

# **Integrated production-distribution planning with make-to-order production system considering Stackelberg competition and discount for a Furniture Company**

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## **Abstract**

Nowadays, in the competitive global market, increasing market share is the main objective of the most manufacturers, however, customization, service speed, customer satisfaction, and environmental problems are vital factors that manufacturers ought to consider to expand their market share. so, the supply chain management can be applied as a proper approach to optimize these factors in whole supply chain to benefit the supply chain members. In this way, the current paper addresses an integrated production and distribution model with combination of Stackelberg competition and Make-to-order production system in different periods. In addition, this model wants to investigate how discounts impact the chain's profits with presence of competition and Make-to-Order production system. This study uses a modified Non-Dominated Sorting Genetic Algorithm II (NSGA-II) approach to solve the medium and large cases model because of the NP-hardness feature. Additionally, the model is applied to Furniture Company to demonstrate its efficacy and validity and results are provided. According to the obtained results, the modified algorithm has better performance in solving model in medium and large-scale cases. The proposed model would be beneficial to increase network efficiency by integrating production-distribution planning.

**Keywords:** Production-distribution, competition, metaheuristic algorithm, Stackelberg competition, environmental problems

## **1-Introduction**

The supply chain (SC) is a system that comprises manufacturers, storehouses, suppliers, distributors, and demand points at various levels which works as an integrated and coordinated system by internal relationship. According to the previous research the supply chain's production and distribution functions are the two most important operational activities, and they ought to be coordinated, integrated, and organized for best performance and decrease the total chain cost.

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(Rafiei et al. 2021). Nowadays, most of the production systems are shifting to small-case production instead of mass production. Additionally, with improved information technology, product life cycles are shortened, customer demands are diversified, changes are fast, and demand forecasting is difficult (Jiang and Rim 2017). It is clear that a manufacturer that can response the customer orders quickly, would enjoy a competitive advantage. Although, providing demand for a wide range of products and decreasing response time is a conflicting requirement for production systems (Gupta and Benjaafar 2004). In this way, Make-to-order (MTO) systems are prosperous business techniques for managing reactionary supply chains with a wide range of products, changeable customer demand, and brief product life cycles (PLC) (Vidyarthi et al. 2009). In a Make-to-Stock (MTS) system, orders are processed from final products inventories held at diverse points along with the network. on the other hand, MTO production is typically used to address the specific demand for a custom product (Chang and Lu 2010). Make-to-Order (MTO) strategy, runs out of stock of finished products and reduces the risk of business obsolesce, this usually means that the customer has a long response time (Vidyarthi et al. 2009).

The MTO strategies provide a wide range of customer-specific, usually, costlier products. Order execution and average response time, as well as average order delay, are the focus of production planning. Faster delivery is also a competitive priority (Soman et al. 2004) many firms including American Standard, Compaq, BMW, General Electric, IBM, and National bicycle, adopt a hybrid strategy (i.e., a combination of MTS and MTO) to meet the dual needs for fast response times and a wide range of products (Vidyarthi et al. 2009). One of the most critical factors in the production of MTO is the on-time delivery of products to customers (Jiang and Rim 2017). Therefore, the MTO production system reduces the service time at the network at least which is one of the challenging issues in this scope owing to its advantages.

Apart from that, Companies are competing with each other to attain their goals in these marketplaces more than before thanks to rapid advancements in information technology and increased worldwide competition (Rafiei et al. 2018). In today's highly competitive market, more and more firms decipher the importance of the that price to attract customers (Wang 2017). So, the competition could play a crucial role in the supply chain to gain more market profit. In this way, Horizontal competition is a competition between chains at the identical level and it has two types: in-chain horizontal competition refers to competition among parts of the one supply chain, and chain-to-chain horizontal competition refers to competition among the final levels of two disparate networks (Rafiei et al. 2018). Moreover, most companies want to consider a discount to encourage their customers to buy more and increase their benefit (Rahimi et al. 2019). So, it is important to make the right decision about this kind of promotion in designing the supply chain. In this paper, we consider quantity discount and investigate its effects on profit.

Supply chain management approach and transportation are broadly developed to increase the availability of commodities and services (Heiko and Darkow 2016). On the other hand, the transportation field is one of the major consumers of the global energy. It uses almost 29% of the global energy consumption, and produce up to 24% of global CO<sub>2</sub> emissions. Therefore, companies are faced with a challenge when it comes to green transportation. The purpose of green transportation is to decrease the emission of CO<sub>2</sub> to reduce environmental issues. Therefore, one of the objective functions of our model is minimizing CO<sub>2</sub> emission by selecting eco-friendly vehicles.

The consideration of above aspects leads some gaps in the previous studies such as considering discount, green transportation and competition within an integrated production-distribution planning. Therefore, the current study presents a multi-objective production-distribution planning problem with consideration MTO production system, discount and competition within a green supply chain management approach in multi-period.

In section 2 literature review is presented. As it is indicated in section 3, a mathematical model is proposed for the presented problem. A solution methodology that takes into account a metaheuristic approach called the modified NSGA-II is developed in section 4. This method is then evaluated in this Section, and a case

study about a furniture company is developed to show the model's application and efficiency. Eventually, section 5 summarizes the results of this paper and offers further research recommendations.

## **2-Literature review**

Diverse studies are conducted in the integrated P-D planning. Van Roy (1989) presented a mathematical model for a petrochemical company with P-D network. Park (2005) presented a P-D planning model and examined the productiveness of their integration by a study which has some factories, multi-retailer, and multi-time period scenarios. In addition, their goal is to maximize the optimization engine's whole net benefit. Gunnarsson et al. (2007) considered a mathematical model for transmission of materials, production, and distribution of commodities of all supply chains of a great European pulp mill firm. Moreover, Khalili et al. (2017) presented a model using mixed stochastic probabilistic programming, with two-stage scenarios for production planning and distribution of commodities across a two-level supply chain, while the active capacity levels of transportation and distribution facility points are at risk. Badhotiya et al. (2019) explained a mixed-integer supply chain network with numerous selling locations and producers for solving production-distribution planning. Their presented model had various objective functions and two-level. Bo et al. (2021) developed a real-world integrated P-D problem. Their proposed problem included several types of commodities that need scheduling on a linear basis across multiple factories, different customers whose demand can be completely or not entirely satisfied, and it was vital to make the optimal decisions in a detached planning horizon to optimize the cost of production, opportunity, shipping, and warehouse.

Besides, Competition is one of the most significant problems in a network and It was conducted in the previous studies by several authors over the past few decades. Adida and DeMiguel (2011) investigated competition in a supply chain in which several producers provide a set of products for several risk-averse retailers who must meet the demand in uncertain situations. In addition, Yue and You (2014) provided a new framework for game-theoretic modeling. It could design and plan a non-cooperative three-level supply network. According to their assumption, the manufacturer was considered as a leader and each supplier or customer was a follower. Wang (2017) considered a two-level supply chain model with a manufacturer as the leader and two retailers as followers under a fuzzy decision-making environment. Moreover, Rafiei et al. (2018) investigated a P-D planning problem in a multi-level supply chain with two objective functions such as optimizing service level and whole chain cost. They also focused on three types of competition: Cournot, Stackelberg, and quality competition. A Stackelberg game framework for a closed-loop supply chain with remanufacturing was developed by Tang et al. (2020). Gao et al. (2021) studied the dual-channel green SC problem as a part of an eco-label policy in a green supply chain and, they propounded the Stackelberg game between the producer and retailer.

Since MTO is used as one of the production systems in a network, it was developed in a wide range of studies in the literature review. Carr and Duenyas (2000) addressed the matter of sequence and admission control in a production system that generates two categories of products. The primary category is MTS which the company has to meet the customer demand contractually. Also, the second category is MTO and therefore the company has the choice to accept or decline a specific order. Soman et al. (2004) presented the literature review about MTS-MTO strategies. They discussed various studies within the base of food processing firms, where compounded MTO-MTS production is very normal in detail. Stecke and Zhao (2007) presented a model for a manufacturing company's using a commit-to-delivery business model and MTO strategy. Also, they presented a mixed-integer programming model (MIP) as well as a minimum cost flow model. Chang and Lu (2010) developed a practical production planning consisting of MTO and MTS production and address two ordinary and specific demands. They analyzed the proposed system by applying the inventory-queue model and solved their model by using the matrix geometry method. Li et al. (2017) proposed a model for planning production system with capacity constraints to maximize benefit. To attain

the approximate optimal solution in the MTO production system, they considered four types of costs. Kim and Vanoyen (2021) presented a problem with admission, production sequencing, and production rate controls for a two-classes MTO manufacturing strategy. The model was formulated as a Markov decision-making process model, and the structural characteristics of the optimal control policies were identified under both discount and average profit conditions.

Only a few studies have examined the effects of quantity discounts on sales to customers. In this area, Rad et al. (2018) have investigated a closed-loop supply chain with environmental concerns, social responsibilities, and customers' demands to maximize customer satisfaction at the lowest cost. also, they offered quantity discounts to the customers to increase demands. Gholamian and Zamani Bjegani (2019) presented an inventory model with one retailer and quantity discount to minimize inventory cost and find optimal purchase price. The items of their supply chain were deteriorating items so retailers needed to encourage customers to buy more by discount quantity.

There is a diversity of methods for solving the given models in the literature review. Researchers adopted various solution methods owing to the features of their models. Some of the considered approaches are presented as follows:

Chiang et al. (2009) adopted a Tabu search approach to modify their obtained solutions. Additionally, a new fitness function was added to the particle-swarm optimization (PSO) algorithm and a hybrid genetic algorithm was proposed by Jolai et al. (2011) to overcome the complexity of the considered model. Abraham et al. (2015) solved their presented model by adopting a genetic algorithm to decrease the time of solving and complexity of the model to achieve feasible solutions. Manavizadeh et al. (2016) solved their integrated production planning with maintenance planning by adopting two meta-heuristics algorithms: simulated annealing and harmony search according to the size and np hardness feature of the model. Moreover, Khalifehzadeh et al. (2017) proposed two new heuristics methods called a concessive variable neighborhood search (CVNS) and ranking genetic algorithm (RGA) to deal with the large instance problems. Also, Jallad et al. (2018) presented a hybrid FA-PSO-based algorithm to attain optimal solutions. A hybrid two-phase solution procedure based on possibility programming was additionally applied by Goodarzian and Hosseini-Nasab (2019) to resolve the presented np-hard problem. Furthermore, Fatemi Ghomi et al. (2021) used a particle swarm optimization algorithm. they used a genetic algorithm operator to solve the mixed-integer np-hard problem. Table (1) presents some conducted studies.

**Table 1.** Some conducted research

Authors(year)	Considered features						
	Objective function	Production-distribution	competition	Multi-objective	discount	MTO	Environmental policy
Gupta and Benjafaar (2004)	Minimizing cost					•	
Stecke and Zhao (2007)	Minimizing cost				•	•	
Chen et al. (2012)	Maximizing profit		•				
Li et al (2017)	Minimizing cost					•	
Mishara and Talati (2018)	Maximizing profit				•		
Sdjadi et al. (2018)	Maximizing profit		•		•		
Rad et all (2018)	minimization economic cost and environmental emissions maximizing customer satisfaction			•	•		•
Rahimi et al. 2019	Maximizing profit Minimizing environmental effects			•	•		•
Goodarzian and Hosseini-Nasab (2019)	Minimizing Cost Maximizing reliability rate	•		•			
Gharaeia and Jolai (2021)	Minimizing tardiness Minimizing cost	•		•			
Manteghi et al (2020)	Maximizing profit Minimizing greenhouse gas Customer health			•			•
Hu et al (2021)	Minimizing cost					•	
Aazami and Saidi-Mehrabad(2021)	Maximizing profit	•	•		•		
Khademi and Niazi(2021)	Minimizing cost Minimizing environmental effects	•		•			•
Gao et al (2021)	Minimizing cost Minimizing environmental effects			•			•
Fatemi Ghomi et al. (2021)	Maximizing profit Minimizing CO2	•		•			•
This paper	Maximizing profit Minimizing CO2 emission	•	•	•	•	•	•

Reviewing the presented literature review reveals some lack of consideration in some aspects of the production-distribution planning problem in the supply chain. In this way, we propose a mixed integer nonlinear problem with multi objective functions in order to maximize profit of the chain and minimize CO<sub>2</sub> with considering Stakelberg competition and discount. The presented model is solved by modified NSGA-II to tackle the NP-hardness of the model.

### 3- Problem definition

The mixed-integer non-linear multi-objective programming model for integrated P-D planning is proposed. The first objective function decreases cost of chain and the second one is related to the competition. The third objective of the model decreasing CO2 emission by the procedure of the project's cost and the project meet deliverable Hickman et al. (1999). This methodology uses equation (1) to calculate the CO2 emissions for the arc  $ij$ :

$$O_{ij}(q, d) = d_{ij} \times \left[ \left( \frac{e_{fl} - e_{el}}{Q} \right) q_{ij} + e_{el} \right] \quad (1)$$

$O_{ij}(q, d)$  is the function that calculates the CO2 emissions for the arc  $ij$  produced by a vehicle with the variable of load  $q$  and the volume capacity  $Q$ .  $e_{fl}$  is the CO2 emissions that are released by a fully-loaded vehicle and  $e_{el}$  is the CO2 emissions that produce by an empty vehicle.

Also, in this study a discount quantity is considered as a promotion to encourage customer to buy more. We considered  $\beta$  as a discount rate and equation (2) - (3) show the changes in demand ( $DC_{at}$ ) and price of commodity ( $RV_t$ ) in each period ( $t$ ) for each customer ( $a$ ).

$$DC_{at} = DC_{at} + \beta * DC_{at} \quad (2)$$

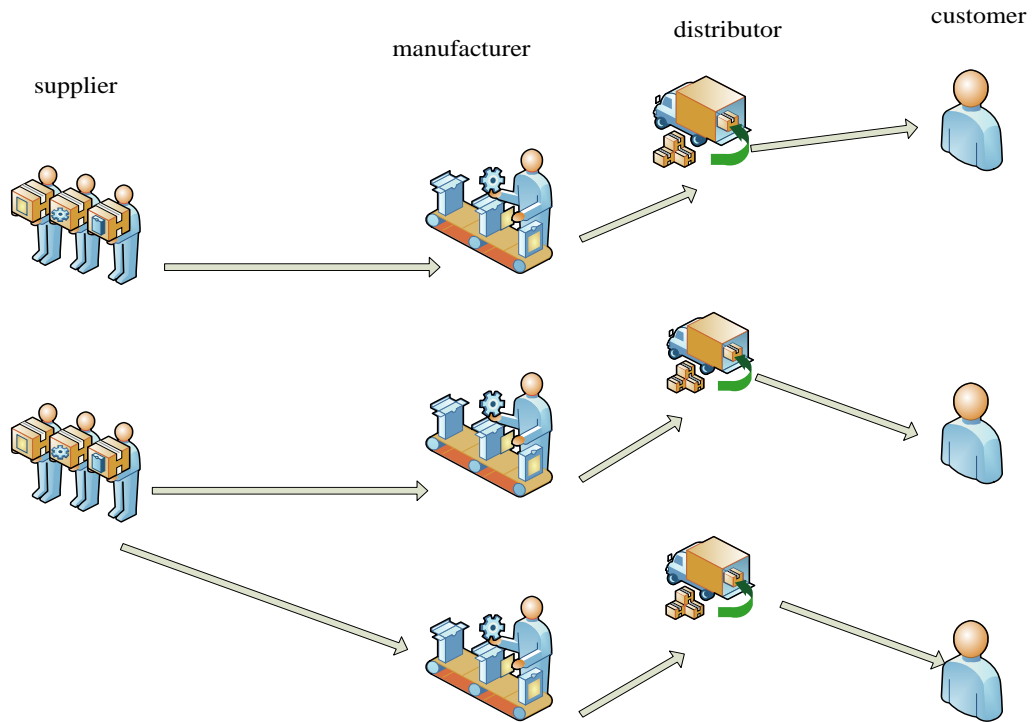
$$RV_t = RV_t * (1 - \beta) \quad (3)$$

The considered assumptions are as follows.

#### 3-1- Assumptions

- There are four echelons including suppliers, manufacturers, distributors, and customers in the supply chain.
- The MTO production system is considered in the manufacturers.
- The vehicles are heterogeneous and the variable and fixed cost are considered for transportation
- The inventory of distributors and suppliers has a certain capacity
- The Stackelberg competition is considered between suppliers.
- The lost sale is allowed in the network
- Lead time is considered for orders so the time of set up, process, and transportation are very important.
- Multi period is considered for designing the model.

The network has three strategic choices to satisfy the demand of customers. 1. The network can opt one vehicle with sufficient capacity (higher cost) and satisfy the demand of customers in one period. 2. The network can choose one vehicle with less capacity (lower cost) and satisfy the remained demand for the next period by holding them as inventory. 3. It can choose satisfying a part of demand and select shortage. The figure (1) shows the network of the model:



**Fig. 1.** The proposed production distribution model

### 3-2- Indices

S	Suppliers
M	Manufacturers
D	Distributors
A	Customers
T	Period of times
I	Type of vehicles $i = 1,2$

### 3-3- Parameters

$CS$	raw materials cost for the suppliers
$CM$	raw materials cost for the manufacturers
$FC_{mt}$	Processing fixed cost in the Fixed cost in the manufacturer $m$ in period $t$
$VC_{mt}$	Processing variable cost in the manufacturer $m$ in period $t$
$ef_i$	the produced CO2 emissions of a fully-loaded vehicle $i$
$el_i$	the CO2 emissions of an empty vehicle (in kg / km)
$Q_i$	Capacity of vehicle $i$
$FCT1_{smti}$	Fixed cost of transportation material from supplier $s$ to manufacturer $m$ in period $t$ with type $i$ vehicle
$VCT1_{smti}$	Variable cost of transportation material from supplier $s$ to manufacturer $m$ in period $t$ with type $i$ vehicle

$FCT2_{mdti}$	Fixed cost of transportation commodities from manufacturer $m$ to distributor $d$ in period $t$ with type $i$ vehicle
$VCT2_{mdti}$	Variable cost of transportation commodities from manufacturer $m$ to distributor $d$ in period $t$ with type $i$ vehicle
$FCT3_{dati}$	Fixed cost of transportation commodities from distributor $d$ to customer $a$ in period $t$ with type $i$ vehicle
$VCT3_{dati}$	Variable cost of transportation commodities from distributor $d$ to customer $a$ in period $t$ with type $i$ vehicle
$CR_{st}$	Holding cost for supplier $s$ in period $t$
$CR'_{dt}$	Holding cost for distributor $d$ in period $t$
$CR''_{mt}$	Holding cost for manufacture $m$ in period $t$
$DC_{at}$	Customer demand $a$ in time $t$
$FRA^*$	Equilibrium flow for the first supplier (for Stackelberg competition)
$FRb^*$	Equilibrium flow for the second supplier (for Stackelberg competition)
$IL_s^{max}$	Maximum level of inventory in supplier $s$ in time $t$
$IL_d^{max}$	Maximum level of inventory in distributor $d$ in time $t$
$CLS_{dt}$	Cost of the lost sale for distributor $d$ in time $t$
$TL_t$	Lead time for the product in time $t$
$LJ_s^{maz}$	Maximum level of inventory for supplier $s$
$LJ_d^{maz}$	Maximum level of products for distributor $d$
$TS_m$	Need time for setup for manufacturer $m$
$TP_m$	Need time for the process for manufacturer $m$
$TT_{imd}^1$	Need time for transportation in arc $(m, d)$ with vehicle $i$
$TT_{ida}^2$	Need time for transportation in arc $(d, a)$ with vehicle $i$
$d_{sm}$	The distance between $(s, m)$
$d'_{md}$	The distance between $(m, d)$
$d''_{da}$	The distance between $(d, a)$
$M1$	Big parameter
$M2$	Big parameter
$M3$	Big parameter

### 3-4- Variables

$FR_{smti}$	The level of raw materials that are shipped from supplier $s$ to manufacturer $m$ in period $t$ with vehicle type $i$
$O_{smit}$	The CO2 emissions for the arc $(s, m)$ from a vehicle $i$ in period $t$
$O'_{mdit}$	The CO2 emissions for the arc $(m, d)$ from a vehicle $i$ in period $t$
$O''_{mdit}$	The CO2 emissions for the arc $(d, a)$ from a vehicle $i$ in period $t$
$P_{mt}$	The level of production in manufacturer $m$ in period $t$
$FP_{mdti}$	The level of products that are shipped from manufacturer $m$ to distributor $d$ in period $t$
$Fd_{dati}$	The level of products that are transformed from distributor $d$ to customer $a$ in period $t$
$LI_{st}$	The level of materials in supplier $s$ in period $t$
$LI'_{dt}$	The level of products in distributor $d$ in period $t$
$LI''_{mt}$	The level of products in manufacturer $m$ in period $t$
$X_{smti}$	1 if materials are transported from supplier $s$ to manufacturer $m$ in period $t$ ; 0 otherwise
$Y_{mdti}$	1 if products are transported from manufacturer $m$ to distributor $d$ in period $t$ ; 0 otherwise
$W_{dati}$	1 if products are transported from distributor $d$ to customer $a$ in period $t$ ; 0 otherwise



$V_{mti}$	1 if manufacturer $m$ selects vehicle $i$ in period $t$
$VV_{dti}$	1 if manufacturer $m$ selects vehicle $i$ in period $t$
$YY_{at}$	1 if the network decides to satisfy demand of period $t-1$ for customer $a$
$LS_{dt}$	Level of the lost sale in distributor $d$ in period $t$
$Sh_{at}$	Level of shortage in customer $a$ in period $t$
$TM_{mdit}$	Total time of production in make-to-order structure that produce by manufacturer $m$ and transport with vehicle $i$ and distributor $d$ in period $t$
$D_{mt}$	Demand of manufacturer $m$ in period $t$

### 3-5- Mathematical model

$$\begin{aligned} \max \quad & \sum_{d=1}^D \sum_{t=1}^T \sum_{a=1}^A \sum_{i=1}^I RV_t \times Fd_{dati} - (\sum_{d=1}^D \sum_{t=1}^T LS_{dt} \times CLS_{dt}) + (\sum_{m=1}^M \sum_{t=1}^T FC_{mt} + \\ & (P_{mt} \times VC_{mt})) + (\sum_{s=1}^S \sum_{t=1}^T LI_{st} \times CR_{st}) + (\sum_{d=1}^D \sum_{t=1}^T LI_{dt} \times CR'_{dt}) + (\sum_{m=1}^M CM \times \\ & \sum_{m=1}^M \sum_{t=1}^T P_{mt} + (\sum_{t=1}^T \sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I FCT1_{smti} \times X_{smti} + \sum_{t=1}^T \sum_{m=1}^M \sum_{s=1}^S \sum_{i=1}^I VST1_{smit} \times \\ & FR_{smti} + (\sum_{d=1}^D \sum_{t=1}^T \sum_{m=1}^M \sum_{i=1}^I Y_{mdti} \times FCT2_{mdti} + \sum_{t=1}^T \sum_{m=1}^M \sum_{d=1}^D \sum_{i=1}^I VCT2_{mdti} \times \\ & P_{mt}) + \sum_{t=1}^T \sum_{i=1}^I \sum_{a=1}^A \sum_{d=1}^D W_{dati} \times FCT3_{dati} + \sum_{t=1}^T \sum_{i=1}^I \sum_{a=1}^A \sum_{d=1}^D VCT3_{dati} \times \\ & FP_{dati} + \sum_{s=1}^S \sum_{m=1}^M \sum_{t=1}^T \sum_{i=1}^I FR_{smti} * CS + \sum_{m=1}^M \sum_{t=1}^T LI''_{mt} * CR''_{mt})) \end{aligned} \quad (4)$$

$$\min \quad \sum_{s=1}^S \sum_{m=1}^M \sum_{t=1}^T \sum_{i=1}^I |FR_{smti} - FRa^*| + \sum_{s=2}^S \sum_{m=1}^M \sum_{t=1}^T \sum_{i=1}^I |FR_{smti} - FRb^*| \quad (5)$$

$$\min \quad Z_3 = \sum_{s=1}^S \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T O_{smit} * X_{smti} + \sum_{d=1}^D \sum_{m=1}^M \sum_{i=1}^I \sum_{t=1}^T O'_{mdit} * Y_{mdti} + \sum_{d=1}^D \sum_{a=1}^A \sum_{i=1}^I \sum_{t=1}^T O''_{dait} * W_{dati} \quad (6)$$

Constraints:

$$\sum_{m=1}^M \sum_{i=1}^I X_{smti} \geq 1 \quad \forall s, t \quad (7)$$

$$\sum_{m=1}^M \sum_{i=1}^I X_{smti} \leq M \quad \forall s, t \quad (8)$$

$$\sum_{d=1}^D \sum_{i=1}^I Y_{mdti} \geq 1 \quad \forall m, t \quad (9)$$

$$\sum_{d=1}^D \text{the } \sum_{i=1}^I Y_{mdti} \leq D \quad \forall m, t \quad (10)$$

$$\sum_{a=1}^A \sum_{i=1}^I W_{dati} \geq 1 \quad \forall d, t \quad (11)$$

$$\sum_{a=1}^A \sum_{i=1}^I W_{dati} \leq A \quad \forall d, t \quad (12)$$

$$LI_{st} \geq LI_{st-1} + \sum_{s=1}^S \sum_{i=1}^I FR_{smti} \quad \forall m, t \quad (13)$$

$$LI_{dt} \geq \sum_{d=1}^D \sum_{i=1}^I Fd_{dati} \quad \forall a, t \quad (14)$$

$$D_{mt} = \sum_{d=1}^D \sum_{i=1}^I FP_{mdti} \quad \forall m, t \quad (15)$$

$$Sh_{at} + \sum_{d=1}^D \sum_{i=1}^I Fd_{dati} = DC_{at} \quad \forall a, t = 1 \quad (16)$$

$$Sh_{at} + \sum_{d=1}^D \sum_{i=1}^I Fd_{dati} = DC_{at} + (DC_{at-1} - \sum_{m=1}^M \sum_{d=1}^D \sum_{i=1}^I Fd_{dati}) * YY_{at-1} * W_{dati} * Y_{mdti} \quad \forall a, t > 1 \quad (17)$$

$$LI''_{mt} = \sum_{i=1}^I \sum_{d=1}^D \sum_{a=1}^A (DC_{at} - Fd_{dati} * YY_{at} * W_{dati} * Y_{mdti}) \quad \forall m, t \quad (18)$$

$$LS_{dt} = \sum_{a=1}^A \sum_{i=1}^I Sh_{at} * W_{dati} \quad \forall d, t \quad (19)$$

$$LI_{st} \leq LI_s^{maz} \quad \forall s, t \quad (20)$$

$$LI'_{dt} \leq LI_d^{maz} \quad \forall d, t \quad (21)$$

$$Y_{mtdi} \leq V_{mti} \quad \forall m, id, t \quad (22)$$

$$W_{dati} \leq VV_{dati} \quad \forall a, id, t \quad (23)$$

$$TT_{mit} = TT_{imd}^1 * V_{mti} \quad \forall m, i, t \quad (24)$$

$$TT'_{di} = TT_{id}^2 * VV_{dati} \quad \forall d, i, t \quad (25)$$

$$TM_{mdit} = TP_m + TS_m + TT_{mit} + TT'_{dit} \quad \forall m, d, t, i \quad (26)$$

$$TM_{mdit} \leq TL \quad \forall d, m, i, t \quad (27)$$

$$O_{mdit} = d_{sm} \times [(ef_i - el_i)/Q_i] FR_{smti} + el_i \quad \forall m, t, i, s \quad (28)$$

$$O'_{mdit} = d'_{md} \times [(ef_i - el_i)/Q_i] FP_{mdti} + el_i \quad \forall m, t, i, d \quad (29)$$

$$O''_{mdit} = d''_{da} \times [(ef_i - el_i)/Q_i] Fd_{dati} + el_i \quad \forall a, t, i, d \quad (30)$$

$$FP_{mdti} \leq M1 * Y_{mdti} \quad \forall m, t, i, d \quad (31)$$

$$FR_{smti} \leq M2 * X_{smti} \quad \forall m, t, i, s \quad (32)$$

$$Fd_{dati} \leq M3 * W_{dati} \quad \forall d, t, i, a \quad (33)$$

$$P_{mt} \leq \sum_{s=1}^S \sum_{i=1}^I FR_{smti} \quad \forall m, t \quad (34)$$

$$X_{smti}, Y_{mdti}, W_{dati}, V_{mti}, VV_{dati} \in \{0,1\} \quad \forall m, t, i, s, a \quad (35)$$

$$FR_{smti}, P_{mt}, FP_{mdti}, FP_{dati}, LI_{st}, LI'_{dt}, LS_{dt}, TM_{mdit}, O_{smit}, O'_{mdit}, O''_{mdit}, Fd_{dati} \geq 0 \quad \forall m, t, i, s, a \quad (36)$$

Equation (4) presents the first objective function that is maximizing total profit of the considered supply chain. The costs of network are equal to the sum of the cost of lost sales, production in manufacturers,

holding, and transportation in different levels. The second objective (equation (5)) represents the competitive objective function that is correlated with Stackelberg competition which is equal to the difference of level of raw materials from suppliers and equilibrium values, and equation (6) shows the third objective function that is minimizing CO2 emission that is obtained from vehicles in different arcs. According to equation (7), the minimum number of times flows from the suppliers to manufacturers, implying that there is at least one material flow from a supplier to a manufacturer. The maximum number of materials flows from suppliers to the manufacturer is specified by equation (8). This must be the same as the total number of manufactures. Constraint (9) shows the minimum number of times of flows from manufacturers to distributors which denotes that there is one flow of products from a manufacturer to the distributor at least. The equation (10) shows the maximum number of times that a product enters the distribution channel from the manufacturer. This number must be equal to the total number of distributors. Constraint (11) and constraint (12) have the same description as constraints (9) and (10), respectively. Constraints (13) to (14) are inventory balance constraints in suppliers and distributors, respectively. The inventories for each supplier's finished products or materials are calculated based on the inventory balance equation at the end of each period. Constraint (15) calculates the demand of manufacturers in each period and equations (16) and (17) show the quantity of shortage and lost sales in customers and distributors respectively. Constraint (18) shows the level of inventory in each manufacturer. Constraint (19) shows the shortage in the distributors, and the maximum capacity of inventory in suppliers and distributors is defined by constraints (20) to (21). Equations (22) and (23) express if manufacturer  $m$  and distributor  $d$  select vehicle  $i$ , transportation occur with this type of vehicle. Constraint (24) and (25) calculate the time of transportation between manufacturer and distributor and distributor and customer respectively. Equation (26) demonstrates the total time for make-to-order production in manufacturers that is equal to process time and setup time. Constraint (27) represents that the total time of make-to-order production in the manufacturer has to be less or equal to the present lead time by the consumer. Constraint (28), (29) and (30) calculates CO2 emissions for the arcs, and Constraints (31) -(33) present the limitation of flow in the network. Constraint (34) states that the maximum of production is equal to the transported material from suppliers.

### 3-6- Stackelberg equilibrium

The current study is considered a Stackelberg competition between the two suppliers. At the Stackelberg competition, participants compete on the price at the same time (Rafiei et al. 2021). Moreover, a leader and a follower are considered in this competition which the first supplier is considered as a leader and the second one is a follower.

$E$	The price of the raw material
$m$	The potential price when the sale is zero
$f_1$	The sale for the first supplier
$f_2$	The sale for the second supplier
$\alpha$	The intensity of the Stackelberg competition
$c_i$	The cost for supplier $i \in \{1,2\}$
$S_i$	The profit of supplier $i \in \{1,2\}$

The equilibrium equations of the Stackelberg competition can be calculated as:

$$E(f_1 + f_2) = m - \alpha(f_1 + f_2) \quad (36)$$

Equation (36) is the function of price, which on  $m$  is the potential price and  $f_1$  and  $f_2$  are the level of sale for each supplier.

$$R_i(f_i) = c_i \times f_i \quad i \in \{1,2\} \quad (37)$$

Equation (37) is the function of the cost. Follower's profit is calculated from equation (38):

$$S_2 = E(f_1 + f_2) \times f_2 - R_i(f_i) = ((m - \alpha(f_1 + f_2)) \times f_2) - (c_2 \times f_2) \quad (38)$$

The above-mentioned equations are given for calculating the follower's profit which the differentiation is done and then set equal to zero to find the values of  $f_2$  that maximizing the suppliers' profit in equations (39)-(40).

$$\partial S_2 / \partial f_2 = 0 \rightarrow m - \alpha f_1 - 2 \alpha f_2 - c_2 = 0 \quad (39)$$

$$f_2 = (m - \alpha f_1 - c_2) / 2\alpha \quad (40)$$

By determining the best follower's strategy according to the leader's strategy, the leader also can determine its strategy in the best profitable way.

$$s_1 = (m - (\alpha(f_1 + f_2)) \times f_1 - (c_1 \times f_1)) \quad (41)$$

$$= m f_1 - \alpha f_1 (m - \alpha f_1 - c_2) / 2\alpha - \alpha f_1^2 - c_1 f_1$$

Equation (41) shows the profit function of the first supplier. We went through all steps to calculate the maximum profit of the follower with considering certain strategies for the follower to get the maximum profit of the leader by equation (42) and equation (43):

$$\frac{\partial s_1}{\partial f_1} = 0 \rightarrow (m - 2\alpha f_1 c_2 - 2c_1) / 4 = 0 \quad (42)$$

$$f_1 = (m + c_2 - 2c_1) / 2\alpha \quad (43)$$

$$f_1^* = (m + c_2 - 2c_1) / 2\alpha \quad (44)$$

$$f_2^* = (m - \alpha f_1 - c_2) / 2\alpha \quad (45)$$

Equations (44) and (45) show  $f_1^*$  (the equilibrium value of sale for the first supplier (leader)) and  $f_2^*$  (the equilibrium value of sale for the second supplier (follower)), respectively.

The equilibrium values are shown in the model as below:

$$f_1^* = FRa^*$$

$$f_2^* = FRb^*$$

#### 4- Solution methodology

In this section, GAMS software and two meta-heuristic algorithms are used to solve the model. In the first section, GAMS software is applied to solve a case study by computer with a Core i5- 6200U CPU and a 2.3 GHz 4GB RAM and describe the behavior of the model. In the next step, to investigate the function of the modified metaheuristic algorithm, the results of metaheuristic algorithms are presented in 5 small instances and 6 large instances, and the results are compared.

#### 4-1- Case study

This section considers a furniture company as an example to illustrate the model's potential. This company has two suppliers for wood, and three manufacturers. Moreover, it collaborates with two distributors. Distributors have two policies about environmental problems. The first distributor uses some eco-friendly vehicles with higher speed, and the second distributor uses some common vehicles. The cost of transportation for the first distributor is more than another one. So, the company has to consider different aspects and opt one of them. The supplier must have a plan for supplying materials, and manufacturers must plan for their MTO production. So, having integrated planning for supplying and production seems to be vital for them. Moreover, some challenges are common for this integration as follows:

**1. Cost of transportation:** Transportation costs constitute a major part of the company's costs, especially for Furniture Company due to the unusual shapes and oversize furniture. Also, spacious vehicles have higher cost, so the manufacturer must decide which one of these alternatives is better: **a:** transfer all customer demand with higher transportation cost. **b:** transfer part of demand in another period and accept the cost of holding. **c:** Satisfy the part of the demand and having shortage.

**2. Environment problems:** environmental policy will help company to observe green environment management for their customers who are concerned about environmental issues such as climate changes.

The company follows promotion strategies, hence, the rate of it should be determined by company. Moreover, competition between suppliers amplify the performance of the company and the reluctance of the company to keep the excess inventory in warehouses due to economic and environmental issues leads the company to use MTO production approach. Also, in this particular company, delivery service is important because in this industry as long as the company does not deliver the furniture to the customers, it has not finished its job, and the quality of delivery can reflect the professionalism of a brand for customers who want a precise time window to change the decoration of their home or workplace. So, considering a lead time is an important factor in P-D planning for furniture company.

In the study, the problem is solved by GAMS software and parameters of table (2). The results are presented as follow:

**Table 2.** The input parameters of GAMS

Parameters	interval	Parameters	Interval	Parameters	interval	parameters	interval
$CM$	12	$FCT1_{smti}$	(15,20)	$CR_{st}$	(20,25)	$CLS_{dt}$	(100,155)
$FC_{mt}$	(10,15)	$VCT1_{smti}$	(6,8)	$CR'_{dt}$	(15,17)	$TL_t$	(40000,45000)
$VC_{mt}$	(5,10)	$FCT2_{mdti}$	(10,17)	$TT_{ida}^2$	(35,50)	$TS_m$	(20,30)
$ef_i$	(20,40)	$VCT2_{mdti}$	(4,8)	$DC_{at}$	(30,40)	$TP_m$	(20,25)
$el_i$	(7,10)	$FCT3_{dati}$	(10,12)	$IL_s^{max}$	(200,240)	$d'_{md}$	(11,17)
$Q_i$	(10,15)	$VCT3_{dati}$	(5,7)	$IL_d^{max}$	(200,220)	$CS$	9
$TT_{imd}^1$	(30,40)	$d_{sm}$	(11,15)	$d''_{da}$	(12,17)	$RV_t$	(70,90)

**Table 3.** The allocation of the manufacturers

Time	Suppliers	Manufacturers
T=1	1	1,3
	2	2
T=2	1	1
	2	2,3
T=3	1	3
	2	1,2

**Table 4.** The allocation of distributors

Time	Manufacturers	Distributors
T=1	1	2
	2	1
	3	1
T=2	1	1
	2	2
	3	1
T=3	1	1
	2	1
	3	1

**Table 5.** The allocation of customers

Time	Distributors	customers
T=1	1	1,2,3,5
	2	4
T=2	1	2,4
	2	1,3,5
T=3	1	1,2,3,4,5,6
	2	-

Table (3) represents the allocation of the manufacturers to the suppliers, which shows that two suppliers have been used and all the producers have been covered by at least one supplier. Moreover, table (4) illustrates the selection of distributors by manufacturers. This table shows manufacturers prefer to use the first distributor more than the second distributor because it is eco-friendly and has higher speed, although it costs more. Moreover, table (5) shows the allocation of customers to distributors that guarantee each customer is covered.

**Table 6.** Output results of GAMS software

Variable	Interval	Variable	interval
$FRP_{smti}$	(0,100)	$Fd_{dati}$	(0,39)
$O_{smit}$	(0,.)	$LI_{st}$	(0,100)
$O'_{mdit}$	(0,1452)	$LI_{dt}$	(0,128)
$O''_{dait}$	(0,736)	$LS_{dt}$	0
$P_{mt}$	(0,128.24)	$Sh_{at}$	0
$FP_{mdti}$	(0,128.24)	$TM_{mdit}$	(0,9721)
$FRP_{smti}$	0	$FRM_{smti}$	0
$LI''_{mt}$	0		

Table (6) shows other variables.  $LS_{dt}$  and  $Sh_{at}$  are zero. So, the network can supply the demands of customers. Moreover, the need time for manufacturing and distribution of commodities by manufacturer  $m$  and distributor  $d$  shows commodities are received in lead time.  $LI_{dt}$  and  $LI_{st}$  illustrate the inventory of suppliers and manufacturers respectively that demonstrate they are not more than the maximum capacity of the warehouses.  $FP_{mdti}$  and  $LI_{dt}$  have the same quantity so it shows commodities don't remain in the warehouse of distributors. The quantity of  $O_{smit}$ ,  $O'_{mdit}$  and  $O''_{mdit}$  show the highest amount of CO2 is produced between the path of producers and distributors. The quantity of  $FRP_{smti}$  and  $FRM_{smti}$  indicate supplier remains in the competition and have not obtained values higher or lower than the adjustment value. Moreover, manufacturers prefer using vehicles with higher volume.

In the following, we investigate the impact of distance between points on the problem. By increasing the distance between points, the cost of transportation increases too. The impacts of these changes in different

intervals are shown in table (7) and figure (2.a). The results show the important rule of distance parameters in decreasing profit and increasing CO2 emission.

**Table 7.** The impacts of distances

	1	2	3	4	5	6
$d_{sm}$	(11,15)	(20,25)	(25,30)	(30,35)	(35,40)	(40,45)
$d'_{md}$	(11,17)	(20,27)	(25,32)	(32,37)	(37,42)	(40,46)
$d''_{da}$	(12,17)	(22,30)	(29,35)	(35,40)	(40,45)	(45,50)
$TT_{ida}^2$	(35,50)	(45,60)	(55,65)	(65,70)	(70,75)	(75,80)
$TT_{imd}^1$	(30,40)	(35,50)	(40,55)	(45,60)	(50,65)	(55,70)
Z1	$7.04*10^4$	$6.95*10^4$	$3.70*10^4$	$2.71*10^4$	$1.24*10^4$	$7.45*10^3$
Z3	26354.38	28451.12	30840.14	40278.26	42145.23	43954.21

Promotion planning is very important for any company. Because it can increase the demand and loyalty of customers. So, in this section, we try to determine the suitable discount rate. Table (8) and figure (2.b) show the results. It seems that the pricing of commodities is fair due to the high costs of the network. Although considering discount increase demand, it reduces the profit and increases the carbon dioxide emission due to the transportation of more commodities. Suppliers also prefer to be out of competition due to the greater needs than the equilibrium amount.

**Table 8.** The impacts of discount rate

	1	2	3	4	5	6	7	8
$\beta$	0,5	10	15	20	25	30	35	40
Z1	$4.96*10^4$	$4.90*10^4$	$3.87*10^4$	$3.63*10^4$	$2.98*10^4$	$2.47*10^4$	$1.84*10^4$	$1.34*10^4$
Z2	0	0	0	101.24	191.62	241.37	312.61	391.47
Z3	27302.74	29768.56	33240.60	34211.24	36814.21	39415.41	42319.28	45343.17

In this study the effect of material prices is consider; the results are shown in table (9) and figure (2.c). The increase in the price of raw materials occurs simultaneously with the increase in the purchase price of materials for the manufacturer, and also this price is effective in determining the amount of equilibrium quantity, so we examine the increase of these 4 parameters simultaneously. Moreover, the price of raw material effects the final price for the customers. The results show that the price increases the first objective function, but from the second interval onwards the second objective function increases (due to unwillingness to supply), and the third objective function decreases due to less production.

**Table 9.** The impacts of price

	1	2	3	4	5	6
CS	9	17	25	32	39	46
CM	12	25	34	38	44	51
$RV_t$	(70,90)	(81,103)	(95,112)	(108,123)	(119,131)	(127,148)
FRa*	100.00	169.14	255.31	343.25	417.39	492.14
FRb*	60.00	110.48	201.14	312.79	397.12	444.18
Z1	$5.39*10^4$	$5.82*10^4$	$6.05*10^4$	$6.13*10^4$	$6.18*10^4$	$6.21*10^4$
Z2	0	0	117.28	168.91	241.86	297.34
Z3	27811	24767	21782	19562	17648	16423

In addition,  $(ef_i)$  and  $(el_i)$  are the parameters that indicate how the vehicles are environmentally friendly. Using environmentally-friendly vehicles usually costs more in short periods, and the older vehicles have lower costs, but they consume more fuel. So, the results show that increasing these two parameters reduces the costs and enhance profit function. If this trend continues, manufacturers produce less because CO2 emission is increased sharply, and decreasing the volume of transported production can help to prevent the environmental problem. Table (10) and figure (3.d) show this trend.

**Table 10.** The impacts of type of vehicles

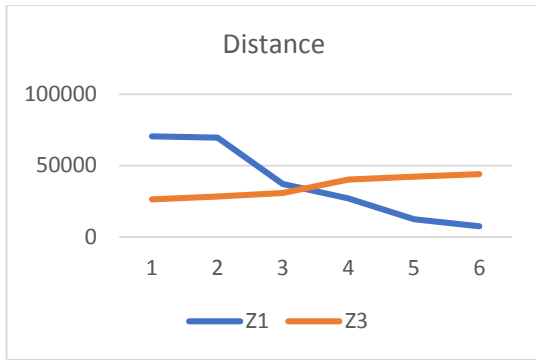
	1	2	3	4	5	6	7
$ef_i,$	(20,40)	(30,50)	(40,60)	(50,70)	(60,80)	(80,90)	(90,100)
$el_i$	(7,10)	(10,12)	(14,17)	(16,19)	(18,21)	(20,24)	(22,27)
$Z1$	$5.28*10^4$	$5.49*10^4$	$6.39*10^4$	$6.61*10^4$	$7.04*10^4$	$7.58*10^4$	$8.12*10^4$
$Z2$	0.00	0.00	0.00	0.00	0.00	97.23	157.28
$Z3$	10900.80	16745.23	22518.39	28374.32	35124.74	42874.19	46791.45

One of the characteristics of vehicles is their capacity, which is very important in furniture because carrying these devices due to the asymmetrical shape requires adequate capacity. In table (11) and Fig (2.e), the effect of capacity on objective function is investigated, which shows that costs. This is mainly due to the fact that the higher capacity car enhances the system's shipping costs, but the objective function of carbon dioxide decreases, which indicates that this policy reduces carbon dioxide. (Of course, in this assumption, the production of carbon dioxide for each unit  $(ef_i)$  and  $(el_i)$  are considered the same). Furthermore, row of choice shows the selection of network for type of vehicle. In first and second column, network decide to choose vehicles with higher capacity. So, all customer demand is satisfied and this policy decreases Co2 emission. In third column, network decides choosing multiple types of vehicles, in the last two columns, network decides to choose vehicle with lower capacity so a part of demand remains in warehouse of manufacturers to transfer them in next period. Moreover, capacity doesn't have effect on the competition.

**Table 11.** The impacts of the capacity of vehicles

	1	2	3	4	5
$Q_i$	1= (13,17)	1= (15,20)	1= (20,25)	1= (25,30)	1= (30,35)
	2= (10,15)	2= (10,15)	2= (10,15)	2 = (10,15)	2= (10,15)
choice	1	1	1,2	2	2
$Ll''_{mt}$	0	0	14	27	27
$Z1$	$4.76*10^4$	$5.48*10^4$	$6.12*10^4$	$5.74*10^4$	$5.74*10^4$
$Z3$	27315	26417	24912	25111	25111





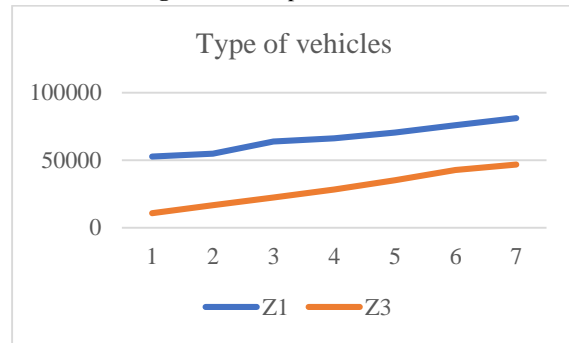
**Fig 2.a.** The impacts of distance



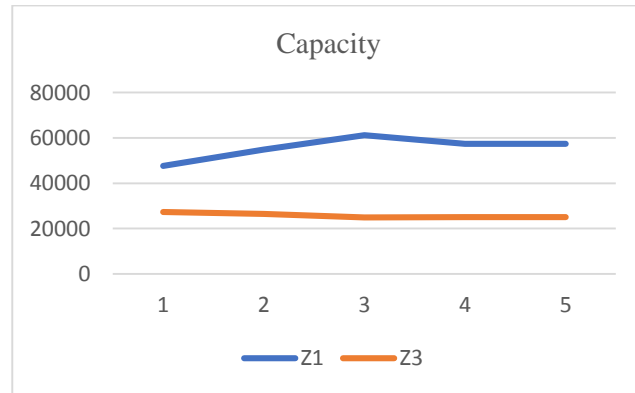
**Fig 2.b.** The impacts of discount



**Fig 2.c.** The impacts of price



**Fig 2.d.** The impacts of type of vehicles



**Fig 2.e.** The impacts of capacity

## 4-2- Managerial insights

The investigation of the model reveals some interesting managerial insights in the context of this furniture company.

1. In this supply chain, paying attention to allocating the right points in terms of their distance from each other is one of the factors that plays an important role in service time, reducing costs and reducing carbon dioxide simultaneously. As a result, it increases customer satisfaction and makes the whole chain more successful.
2. These days, according to the dynamic condition of the market, the manufacturers should improve the accuracy of demand prediction and pay attention to the supply chain infrastructure (such as the capacity of a warehouse) to reduce shortages and increase customer satisfaction.

3. Controlling the price of raw materials can be an incentive for suppliers to remain in competition and produce more to meet customer demand by the manufacturer and reduce the risk of supply-demand mismatch.
4. In this case, because the price is considered fair, the discount reduces profits, but if the company intends to consider a discount to increase customer or increase loyalty, a minimum discount) for example about (5-10) percent could be useful.
5. Organizations are putting a lot of effort into integrating green transportation into their networks as a result of the growing environmental problems. In this industry green transportation can be achieved by choosing vehicles that produce less CO2 emission and have adequate capacity.
6. Properly regulating the number of manpower and optimizing production costs is very important for an organization and should be planned based on delivery time. If these costs are too high for each production unit, it will lead to the system's unwillingness to produce and lack of goods.
7. The choice of machine capacity depends on their cost, but the results show that the method in which both types of capacity are used has been more environmentally friendly. Also, in this network, some factors such as distance, type of vehicles, price, and discount rate can effect on competition.

## 5- Metaheuristic algorithms

Since N-hard problems don't have an analytical solution, especially in large instances, meta-heuristic algorithms are adopted to solve them. One of the most common approaches among meta-heuristic algorithms is the Genetic algorithm. Scalability and robustness are important features that increase the popularity of the genetic algorithm. This proposed model is a multi-objective problem so we use NSGA-II that is a version of a genetic algorithm for solving this type of models. This algorithm was developed by Deb et al. (2002), and it is very popular for solving multi-objective problems and based on the Pareto solution. Traditional genetic algorithm and NSGA-II are easy to converge and fall into the local optimum so many studies try to improve them with a different approach such as applying different crossover and mutation, fuzzy roulette wheel selection mapped Gantt chart (Bandyopadhyay et al. 2013; Thammano and Teekeng 2015; Worapradya and Thanakijkasem 2014). Moreover, in this area Viana et al. (2020) used a multi crossover and mutation to modify the genetic algorithm also Li et al. (2017) could improve the genetic Algorithm by an IGA with different phases. So, in this study, a new modified NSGA\_II is developed to enhance diversity and quality of Pareto solution by strengthen search ability. This modified NSGA\_II has two phases which in each of them different operators with different possibility are used. Figure (5) illustrates the algorithm's flow chart:

The modified NSGA-II algorithm has some steps as follow:

1. **Chromosome encoding:** this section is applied to make a feasible solution in space of the algorithms. This encoding is done as figures (3): In the first step a permutation from manufacturers, distributors and customers is create. Then, points of the next level are assigned to pervious level randomly. For example, for assigning manufacturers to suppliers, In the first step, (S-1) numbers between (0, M-1) are chosen randomly and manufacturers separate according to the numbers. So, S parts are made, then each S parts assign to each S supplier randomly (S is set of suppliers and M is set of manufacturers). For instance, when the problem has 5 manufacturers and 2 suppliers, a chromosome and assigning demand points for first level (between supplier and manufacturers) are as below:

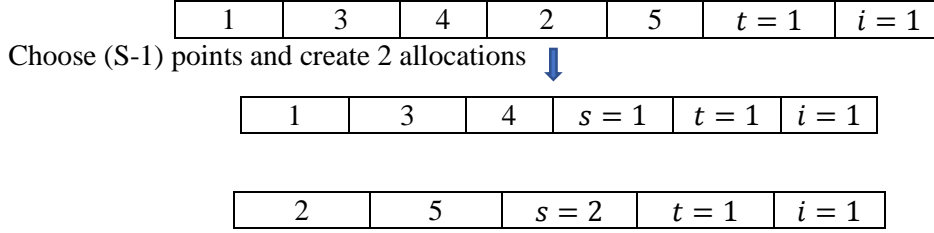


Fig. 3. Chromosome encoding

2. **Initial population producing:** The initial population is generated randomly. Because random choice has a significant impact on the solution's convergence and quality.
3. **Calculation evaluation factors:** At first, the objective functions of members are calculated. Then, two factors of NSGA\_II (non-dominated rating and crowding distance) are calculated. These factors determine the rank of members. the first factor indicates the predominance of answers and the second shows the average distance of its two neighboring solutions. So, the members can have a rank according to the non-dominated rating. When rating factor is the same between two members, the member located in a less crowded region is selected.
4. **Product offspring:**

Firstly, a random parent population is made and modified NSGA\_II operators of NSGA\_II are applied in two different phases as follow:

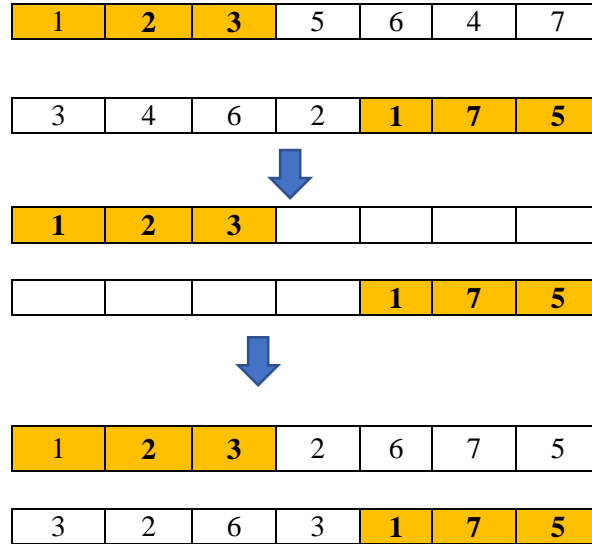
Three operators are used for each phase with the different possibility to avoid falling into local optimal solution and increases the quality of the population. The applied operators are selection operator, Crossover, and mutation. In the first phase the possibility of selection operator is about (70%) to save better solution. In the second phase sum of the possibility of crossover and mutation is about (70%) because these factors can produce more new solutions and eliminate the premature convergence. In the following, each of these operators is explained:

- a. **Selection operator:** This operator was applied by Li et al. (2017) for the genetic algorithm. The stochastic tournament strategy and the elite preservation strategy are used in this operator. In this paper, binary tournament selection in selection operator to find better parents and save them in offspring set.
- b. **Crossover:** The purpose of crossover is to generate better children by two selected parents. In this paper, two-point crossover is used and repeated for RC times. This method is developed by Viana et al. (2020) for the first time to increase the quality of children. This crossover is applied as table (12):

Table 12. The process of crossover

$P = (P1, P2)$	<b>select parents</b>
$F1=F(P1)$ $F2=F(P1)$	<b>evaluate parents</b>
<hr/>	
<b>For <math>i = 1: R_c</math></b>	
<b>CR: crossover</b>	
$(c_{i,1}, c_{i,2}) = CR(P1, P2)$	
$F'1=F(c_{i,1})$	<b>evaluate children</b>
$F'2=F(c_{i,2})$	
<b>if at least one child is better than the parents this loop stops</b>	
<b>else this loop continues until <math>i=RC</math></b>	
<b>end</b>	

The crossover of this study is a two-point crossover. This crossover is that the first two points from each parent are randomly selected and the points that are between these two points are passed directly to the children, then the remaining points from the second parent are transferred to the first child and the remaining points from the first parent are transferred to the second child. Figure (4) shows this crossover.



**Fig 4.** two-points Crossover

- c. **Mutation:** this operator tries to change one parent and produce a useful child to increase diversity and improve the quality of answers. This process is shown in table (13). In this paper are used three kinds of mutation (swap, invers and insert).

The process of mutation is as follow:

**Table 13.** The process of Mutation

<b>P = P1</b>	<b>select parents</b>
<b>F1=F(P1)</b>	<b>evaluate parents</b>
<b>For <math>i = 1: R_m</math></b>	
<b>MU= {mu1, mu2, mu3} mutation</b>	
<b>Select randomly between the members of MU</b>	
<b>If m=mu1</b>	
$(c_{i,1}) = mu1(P1)$	
<b>F'1=F (<math>c_{i,1}</math>)</b>	<b>evaluate children</b>
<b>elseif m=mu2</b>	
$(c_{i,1}) = mu2(P1)$	
<b>F'1=F (<math>c_{i,1}</math>)</b>	<b>evaluate children</b>
<b>elseif m=mu3</b>	
$(c_{i,1}) = mu3(P1)$	
<b>F'1=F (<math>c_{i,1}</math>)</b>	<b>evaluate children</b>
<b>if child is better than parent this loop stops</b>	
<b>else this loop continues until <math>i = R_m</math></b>	
<b>end</b>	

In the first phase and step, by selection operator better parents (according to their rank) are saved and crossover and mutation are applied on the solution with worth rank. Then, offspring set with children and parent with better rank is made. In the next step, the offspring set of first phase is

transformed to second phase and this process is repeated on offspring set of first phase with higher possibility for mutation and crossover that make new solution more.

5. **Update the population and archive:** In the next step, offspring set and initial population are combined. Then, this mixed population is sorted according to the nondomination factors. The best non-dominated members are saved in an archive to make Pareto front. Finally, for the next repetition, a population is updated by solutions of better fronts and the extra population is eliminated.
6. **Stopping condition:** This loop repeats but a number for iteration must be considered to avoid an endless loop of the algorithm. The algorithm will continue until it meets this number.

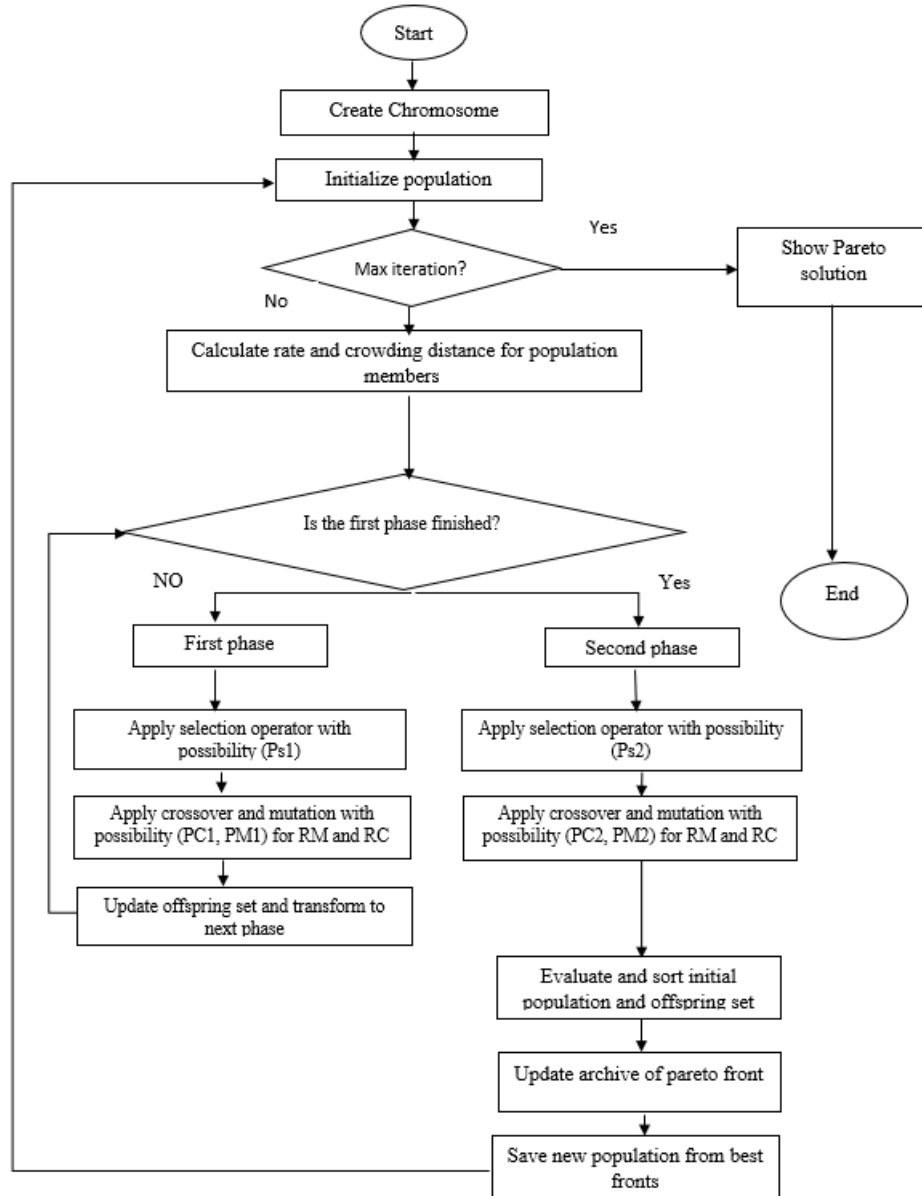


Fig. 5. The process of the modified algorithm

### 5-1- Numeral example

In this section, NSGA\_II and modified NSGA-II are coded in MATLAB, and the results are compared with the results of the GAMS software. For evaluation of algorithms, we use the first objective function, and the problem is solved as a single objective problem in GAMS software by the weighted sum approach. For investigation of results, we use equation (46) to calculate the proximity of the answer to the optimal answer.

$$GAP\% = \frac{meta z_1 - gams z_1}{gams z_1} * 100 \quad (46)$$

After coding the algorithms in MATLAB software, the parameters of the algorithms must be entered. Taguchi (1986) was developed a systematic method for design parameters and we use it for the parameters of NSGA-II and modified-NSGA-II. This method calculates the quality values of experiments with different level combinations. Then, these values are transformed in signal-to-noise ratio (S/N) (Moattar Hussein et al. 2015). Also, this method takes into account three types of objective functions (the smaller-the-better, the larger-the-better, and the nominal-the-best type) that each with its own S/N ratio. We use the smaller the better in equation (47)-(48) to obtain the optimal level combination for this problem and levels and the optimal level for each algorithm are shown in table (14). These levels were recommended by Rahimi et al. (2019) and Li et al. (2017).

$$S/N = -10 \log_{10}(MSD) \quad (47)$$

$$MSD = Y_1^2 + Y_2^2 + \dots + Y_n^2 / N \quad (48)$$

**Table 14.** Parameter analysis

	NSGA_II	modified NSGA-II		Optimal level for NSGA-II	Optimal level for modified NSGA-II			
		Phase1	Phase2		Phase1	Phase2		
Iteration	50	50		120	120			
	80	80						
	100	100						
	120	120						
Size of population	30	30		50	50			
	40	40						
	50	50						
Crossover ratio		Phase1	Phase2	0.6	0.1	0.3		
	0.4	0.05	0.2					
	0.5	0.1	0.3					
	0.6		0.4					
Mutation ratio		Phase1	Phase2	0.4	0.1	0.4		
	0.3	0.05	0.2					
	0.4	0.1	0.3					
	0.5	0.2	0.4					
Selection operator ratio	Not used	0.7	0.1	Not used	0.8	0.3		
		0.8	0.2					
		0.9	0.3					
Iteration of Crossover	Not used	5	5	Not used	10	10		
		8	8					
Iteration of Mutation			10		10		10	10
			5		5			
		8	8					
		10	10					

The problems of this section are shown in table (15). We stop increasing the sizes of problems because GAMS cannot solve them because of the limitation of GAMS in solving large sizes No.5.

**Table 15.** The size of problems

Problem code	size
1	$(s = 2, m = 2, d = 2, a = 4, t = 3, i = 2)$
2	$(s = 2, m = 3, d = 3, a = 6, t = 3, i = 2)$
3	$(s = 2, m = 4, d = 4, a = 8, t = 3, i = 2)$
4	$(s = 2, m = 6, d = 5, a = 12, t = 4, i = 3)$
5	$(s = 2, m = 8, d = 6, a = 15, t = 5, i = 3)$
6	$(s = 2, m = 10, d = 8, a = 20, t = 5, i = 3)$
7	$(s = 2, m = 12, d = 10, a = 25, t = 6, i = 3)$
8	$(s = 2, m = 14, d = 12, a = 30, t = 6, i = 3)$
9	$(s = 2, m = 16, d = 14, a = 33, t = 6, i = 3)$
10	$(s = 2, m = 20, d = 18, a = 40, t = 6, i = 3)$

**Table 16.** The investigation of quality

Problem code	No.experiment	Quality of answers						Time (s)		
		NSGA-II			MNSGA-II			GAMS	NSGA_II	M NSGA-II
		Max GAP	Min GAP	Ave.GAP	Max GAP	Min GAP	Ave.GAP			
1	1	0.29	0.21	0.25	0.089	0.045	0.067	210	31.27	35.41
	2	0.32	0.14	0.23	0.11	0.037	0.073	212	32.52	35.64
	3	0.38	0.12	0.25	0.10	0.00	0.05	210	31.25	36.27
	4	0.21	0.15	0.18	0.15	0.09	0.12	211	30.12	36.99
2	1	0.51	0.31	0.41	0.12	0.039	0.07	317	37.78	43.69
	2	0.34	0.25	0.29	0.02	0.00	0.01	315	38.25	43.74
	3	0.38	0.27	0.32	0.16	0.055	0.10	318	38.36	43.21
	4	0.29	0.24	0.26	0.26	0.00	0.13	316	37.96	44.01
3	1	0.49	0.37	0.43	0.20	0.07	0.13	451	46.35	53.88
	2	0.52	0.40	0.46	0.24	0.13	0.18	455	46.37	54.21
	3	0.48	0.32	0.40	0.31	0.23	0.27	454	46.36	54.13
	4	0.54	0.35	0.59	0.34	0.09	0.21	455	47.95	54.12
4	1	0.67	0.34	0.50	0.41	0.24	0.32	611	55.96	63.12
	2	0.71	0.43	0.57	0.39	0.19	0.29	617	56.35	63.63
	3	0.73	0.40	0.56	0.43	0.27	0.35	614	56.57	63.19
	4	0.62	0.41	0.51	0.41	0.25	0.33	614	56.69	63.54
5	1	-	-	-	-	-	-	-	62.39	66.21
	2	-	-	-	-	-	-	-	61.95	66.39
	3	-	-	-	-	-	-	-	62.24	66.18
	4	-	-	-	-	-	-	-	62.27	66.27
6	1	-	-	-	-	-	-	-	85.29	90.13
	2	-	-	-	-	-	-	-	85.21	90.15
	3	-	-	-	-	-	-	-	85.25	90.15
	4	-	-	-	-	-	-	-	85.28	90.15

We solve 6 problems and repeat each of them four times and report the minimum, maximum, and average GAP of these repetitions. The results are shown in table (16). These numbers demonstrate the average of the GAP for modified NSGA\_II is between 0.01 and 0.45 that is less than the average of answers for NSGA\_II (between 0.18 and 0.76). So, this shows the quality of answers improved by the modified algorithm. Moreover, these GAPs are increasing with increasing the size of the problem, but GAP for

NSGA\_II is less than 0.76 and in modified NSGA\_II is less than 0.45, so both of them reliable for solving the problem. Also, the time of solving for modified NSGA\_II is more than NSGA\_II. Due to the quality of the answers, this small difference can be inconsequential. As a result, the high solution time of GAMS and the inability to solve even moderate problems such as Problems 5 and 6 caused we prefer the meta-heuristic solution method for solving moderate and big problems.

In the next step, we use an approach that the results of two different metaheuristics in medium and large sizes were evaluated by [56]. In this method, the non-dominated solutions of two algorithms are saved in an archive, and then rank, crowded distance, and the ratio of non-dominated solutions are calculated for the results of each algorithm. The results are shown in table (17). This table also represents the number of non-dominated solutions for each algorithm.

$$RN_{NSGA_{II}(I-NSGA_{II})} = \left| \{x \in D_{NSGA_{II}(I-NSGA_{II})}\} \cap D_{ND} \right| / D_{NSGA_{II}(I-NSGA_{II})} \quad (49)$$

Equation (49) calculates the ratio of non-dominated solution for NSGA\_II and modified NSGA\_II called  $RN$ .  $D_{(NSGA_{II})}$  demonstrates the set of non-dominated solutions of NSGA-II and  $D_{(I-NSGA_{II})}$  denotes the set of non-dominated solutions of modified NSGA\_II, then the union of these set is  $D$  and  $D_{ND}$  saves the set of non-dominated solution of  $D$ . The results show that the solutions of the modified algorithm can dominate the solution of NSGA\_II more. Moreover, the number of non-dominated solutions in this algorithm is more than the number of NSGA\_II. So, this algorithm has better operation in large problems.

**Table 17.** The comparison of NSGA\_II and modified NSGA\_II

Problem code	No. experiment	Number of non-dominated solution		The average ratio of non-dominated solution	
		NSGA-II	Modified NSAG-II	NSGA-II	Modified NSAG-II
6	1	11	14	0.64	1
	2	10	16	0.71	1
	3	12	17	0.59	0.82
	4	14	16	0.75	0.94
	Ave	11.75	15.75	0.67	0.94
7	1	9	17	0.54	0.75
	2	10	17	0.67	0.82
	3	12	19	0.72	1
	4	10	21	1	0.88
	Ave	10.25	18.5	0.73	0.86
8	1	15	18	0.52	0.72
	2	13	17	0.68	1
	3	13	19	0.74	1
	4	13	18	0.88	0.95
	Ave	13.5	18	0.70	0.91
9	1	14	16	0.74	1
	2	14	16	1	1
	3	12	15	1	1
	4	15	18	0.91	1
	Ave	13.75	16.25	0.91	1
10	1	12	17	0.85	1
	2	17	17	0.76	1
	3	14	19	0.53	1
	4	11	16	0.71	0.82
	Ave	13.5	17.25	0.71	0.95



## 6- Conclusions

A multi-echelon supply chain with a Stackelberg competition between suppliers is presented in this study. The obtained results showed that the model remained in Stackelberg competition at a specific level. After this level of inventory cost, suppliers decided to supply the amount of raw material that was different from the competitive value. In addition, the model tends to maximize profit and minimize CO2 emissions. The type of ordering in this issue is made to order that this type of ordering in today's world due to the increase in personalization. To solve the model, it is primarily solved a case study with GAMS software and the results showed the behavior of the model. However, according to the complexity of the NP-Hard model, we applied two meta-heuristic algorithms. The results demonstrated that the modified algorithm has better solutions from NSGA-II, and the number of non-dominated solutions in this algorithm is more than the number of NSGA-II. Further research is recommended to focus on the following areas to overcome the limitations of this study. For example, different types of competition, the mixture of make to order and make to stock for ordering, routing problems maybe good ideas. Moreover, attending to the risk concept or other uncertainty approaches, such as robust or fuzzy optimization approaches, could help develop the study. In addition, using other objective functions could be suitable decision to consider real-world problems. The proposed algorithm can enhance diversity during evolution process. But the running time is a little more than NSGA-II. So, future research could explore the application of other heuristic and metaheuristic algorithm such as MOPSO (Multi objective particle swarm optimization) or Tabu search to enhance convergence, and decrease running time. It is also possible to use some methods such as applying an archive to save dominated solutions to use in other iteration to enhance diversity.

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