as a business strategy. To improve quality of products or services, there are different tools and methods available that could be used. Statistical process control is a powerful set of problem solving tools, playing an important role in creation of stability in process and optimizing its capability through reduction of variability (Montgomery, 1991).

In most applications of statistical process control, quality of a product or process can be described by a quality characteristic with a certain distribution and/or in general by several quality characteristics with a multivariate distribution. Parameters of distribution can be monitored through control charts. Sometimes quality of a product or process can be described by a regression relationship between a response variable and one or several explanatory variables (Noorossana et al., 2011). This relationship is called "profile" by researchers and can be a linear, non-linear and even more complex relationship. Profile monitoring is one of the applied subjects recently developed by researchers. Profile monitoring would be performed in two phases, using control charts with different goals. In phase I, we are aimed

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ISSN: 1735-8272,

at obtaining information about process dispersion during time, evaluating process stability and estimating model's parameters in the in-control process. However, in phase II the process is monitored online and the goal is quick detection of shifts in parameters (Montgomery, 2009). Some methods have been proposed by Kang and Albin, (2000), Kim et al., (2003), Noorossana et al., (2004), Mahmoud and Woodall, (2004), Zou et al., (2006), Mahmoud et al., (2007), Nassar and Abdel-Salam, (2021), Yeganeh and Shadman, (2021), Ahmadi and Shahriari, (2022), Song et al., (2022), Zhou and Qiu, (2022), Abbasi et al., (2021), Liu et al., (2022) and Moheghi et al., (2022) to monitor simple linear profile in phase I or II. Multiple linear profiles have been studied by Mahmoud, (2008), Jensen et al., (2008), Amiri et al., (2012), Fallahdizcheh and Wang, (2022) and Li and Tsai, (2022). To monitor polynomial profiles in phases I and II, some methods have been proposed by Kazemzadeh et al., (2008), Zou et al., (2007), Abdella et al., (2016) and Atashgar and Abbassi, (2020). Monitoring non-linear profiles has been studied by researchers as Williams et al., (2007), Walker and Wright, (2002), Williams et al., (2003), Xin et al., (2020) and Nasiri Boroujeni et al., (2022).

Sometimes quality of a process or product can be simultaneously described by several profiles in which response variables are dependent. Under such circumstances, where dependence between response variables would not be taken into consideration and profiles would be monitored separately; the results could be deceptive. To solve the problem, considering multivariate structure for response variables is unavoidable where simple refers to one explanatory variable and multivariate refers to the response variables. An application of this profile has been shown by Noorossana et al., (2004) via a hydraulic press machine in an automotive industrial group.

Multivariate simple linear profile in phase II has been studied by some researchers. Three methods have been implemented by Noorossana et al., (2010) to monitor multivariate simple linear profiles; and applicability of these methods was illustrated using a real case of calibration application. In the First method, they used a MEWMA control chart, developed by Lowry et al., (1992), to monitor all profile parameters. In the second recommended method, differences between the reference profile and the sample profile has been monitored through development of EWMA/R method proposed by (Kang and Albin, 2000). In this method, calledMEWMA/χ^2, MEWMA and χ^2 control charts have been used to monitor mean vector of errors and process variability. The third method, recommended by Noorossana et al., (2010), is considered as a development of the EWMA-3 method proposed by (Kim et al., 2003). In this method, called MEWMA-3, three separate MEWMA control charts would be used to detect shifts in the y-intercepts and slopes vectors and profiles variability. These three methods have been compared using simulation and based on ARL metric in a numerical example for two-variable simple linear profile.

Zou et al., (2012) referred to a problem, that when the profile parameter dimensionality is large, the detection ability of the procedures commonly used T^2-type charting statistics is likely to decline substantially. To solve the problem, using a variable-selection-based multivariate control scheme has been recommended to transformations of estimated profile parameters. Zou et al., (2012) showed that the proposed control chart is capable of detecting various shifts of profile and provides diagnostic aid.

A maximum likelihood estimator has been developed by Ayoubi et al., (2014) to estimate change point in the mean of response variables in multivariate linear profile which can be used with no prior knowledge about type of the shift created. Their simulation results showed that this estimator performs well in detection of various shifts.

Independence is one of the assumptions in profile monitoring methods that violating it can lead to deceptive results. Monitoring phase II of multivariate simple linear profiles has been studied by Soleimani et al., (2013) where observations within profile are dependent and ARMA (1, 1) structure is used. Considering three monitoring methods of MEWMA, MEWMA/χ^2, and MEWMA-3 proposed by Noorossana et al., (2010) and based on ARL metric, it was shown that autocorrelation structure has considerable effect on performance of these monitoring methods. They proposed a remedial measure based on a transformation method for removing the autocorrelation structure, which made the considered methods perform similar to the state with assumption of independence. In the presence of within profile autocorrelation, two methods have been provided by Rahimi et al., (2019). If the correlation between response variables is small, their methods are not able to detect the shifts quickly. Soleimani and Noorossana, (2014) considered presence of between profile autocorrelation and using three method based on time series, they eliminated autocorrelation effect and studied performance of MEWMA/χ^2, MEWMA-3, and T^2 monitoring methods. Monitoring correlated multivariate linear profiles and generalized linear profiles have been studied by Amiri et al., (2018). Three methods have been introduced by them for the simultaneous monitoring. For this purpose, they used the MEWMA control chart, which performed well in detecting small shifts. To monitor generalized linear profile in phase II, machine learning techniques have been used by Yeganeh et al., (2022); and, a control chart based on artificial neural network has been used to improve performance of control chart to monitor parameters of the profile. Univariate and multivariate linear profiles have been monitored by Malela-Majika et al., (2022) through usage of a triple EWMA control chart; and using a numerical example, performance of the method has been studied. In the study performed by Haq, (2020), the two MEWMA and Adaptive MEWMA control charts have been proposed to monitor simple and multivariate linear profiles; and comparing performance of the methods with some existing ones, it was shown that recommended methods have better performance. The effect of correlation between response variables was not investigated in their proposed methods.

In monitoring phase II of a process using control charts, process parameters are considered as known. This assumption makes easier development and evaluation of control charts; however, parameters are mostly unknown in reality and control charts should be designed through estimated parameters of in-control process in phase I. Effect of parameter estimation in monitoring multivariate simple linear profile has been studied by Yazdi et al., (2019). MEWMA, MEWMA-3, and MEWMA$/\chi^2$ monitoring methods proposed by Noorossana et al., (2010) in case of using estimated parameters have been compared by them, using AARL, SDARL, and CVARL metrics. They showed that parameters estimation is highly effective on performance of in-control and out-of-control performance of monitoring methods. In this study, the value of correlation between response variables was considered constant and no sensitivity analysis was performed on it. Different values of this coefficient may have different results.

Kordestani et al., (2020) considered the performance of ordinary regression estimators in the presence of outliers and introduced three robust estimators to estimate parameters of multivariate simple linear profiles. They showed that recommended methods have proper performance compared to OLS estimator in phases I and II, but they did not compare their proposed method with other estimators. Haq et al., (2021) propose two control charts, maximum multivariate EWMA (MaxMEWMA) and maximum double MEWMA (MaxDMEWMA), for monitoring the parameters of a multivariate simple linear profile using the individual observations. They increased the sensitivities of the charts with the variable sampling interval feature. Using Mont Carlo simulation, they concluded that the MaxDMEWMA chart is slightly more sensitive than the MaxMEWMA chart. In their study, the VSI based charts are more sensitive than that based on the FSI, as expected. Yeganeh et al., (2021) used an Exponentially Weighted Moving Average Range (EWMAR) control chart to monitor the linear profiles with a combination of run rules to enhance the performance of the chart in detecting out-of-control conditions. They used the proposed method for monitoring all types of linear profiles (simple, multiple and multivariate) and investigated its performance. Karavigh and Amiri, (2022) developed three control charts, MEWMA, $MEWMA/\chi^2$ and DMEWMA, equipped with proposed runs rules schemes to improve the performance of control charts in monitoring multivariate simple linear profiles. The results showed that their proposed schemes had a better performance compared to previous charts for detecting shifts.

Improving profile monitoring methods, currently considered as one of the advanced statistical quality control bases, have been considered by many researchers; because by reducing errors and reworks and eliminating losses and wastes, quality improves and high costs are avoided. In this study, a new method for monitoring phase II of multivariate simple linear profile is proposed that is able to detect various shifts faster than existing methods and improves profile monitoring. Comparing performance of the proposed method to existing methods, shows its proper performance.

The organization of the paper is as follows: in section 2, multivariate simple linear profile is discussed. In section 3, MHWMA/EWMA method is presented to monitor the multivariate simple linear profile. Performance of recommended method is studied numerically based on ARL metric and is compared to one existing method in section 4. Finally in section 5, the conclusion remarks are provided.

2-Multivariate simple linear profile

In some practical applications, the quality of a process or product can be characterized by a multivariate simple linear profile where several correlated response variables are considered as a linear functions of an explanatory variable.

Assume that in the k^{th} random sample, we have n fixed values for explanatory variable (sample size) and for each value of it, p corresponding values are available for response variables. So, for the k^{th} sample, n observations are available as $(x_i, y_{i1k}, y_{i2k}, \dots, y_{ipk})$ where $i = 1,2, \dots, n$ and $k = 1,2, \dots$. x_i is the i^{th} value of explanatory variable and $y_{i1k}, y_{i2k}, \dots, y_{ipk}$ are corresponding values of response variables.

In the k^{th} sample, a model showing the relationship between response variables and explanatory variable is as follows:

$$Y_k = XB + E_k \tag{1}$$

Or in the matrix form:

$$\begin{bmatrix} y_{11k} & y_{12k} & \cdots & y_{1pk} \\ y_{21k} & y_{22k} & \cdots & y_{2pk} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n1k} & y_{n2k} & \cdots & y_{npk} \end{bmatrix} = \begin{bmatrix} 1 & x_1 \\ 1 & x_2 \\ \vdots & \vdots \\ 1 & x_n \end{bmatrix} \begin{bmatrix} \beta_{01} & \beta_{02} & \cdots & \beta_{0p} \\ \beta_{11} & \beta_{12} & \cdots & \beta_{1p} \end{bmatrix} + \begin{bmatrix} \varepsilon_{11k} & \varepsilon_{12k} & \cdots & \varepsilon_{1pk} \\ \varepsilon_{21k} & \varepsilon_{22k} & \cdots & \varepsilon_{2pk} \\ \vdots & \vdots & \ddots & \vdots \\ \varepsilon_{n1k} & \varepsilon_{n2k} & \cdots & \varepsilon_{npk} \end{bmatrix} \tag{2}$$

Y_k is a $n \times p$ matrix of response variables for k^{th} sample, where each row is corresponding to one of the values of explanatory variable, including p value for response variables. X is a $n \times 2$ matrix of explanatory variable, fixed from sample to sample. $E_k = (\varepsilon_{1k}, \varepsilon_{2k}, \dots, \varepsilon_{nk})'$ is a $n \times p$ matrix of error terms for the k^{th} sample. ε_{ik} is a $1 \times p$ vector, i^{th} row of E_k matrix, which follows a multivariate normal distribution with mean vector of zeroes and $p \times p$ covariance matrix (Σ). B is a $2 \times p$ matrix of known parameters, where the least squares estimator of it ($\hat{B}_k$) is given by:

$$\hat{B}_k = (\hat{\beta}_{0k}, \hat{\beta}_{1k})^T = (X^T X)^{-1} X^T Y_k \tag{3}$$

The elements of the $\hat{B}_k$ matrix are as follows:

$$\hat{\beta}_{0jk} = \bar{y}_{.jk} - \hat{\beta}_{1jk}\bar{x} \qquad \hat{\beta}_{1jk} = \frac{S_{xy(j)}}{S_{xx}} \qquad j = 1,2, \dots, p \tag{4}$$

where $\bar{y}_{.jk} = \frac{1}{n}\sum_{i=1}^{n} y_{ijk}$, $S_{xy(j)} = \sum_{i=1}^{n}(x_i - \bar{x})y_{ijk}$, $\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$, and $S_{xx} = \sum_{i=1}^{n}(x_i - \bar{x})^2$.

3-Proposed method

In this section, a new method is presented for monitoring phase II of multivariate simple linear profile. In this method, called MHWMA/EWMA, we propose the use of multivariate homogeneously weighted moving average (MHWMA) control chart, proposed by Adegoke et al., (2019), with an EWMA control chart based on generalized variance, proposed by Yeh et al., (2006).

Assume that for the k^{th} random sample collected over time, $\bar{\varepsilon}_k = (\bar{\varepsilon}_{1k}, \bar{\varepsilon}_{2k}, \dots, \bar{\varepsilon}_{pk})^T$ is a $1 \times p$ vector of average error where $\bar{\varepsilon}_{jk} = \frac{1}{n}\sum_{i=1}^{n} \varepsilon_{ijk}$, $j = 1,2, \dots, p$. It can be shown that for the in-control process, the average error vectoror ($\bar{\varepsilon}_k$) follows a multivariate normal distribution with mean vector of zeros and covariance matrix of $\Sigma_{\bar{\varepsilon}} = \frac{1}{n}\Sigma$.

To monitor mean vector of errors, MHWMA control chart is used. For k^{th} sample, $1 \times p$ mean vector of errors weighted moving average is defined as:

$$H_k = w\bar{\varepsilon}_k + (1 - w)\bar{\bar{\varepsilon}}_{k-1} \qquad k = 1,2, \dots \tag{5}$$

H_k is a $1 \times p$ vector, where w is smoothing parameter($0 < w \leq 1$), and $\bar{\bar{\varepsilon}}_{k-1}$ is mean vector of errors in previous samples up to k-1th sample. $\bar{\bar{\varepsilon}}_0$ is considered as equal to zero vector. It can be shown that when the process is in-control, H_k has multivariate normal distribution with zero as mean vector and covariance matrix of Σ_{H_k}.

Σ_{H_k} is a $p \times p$ matrix calculated as:

$$\Sigma_{H_k} = \begin{cases} w^2\Sigma_{\bar{\varepsilon}} & k = 1 \\ w^2\Sigma_{\bar{\varepsilon}} + (1-w)^2\dfrac{\Sigma_{\bar{\varepsilon}}}{k-1} & k \geq 2 \end{cases} \tag{6}$$

So, the k[th] chart statistic for the MHWMA chart denoted by T_k^2 is given by:

$$T_k^2 = (H_k)'\Sigma_{H_k}^{-1}(H_k) \qquad k = 1,2,\ldots \tag{7}$$

The MHWMA control chart gives an out-of-control signal when $T_k^2 > h_M$, where h_M (> 0) is chosen to achieve the desired in-control ARL.

In order to monitor the process variability, we use the EWMA control chart based on generalized variance as follows:

$$Q_k = \sqrt{\frac{n-1}{2p}\ln\frac{|S_k|}{|\Sigma|}} \tag{8}$$

Where, S_k is the covariance matrix of errors in k[th] sample. Considering ε_{ik}~$\mathrm{N}_p(0,\Sigma\ )$, it can be suposed that Q_k is asymptotically distributed as $N(0,1)$. If Σ changes to Σ_1 , Q_k is asymptotically distributed as $(\ln\frac{|\Sigma_1|}{|\Sigma|}, 1)$. In other words, the shift in covariance matrix now changes into a mean shift in Q_k. So, an univariate EWMA control chart can be used for detecting mean shifts in Q_k. The k[th] chart statistic, denoted by G_k, is given by :

$$G_k = \lambda Q_k + (1-\lambda)G_{k-1} \qquad k = 1,2,\ldots \tag{9}$$

Where, λ is smoothing parameter$(0 < \lambda \leq 1)$ and $G_0 = 0$. Control limits of the chart is calculated as follows:

$$\pm L \times \sqrt{\frac{\lambda}{2-\lambda}[1-(1-\lambda)^{2k}]} \tag{10}$$

Where L is chosen to control the ARL of the chart.

If Σ is not known, it can be estimated by $\bar{S}$. Yeh et al., (2006) modified the control chart in this case.

4-Simulation studies

In this section, ARL performance of recommended method is studied using simulation. For this purpose, control charts are designed to have an overall in-control ARL of 200. Smoothing parameters in MHWMA and EWMA control charts are set equal to 0.03 and 0.1 respectively. It was shown by Lucas and Saccucci, (1990) that the smaller smoothing parameters lead to quicker detection of smaller shifts. In this simulation study, each ARL value is estimated using 10000 replications.

To show the performance of the proposed method in terms of the ARL metric, the following multivariate simple linear profiles are considered:

$$\begin{aligned} Y_1 &= 3 + 2x + \varepsilon_1 \\ Y_2 &= 2 + 1x + \varepsilon_2 \end{aligned} \tag{11}$$

Where x_i values are set equal to 2, 4, 6, and 8 (with $\bar{x} = 5$). The vector of error terms $(\varepsilon_1, \varepsilon_2)$ follows a bivariate normal random distribution with a mean vector of zeroes and covariance matrix of $\Sigma = \begin{bmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_1^2 \end{bmatrix}$, where $\sigma_1^2 = \sigma_2^2 = 1$. To investigate the effect of correlation between profiles, the values of ρ equal to 0.1, 0.5, and 0.9 are considered in the simulation studies for individual shifts. The upper control limit of MHWMA control chart (h_M) is set equal to 6.98 yielding an in-control ARL of approximately 400. In addition, for the EWMA control chart, L parameter is set equal to 8.75 which gives an in-control ARL of 400. So, the overall in-control ARL is almost equal to 200. It has to be noted that different combinations of in-control ARL lead to an overall ARL of 200. However in this paper, in-control ARL for both control charts is considered as similar, showing similar importance of detecting shifts in the profile parameters and process variability. Also, the performance of the proposed control chart scheme is compared with the existing MEWMA method, proposed by Haq, (2020).

Table 1, shows the out-of-control ARLs under different shift values in the intercept of first profile (β_{01}) in units of σ_1. It is observed that the MHWMA/EWMA method performs well in detecting shifts. Comparing the results shows that MHWMA/EWMA method is superior in comparison with the MEWMA method. Also, performance of the MHWMA/EWMA method improves when the value of ρ increases. Similar results are obtained for sustained shifts in the intercept of the second profile.

Table 1. Out of control ARL values when β_{01} shifts to $\beta_{01} + \delta\sigma_1$

ρ	Method	δ									
		0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
0.1	MHWMA/EWMA	29.4	9.9	5.4	3.7	2.7	2.1	1.7	1.4	1.2	1.1
	MEWMA	71.4	20.6	9.7	6.2	4.6	3.7	3.1	2.7	2.4	2.1
0.5	MHWMA/EWMA	24.4	8.0	4.5	3.0	2.3	1.7	1.4	1.2	1.0	1.0
	MEWMA	57.8	15.8	7.8	5.1	3.8	3.1	2.6	2.3	2.1	1.9
0.9	MHWMA/EWMA	8.2	3.1	1.7	1.2	1.0	1.0	1.0	1.0	1.0	1.0
	MEWMA	16.0	5.2	3.1	2.3	2.0	1.7	1.4	1.1	1.0	1.0

Table 2 presents a comparison of the performance of the proposed method with the existing MEWMA method under various shifts in the slope of the first profile (β_{11}) in units of σ_1. Investigating the out-of-control ARL values shows that MHWMA/EWMA method is capable of detecting shifts and it can signal quickly when a sustained shift is occurred. This method performs uniformly better than the other competing method. According to the ARL results, the performance of the control chart scheme improves as the value of ρ increases. Again, similar results are obtained for the sustained shifts in β_{12} (the slope of the second profile).

Table 2. Out of control ARL values when β_{11} shifts to $\beta_{11} + \delta\sigma_1$

ρ	Method	δ									
		0.025	0.05	0.075	0.1	0.125	0.15	0.175	0.2	0.225	0.25
0.1	MHWMA/EWMA	57.2	21.1	11.1	7.1	5.2	3.9	3.2	2.7	2.3	2.1
	MEWMA	112.7	24.4	19.6	11.4	7.9	6.0	4.8	4.1	3.5	3.1
0.5	MHWMA/EWMA	47.3	17.2	9.0	5.7	4.3	3.3	2.7	2.3	1.9	1.7
	MEWMA	98.0	32.8	15.0	9.0	6.4	4.9	4.1	3.5	3.0	2.7
0.9	MHWMA/EWMA	16.6	5.8	3.3	2.3	1.7	1.3	1.1	1.0	1.0	1.0
	MEWMA	33.2	9.1	5.0	3.5	2.7	2.3	2.0	1.9	1.7	1.5

Out-of-control ARL values when there is a shift in σ_1 (standard deviation of the first profile) are shown in the table 3. According to the results, MHWMA/EWMA has almost proper performance in detecting moderate and large shifts of σ_1 . However, small shifts in process variability are detected with delay. Comparison of the proposed method with that of the existing method shows that MEWMA method has better performance than MHWMA/EWMA method in detecting shifts of standard deviation. Increasing of ρ leads to the improvement of the performance.

Table 3. Out of control ARL values when σ_1 shifts to $\delta\sigma_1$

ρ	Method	δ									
		1.2	1.4	1.6	1.8	2.0	2.2	2.4	2.6	2.8	3.0
0.1	MHWMA/EWMA	137.5	90.0	59.3	38.9	25.8	18.2	13.3	10.2	8.1	6.3
	MEWMA	52.1	18.6	9.39	6.6	4.9	3.9	3.3	2.8	2.5	2.2
0.5	MHWMA/EWMA	135.5	86.3	54.0	35.2	22.5	15.5	11.2	8.6	6.6	5.4
	MEWMA	50.2	17.2	9.1	6.0	4.5	3.6	3.0	2.6	2.3	2.0
0.9	MHWMA/EWMA	115.3	53.1	25.0	12.9	7.7	5.1	4.0	3.1	2.6	2.4
	MEWMA	32.5	9.4	4.9	3.3	2.4	2.0	1.7	1.5	1.3	1.2

Tables 4-7 represent the out-of-control ARLs under simultaneous shifts. These shifts are defined as: 1) shifts in the intercepts of both profiles; 2) shifts in the slopes of both profiles; 3) shifts in the intercept and slope of the first profile; and, 4) shifts in standard deviations of both profiles. Obtained results in tables 4-6 show that MHWMA/EWMA method performs well and is better than the competing method in detecting simultaneous shifts in regression line parameters. In addition, considering ARL values in table 7, it can be concluded that the MEWMA method outperforms the proposed method in simultaneous shift detection of two standard deviations.

Table 4. Out of control ARL values under combinations of intercepts shifts from $\beta_{01}$to $\beta_{01}+\lambda\sigma_1$ and β_{02} to $\beta_{02}+\delta\sigma_2$

λ	Method	δ				
		0.1	0.2	0.3	.4	0.5
0.1	MHWMA/EWMA	63.7	30.2	15.5	9.5	6.6
	MEWMA	130.9	72.1	35.0	19.3	12.3
0.2	MHWMA/EWMA	30.3	24.4	15.6	10.1	7.1
	MEWMA	72.2	57.8	35.0	20.7	13.5
0.3	MHWMA/EWMA	15.1	15.6	12.8	9.4	7.0
	MEWMA	35.1	35.1	27.5	19.3	13.4
0.4	MHWMA/EWMA	9.4	10.0	9.3	8.1	6.5
	MEWMA	19.3	20.8	19.3	15.8	12.3
0.5	MHWMA/EWMA	6.5	7.1	7.1	6.5	5.7
	MEWMA	12.3	13.4	13.4	12.3	10.5

Table 5. Out of control ARL values under combinations of slopes shifts from $\beta_{11}$to $\beta_{11} + \lambda\sigma_1$ and β_{12} to $\beta_{12} + \delta\sigma_2$

λ	Method	δ				
		0.02	0.04	0.06	0.08	0.1
0.02	MHWMA/EWMA	62.7	30.0	15.3	9.4	6.6
	MEWMA	122.2	63.0	29.5	16.2	10.5
0.04	MHWMA/EWMA	30.4	24.1	15.3	10.1	7.2
	MEWMA	62.7	49.8	29.4	17.5	11.4
0.06	MHWMA/EWMA	15.6	15.7	12.8	9.5	7.1
	MEWMA	29.5	29.5	23.1	16.1	11.4
0.08	MHWMA/EWMA	9.5	10.2	9.5	8.1	6.5
	MEWMA	16.2	17.5	16.2	13.2	10.5
0.1	MHWMA/EWMA	6.6	7.1	7.1	6.6	5.8
	MEWMA	10.4	11.4	11.4	10.5	9.0

Table 6. Out of control ARL values under combinations of intercept and slope shifts from $\beta_{01}$to $\beta_{01} + \lambda\sigma_1$ and β_{11} to $\beta_{11} + \delta\sigma_1$

λ	Method	δ				
		0.02	0.04	0.06	0.08	0.1
0.1	MHWMA/EWMA	24.3	12.6	8.0	5.8	4.5
	MEWMA	55.5	25.4	14.3	9.5	7.0
0.2	MHWMA/EWMA	12.8	8.1	5.8	4.5	3.6
	MEWMA	27.0	15.0	9.9	7.3	5.8
0.3	MHWMA/EWMA	8.0	5.8	4.5	3.6	3.1
	MEWMA	15.7	10.2	7.5	5.9	4.9
0.4	MHWMA/EWMA	5.7	4.5	3.6	3.0	2.6
	MEWMA	10.4	7.7	6.0	5.0	4.2
0.5	MHWMA/EWMA	4.5	3.6	3.0	2.6	2.3
	MEWMA	7.7	6.1	5.0	4.3	3.7

Table 7. Out of control ARL values under combinations of standard deviations shifts from σ_1 to $\lambda\sigma_1$ and σ_2 to $\delta\sigma_2$

λ	Method	δ				
		1.1	1.2	1.3	1.4	1.5
1.1	MHWMA/EWMA	140.4	115.3	95.0	74.2	61.3
	MEWMA	57.4	32.5	20.2	13.9	10.3
1.2	MHWMA/EWMA	115.8	100.2	82.2	67.1	54.1
	MEWMA	32.6	21.5	15.2	11.3	8.8
1.3	MHWMA/EWMA	95.1	81.2	69.0	57.0	47.8
	MEWMA	20.3	15.1	11.6	9.2	7.6
1.4	MHWMA/EWMA	76.2	67.9	56.3	46.9	40.5
	MEWMA	13.9	11.2	9.2	7.7	6.6
1.5	MHWMA/EWMA	61.5	55.4	45.2	40.1	33.7
	MEWMA	10.3	8.8	7.5	6.6	5.7

5-Case study

In this section, performance of the proposed method in a real case of calibration in a 1600-ton hydraulic press in an automotive industrial group, discussed by (Noorossana et al., 2010), is studied.

In this case, they modeled the relationship between the nominal force (x) and the real forces measured in four cylinders (Y_1, Y_2, Y_3, Y_4) as multivariate simple liner profiles.

In-control model obtained from the phase I is as follows:

$$\begin{aligned} Y_1 &= -8.5 + 0.87x + \varepsilon_1 \\ Y_2 &= -5.8 + 0.95x + \varepsilon_2 \\ Y_3 &= 3.2 + 1.04x + \varepsilon_3 \\ Y_4 &= 13.6 + 1.09x + \varepsilon_4 \end{aligned} \tag{12}$$

Where, $(\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$ is a multivariate random normal vector with zero mean vector and covariance matrix of Σ:

$$\Sigma = \begin{bmatrix} 80.0 & 89.6 & 45.1 & 25.3 \\ 89.6 & 122.1 & 71.5 & 29.1 \\ 45.1 & 71.5 & 189.0 & -28.8 \\ 25.3 & 29.1 & -28.8 & 84.4 \end{bmatrix} \tag{13}$$

Each sample consists of 11 values for nominal force as 50, 80, 110, 140, 170, 200, 230, 260, 290, 320 and 350 and there are four real force values (Y_{i}) for each nominal force value. Initially, we generated 10 in-control multivariate simple linear profile observations and computed the in-control MHWMA and EWMA statistics; and then, in next 5 sample, we generated out-of-control data where the value of β_{01} was shifted from -8.5 to -4.

Figures 1 and 2 show performance of MHWMA/EWMA in detecting considered shift. Control limits of charts, shown on the figures, are set to achieve an in-control ARL of 200. Figure 1 shows that MHWMA chart signals on the 11$^{\text{th}}$ sample, immediately after the shift occurred. According to the figure 2, EWMA chart is unable to detect shifts in β_{01}, as expected. It is worth mentioning that after occurring the shift, 6 consecutive points fall on the same side of the centerline and, according to Western Electric

Rules, continuation of this procedure up to 8 points, provides a justification for the out-of-control signal.

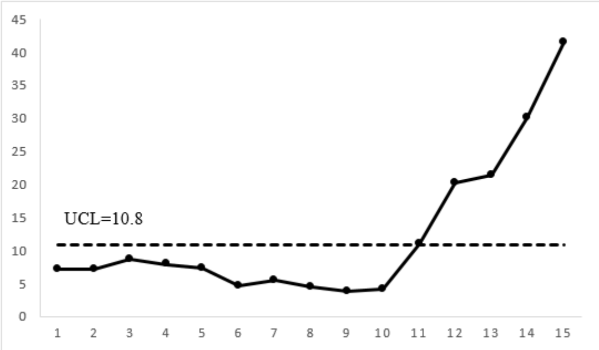

Fig 1. MHWMA chart for the calibration application data under shift from $\beta_{01} = -8.5$ to $\beta_{01} = -4$ in the 11^{th} sample

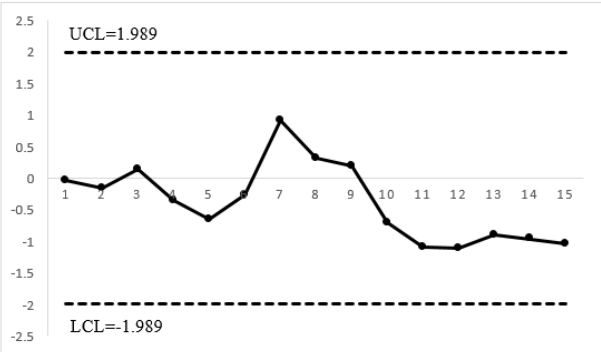

Fig 2. EWMA based on generalized variance chart for the calibration application data under shift from $\beta_{01} = -8.5$ to $\beta_{01} = -4$ in the 11^{th} sample

6-Conclusion and recommendations

In this paper, a monitoring method, called MHWMA/EWMA, has been presented to monitor multivariate simple linear profile in phase II. The performance of the proposed method was investigated through simulation studies via ARL metric.

The obtained results showed that MHWMA control chart performs well in detecting shifts of profile parameters and detects different shift values in the intercept and the slope of the multivariate simple linear profile quickly. Comparing this method with the existing MEWMA method, proposed by (Haq, 2020), showed superior performance of the MHWMA/EWMA method. Detecting shifts as quickly as possible in a process, prevents the production of defective products. Therefore all costs associated with poor quality or product failure, including rework, scrap, warranty costs and other costs incurred in preventing or resolving quality problems, are reduced. Also investigating out-of-control ARL values showed that MHWMA/EWMA method is not highly capable of detecting process variability shift, compared to existing method. We also showed the use of the MHWMA/EWMA method under a simulated data set based on a calibration case in an automotive industrial group. The illustrative example showed the appropriate performance of the proposed method in detecting shifts in the parameters of simple linear regression profiles.

For future research in this area, another chart can be used to monitor process variability to improve the performance of this method in detecting shifts of standard deviation. Also using Variable Sample Size (VSS) and Variable Sampling Interval (VSI) features, the performance of this method may be improved.

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