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Automotive supply chain with government intervention: A game theory approach

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Abstract

The automotive industry is one of the most competitive industries globally. Hence, a proper pricing policy is vital to be able to compete. This paper develops an automotive supply chain consisting of one manufacturer and one retailer, with two types of one product (Basic and Premium). The Premium type is equipped with extra features and technologies to convince customers to purchase it at a higher price, while the Basic one without these features is sold at a lower price. Furthermore, the manufacturer is intervened by the government regulations, which results in receiving subsidies or paying taxes. This paper develops a mathematical model to maximize profit in both centralized and decentralized systems. The model examines the effects of the feature and technology level, as well as advertisements on vehicle pricing. To examine the model, several numerical examples are applied. The results reveal that the model can increase the manufacturer's profit by considering the government benefits, as well as the retailer's profit. Finally, some managerial implications are developed by a sensitivity analysis of the main parameters.

Keywords: Automotive industry, supply chain management, pricing theory, game theory, government intervention, technology level, advertisement.

1- Introduction

Due to rapid technological development, industrialization, and the diversity of customers' expectations, the automotive industry has become more competitive and dynamic in recent years. Hence, automotive supply chains have been modulated from independent firms to competitive supply chains. Pricing is a practical approach to achieving profitability throughout the supply chain (Cachon 2003). Except for other factors, a 5% change in price will lead to about a 22% improvement in profitability. On the other hand, lower than 15% of manufacturers could systematically investigate the pricing issue (Clancy and Shulman 1993). Finding proper pricing strategies is the key to attaining the high profitability of supply chains (Fang et al., 2020).

Since in today's modern life, an automobile has become a necessity, auto manufacturers consider customers' requirements to get a higher market share which leads to higher profitability. Due to the significant effects of features and technology on customer satisfaction, many well-known car companies have invested in improving these two items.

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For instance, by improving the features of the products, Volkswagen has been able to gain considerable market share over its competitors in India (Gupta, 2013). Furthermore, a study of the Norwegian car market shows the significance of the impact of car specifications and features on its demand (Zhang et al., 2016). The extra features can be considered as sunroofs, driver medical seats, and dual-zone air conditioner, while the technologies consist of keyless entry, keyless start-stop, connected system, etc.

The impact of advertising as a factor affects the selling rate in today's automotive industry, where a new product enters the market in the presence of a variety of media, internet, and billboard advertising more than before (Pedros-Perez et al., 2019, Dahiya and Gayatri, 2018, Murry, 2014). For example, Nissan and Tata Motors have successfully used ads on social media to sell two products of their brand (Tybout and Fahey, 2017).

Each country with an oligopoly market of automotive usually has special regulations for auto manufacturers, such as investment incentives, tariffs, and quantitative restrictions. Accordingly, owing to the significant direct and indirect government intervention in vehicle pricing, it is essential to develop a framework that can find appropriate answers to the following questions:

- What are the effects of the feature and technology level on the pricing of the Premium class?
- What is the equilibrium point for the combination of price, feature, and technology?
- What is the effect of the advertisement level on optimum pricing of the Premium class?
- What is the effect of the government intervention (both economic and social) on the profitability of both manufacturer and retailer (in the form of tax and subsidy)?

To summarize our research, we consider one manufacturer and one retailer with two classes of product types in a supply chain. The classes are distinguished via the features and technologies level (FTL). Customers choose FTL depending on their purchasing power and tendency to have a well-equipped vehicle. Direct/indirect government intervention in automotive industries is considered in both economic and social forms, which is implemented via subsidy-taxation policy to the manufacturer to develop the Premium type of the vehicle. A novel mathematical model for the automotive supply chain with presuming government intervention is represented. The main contributions of the current research are:

- Presenting a novel mathematical game model for an automotive supply chain with two classes of product types (Basic and Premium) under government intervention.
- Considering that these two classes of products are distinguished via the Features and Technologies level (FTL).
- Analyzing the effect of customers' purchasing power and tendency to choose a well-equipped vehicle.
- Proposing Direct/indirect government intervention in automotive industries through government domination and infiltration.
- Implementing the government intervention in the automotive supply chain via the policy of subsidy-taxation to the manufacturer.

It should be noted that although the game theory has been applied to develop various kinds of supply chains (Zhao et al. 2020; Hadi et al. 2020), it has rarely been used in automotive supply chains. Moreover, the effects of FTL on differentiating product types as well as the effect of advertisement have not been investigated in the literature simultaneously. This article develops a mathematical model to investigate the effect of these factors as well as government regulations on the automotive supply chain. The remainder of this article is organized as follows. Section 2 reviews the related literature. Section 3 contains the model formulation. Section 4 represents the government intervention. Section 5 contains some numerical examples with sensitivity analysis before/after government intervention. Some managerial insights are represented in section 6. Finally, section 7 represents the conclusions.

2- Literature review

This research is related to three different literature areas: pricing in the supply chain coordination, automotive supply chain, and government intervention.

2-1-Pricing in the supply chain coordination

Pricing in the automotive industry has been examined by some researchers. Huang and Huang (2010) developed a price coordination problem with deterministic demand in a three-stage supply chain using the Stackelberg game approach through three scenarios, centralized, decentralized, and semi-centralized. They revealed that the centralized system would lead to higher performance. They also showed that in a monopoly market, the integration of the retailer and the manufacturer might not increase their profits. Mirzapour et al. (2011) considered a supply chain with multiple manufacturers, multiple suppliers, and multiple customers. They developed a multi-objective programming model for the Aggregate Production Planning (APP) problem with a multi-product, multi-cycle, multi-factory model with uncertain cost and demand parameters. Pezeshki et al. (2013) considered both pricing and capacity reservation problems in a decentralized two-echelon supply chain, including one supplier and multiple retailers where products have a short life period. They developed a non-linear model, where the supplier decided based on the capacity and the production time, and the retailer made decisions regarding price and order quantity. Xu and Lu (2013) developed a newspaper boy pricing model considering uncertainty in the supply chain and examined the effect of stochastic performance on optimization decisions and expected profit. In their model, both pricing and inventory decisions are studied. They showed that with lower levels of yield, the optimal price is lower, but the profit is higher. Moreover, Hsieh et al. (2014) examined a supply chain, including multiple manufacturers and one retailer. Two systems of centralized and decentralized were considered to find equilibrium price and quantity. Each manufacturer produced one different exchangeable product, which all were sold through one retailer. The results indicated that in the centralized model, both the overall optimal profit and the profit of each supply chain member were higher than the decentralized one. Giri and Bardhan (2017) considered a decentralized three-echelon supply chain with one supplier, one manufacturer, and one retailer. They developed a model to examine the impact of the channel parameters on the optimal strategies. Qing et al. (2017) examined a monopolistic supplier’s capacity-allocation problem with dual channels using a game theory approach. Qi et al. (2017) developed a pricing model in a two-stage decentralized supply chain based on a Make-to-Order (MTO) system consisting of one supplier and two retailers and examined the pricing decisions through a game theory approach. The proposed model found the optimal pricing strategies for the supplier and the retailer. Xu et al. (2017) examined production and pricing in an MTO supply chain consisting of a downstream retailer and an upstream manufacturer. The upstream manufacturer produced two MTO-based products. Moreover, it determined the wholesale price to respond to the price-sensitive demand by the retailer. Zhao et al. (2017) considered a pricing and capacity allocation problem related to a duopoly. The model studied how the pricing decisions of substitutable perishable products and capacity allocation decisions depend on market conditions. Venegas and Ventura (2018) developed a game theory model for a decentralized supply chain with one supplier and one buyer. Modak and Kelle (2019) proposed a mathematical model to maximize profit in both centralized and decentralized systems for a supply chain consisting of one manufacturer and one retailer with two sale channels in which the demand was stochastic and a function of both price and delivery time. Honarvar et al. (2020) examined pricing policies and lead time in a supply chain with a given production capacity, one MTO manufacturer, and two sale channels; a retailer and an online channel. Customers were divided into different classes with different demands based on their price sensitivity and deadline. A non-linear programming model was developed for centralized and decentralized scenarios, and a PSO metaheuristic algorithm was applied to find answers. Furthermore, Wang and Song (2020) considered a two-stage two-channel supply chain with stochastic demand consisting of one manufacturer and one retailer, with two types of products (green and not green). They developed a Steckelberg model for the pricing. Salehi et al. (2020) developed a three-stage supply chain with two sales channels and uncertain capacity. A two-channel supply chain with one supplier was investigated by Yan et al. (2020). They developed a mixed-integer non-linear programming (MINLP) model in the presence of limited investment and one web-based retailer. An equilibrium price and quantity and the profit were calculated. Wang et al. (2021) proposed a game model for a price-dependent stochastic demand supply chain with two suppliers and one retailer, in which two types of substitutable products are sold. Raza and Govindaluri (2022) investigated a supply chain with one manufacturer and one retailer, where the model focused on pricing and market segmentation issues.

FTL and advertisement have not been considered simultaneously in the previous studies. Here, we examine the optimal pricing of the automotive supply chain via FTL differentiation as well as the advertisement level.

2-2-Automotive supply chain

Despite the considerable level of cooperation and competition in the automotive supply chain, little research has been done by applying the game theory approach in this context. Biller et al. (2005) developed a model to optimize the coordination between production and inventory decisions using dynamic pricing, where a greedy algorithm was developed with a concave profit function. Qu and Williams (2008) considered a reverse vehicle supply chain, where the pricing and production planning are formulated through a non-linear programming model. Olugu and Wong (2012) proposed a fuzzy rule-based system for evaluating the capability of a closed-loop supply chain for vehicles. Günther et al. (2015) examined the role of electric cars on the improvement of sustainability in an automotive supply chain. Vanalle et al. (2017) considered a vehicle supply chain based on the Brazilian market and investigated the three approaches to supply chain management (pressures, practices, and performance) on it. Al-Doori (2019) assessed the effect of members' collaboration in an automotive supply chain on their performance. Hadi et al. (2020) considered a green vehicle supply chain with government intervention and two types of substituted products. Their results obtained by solving the proposed mathematical model indicated that the government environmental policies had a considerable effect on both the governmental-related revenue as well as the benefits of supply chain members. Ma et al. (2021) developed a mathematical model for an electric vehicle manufacturer considering carbon emission reduction and government intervention in which the government had preferential policies on consumption subsidies and exemption from consumption tax. Furthermore, the development of electric vehicles was supported by the government by establishing charging piles. Moreover, Fander and Yaghoubi (2021) investigated the interaction of an automotive supply chain and a fuel supply chain with each other based on the Iran automotive market. Li and Li (2022) formulated a coordination model for a new energy vehicle supply chain with financial constraints and demand uncertainty.

This research addresses both FTL and advertisement levels in the pricing model, which has not been addressed in the literature.

2-3- Government intervention

Government intervention is defined as social and economic supervision that reassures of government to influence the decisions of players in a specific area in order to manage the market. Here, we reviewed research works that consider government intervention in supply chains.

Hafezalkotob (2015) considered government intervention on a green supply chain for a three-tier distributed planning consisting of one manufacturer and one retailer. The supply chain represents various kinds of one product, which are partially substitutable in the market. A game theory approach was used to model the government intervention in the supply chain. The results illustrated that the government had both a social and environmental impact on the profit of supply chain members. Mahmoudi and Rasti-Barzoki (2018) formulated the conflict between the objectives of the government and producers using a game theory approach. Sinayi and Rasti-barzoki (2018) developed a two-level supply chain, where the government had the role of leader for the whole supply chain. The model considered the price and greening level of the product as the main variables. The government collected taxes and received a subsidy on the final price of the product. They found that the cooperation between the manufacturer and the retailer led to producing a greener product that increased the profit of the whole supply chain. Giri et al. (2019) considered a two-echelon green supply chain consisting of two competitive manufacturers and one retailer, considering the indirect taxes, selling prices, and also greening levels. Each of the manufacturers produced one of two green substitutable products, and retailers sell green products to satisfy customers' demands. The model investigated the government intervention in the decisions of chain members. The government intervened in the supply chain by taxes and penalties on the trading price to encourage manufacturers to produce green products. Moreover, Daniel and Usman (2020) examined the effect of government policies on developing the Nigerian automotive industry. Mahmoudi et al. (2021) investigated the effect of the presence of third-party logistics companies in multi-channel supply chains with green and non-green products under government intervention. Xie et al. (2022) investigated a green supply chain, in which the impacts of considering government intervention and public-private partnerships simultaneously on ecological modernization are analyzed.

Furthermore, some researchers considered the effect of government intervention on new energy vehicles. Xiong and Huang (2018) examined both "government purchasing" and "consumption subsidy" for producing new energy vehicle manufacturers. Based on the developed model, government purchasing was more effective than consumption subsidy. Yang et al. (2019) considered the development of a new energy vehicle market, which is supported by the government. They developed a Cournot duopoly model considering government intervention to find the optimum prices. Lu et al. (2020) investigated the subsidy effect on battery electric vehicles (BEV). They showed that incentive policy could considerably have an impact on the adaptation of BEVs. Zhao et al. (2020) considered a closed-loop supply chain for the new energy vehicle market with different kinds of government none financial subsidies. They applied a game theory methodology. Furthermore, Zhang and Yousaf (2020) considered government intervention in the form of subsidies or taxes. They found that government intervention could increase the performance of the supply chain.

Although government intervention has been addressed in the literature, the economic and social impact of the government intervention on supply chains in the form of tax and subsidy has not been considered yet simultaneously. Therefore, the mentioned gap is a suitable ground that is conducted in this research.

2-4- The research gap

Although some research works have been conducted on the application of pricing in the production supply chain, little research has been conducted on the impact of pricing and technology and advertisement level simultaneously on the automotive supply chain profitability. Furthermore, few researchers have focused on the effect of government intervention in the form of taxes and subsidies in an automotive supply chain. Table 1 summarizes the literature on the related area of research.

Table 1. A brief comparison of most relevant studies

Authors	Supply chain		Multi-Products	Pricing	Technology Level	Government Intervention		Game Theory	
	Automotive	Others				Subsisity	Tax	Nash	Stackelberg
Huang & Huang (2010)	•			•				•	•
Hsieh et al. (2014)		•	•	•				•	•
Hafezalkotob (2015)		•	•	•		•	•		•
Mahmoudi & Rasti-Barzoki (2018)		•	•	•		•	•	•	•
Modak & Kelle (2019)		•		•				•	•
Yan et al. (2020)		•	•	•				•	•
Raza & Govindullari (2020)		•	•	•	•				•
Hadi et al. (2020)	•			•		•	•	•	•
Zhang &Yousaf (2020)		•		•				•	•
Zhao et al. (2020)	•			•		•	•	•	•
Current Study	•		•	•	•	•	•	•	•

This research considers the effect of government intervention and technology and advertising level on the profitability of supply chain members.

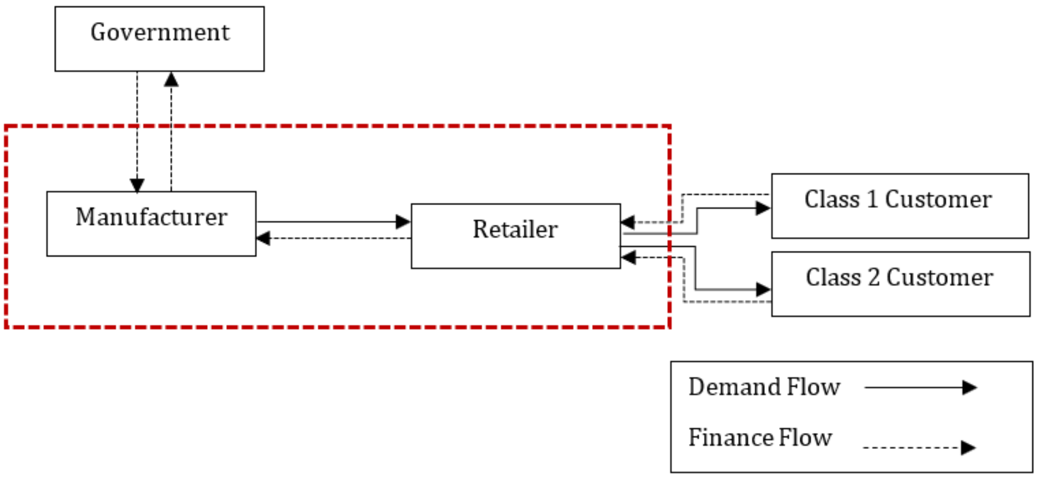


Fig 1. Model presentation

3-Model formulation

We consider a two-level automotive supply chain consisting of one manufacturer and one retailer with a product supplied in two different types, called Premium and Basic. Customers with a limited budget choose the Basic type, while the other ones with a higher budget prefer the Premium one. Due to the higher profit margin of the Premium type in comparison with the Basic one, the manufacturer is willing to promote the sale of the Premium type, and the retailer uses advertisements to persuade customers to buy this type of product. Fig. 1 represents a schematic representation of the model. Simultaneously, the manufacturer aims to maximize its profit via defining FTL, and the retailer endeavors to optimize the price of two types of the product and the advertisements level for the Premium type.

Table 2. Notations

Parameters	
D_i	Customer demand of class i, $i = 1,2$ (Premium and Basic; respectively)
a_i	The market potential for product type i, $i = 1,2$
W_i	Purchasing price of product type i from the manufacturer by the retailer, $i = 1,2$
C_i	The production cost of product type i for the manufacturer, $i = 1,2$
A_i	Retailer general and administrative cost of product i, $i = 1,2$
δ	Government intervention level
Decision Variables	
P_i	The sales price of product type i, $i = 1,2$
S_1	Advertisement level for product type 1 (Premium type)
L_1	Feature and technology level for product type 1 (Premium type)
Coefficients	
β_i	Price elasticity coefficient of demand for product type i, $i = 1,2$
τ_i	Competitive price elasticity coefficient of demand for product type i, $i = 1,2$
γ_i	Advertisement elasticity of demand for product type i, $i = 1,2$
ρ_i	Feature and technology elasticity of demand for product type i, $i = 1,2$
k	The coefficient of advertisement cost
h	The coefficient of features and technologies cost for the manufacturer
h'	The coefficient of features and technologies cost for the government
θ_1	The coefficient of the economic utility of government
θ_2	The coefficient of the social utility of government

The main assumptions of the model are represented as follows:

- The total market demand for the proposed automotive supply chain is divided into two categories; type 1 (Premium) and type 2 (Basic). All demand types are assumed to be deterministic. Demands for two types of product are affected by four endogenous decision variables (P_1, P_2, S_1, and L_1). In this research, it is supposed that the demand functions of product type 1 and type 2 (D_1, D_2) follow equation (1) and equation (2), which is similar to what Modak and Kelle (2019) used in their research.

$$D_1 = a_1 - \beta_1 P_1 - \tau_1 (P_1 - P_2) + \rho_1 L_1 + \gamma_1 S_1 \tag{1}$$

$$D_2 = a_2 - \beta_2 P_2 - \tau_2 (P_2 - P_1) - \rho_2 L_1 - \gamma_2 S_1 \tag{2}$$

  Where β_i , τ_i, ρ_i, and γ_i are positive coefficients.
- The demand for product type Premium will decrease when the FTL decrease and this demand are transferred to the other class (Basic) because the customers may not be satisfied with the combination of low FTL and high price. Conversely, by increasing the FTL, the demand for the Basic type of product will decrease.
- To formulate the FTL and advertisement level of the Premium type, a quadratic relation is exploited that has been employed by several researchers in the literature (see Bai et al. 2015; Bai et al. 2017).
- The government intervenes in the supply chain through two approaches. First, subsidizing during the loss period to support the manufacturer, and second, receiving income tax during its profitability period. This can be seen in some oligopolistic automotive markets with limited numbers of significant players in some countries (Fander and Yaghoubi, 2021).
- The shortage is not acceptable, which means all demands of customers have to be satisfied (Rasti-Barzoki and Moon, 2020).

The manufacturer decides on FTL (L_1), while the retailer decides on the advertisement level (S_1) and prices ($P_1$and P_2). Initially, by increasing FTL, the demand for the Premium type will increase due to adding attractiveness. On the other hand, in case of higher cost for increasing FTL, some portion of demand for Premium type will be decreased. Thus, finding the optimum equipment level for FTL will result in boosting the total profit.
Furthermore, the advertisement makes the Premium type more appealing to the customers and increases the demand. A trade-off between the yields gained by the advertisement and the cost of the advertisement guarantees maximum profit.

In this research, the above-described model will be investigated through a game theory approach. In the following sections, the model will be examined in two systems, decentralized and centralized. Also, in the decentralized system, two strategies of Stackelberg and Nash will be studied. In a Nash game model, members have equal power with each other, relationships are horizontal, and decision-making among members occurs simultaneously. But in a Stackelberg game model, the relationships of the members are vertical, and at first, it is assumed that the leader has announced his decision and the follower makes a decision based on the leader's decision.

3-1- Decentralized system

Both the manufacturer and retailer tend to maximize their profit regardless of the other party's profit. The retailer decides on the selling prices of two product types ($P_1$and P_2) by the advertisement level of product type 1 (S_1), while the manufacturer determines the FTL of Premium type (L_1). The decentralized models of Stackelberg and Nash games are presented in sections 3-1-1 and 3-1-2, respectively.

3-1-1- Stackelberg strategy

Here, the manufacturer plays the role of a leader and the retailer as a follower. The profit model of the retailer in a decentralized system $\left(\Pi_r^{ds}(P_1,\ P2, S_1)\ \right)$ is as follows (superscription ds represents decentralized considering Stackelberg strategy).

$$\Pi_r^{ds}(P_1, P_2, S_1) = \sum_{i=1}^{2}(P_i - W_i)D_i - \sum_{i=1}^{2} A_i D_i - k\frac{S_1^2}{2} \tag{3}$$

$$\begin{aligned} &s.t \\ &P_i \geq W_i \end{aligned} \tag{4}$$

In equation (3), the first clause indicates the retailer's revenue, the second one states the general and administrative costs for the retailer, and the third one refers to the advertisement cost for the Premium type, where k is a positive coefficient of the advertisement cost.

Theorem 3.1. The decentralized profit function of the retailer $\left(\Pi_r^{ds}(P_1,\ P2, S_1)\right)$ is a concave function if $\max\{12\tau_1\tau_2, 4\beta_1\beta_2\} > (\tau_1 - \tau_2)^2$and $k > (\gamma_1 - \gamma_2)^2$.

Proof: The Hessian matrix of the profit function (see Appendix A) is negative semidefinite if the assumptions hold. Based on the model assumption, the price elasticity coefficient for the product is higher than that of the competitive one ($\beta_i > \tau_i$). The first principal minor determinant of profit function (Π_r^{ds}) is $\left(-2(\beta_1 + \tau_1)\right) < 0$. Besides, the second principal minor determinant of profit function (Π_r^{ds}) is positive $(4\beta_1\beta_2 + 4\beta_1\tau_2 + 4\beta_2\tau_1 - \tau_2^2 + 2\tau_1\tau_2 - \tau_1^2) > 0$ and the third principal minor determinant is negative $(-4\beta_1\beta_2 k - 4\beta_1\tau_2 k - 4\beta_2\tau_1 k + k\tau_2^2 - 2k\tau_1\tau_2 + k\tau_1^2 + 2\beta_2\gamma_1^2 + 2\beta_1\gamma_2^2 + 2\tau_2\gamma_1^2 - 2\tau_2\gamma_1\gamma_2 - 2\tau_1\gamma_1\gamma_2 + 2\tau_1\gamma_2^2) < 0$ due to the model assumption. ■

Based on the concavity of the retailer's profit function, the optimal values of decision variables (P_1, P_2, and S_1) are obtained by setting the first-order partial derivatives of equation (3) equal to zero.

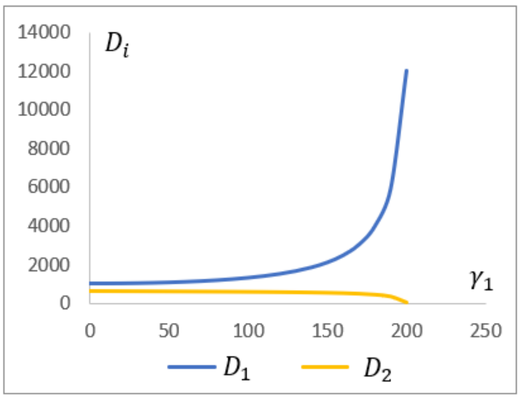
$$P_1^{ds}(L_1) = (T_1 + (k(\tau_1 - \tau_2)(\beta_2 + \tau_2) + (\beta_2 + \tau_2)\gamma_1\gamma_2 - \tau_1\gamma_2^2)A_2)/T_2 \tag{5}$$

$$P_2^{ds}(L_1) = (k\tau_1(a_1 - 2L_1\rho_2) + kL_1\rho_1(\tau_1 + \tau_2) + T_3)/T_2 \tag{6}$$

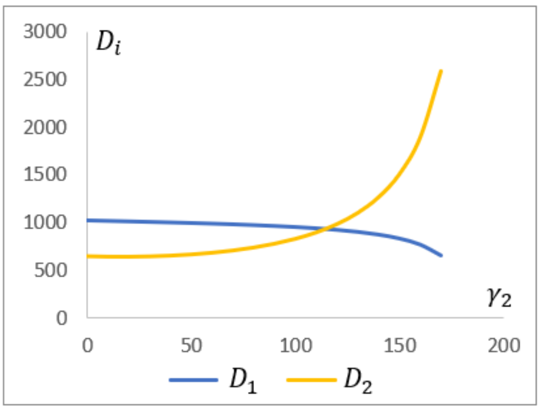
$$S_1^{ds}(L_1) = (-(\gamma_1(-2L_1\rho_1(\beta_2 + \tau_2) + \tau_2(L_1\rho_2 - a_2)) + T_4)/T_2) \tag{7}$$

The values of ($T_1 - T_4$) are provided in Appendix B.

The effect of changes in advertisement elasticity of demand is shown in figure 2. The advertisement elasticity of demand for product type 1(2) has a positive effect on the demand value of product type 1(2), while it decreases the demand of product type 2(1).



(a) Effect of γ_1 on D_i



(b) Effect of γ_2 on D_i

Fig 2. Demands vs. advertisement elasticity of demand

Furthermore, the manufacturer's profit function in a decentralized system ($\Pi_m^{ds}(L_1)$) is shown in equation (8).

$$\Pi_m^{ds}(L_1) = \sum_{i=1}^{2}(W_i - C_i)D_i - h\frac{L_1^2}{2} \tag{8}$$

$$s.t$$
$$W_i \geq C_i \tag{9}$$

In equation (8), the first and second terms indicate the manufacturer's revenue and the cost imposed by FTL, respectively.

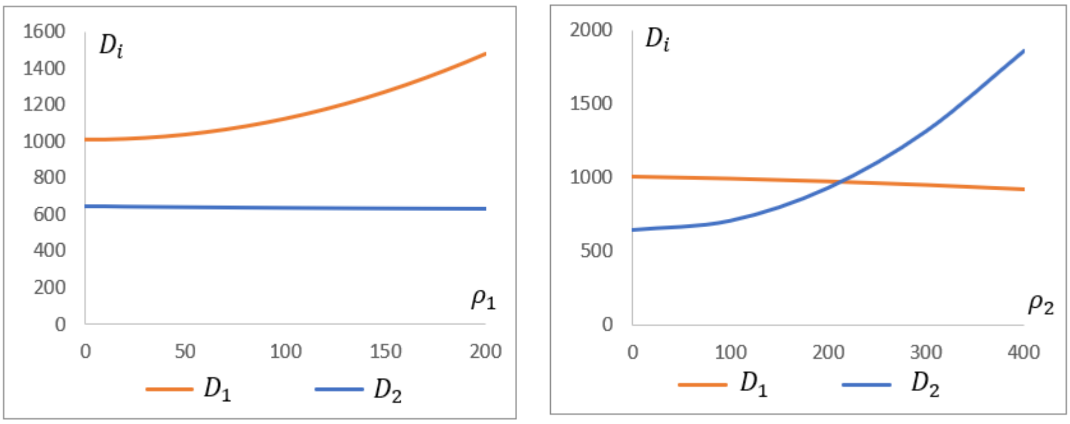
Theorem 3.2. The decentralized profit function of the manufacturer ($\Pi_m^{ds}(L_1)$) is concave concerningL_1.

Proof: By determining the second-order derivative of the profit function, it is obvious that a positive h guarantees the concavity condition. ■

Based on theorem 3.2., the optimum FTL is obtained by setting the first-order derivative of the corresponding profit function equal to zero, which is shown in equation (10). The value of the substitute variables T_2 and T_5 are provided in Appendix B.

$$L_1^{ds} = \Big(T_5 + \big(k(\tau_1 - \tau_2)(\rho_2\tau_1 - \rho_1\tau_2) + (\rho_2\gamma_1 - \rho_1\gamma_2)(\tau_2\gamma_1 - \tau_1\gamma_2)\big)(C_1 - C_2 - W_1 + W_2)\Big) / hT_2 \tag{10}$$

Figure 3 illustrates the effect of changes in feature and technology elasticity of demand versus the demand for both product types. Although an increase in $\rho_1(\rho_2)$ decreases $D_2(D_1)$, it causes a significant increase in $D_1(D_2)$.



(a) Effect of $\rho_1$on D_i (b) Effect of ρ_2 on D_i

Fig 3. Demands vs. feature and technology elasticities of demand

3-1-2-Nash strategy

This strategy deals with a situation in which the manufacturer and the retailer have equal power, and both endeavor to maximize their profits separately. The profit functions of the retailer $\Pi_r^{dn}(P_1,\ P2, S_1)$, and the manufacturer $\Pi_m^{dn}(L_1)$ are the same as in the Stackelberg strategy (equation (3) and equation (8); respectively, for retailer and manufacturer).

Theorem 3.3. The decentralized profit function of the retailer $\left(\Pi_r^{dn}(P_1,\ P2, S_1)\right)$ is concave.

Proof. The proof of theorem 3.3 is similar to the proof of theorem 3.1. ■

Theorem 3.4. The decentralized profit function of the manufacturer ($\Pi_m^{dn}(L_1)$) is concave in L_1.

Proof. The proof of theorem 3.4 is similar to that of theorem 3.2. ■

Based on theorem 3.3, the retailer's profit function is concave. To obtain the optimum values of decision variables (P_1^{dn}, P_2^{dn}, S_1^{dn}, L_1^{dn}), a set of equations was obtained by setting the first-order partial derivatives of $\Pi_r^{dn}(P_1^{dn},\ P_2^{dn},\ S_1^{dn})$ and $\Pi_m^{dn}(L_1)$equal to zero are solved**.**
Equations (11-14) give closed forms of optimal values of decision variables. dn is a superscription to demonstrate a decentralized system considering the Nash strategy.

$$P_1^{dn} = (T_6 + h(k(\tau_1 - \tau_2)(\beta_2 + \tau_2) + (\beta_2 + \tau_2)\gamma_1\gamma_2 - \tau_1\gamma_2^2)A_2)/hT_2 \tag{11}$$

$$P_2^{dn} = (T_7 + a_1 hk(\tau_1 + \tau_2) - C_1 k\rho_1^2(\tau_1 + \tau_2))/hT_2 \tag{12}$$

$$S_1^{dn} = (T_8 + W_2\gamma_1(\tau_1(\beta_2 h + \rho_2^2) + \tau_2(h(\tau_1 - \tau_2 - \beta_2) + \rho_2^2)))/hT_2 \tag{13}$$

$$L_1^{dn} = \frac{-C_1\rho_1 + C_2\rho_2 + \rho_1 W_1 - \rho_2 W_2}{h} \tag{14}$$

Substitute variables T_2 and T_6 to T_8 (see appendix B) are used to summarize the equations.

3-2-Centralized system

The contrary to what we assumed in a decentralized system, here, both manufacturer and retailer cooperate to gain maximum total profit. The profit function in the centralized system $\Pi^c(P_1,\ P2, L_1,\ S_1)$ is as shown in equation (15).

$$\Pi^c(P_1, P_2, L_1, S_1) = \sum_{i=1}^{2}(P_i - C_i)D_i - \sum_{i=1}^{2}A_i D_i - k\frac{L_1^2}{2} - h\frac{S_1^2}{2} \tag{15}$$

$$s.t$$
$$P_i \geq W_i \geq C_i \tag{16}$$

In equation (15), the first and second terms represent income and general and administrative costs, while the third and fourth ones refer to FTL and advertisement cost for Premium type, respectively.

Theorem 3.5. The centralized profit function $\left(\Pi^c(P_1,\ P2, S_1,\ L_1)\right)$ is a concave function provided that $\max\{12\tau_1\tau_2, 4\beta_1\beta_2\} > (\tau_1 - \tau_2)^2,\ k > (\gamma_1 - \gamma_2)^2$, and $hk > h + k$.

Proof: The Hessian matrix of the profit function is negative semidefinite if the assumptions hold (see Appendix C). The three first principal minor determinants of the profit function (Π^c) is the same as what we have in theorem 3.1. Furthermore, the fourth minor determinant is positive $\Big((4\beta_1\beta_2 hk + 4\beta_1 hk\tau_2 - 2\beta_1 h\gamma_2^2 - 2\beta_1 k\rho_2^2 + 4\beta_2 hk\tau_1 - 2\beta_2 h\gamma_1^2 - 2\beta_2 k\rho_1^2 - hk\tau_2^2 + 2hk\tau_1\tau_2 - hk\tau_1^2 - 2h\tau_2\gamma_1^2 + 2h\tau_2\gamma_1\gamma_2 + 2h\tau_1\gamma_1\gamma_2 - 2h\tau_1\gamma_2^2 - 2k\tau_1\rho_2^2 + 2k\rho_1\rho_2\tau_2 + 2k\rho_1\rho_2\tau_1 - 2k\rho_1^2\tau_2 + \gamma_1^2\rho_2^2 - 2\rho_1\rho_2\gamma_1\gamma_2 + \rho_1^2\gamma_2^2) > 0\Big)$ due to the aforementioned assumptions. ■

The optimal values of decision variables can be obtained by setting the first-order partial derivatives of the profit function equal to zero. Equations (17-20) give closed forms of optimal values of decision variables. Here, superscription c refers to a centralized system.

$$P_1^c = \Big(T_{11} + hk\tau_2\big(a_2 + C_1\tau_1 + C_2(\tau_1 - \tau_2 - \beta_2)\big)\Big)/(hkT_9 + T_{10}) \tag{17}$$

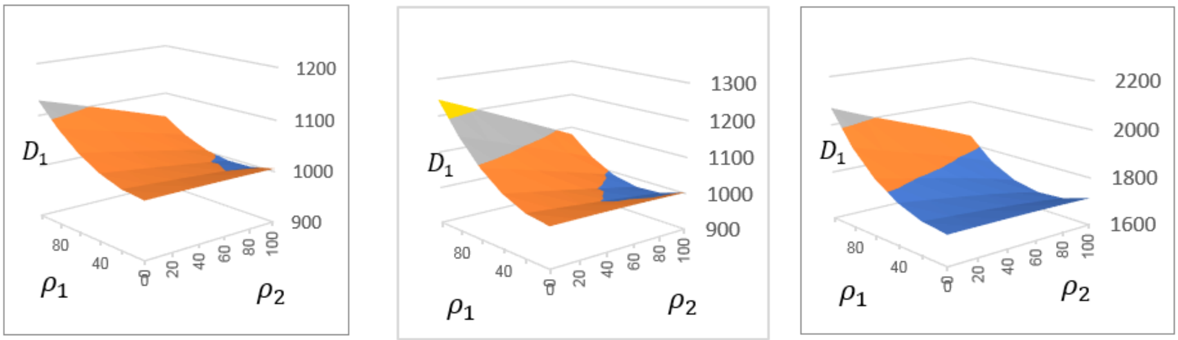
$$P_2^c = \Big(T_{12} + \rho_1\rho_2\big(k(\tau_1(C_2 + C_1) + 2C_2\tau_2 - a_1)\big)\Big)/(hkT_9 + T_{10}) \tag{18}$$

$$S_1^c = \left(T_{13} + a_1(\gamma_1(2h(\beta_2 + \tau_2) - \rho_2^2) - h(\tau_1 + \tau_2)\gamma_2)\right)/(hkT_9 + T_{10}) \quad (19)$$

$$L_1^c = \left(T_{14} + k\rho_1\tau_2(\beta_2 C_2 - a_2 + \rho_1\tau_2(C_2 - C_1))\right)/(hkT_9 + T_{10}) \quad (20)$$

Variables $(T_9 - T_{14})$ are provided in appendix B.

Demand for product type 1 (D_1) as a function of feature as well as technology elasticities of demand (ρ_1, ρ_2) for three different scenarios are illustrated in figure 4.



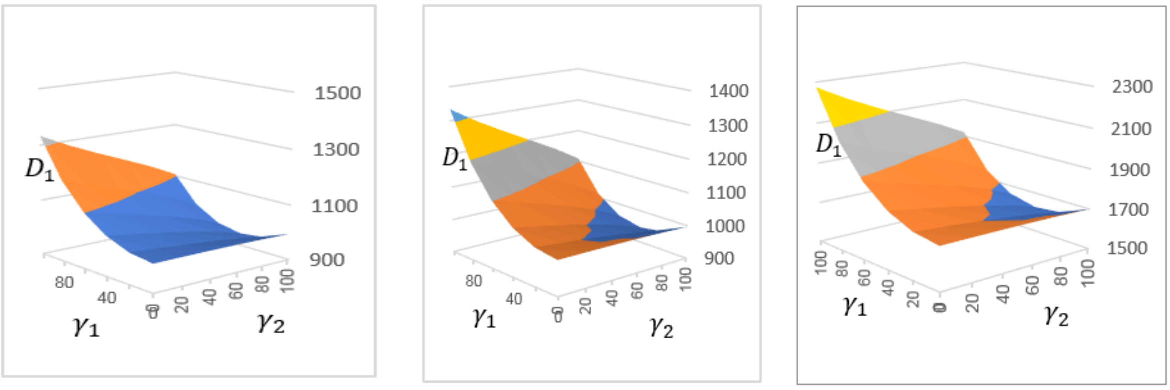
(a) Demand sensitivity to (ρ_1, ρ_2) in the Stackelberg scenario

(b) Demand sensitivity to (ρ_1, ρ_2) in Nash scenario

(c) Demand sensitivity to (ρ_1, ρ_2) in the centralized model

Fig 4. Demand for product type 1 (D_1) vs. feature and technology elasticities (ρ_1, ρ_2).

Demand for product type 1 (D_1) is illustrated against advertisement elasticities (γ_1, γ_1) for three different scenarios in figure 5.



(a) Demand for product type 1 vs. advertisement elasticities (Stackelberg scenario)

(b) Demand for product type 1 vs. advertisement elasticities (Nash scenario)

(c) Demand for product type 1 vs. advertisement elasticities (centralized model)

Fig 5. Demand for product type 1 (D_1) vs. advertisement elasticities (γ_1, γ_2)

4- Government intervention in the automotive supply chain (Stackelberg strategy)

Government intervention in the automotive industry is employed to support the manufacturer or pursuit some other strategic goals. Here, we study the situation in which the government gives subsidies to the manufacturer to encourage it to develop both product types. On the other hand, the government receives some portion of the manufacturer's income as tax. Government plays the role of leader, and the manufacturer is known as a follower, i.e., the manufacturer/follower reacts to the government/leader's strategies. Moreover, the manufacturer has its leadership over the retailer.

As a result, the retailer decides on the sale price of the Premium and Basic types (P_1 and P_2; respectively) and the advertisement level of the Premium one (S_1). The manufacturer determines FTL (L_1), while the government decides on the intervention rate (δ). The government contribution to costs and revenues is %100 δ and $\%100(1-\delta)$, respectively. The total profit function for the retailer and manufacturer are as shown in equation (3) and equation (21), respectively, while equation (23) shows indicates the government utility. Superscription ds^* refers to the decentralized system considering Stackelberg with government intervention, and h is a positive coefficient.

$$\Pi_m^{dS^*}(L_1) = \sum_{i=1}^{2}(\delta W_i - (1-\delta)C_i)D_i - (1-\delta)h\frac{L_1^2}{2} \tag{21}$$

$$\begin{aligned} &s.t \\ &W_i \geq C_i \end{aligned} \tag{22}$$

$$\Pi_g^{dS^*}(\delta) = \theta_1\left[\sum_{i=1}^{2}((1-\delta)W_i - \delta C_i)D_i - \delta h\frac{L_1^2}{2}\right] + \theta_2[h' L_1] \tag{23}$$

In equation (21), the first terms indicate the portion of income and costs that the manufacturer should pay as income tax and also the subsidies that are received from the government. The second term shows the portion of costs related to FTL. In equation (23), both economic and social utilities of government are considered.

Theorem 4.1. The decentralized profit function of the manufacturer ($\Pi_m^{dS^*}(L_1)$) is concave in L_1.

Proof. Given that the second-order derivative of the profit function is non-negative, it is simply proved. ■

Based on theorem 4.1, the optimal values of P_1, P_2, S_1, and L_1 are calculated as shown in equations (24-27).

$$P_1^{gs} = \left(T_{16} + k\tau_1\left(a_2(\tau_2+1) - L_1\rho_2 + \tau_2(W_1+W_2)\right)\right)/(kT_9 - T_{15} - 4\tau_1\gamma_2^2) \tag{24}$$

$$P_2^{gs} = \left(T_{17} + k(\tau_1(2\beta_2 W_2 - \tau_1 W_1) + \beta_1)\right)/(kT_9 - T_{15} - 4\tau_1\gamma_2^2) \tag{25}$$

$$S_1^{gs} = \left(T_{18} + a_1\gamma_2(\tau_1(\beta_1 + \tau_1 - 1) - (1+\beta_1+\tau_1)\tau_2)\right)/(kT_9 - T_{15} - 4\tau_1\gamma_2^2) \tag{26}$$

$$\begin{aligned} L_1^{gs} = \Big(&\left(C_2 + C_1(\delta-1) - \delta(C_2 - W_1 + W_2)\right)\left(k(\tau_1-\tau_2)(\rho_2\tau_1 - \rho_1\tau_2)\right. \\ &\left.+ (\rho_2\gamma_1 - \rho_1\gamma_2)(\tau_2\gamma_1 - \tau_1\gamma_2)\right) + T_{19}\Big)/(h(\delta-1)T_2) \end{aligned} \tag{27}$$

The optimal values of P_1, P_2, and S_1 are determined as a function of L_1. The values of the substitute variables ($T_{15} - T_{19}$) are provided in appendix B.

5-Case Study

In this section, we give some examples based on an Iranian automotive company [SaipaCorp.com]. Also, a sensitivity analysis of important parameters is implemented.

5-1-Numerical examples of models without government intervention

To illustrate the results of the proposed automotive supply chain model, ten numerical examples are presented in this section. The data is based on the information provided by the interviewed Iranian automotive experts [SaipaCorp.com], see table 2. The market potential of vehicles with market price of around 10,000 USD up to 15,000 USD is about 2,000 units to 10,000 units in Iran's automotive market ($a_1 = 5000, a_2 = 3000$) (Zaefarian et al., 2018, www.oica.net, www.mimt.gov.ir). Furthermore, the production cost of products for the market price of around 10,000 USD up to 15,000 USD is about 8,000 USD up to 11,000 USD ($C_1 = 90, C_2 = 80$) (Fander and Yaghoubi, 2021). The others are conducted based on Iranian automotive experts as follows [SaipaCorp.com]: $h = 3000$, $k = 2000, W_1 = 230, W_2 = 180, A_1 = 60$, and $A_2 = 50$.

In example #1, to prove the concavity of retailer profit function (Π_r^{ds}) in the decentralized model, the following conditions of the Hessian matrix exists.

$$H = \begin{bmatrix} -22 & 2.5 & 25 \\ 2.5 & -19 & -15 \\ 25 & -15 & -2000 \end{bmatrix}$$

First, all arrays of the main diagonal have negative values. Hence, the ability to move between the main variables has existed. Second, the first minor has a negative value ($a_1 = -22 < 0$), the second minor determinant has a positive value (a_2=411.75>0), and the third minor determinant also has a negative value ($a_3 = -808{,}550 < 0$). Hence, the concavity of the retailer profit function (Π^r) is proved. Furthermore, to prove the concavity of the manufacturer profit function (Π_m^{ds}), the second derivative of the manufacturer profit function than the decision variable is lower than zero ($-h = -3{,}000 < 0$).

On the other hand, to prove the concavity of manufacturer profit function (Π_m^{ds}) in the decentralized model in Example #1, the following conditions of the Hessian matrix exists.

$$H = \begin{bmatrix} -22 & 2.5 & 25 & 15 \\ 2.5 & -19 & -15 & -8 \\ 25 & -15 & -2000 & 0 \\ 15 & -8 & 0 & -3000 \end{bmatrix}$$

First, all arrays of the main diagonal have negative values. Second, (9), (10), and (24), respectively, the first minor of the Hessian matrix has a negative value ($a_1 = -22 < 0$), the second minor determinant has a positive value (a_2=411.75>0), the third minor determinant has a negative value ($a_3 = -808{,}550 < 0$) and also the fourth minor determinant has a positive value ($a_4 = 24.155 \times 10^8 > 0$). Therefore, the concavity of profit function (Π) is proved.

Table 3. Parameter values for the numerical experiments (100 USD)

No.	β_1	β_2	τ_1	τ_2	γ_1	γ_2	ρ_1	ρ_2
1	10	8	1	1.5	25	15	15	8
2	8	9	2	1	22	16	16	7
3	9	12	5	3	20	14	12	10
4	10	8	3	2	30	13	20	5
5	10	9	3	4	25	10	10	9
6	12	11	4	3	32	20	12	7
7	9	7	6	1	16	10	15	10
8	15	14	7	5	10	9	12	11
9	12	7	6	4	11	5	17	13
10	14	12	3	1	20	15	14	11

The obtained value of decision variables in the Nash strategy in example 1 is P_1=392.742, P_2=308.577, S_1=0.605, and L_1=0.433. The total profit function (Π=Π_r+Π_m) is 359,918.107. For the

Stackelberg strategy of the decentralized model, the value of decision variables are P_1=392.608, P_2=308.647, S_1=0.693, and L_1=0.230, and the value of the total profit function is 359,794.756. Moreover, for the centralized model, obtained decision variables are P_1=324.317, P_2=256.477, S_1=1.230, and L_1=0.534, with the total profit function value of Π=429,155.520. The results represent that the total profit value for the centralized model is higher than that in the decentralized models (both Stackelberg and Nash strategies).

Table 4 illustrates the results of the numerical examples without government intervention, including the values of decision variables, the values of demand functions, and also profit functions.

Table 4. The results of the numerical examples before government intervention

No.			P_1	P_2	S_1	L_1	D_1	D_2	Π_r	Π_m	Π
1	Decentralized	Nash	392.7	308.6	0.7	0.4	1012.3	643.7	154104.8	205813.3	359918.1
		Stackelberg	392.6	308.6	0.7	0.2	1010.7	644.5	153919.3	205875.5	359794.8
	Centralized		324.3	256.5	1.2	0.5	1727.8	1027.2	-	-	429155.5
2	Decentralized	Nash	429.9	299.5	1.0	0.5	1329.7	415.4	213961.0	227297.9	441258.9
		Stackelberg	429.7	299.6	1.0	0.2	1327.5	416.5	213483.8	227409.4	440893.2
	Centralized		358.5	252.4	1.3	0.8	1962.3	807.4	-	-	505172.9
3	Decentralized	Nash	375.6	266.5	0.6	0.2	1088.8	119.1	97173.1	164267.1	261440.2
		Stackelberg	375.5	266.5	0.6	0.1	1088.1	119.8	97083.5	164294.6	261378.1
	Centralized		303.4	220.4	0.9	0.3	1876.3	588.7	-	-	340043.1
4	Decentralized	Nash	382.0	316.3	0.8	0.8	1023.1	586.8	144043.8	201027.6	345071.4
		Stackelberg	381.7	316.3	0.8	0.4	1019.1	587.9	143493.2	201257.9	344751.1
	Centralized		311.5	269.2	1.5	0.8	1820.8	907.3	-	-	416902.0
5	Decentralized	Nash	383.5	299.8	0.8	0.2	936.2	626.9	130607.5	193713.2	324320.7
		Stackelberg	383.5	299.8	0.8	0.1	935.8	627.2	130586.1	193720.5	324306.6
	Centralized		315.4	247.4	15	0.2	1680.8	1029.4	-	-	396568.1
6	Decentralized	Nash	338.2	264.9	0.4	0.3	666.2	295.6	42219.7	122655.9	164875.6
		Stackelberg	338.1	264.9	0.4	0.2	665.1	296.2	42163.0	122699.2	164862.2
	Centralized		267.1	216.8	1.0	0.3	1621.2	745.1	-	-	254229.3
7	Decentralized	Nash	389.1	363.7	0.1	0.4	1352.9	474.8	197538.3	236687.4	434225.7
		Stackelberg	389.0	363.8	0.1	0.1	1351.2	476.4	197498.5	236795.4	434293.9
	Centralized		315.5	334.0	0.3	0.1	2278.5	639.0	-	-	507332.3
8	Decentralized	Nash	296.4	234.1	0.0	0.2	120.3	32.5	900.7	20033.3	20943.0
		Stackelberg	296.4	234.1	0.0	0.1	119.6	33.1	897.1	20052.8	20949.9
	Centralized		225.0	187.3	0.1	0.1	1363.4	564.0	-	-	134557.3
9	Decentralized	Nash	350.6	331.4	0.1	0.4	684.9	751.9	117725.1	170878.9	288603.9
		Stackelberg	350.5	331.5	0.1	0.1	683.2	753.3	117786.2	170945.9	288732.1
	Centralized		279.4	287.3	0.3	0.1	1699.5	954.5	-	-	369970.0
10	Decentralized	Nash	314.3	245.1	0.1	0.3	398.4	123.1	11530.0	67964.1	79494.1
		Stackelberg	314.3	245.1	0.1	0.1	397.2	124.1	11500.2	68008.3	79508.5
	Centralized		242.1	200.0	0.4	0.2	1495.2	634.0	-	-	181852.8

5-2- Numerical examples of models with government intervention

In this section, ten numerical examples are employed to represent the results of government intervention in the automotive supply chain. The parameters of these examples are the same as what we have in table 2. The coefficients of $\delta, h', \theta_1, and\ \theta_2$ for ten examples are equal to what is illustrated in table 4. These values are also obtained based on the information provided by Iranian experts.

Table 5. Parameter values for the numerical examples with government intervention

No.	δ	h'	θ_1	θ_2
1	0.75	70,000	0.4	1.0
2	0.75	70,000	0.4	1.0
3	0.75	70,000	0.4	1.0
4	0.80	60,000	0.5	0.8
5	0.80	60,000	0.5	0.8
6	0.80	60,000	0.5	0.8
7	0.80	60,000	0.5	0.8
8	0.85	80,000	0.6	0.9
9	0.85	80,000	0.6	0.9
10	0.85	80,000	0.6	0.9

Moreover, table 6 represents the numerical results of ten examples in presence of government intervention.

Table 6. The results of the numerical examples after government intervention

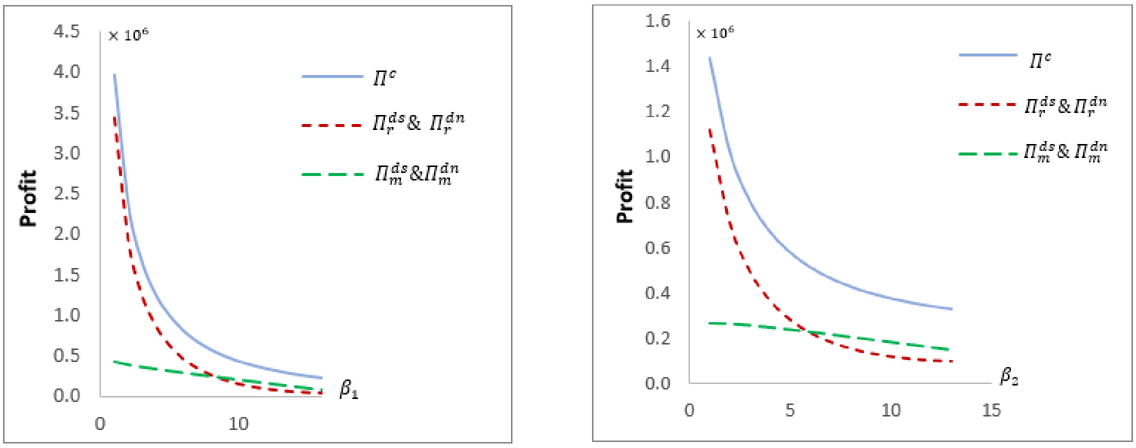
No.	P_1	P_2	S_1	L_1	D_1	D_2	$\Pi_r^{dS^*}$	$\Pi_m^{dS^*}$	Π	$\Pi_g^{dS^*}$
1	393.1	308.4	0.700	0.943	1016.2	641.7	154572.0	225901.2	380473.1	57727.3
2	430.3	299.4	0.988	0.990	1333.5	413.5	214797.5	247207.0	462004.5	61035.6
3	375.6	266.4	0.601	0.335	1089.5	118.5	97244.7	177003.7	274248.4	18308.2
4	383.1	316.2	0.836	2.175	1037.4	582.6	146042.8	245358.0	391400.8	79930.2
5	383.6	299.7	0.821	0.506	937.9	625.4	130712.0	235675.0	366386.9	3200.9
6	338.4	264.8	0.426	0.870	669.4	293.5	42401.8	148467.7	190869.5	28479.5
7	389.2	363.6	0.125	0.488	1353.7	474.0	197556.6	285323.3	482879.9	-935.6
8	296.5	234.0	0.014	0.548	122.3	30.5	912.4	26493.3	27405.7	35386.7
9	350.9	331.1	0.082	1.120	691.0	746.7	117509.2	230766.8	348276.0	43928.3
10	314.5	244.9	0.134	0.853	402.2	119.8	11630.3	89921.6	101551.9	47757.7

5-3- Sensitivity analysis

Here, the sensitivity analysis of the parameters of the model is performed. We employ example 1 to analyze the impact of the change in parameters on the optimal solutions.

Figure 6(a) illustrates the changes in the profit functions when β_1 changes. The chart represents that an increase in β_1 decreases the amount of profit. Moreover, the profit functions of the retailer are more sensitive than those of the manufacturer concerning changes in β_1. Furthermore, the profit functions of the retailer and the manufacturer for the centralized/decentralized model and both strategies of Nash and Stackelberg are illustrated in figure 6(b) for β_2. The same explanations as given for figure 6(a) will be applied here. The main finding of this figure is as follows:

- The sensitivity to the price is very high in this model, and this issue can be deduced from figure 6. Therefore, in order to increase profitability and reduce the sensitivity of the model to the price, an optimal balance point must be created between the price and the level of technology.

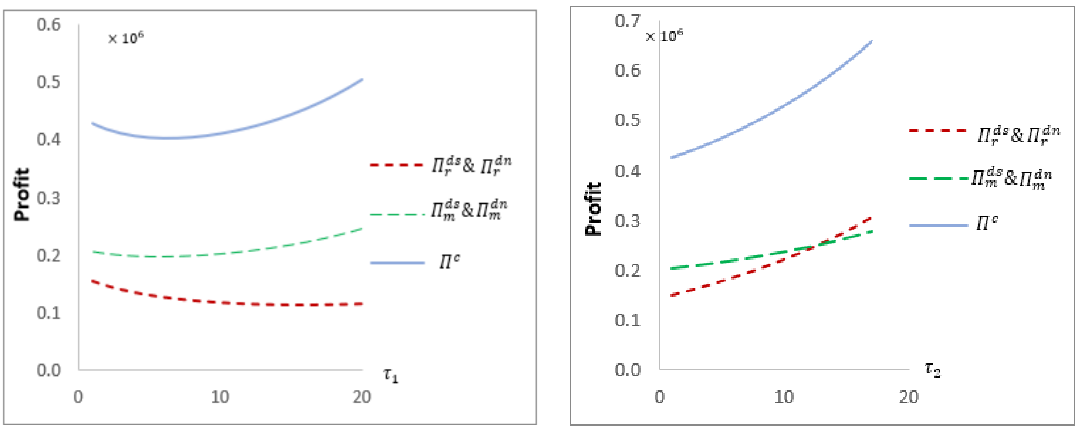


(a) concerning β_1 (a) concerning β_2

Fig 6. Sensitivity analysis of profit functions for β_i

The results of a sensitivity analysis to show the impact of competitive price coefficient (τ_i) on the profit functions are displayed in Fig. 7. The profit functions of the retailer and the manufacturer for the centralized/decentralized model (both of the Nash and Stackelberg strategies) for τ_1 are illustrated in figure 7(a). The trend for both Nash and Stackelberg's strategies is highly similar to each other. The profit of the manufacturer increases when the competitive price coefficient for the Premium type increases. Conversely, retailer's profit decreases when τ_1 increases. Figure 7(b) illustrates the results of a sensitivity analysis of profit functions for the centralized/decentralized model (both of the Nash and Stackelberg strategies) for τ_2. When the competitive price coefficient for the Basic type increases, the profit of both the manufacturer and the retailer increases. The significant insight can be derived from figure 7 as follows:

- Considering that the price of premium cars is higher than basic cars, by increasing the sensitivity of the competitive price, it is possible to reduce the demand and profit for premium cars. Therefore, in order to prevent the decrease in demand, the technology level of the premium car should be increased in proportion to its price.



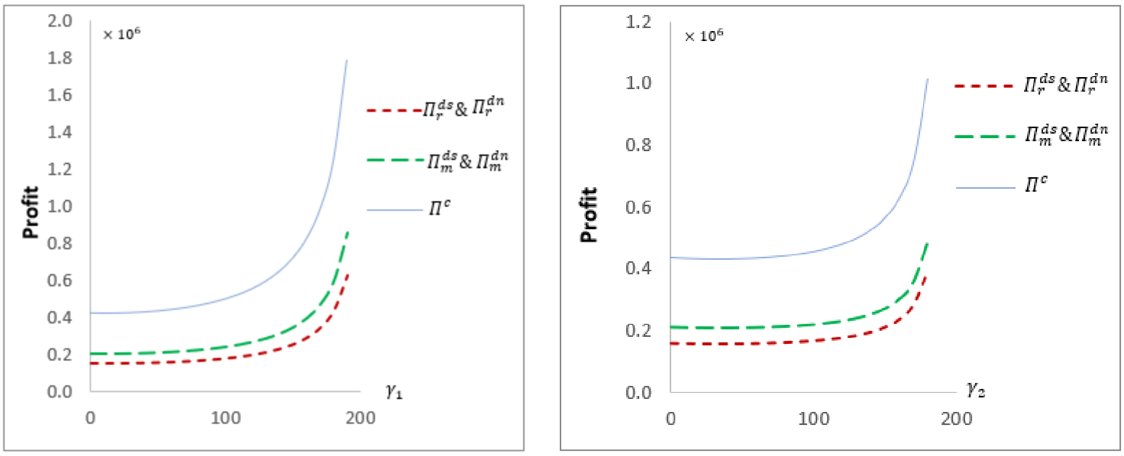
(a) Sensitivity analysis concerning τ_1 (b) Sensitivity analysis concerning τ_2

Fig 7. Sensitivity analysis of profit functions for τ_i

The sensitivity analysis of profit functions for the advertisement elasticity of demand for product types (γ_i) is displayed in Fig. 8. The profit functions of the retailer and the manufacturer for the centralized model and decentralized model (both of the Nash and Stackelberg strategy) for γ_1 are illustrated in figure 8(a). For two strategies (Nash/Stackelberg), figure 8(a) displays that an increase in the advertisement elasticity of demand (γ_1) will increase profit amounts. The graph shows that when

γ_1 is less than 150, the increase in the profit functions is slight, although, for higher amounts of γ_1, the profit functions rise sharply. Moreover, the profit functions of the retailer and the manufacturer for the centralized model and decentralized model (both of the Nash and Stackelberg strategies) for γ_2 are illustrated in figure 8(b). Results similar to those given for figure 8(b) will be applied here. Some managerial findings can be considered in figure 8 as follows:

- As can be expected, advertising will have a positive effect on sales and, subsequently, profits. But it should be noted that the advertising costs do not exceed the profit and do not cause the organization to suffer.

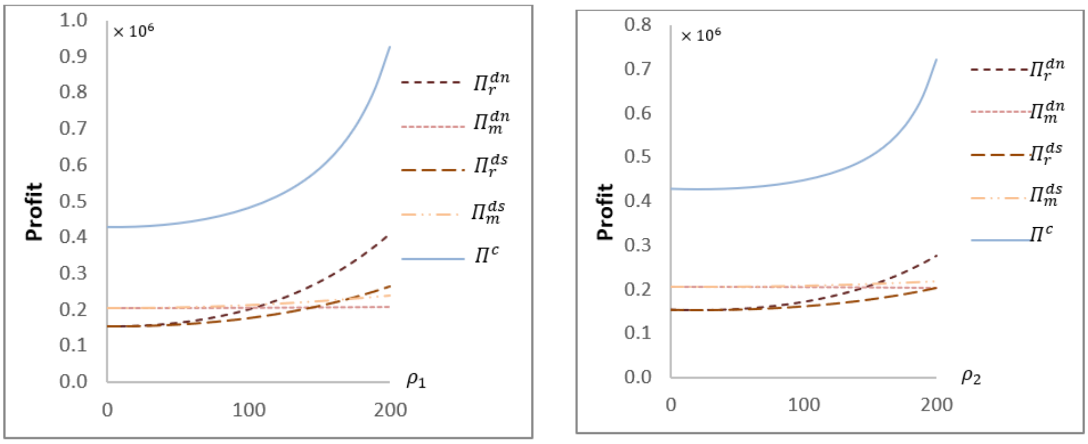


(a) Sensitivity analysis concerning γ_1 (b) Sensitivity analysis concerning γ_2

Fig 8. Sensitivity analysis of profit functions for γ_i

Feature and technology level also affects demand and profit amounts. A sensitivity analysis of profit amounts for FTL elasticity of demand (ρ_i) is shown in figure 9. The profit functions of the retailer and the manufacturer for the centralized model and decentralized model (Nash/Stackelberg strategy) for ρ_1 are illustrated in figure 9(a). The results represented in figure 9(a) show that the trend of changes in retailer's profit for the centralized model (Nash/Stackelberg strategy) is increasing, while this trend for manufacturer's profit is smoother. Furthermore, the profit functions of the retailer and the manufacturer for the centralized model and decentralized model (Nash/Stackelberg strategy) for ρ_2 are illustrated in Fig. 9(b). Interpretation similar to what given for ρ_1 applies to ρ_2. The main finding of this figure 9 is as follows:

- Since FTL is a positive factor in demand, by increasing the sensitivity of this variable, the amount of demand and thus the profit of members increases. More importantly, if FTL sensitivity is significantly greater than price sensitivity, all demand for basic cars will shift toward premium cars.



(a) Sensitivity analysis concerning ρ_1 (b) Sensitivity analysis concerning ρ_2

Fig 9. Sensitivity analysis of profit functions for ρ_i

One of the critical factors in the proposed model is the government intervention rate. Fig. 10 represents the impact of this factor on the amounts of profit functions. The chart illustrates the changes in the amounts of profit functions when the government rate of intervention (δ) changes from zero to 0.95. The graph associated with the total profit earned by both manufacturer and the retailer ($\Pi_r^{dS^*} + \Pi_m^{dS^*}$) increases linearly when the rate of intervention increases. The government utility decreases slowly to about 0.5, but for the rate of interventions greater than about 0.7, it rises dramatically. The graph illustrated that before $\delta = 0.3$, the government utility is higher than the total profit. However, for $\delta \geq 0.3$, the total profit is higher than the government utility, i.e., the utility earned by the government has a concave shape, and once it reaches its minimum, it starts to grow dramatically. Some managerial findings can be considered in figure 10 as follows:

- As it is evident in the figure, the government is willing to cooperate when the level of its involvement in the decisions and profit and loss of the car supply chain increases from a basic level, and the government will not enter into this cooperation until then.

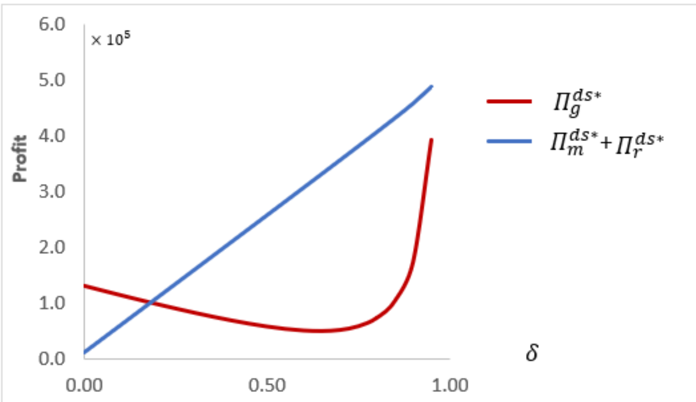


Fig 10. Sensitivity analysis of profit functions with respect to δ

6- Managerial insights

- Even though the higher price elasticity of both product types results in lower profit values of both manufacturer and retailer, the retailer seems to be affected more than the manufacturer.
- Having a higher price elasticity of the Basic type will cause higher profitability for both manufacturer and retailer than having higher price elasticity of the Premium type.

- A Higher FTL coefficient leads to higher profit amounts of the manufacturer and retailer. This increase is higher for the retailer than the manufacturer.
- Regarding the relationship between government utilities and the supply chain profitability, It is intuitively perceptible that total profit increments when the government intervention rate increases. As the government intervention causes lower prices, the amounts of demands increase, which leads to higher total profit, which in turn increases the government utility.
- The economic and social utilities of government have opposite trends. Having low government intervention, the economic utility of the government will be higher than its social utility. Moreover, with a high government intervention rate, the social utility of the government will be much higher than its economic utility.
- A low advertisement level has a very low impact on the profit amounts of the manufacturer and the retailer, while a high amount of advertisement has a considerable impact on the profit amounts.

7- Conclusions

The automotive industry is one of the most competitive industries in the world, where optimal pricing is a critical factor of success in this industry. In recent years, new pricing methodologies are employed for different products.

This research has considered an automotive supply chain, consisting of one manufacturer, and one retailer, with two types of one product (Basic and Premium). The differences between the two product types are FTL (feature and technology level), and the advertisement, where only the Premium type enjoys them. Depending on the pricing of the Premium type, the customer class 1 decides to buy it or switch to the Basic type, and vice versa. Furthermore, supposing an oligopolistic market automotive, the government intervenes on the manufacturer by giving subsidies and receiving taxation. To solve the model, a game theory approach is developed based on centralized and decentralized (Nash and Stackelberg) models. The results illustrate that by applying government intervention, the government will earn both economic and social utilities, and both the retailer and the manufacturer will increase their profit.

This study can be reinforced by considering non-deterministic demand or non-deterministic production rates. Furthermore, a generalization in the number of players can be regarded as an improvement in the model. Moreover, government intervention could be supposed to be dynamic.

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Appendix A

Hessian matrix (H) for theorem 3.1 is as follows:

$$H(P_1, P_2, S_1) = \begin{bmatrix} \frac{\partial^2 \pi^r}{\partial P_1^2} & \frac{\partial^2 \pi^r}{\partial P_1 \partial P_2} & \frac{\partial^2 \pi^r}{\partial P_1 \partial S_1} \\ \frac{\partial^2 \pi^r}{\partial P_2 \partial P_1} & \frac{\partial^2 \pi^r}{\partial P_2^2} & \frac{\partial^2 \pi^r}{\partial P_2 \partial S_1} \\ \frac{\partial^2 \pi^r}{\partial S_1 \partial P_1} & \frac{\partial^2 \pi^r}{\partial S_1 \partial P_2} & \frac{\partial^2 \pi^r}{\partial S_1^2} \end{bmatrix} = \begin{bmatrix} -2(\beta_1 + \tau_1) & \tau_1 + \tau_2 & \gamma_1 \\ \tau_1 + \tau_2 & -2(\beta_2 + \tau_2) & -\gamma_2 \\ \gamma_1 & -\gamma_2 & -\kappa \end{bmatrix}$$

Appendix B

$$\begin{aligned} \boldsymbol{T_1} = \Big(&\boldsymbol{k}\Big(\boldsymbol{2L_1\rho_1(\beta_2 + 2\tau_2) + \tau_1(a_2 - L_1\rho_2) + \tau_2(a_2 - L_1\rho_2) + 2a_1(\beta_2 + \tau_2)} \\ &\boldsymbol{+ 2\beta_1\beta_2(W_1 + A_1) + W_1\big(2\beta_2\tau_1 - \tau_1^2 + 2\beta_1\tau_2\big)} \\ &\boldsymbol{+ \tau_1\tau_2(W_1 + W_2 + A_1) + W_2\big(\beta_2(\tau_1 - \tau_2) - \tau_2^2\big)} \\ &\boldsymbol{+ A_1\big(2\beta_2\tau_1 - \tau_1^2 + 2\beta_1\tau_2\big)\Big)} \\ &\boldsymbol{- \gamma_2^2\big((a_1 + L_1\rho_1) - W_1(\beta_1 + 2(\beta_2 + \tau_2) + \tau_1) - \tau_1 W_2\big)} \\ &\boldsymbol{+ -2\gamma_1^2 A_1(\beta_2 + \tau_2)} \\ &\boldsymbol{+ \gamma_1\gamma_2(A_1(2\tau_1 + \tau_2) + W_1(2\tau_1 + \tau_2) + W_2(\beta_2 + \tau_2) - a_2} \\ &\boldsymbol{+ L_1\rho_2) - \gamma_2^2 A_1(\beta_1 + \tau_1)\Big)} \end{aligned} \tag{A.1}$$

$$\begin{aligned} \boldsymbol{T_2} = \big(&\boldsymbol{4\beta_1\beta_2 k + 4\beta_2 k\tau_1 - k\tau_1^2 + 4\beta_1 k\tau_2 + 2k\tau_1\tau_2 - k\tau_2^2 - 2\beta_2\gamma_1^2 - 2\tau_2\gamma_1^2} \\ &\boldsymbol{+ 2(\tau_1 + \tau_2)\gamma_1\gamma_2 - 2(\beta_1 + \tau_1)\gamma_2^2\big)} \end{aligned} \tag{A.2}$$

$$\begin{aligned} \boldsymbol{T_3} = \Big(&\boldsymbol{2a_2 k(\beta_1 + \tau_1) + a_1 k\tau_2 - k\tau_1^2(W_1 + A_1) + k\tau_1\tau_2(W_1 + W_2 + A_1)} \\ &\boldsymbol{+ \gamma_1^2\big(L_1\rho_2 - a_2 - \tau_2(W_1 - A_1) - W_2(\beta_2 - \tau_2)\big)} \\ &\boldsymbol{+ \gamma_1\gamma_2\big(\tau_1(A_1 + W_1) + W_2\big(\tau_1\big(1 - 2\gamma_2^2 + 2\beta_2 k\big) + 2\tau_2\big) - a_1} \\ &\boldsymbol{- L_1\rho_1 - k\tau_2^2\big) + \Big(k(\tau_1 - \tau_2)\tau_2 + \beta_2\big(2k\tau_1 - \gamma_1^2\big)\Big)A_2} \\ &\boldsymbol{- (\gamma_1 - 2\gamma_2)(\tau_2\gamma_1 - \tau_1\gamma_2)A_2} \\ &\boldsymbol{+ \beta_1\gamma_2\big(\gamma_1(W_1 + A_1) - 2\gamma_2(W_2 + A_2)\big)} \\ &\boldsymbol{+ \beta_1 k\big(-2L_1\rho_2 - \tau_1(W_1 + A_1) + 2\beta_2(W_2 + A_2)} \\ &\boldsymbol{+ \tau_2(W_1 + 2W_2 + A_1 + 2A_2)\big)\Big)} \end{aligned} \tag{A.3}$$

$$
\begin{aligned}
T_4 = \tau_1\gamma_1 \Big(& (L_1\rho_2 - a_2 + 2\beta_2 A_1) - 2a_1(\beta_2+\tau_2)\gamma_1 \\
& + \beta_1\beta_2(2W_1\gamma_1 - 2W_2\gamma_2 + 2\gamma_1 A_1) + W_1\gamma_1(2\beta_2\tau_1 + 2\beta_1\tau_2 - \tau_2^2) \\
& + W_2\gamma_1(\beta_2(\tau_2-\tau_1) + \tau_2^2) + \tau_1\gamma_2(2a_2 + L_1(\rho_1 - 2\rho_2) - \beta_1 A_1) \\
& + \big(a_1(\tau_1+\tau_2) + L_1\rho_1\tau_2 + 2\beta_1(a_2 - L_1\rho_2)\big)\gamma_2 \\
& + W_1\gamma_2(\beta_1\tau_2 - \beta_1\tau_1 - \tau_1^2) - W_2\gamma_2 - W_2\gamma_1 + \gamma_2 A_1 + \gamma_1 A_1) \\
& + W_2\gamma_2(\tau_1^2 - 2\beta_2\tau_1 - 2\beta_1\tau_2) \\
& + A_1\Big(\gamma_1\big(2\beta_1(1-\tau_2)\big) + \gamma_2(\beta_1\tau_2 - \tau_1^2)\Big) \\
& - \big((\tau_1-\tau_2)(\beta_2+\tau_2)\gamma_1 + 2\beta_1(\beta_2+\tau_2)\gamma_2 \\
& + \tau_1(2\beta_2 - \tau_1 + \tau_2)\gamma_2\big)A_2 + \tau_1\tau_2(W_1(\gamma_2+\gamma_1))\Big)
\end{aligned}
\tag{A.4}
$$

$$
\begin{aligned}
T_5 = \beta_1 \Big(& \big(k\big(\rho_2\tau_1(C_1 - W_1) + \rho_2\tau_2(2C_2 + W_1 - 2W_2) - C_1(2\rho_1+\rho_2)\tau_2 \\
& + 2\rho_1\tau_2 W_1 + 2\beta_2(\rho_1(W_1 - C_1) + \rho_2(C_2 - W_2))\big) \\
& + \rho_2(-C_1 + W_1)\gamma_1\gamma_2 + \rho_1(C_1 - W_1)\gamma_2^2\big) \\
& + \beta_2\Big(\rho_1\tau_1(2k(W_1 - C_1) - kW_2) + W_2(\rho_1(k\tau_2 - \gamma_1\gamma_2) + \rho_2(\gamma_1^2 \\
& - 2k\tau_1)) + C_2\big(2k\rho_2\tau_1 + k\rho_1(\tau_1-\tau_2) + \gamma_1(-\rho_2\gamma_1 + \rho_1\gamma_2)\big)\Big)\Big)
\end{aligned}
\tag{A.5}
$$

$$
\begin{aligned}
T_6 = \Big(& -2C_1k\rho_1^2(\beta_2+\tau_2) + a_2hk(\tau_1+\tau_2) - C_2k\rho_2^2(\tau_1+\tau_2) \\
& + \rho_1\rho_2\big(k\tau_2(C_1 + 2(C_2 - W_2) - W_1) - C_1\gamma_1\gamma_2 + 2\beta_2 k(C_2 - W_2) \\
& + k\tau_1(C_1 - W_1) + \gamma_2^2(W_2 - C_2)\big) \\
& + W_1\big(2k\beta_2(h(\beta_1+\tau_1) + \rho_1^2) - hk\tau_1^2 + k\tau_2(2\beta_1 h + 2\rho_1^2 + h\tau_1)\big) \\
& + W_2\Big(hk\big(\beta_2(\tau_1-\tau_2) + \tau_2(\tau_1-\tau_2)\big) + k\rho_2^2(\tau_1+\tau_2)\Big) \\
& - 2hW_1\gamma_1^2(\beta_2+\tau_2) + \gamma_1\gamma_2\Big(C_2\rho_2^2 + W_1\big(\rho_1\rho_2 + h(2\tau_1+\tau_2)\big)\Big) \\
& + W_2\big(h\big((\beta_2+\tau_2) - \rho_2^2\big) - a_2 h\big) \\
& + \gamma_2^2\big(\rho_1^2(C_1+1) - W_1\big(h(\beta_1+\tau_1)\big) - h\tau_1 W_2\big) \\
& + a_1 h(2k(\beta_2+\tau_2) - \gamma_2^2) \\
& + A_1\Big(h\big(2\beta_1\beta_2 k + k\tau_1(2\beta_2 - \tau_1) + k\tau_2(2\beta_1 + \tau_1) \\
& - 2\gamma_1^2(\beta_2+\tau_2) + \gamma_1\gamma_2(2\tau_1+\tau_2) - \gamma_2^2(\beta_1+\tau_1)\big)\Big)\Big)
\end{aligned}
\tag{A.6}
$$

$$
\begin{aligned}
T_7 = \Big(& C_2\rho_2^2(\gamma_1^2 - 2k\tau_1) \\
& + \rho_1\rho_2\big((k\tau_2(C_2 - W_2) + k\tau_1(2C_1 + C_2 - 2W_1 - W_2) \\
& + \gamma_1^2(W_1 - C_1)\big) + W_1\big(k\tau_1(\rho_1^2 - h\tau_1) + \tau_2(k\rho_1^2 - h\gamma_1^2)\big) \\
& + hk\tau_1\tau_2(W_1 + A_1) \\
& + W_2\big(2h\tau_1(k\beta_2 - \gamma_2^2) + \rho_2^2(2k\tau_1 - \gamma_1^2) + hk\tau_2(\tau_1 - \tau_2) \\
& - h\gamma_1^2(\beta_2 + \tau_2)\big) + a_2h(2k(\beta_1 + \tau_1) - \gamma_1^2) \\
& + \gamma_1\gamma_2(W_1(h\tau_1 - \rho_1^2) + \rho_1\rho_2(W_2 - C_2) + h\tau_1(W_2 + A_1) \\
& + h(2\tau_2W_2 - a_1) + C_1\rho_1^2) - hA_1(\tau_2\gamma_1^2 + k\tau_1^2) \\
& + h\big(k(\tau_1 - \tau_2)\tau_2 + \beta_2(2k\tau_1 - \gamma_1^2) - (\gamma_1 - 2\gamma_2)(\tau_2\gamma_1 - \tau_1\gamma_2)\big)A_2 \\
& + \beta_1\big(2k(\rho_1\rho_2(C_1 - W_1) + hW_2(\beta_2 + \tau_2) + \rho_2^2(W_2 - C_2)\big) \\
& + h\big(kW_1(\tau_2 - \tau_1) - 2W_2\gamma_2^2 + kA_1(\tau_2 - \tau_1) + \gamma_1\gamma_2(A_1 + W_1) \\
& + 2\big(k(\beta_2 + \tau_2)\big) - \gamma_2^2)A_2\big)\Big)
\end{aligned}
\tag{A.7}
$$

$$
\begin{aligned}
T_8 = \Big(& \rho_1\rho_2(2\beta_2C_2\gamma_1 + C_1\tau_1\gamma_1 - \tau_1W_1\gamma_1 + C_1\tau_2\gamma_1 + 2C_2\tau_2\gamma_1 - 2\beta_2W_2\gamma_1 \\
& - 2\beta_1C_1\gamma_2 - 2\tau_2W_2\gamma_1 - 2C_1\tau_1\gamma_2 - C_2\tau_1\gamma_2 - C_2\tau_2\gamma_2 + 2\beta_1W_1\gamma_2 \\
& + 2\tau_1W_1\gamma_2 + W_2\gamma_2(\tau_1 + \tau_2) - \tau_2W_1\gamma_1) \\
& + \gamma_1\big(a_2h(\tau_1 + \tau_2) - C_2\rho_2^2(\tau_1 + \tau_2) - 2C_1\rho_1^2(\tau_2 + \beta_2)\big) \\
& + W_1\gamma_1\Big(2\Big((\rho_1^2\big((\beta_2 + \tau_2) - h(\beta_1(\beta_2 + \tau_2))\big) \\
& + (\tau_1(2\beta_2 + \tau_2) + \tau_2^2))\Big)\Big) \\
& + \gamma_2\Big(2\rho_2^2\big(C_2(\beta_1 + \tau_1)\big) - 2a_2h(\tau_1 + \beta_1) + C_1\rho_1^2(\tau_1 + \tau_2)\Big) \\
& + W_1\gamma_2\big(\tau_1(h(\beta_1 + \tau_1) - \rho_1^2) - \tau_2(h(\beta_1 + \tau_1) + \rho_1^2)\big) \\
& + W_2\gamma_2\big(\tau_1(2(\beta_2h - \rho_2^2) - h\tau_1) - 2\beta_1\rho_2^2 \\
& + h(2\beta_1(\tau_2 + \beta_2) + \tau_1\tau_2)\big) + a_1h(2(\beta_2 + \tau_2)\gamma_1 - (\tau_1 + \tau_2)\gamma_2) \\
& + A_1\Big(h\gamma_2(\beta_1(\tau_1 - \tau_2) + \tau_1^2) + h\gamma_1\big(\tau_2(\tau_2 - 2\beta_1) - 2\beta_2(\beta_1 + \tau_1)\big) \\
& - h\tau_1\tau_2(\gamma_1 + \gamma_2)\Big) \\
& + h\big((\tau_1 - \tau_2)(\beta_2 + \tau_2)\gamma_1 + 2\beta_1(\beta_2 + \tau_2)\gamma_2 \\
& + \tau_1(2\beta_2 - \tau_1 + \tau_2)\gamma_2\big)A_2\Big)
\end{aligned}
\tag{A.8}
$$

$$
T_9 = 4\beta_1\beta_2 + 4\beta_1\tau_2 + 4\beta_2\tau_1 - \tau_2^2 + 2\tau_1\tau_2 - \tau_1^2 \tag{A.9}
$$

$$
\begin{aligned}
T_{10} = \rho_2^2 & (-2k(\beta_2 + \beta_1 + \tau_1) + \gamma_1^2) + 2k\rho_1\rho_2(\tau_1 + \tau_2) - 2k\rho_1^2\tau_2 \\
& - 2h\gamma_1^2(\beta_2 + \tau_2) + 2\big(-\rho_1\rho_2 + h(\tau_1 + \tau_2)\big)\gamma_1\gamma_2 \\
& + \big(\rho_1^2 - 2h(\beta_1 + \tau_1)\big)\gamma_2^2
\end{aligned}
\tag{A.10}
$$

$$
\begin{aligned}
T_{11} = {} & -2\beta_2 C_1 k(\rho_1^2 + \gamma_1^2) \\
& + k\rho_1\rho_2\big(\beta_2 C_2 - a_2 + 2C_1\tau_1 + \tau_2(C_1 + C_2) + A_1(2\tau_1 + \tau_2)\big) \\
& + k\tau_1\big(h\big(a_2 - C_1\tau_1 + \beta_2(2C_1 + C_2)\big) - \rho_2^2(C_1 + C_2)\big) \\
& + \gamma_1^2\big(C_1\rho_2^2 - 2C_1 h(\beta_2 + \tau_2)\big) \\
& + \gamma_1\gamma_2\big(h(C_2(\beta_2 + \tau_2) - a_2 + (C_1 + A_1)(2\tau_1 + \tau_2)\big) - 2\rho_1\rho_2(C_1 \\
& + A_1)) + \gamma_2^2\big(C_1\rho_1^2 - h\tau_1(C_1 + C_2)\big) \\
& + a_1(h(2k(\beta_2 + \tau_2) - \gamma_2^2) - k\rho_2^2) + A_1(\tau_1(h(2\beta_2 k - k\tau_1 - \gamma_2^2) \\
& - k\rho_2^2) + \gamma_1^2\big(\rho_2^2 + h(\tau_2(k\tau_1 - 2) - 2\beta_2)\big) \\
& + \rho_1^2\big(\gamma_2^2 - 2k((\tau_2 + \beta_2)\big) \\
& + \beta_1(k(2h(\beta_2 + \tau_2) - \rho_2^2) - h\gamma_2^2)(C_1 + A_1) \\
& + (k(\rho_1\rho_2(\beta_2 + \tau_2) - \rho_2^2\tau_1 + \beta_2 h(\tau_1 - \tau_2) + h(\tau_1 - \tau_2)\tau_2) \\
& + h(\beta_2 + \tau_2)\gamma_1\gamma_2 - h\tau_1\gamma_2^2)A_2
\end{aligned}
\tag{A.11}
$$

$$
\begin{aligned}
T_{12} = {} & -k\tau_1\big(2C_2\rho_2^2 + h(a_1 + C_1\tau_1 + 2\beta_2 C_2)\big) \\
& + \tau_2\left(k\left(h\big(a_1 + C_1\tau_1 + C_2(\tau_1 - \tau_2)\big) - \rho_1^2(C_1 + C_2)\right)\right) \\
& + \gamma_1^2\big(C_2\big(\rho_2^2 - h(\tau_2 + \beta_2)\big) - C_1 h\tau_2\big) \\
& + a_2(k(2h(\beta_1 + \tau_1) - \rho_1^2) - h\gamma_1^2) \\
& + \gamma_1\gamma_2\left(C_1 h\tau_1 + C_2\big(h\big((\tau_1 + 2\tau_2) - 2\rho_1\rho_2\big) - a_1\big)\right) \\
& + C_2(\gamma_2^2(\rho_1^2 - 2h\tau_1) - \beta_2 k\rho_1^2) \\
& + A_1\big(k(\tau_1(\rho_1\rho_2 - h\tau_1) - \rho_1^2\tau_2) + h(\tau_1(k\tau_2 + \gamma_1\gamma_2) - \tau_2\gamma_1^2)\big) \\
& + \big(k\big(-2\rho_2^2\tau_1 - \rho_1^2\tau_2 + h(\tau_1 - \tau_2)\tau_2 + \rho_1\rho_2(\tau_1 + 2\tau_2)\big) \\
& + (\rho_2^2 - h\tau_2)\gamma_1^2 - \beta_2(k(\rho_1^2 - 2h\tau_1) + h\gamma_1^2) \\
& + \big(-2\rho_1\rho_2 + h(\tau_1 + 2\tau_2)\big)\gamma_1\gamma_2 + (\rho_1^2 - 2h\tau_1)\gamma_2^2\big)A_2 \\
& + \beta_1\left(-2C_2 k\rho_2^2 + 2C_2 hk\tau_2 - 2C_2 h\gamma_2^2\right. \\
& + C_1\big(k\big(\rho_1\rho_2 + h(-\tau_1 + \tau_2)\big) + h\gamma_1\gamma_2\big) \\
& + A_1\big(k\big(\rho_1\rho_2 + h(\tau_2 - \tau_1)\big) + h\gamma_1\gamma_2\big) - 2(k(\rho_2^2 - h\tau_2) + h\gamma_2^2)A_2 \\
& \left. + 2\beta_2 hk(C_2 + A_2)\right)
\end{aligned}
\tag{A.12}
$$

$$
\begin{aligned}
T_{13} = {} & \rho_1\rho_2\big(\gamma_1(C_2(\beta_2+\tau_2)-a_2-C_1)+\gamma_2(\tau_1(C_2-C_1)+1)\big) \\
& + \gamma_1\Big(\tau_1\Big(h\big(a_2+\beta_2(C_2-2C_1)\big)+\rho_2^2(C_1-C_2)\Big) \\
& + h\tau_2\big(a_2-\beta_2C_2+\tau_1(C_2-C_1)+\tau_2(C_1-C_2)\big)\Big) \\
& + \gamma_2\Big(\rho_1^2\big(a_2-\beta_2C_2+\tau_2(C_1-C_2)\big)-2a_2h\tau_1 \\
& + h\tau_1\big(C_2(2\beta_2-\tau_1+\tau_2)+C_1(\tau_1-\tau_2)\big)\Big) \\
& + \gamma_1A_1\big(\rho_2^2\tau_1+\gamma_1(\tau_2(h(\tau_2-\tau_1)-\rho_1\rho_2)-2\beta_2h\tau_1) \\
& + \gamma_2(\tau_1(h(\tau_1-\tau_2)-\rho_1\rho_2)+\rho_1^2\tau_2)\big) \\
& + (\rho_1\rho_2(\beta_2+\tau_2)-\rho_2^2\tau_1+\beta_2h(\tau_1-\tau_2)+h(\tau_1-\tau_2)\tau_2)\gamma_1A_2 \\
& + \big(\rho_1\rho_2\tau_1-\beta_2(\rho_1^2-2h\tau_1)-\rho_1^2\tau_2+h\tau_1(-\tau_1+\tau_2)\big)\gamma_2A_2 \\
& + \beta_1\Big(2h\gamma_2(2C_2\tau_2-a_2) \\
& + C_1\Big(\rho_2^2\gamma_1-\rho_1\rho_2\gamma_2+h\big(\tau_1\gamma_2-h\tau_2(2\gamma_1+\gamma_2)\big)\Big) \\
& + A_1\big(\gamma_1(\rho_2^2-2h\tau_2)+\gamma_2(h(\tau_1-\tau_2)-\rho_1\rho_2)\big) \\
& + 2h\Big(\tau_2\gamma_2A_2+\beta_2\big(-\gamma_1(C_1+A_1)+\gamma_2(C_2+A_2)\big)\Big)\Big)
\end{aligned}
\tag{A.13}
$$

$$
\begin{aligned}
T_{14} = {} & \tau_1(k\rho_1(\beta_2(2C_1-C_2)-a_2)+2k\rho_2(a_2-\beta_2C_2))+\tau_1^2\big(\rho_2(C_2-C_1)\big) \\
& + \rho_1\tau_1\gamma_2^2(C_2-C_1)+\tau_1\tau_2\big(k(C_1-C_2)(\rho_1+\rho_2)\big) \\
& + \rho_2\gamma_1^2(\beta_2C_2+\tau_2(C_2-C_1)-a_2) \\
& + \gamma_1\gamma_2\big(\rho_2\tau_1(C_1-C_2)+\rho_1(\tau_2(C_1-C_2)+a_2-\beta_2C_2)\big) \\
& + a_1\Big(k\big(\rho_2(\tau_1+\tau_2)-2\rho_1(\beta_2+\tau_2)\big)+\gamma_2(-\rho_2\gamma_1+\rho_1\gamma_2)\Big) \\
& + A_1\Big(\rho_1\big(\tau_1(k(2\beta_2+\tau_2)-\gamma_2^2)+\tau_2(\gamma_1\gamma_2-k\tau_2)\big) \\
& + \rho_2\Big(-\tau_2\gamma_1^2+\tau_1\big(\gamma_1\gamma_2+k(\tau_2-\tau_1)\big)\Big)\Big) \\
& + \Big(k(\tau_1-\tau_2)(\rho_2\tau_1-\rho_1\tau_2)+(\rho_2\gamma_1-\rho_1\gamma_2)(\tau_2\gamma_1-\tau_1\gamma_2) \\
& + \beta_2\big(-k(\rho_1+2\rho_2)\tau_1+k\rho_1\tau_2+\gamma_1(\rho_2\gamma_1-\rho_1\gamma_2)\big)\Big)A_2 \\
& + \beta_1\Big(\rho_2\big(k\big(2a_2-C_1\tau_1+\tau_2(C_1-2C_2)\big)+C_1\gamma_1\gamma_2\big) \\
& + C_1\rho_1(2k\tau_2-\gamma_2^2)+A_1\big(\rho_1(2k\tau_2-\gamma_2^2)+\rho_2(k(\tau_2-\tau_1)+\gamma_1\gamma_2)\big) \\
& - 2k\rho_2\tau_2A_2+2\beta_2k\big(\rho_1(C_1+A_1)-\rho_2(C_2+A_2)\big)\Big)
\end{aligned}
\tag{A.14}
$$

$$
T_{15} = 2\beta_1\gamma_2^2+2\beta_2\gamma_1^2+2\tau_2\gamma_1^2-2\tau_2\gamma_1\gamma_2-2\tau_1\gamma_1\gamma_2+2\tau_1\gamma_2^2 \tag{A.15}
$$

$$\begin{aligned}T_{16} = &-k\tau_1^2W_1 - k\tau_2^2(W_2 + a_2)\\
&+ a_1(-k\tau_2^2 + 2k\beta_2(1 + \beta_1 + \tau_1) + k\tau_2(2 + 2\beta_1 + \tau_1)\\
&- 2\gamma_1^2(\beta_2 - \tau_2) + (2\tau_1 + \tau_2)\gamma_1\gamma_2 - (1 + \beta_1 + \tau_1)\gamma_2^2) + \gamma_1\gamma_2(L_1\rho_2\\
&+ a_2(\tau_2 - 1) + 2\tau_1W_1 + \tau_2(W_1 + W_2)) - \gamma_2^2(L_1\rho_1 + \beta_1W_1\\
&+ \tau_1(a_2 + W_1 + W_2))\\
&+ \beta_2\left(k\big((2L_1\rho_1 + a_2(\tau_1 - \tau_2) + 2(\beta_1 + \tau_1)W_1 + (\tau_1 - \tau_2)W_2)\right.\\
&\left.+ \gamma_1(-2W_1\gamma_1 + (a_2 + W_2)\gamma_2)\big)\right)\\
&+ \tau_2\left(k\big(a_2 + L_1(2\rho_1 - \rho_2)\big) + 2W_1(\beta_1k - \gamma_1^2)\right)\end{aligned} \tag{A.16}$$

$$\begin{aligned}T_{17} = &k\tau_1\big(a_1(1 - \tau_1) + 2a_2(\beta_2 + 1) + L_1(\rho_1 - 2\rho_2)\big)\\
&+ k\tau_2\big(a_1 + \tau_2(L_1\rho_1 - a_2 - W_2)\big) + 2a_2\beta_1k(1 + \beta_2 + \tau_2)\\
&+ +\gamma_1^2(L_1\rho_2 - \tau_2(a_1 + W_1 + W_2) - a_2(\tau_2 + \beta_2 + 1) - \beta_2W_2)\\
&+ \gamma_1\gamma_2(-L_1\rho_1 + a_1(-1 + \beta_1 + \tau_1) + a_2(\tau_1 + 2\tau_2) + (\beta_1 + \tau_1)W_1\\
&+ (\tau_1 + 2\tau_2)W_2) - 2(\beta_1 + \tau_1)(a_2 + W_2)\gamma_2^2\\
&+ k\tau_1\tau_2(a_1 + a_2 + W_1 + W_2)\end{aligned} \tag{A.17}$$

$$\begin{aligned}T_{18} = &a_1\big(-2\beta_2(\beta_1 + \tau_1 - 1) + \tau_2(2 - 2\beta_1 - \tau_1 + \tau_2)\big)\gamma_1\\
&+ \gamma_1\big(a_2\tau_1 + a_2\tau_2 + 2L_1\rho_1\tau_2 - L_1\rho_2(\tau_1 + \tau_2) - 2\beta_1\tau_2W_1\\
&+ \tau_2(\tau_1 - \tau_2)(a_2 - W_1 + W_2)\\
&+ \beta_2(2L_1\rho_1 + a_2(\tau_1 - \tau_2) - 2(\beta_1 + \tau_1)W_1 + (\tau_1 - \tau_2)W_2)\big)\\
&+ \gamma_2\left(L_1(\tau_1(2\rho_2 - \rho_1) - \rho_1\tau_2)\right.\\
&+ a_2\big(2\beta_1(-1 + \beta_2 + \tau_2) + \tau_1(-2 + 2\beta_2 - \tau_1 + \tau_2)\big) + \tau_1^2W_1\\
&- \tau_1\tau_2W_1 + \tau_1(2\beta_2 - \tau_1 + \tau_2)W_2\\
&\left.+ \beta_1(2L_1\rho_2 + W_1(\tau_1 - \tau_2) + 2(\beta_2 + \tau_2)W_2)\right)\end{aligned} \tag{A.18}$$

$$\begin{aligned}T_{19} = &\beta_2\left(-2C_1k\rho_1\tau_1(\delta - 1)\right.\\
&+ C_2(\delta - 1)(k(2\rho_2\tau_1 + \rho_1(\tau_1 - \tau_2)) + \gamma_1(\rho_1\gamma_2 - \rho_2\gamma_1))\\
&+ \delta\left(k\big(W_2(2\rho_2\tau_1 - \rho_1\tau_2) + \rho_1\tau_1(W_2 - 2W_1)\big)\right.\\
&\left.\left.+ W_2\gamma_1(-\rho_2\gamma_1 + \rho_1\gamma_2)\right)\right)\\
&+ \beta_1\left(2\beta_2k\left(C_2\rho_2(\delta - 1) + C_1\big(\rho_1(\delta - 1)\big) + \delta(\rho_2W_2 - \rho_1W_1)\right)\right.\\
&+ k\left(2C_2\rho_2(\delta - 1)\tau_2 - 2\rho_1\delta\tau_2W_1\right.\\
&+ \rho_2\delta\big((W_1(\tau_1 - \tau_2) + 2\tau_2W_2) - W_1\gamma_1\gamma_2\big)\Big) + \rho_1\delta W_1\gamma_2^2\\
&\left.+ C_1(\delta - 1)\big(k(\rho_2\tau_1 - (2\rho_1 + \rho_2)\tau_2) + \gamma_2(\rho_1\gamma_2 - \rho_2\gamma_1)\big)\right)\end{aligned} \tag{A.19}$$

Appendix C

Hessian matrix of theorem 3.5 corresponding to the total profit function in centralized mode (H) is obtained as below:

$$H(P_1, P_2, S_1, L_1,) = \begin{bmatrix} \frac{\partial^2 \pi^r}{\partial P_1^2} & \frac{\partial^2 \pi^r}{\partial P_1 \partial P_2} & \frac{\partial^2 \pi^r}{\partial P_1 \partial S_1} & \frac{\partial^2 \pi^r}{\partial P_1 \partial L_1} \\ \frac{\partial^2 \pi^r}{\partial P_2 \partial P_1} & \frac{\partial^2 \pi^r}{\partial P_2^2} & \frac{\partial^2 \pi^r}{\partial P_2 \partial S_1} & \frac{\partial^2 \pi^r}{\partial P_2 \partial L_1} \\ \frac{\partial^2 \pi^r}{\partial S_1 \partial P_1} & \frac{\partial^2 \pi^r}{\partial S_1 \partial P_2} & \frac{\partial^2 \pi^r}{\partial S_1^2} & \frac{\partial^2 \pi^r}{\partial S_1 \partial L_1} \\ \frac{\partial^2 \pi^r}{\partial L_1 \partial P_1} & \frac{\partial^2 \pi^r}{\partial L_1 \partial P_2} & \frac{\partial^2 \pi^r}{\partial L_1 \partial S_1} & \frac{\partial^2 \pi^r}{\partial L_1^2} \end{bmatrix}$$

$$= \begin{bmatrix} -2(\beta_1 + \tau_1) & \tau_1 + \tau_2 & \gamma_1 & \rho_1 \\ \tau_1 + \tau_2 & -2(\beta_2 + \tau_2) & -\gamma_2 & -\rho_2 \\ \gamma_1 & -\gamma_2 & -k & 0 \\ \rho_1 & -\rho_2 & 0 & -h \end{bmatrix}$$