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Malmquist Productivity Index using the concept of loyalty factor in bank branches

Parisa Nankali[1], Mohammad Reza Alirezaee[2*], Fatemeh Rakhshan[2]

[1]*Behin-Cara-Pajoh Operations Research Institute (BCaP), Tehran, Iran*
[2]*School of Mathematics, Iran University of Science and Technology, Tehran, Iran*

p_nankali68@yahoo.com, mralirez@iust.ac.ir, rakhshan_20@yahoo.com

Abstract

In this study, we introduce a new concept as loyalty factor of bank branches customers. Data Envelopment Analysis weight restrictions is used to develop a new loyalty model and define the loyalty factor. Assurance region weight restrictions are attached to basic data envelopment analysis models using some predefined loyalty codes based on services quality and in special, e-banking. This model enhances the discrimination power of decision-making units. Using the proposed loyalty factor, we extend Malmquist productivity index to determine the contribution of loyalty factor changes in two times on the productivity changes. The presented method is implemented in a real world case study from 177 Iranian bank branches in 2018-2019 to approve its applicability. The results for both traditional and extended Malmquist index are analyzed.

Keywords: Data Envelopment Analysis, loyalty, bank branch, efficiency, weight research

1-Introduction

In the last two decades, many organizations have begun to realize the importance of customer satisfaction and loyalty. Customer satisfaction is a key component of a firm's marketing strategy and tactics. A small increase in the number of loyal customers can result in a substantial growth in profitability. Loyal customers may generate more profit if they stay with the firm for a longer period of time (Kim & Cha, 2002). Customers' loyalty can increase sales and profit in addition to reducing costs. Another significant issue that makes us place more emphasis on customer satisfaction and loyalty is that higher customer satisfaction can lead to more profitability. Bayraktar et al. (2012) measured the impact of customer satisfaction and loyalty on mobile phone brands through a questionnaire. Perceived value, service quality, customer satisfaction, and brand trust significantly influence customer loyalty (Pasha and Waleed, 2016). The authors used a Data Envelopment Analysis (DEA) model in which customer loyalty and satisfaction are the outputs.

The aim of the present study, however, is to develop a model to calculate a new factor, named the Loyalty Factor (LF) in a bank. No mathematical model has thus far been proposed to measure loyalty and only some studies, as mentioned above, have examined the organization's efficiency in terms of ensuring customer satisfaction and loyalty using questionnaires. Loyalty programs are widely used by organizations as a structured ‘customer relationship management’ tool to build and extend customer-supplier relationship (Alshurideh et al., 2020). The performance of banks aimed at achieving customer satisfaction and loyalty cannot be left to the experiences that are led by chance.

*Corresponding author
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There must be a sufficient understanding of customers' requirements, then work to create the necessary grounds to achieve experiences that rise to the level of excellence compared to customer expectations (Raafat, 2022). The advantage of the model presented in this paper is that it eliminates the need to construct questionnaires to survey users and experts. The methodology used in this study to evaluate the LF is based on the quality of service that customers receive in the bank. Service quality has a considerable impact on customer satisfaction (Cejas, 2006) and gives organizations a strong competitive advantage in the market (Yang et al., 2011). Supriyanto et al. (2021) provide valuable measures on how school and bank organization leaders or managers enhance organization performance, empower their members effectively through providing them with high job satisfaction, and increased loyalty to their jobs. Azizah and Puspito (2021) aimed to determine the effect of satisfaction on loyalty among bank customers in Indonesia. They used a face-to-face interview method among 1910 bank customers in eight Metropolis in Indonesia. Venugopal et al. (2020) examined how dimensions of e-service quality such as service quality, information quality, and system quality affects the satisfaction and loyalty of bank customers in India.

The core idea for the present study, inspired by DEA models, is to measure the customer loyalty of bank branches. DEA is a nonparametric approach, applied to operations research, which makes use of mathematical programming to measure the efficiency of multi-input, multi-output DMUs empirically. In most cases, basic DEA models do not sufficiently distinguish between the efficiency of Decision Making Units (DMUs). Thus, a number of tools have been developed to improve discrimination power of DEA models. A relatively popular example of these tools, most commonly used by decision-makers, is weight restriction (Podinovski & Thanassoulis, 2007). Weight restriction helps reflect the views and/or information that modelers or managers may have had about the significance of each input and output. In some cases, weight restriction also indicates the way a particular relation between outputs and inputs, such as those of cost and price, is imposed. For instance, Camanho and Dyson (2005) introduced the concept of Cost Efficiency (CE) using weight restriction.

Productivity, in general, refers to a combination of several measures of efficiency or an entity's actions. Productivity analysis is an important research area in DEA. The Malmquist Index (MI) evaluates changes in a DMU's productivity during the period between two points in time. We expand the MI by adding in customer loyalty as an influential factor. The MI, is now a popular index that measures changes in productivity over time and has found widespread use in input consumption analysis. Two-component decomposition of MI consists of two components of technology change (TC) and efficiency change (EC), over two time periods. The three-component decomposition of the MI considers both Constant Returns to Scale (CRS) and Variable Returns to Scale (VRS) technologies and the decomposition's components consisted of Pure Efficiency Change (PEC), Scale Efficiency Change (SEC), and TC. These decompositions were conducted using basic CCR and BCC models. In this context, two other studies may also be cited in which two new basic technologies were developed. In order to improve the efficiency of the production possibility set (PPS), Alirezaee and Afsharian (2010) presented an extended Malmquist index (EMI) by adopting some technology trade-offs in addition to a pair of models. Evaluating the behavior of DMUs by examining the impact of strategies imposed on these units, Alirezaee and Rajabi Tanha (2015) presented a balance model to evaluate the balance factor while developing a different EMI. Rakhshan and Alirezaee (2020) investigated ethics factor on productivity performance of bank branches.

In this paper, first, a new model is introduced to calculate the LF of bank branches. Then, a new decomposition of the MI is developed by extracting the LF changes (LFC). The decomposition consists of four components: PEC, SEC, LFC, and TC. The rest of the paper is organized as follows. Section 2 introduces the concept of customer LF in bank branches. Section 3 presents the model developed in this research, which calculates the LF using weight restrictions. Section 4 describes the method proposed to calculate the EMI using the LF to determine the impact of LFC on the growth or decline of productivity. Section 5 presents the case study that serves as a real-world example to verify the applicability of the proposed method. Lastly, section 6 provides a brief conclusion to the research, including some recommendations for future research.

2-Customer loyalty in banks

First, we have listed five reasons why customer loyalty is vital to the business. Loyal customers keep marketing costs down, loyal customers serve as brand advocates, loyal customers leave fantastic reviews, and loyal customers are more likely to buy additional products. More Loyal Customers Mean Higher Profits. Before presenting the loyalty measurement model, we present a brief definition of the concept of customer loyalty and mention the factors that influence customer loyalty in banks. Loyalty refers to the commitment displayed by customers to keep buying a product or frequenting an establishment in a consistent manner over a prolonged period. An increasing number of studies indicates that ensuring customer satisfaction often leads to favorable word-of-mouth, loyalty, repeated instances of purchase, and higher profit in the long term (Wirtz, 2003). Loyalty is regarded as the outcome of the interaction between behavior and attitude. The extent of loyalty is largely dictated by the strength of the connection between prolonged patronage and the consumer's attitude. Based on said connection, the authors defined four levels of loyalty: no loyalty, spurious loyalty, latent loyalty, and loyalty (see figure 1).

		Patronage	
		High	**Low**
Attitude	**High**	Loyalty	Latent loyalty
	Low	Spurious loyalty	No loyalty

Fig 1. Loyalty Matrix

Loyalty could help profitability when the customers' (repeat purchase) behavior and attitude are aligned. Latent loyalty refers to consumers who hold a positive attitude toward the organization but whose purchasing behavior is not unstable and whose choices affect the supply volume, inventory status, etc. In spurious loyalty, the repeat purchase pattern is rather coincidental, as customers may have done continuous business with the same organization only because of the lack of better options elsewhere, attractive offers, better accessibility, or other factors. In other words, the consumer may sometimes be loyal, but would have no qualms about relinquishing their current choice(s) in favor of rival organizations. Lastly, 'no loyalty' occurs when attitude and repeated purchase are both at a low level. The factors influencing customer loyalty are defined in figure 2.

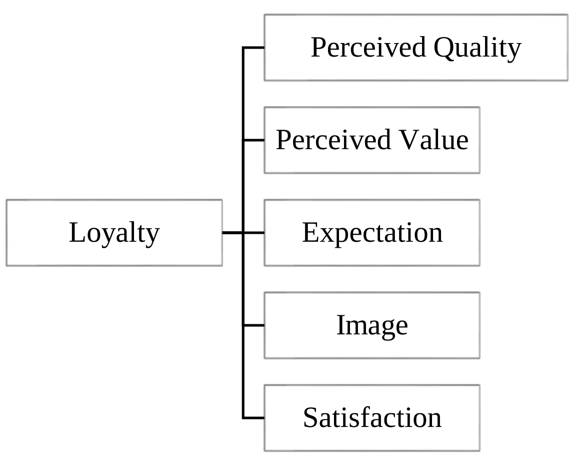

Fig 2. Factors influencing customer loyalty

Perceived Quality: The customer's opinion about the excellence and suitability of a service or product.

Perceived Value: The customer's judgment of the usefulness of a service or product according to their judgment of whether what they receive is, in simple terms, worth what they give for it.

Expectation: Relates to perceived value (i.e. performance). It refers to the extent to which consumers have learned from their experiences in the past, according to which they project the performance level that their purchases are likely to deliver. Expectation also predicts the ability of an organization to deliver in the future, and arguably affects the level of satisfaction.

Image: Refers to the perception a consumer has accumulated in their memory of an organization based on their past interactions. Given the transaction-centric nature of consumer satisfaction, many authors have claimed that corporate image is determined by customers' long-term (dis)satisfaction with organizations' products and/or services.

Satisfaction: Refers to the gratification that a customer experiences with the quality of the product or service they have purchased. In wide range of services, customer satisfaction considerably influences the decision to re-purchase. Sales opportunity in future directly depends on the level of satisfaction and trust that consumers experience during in-person sales encounters.

In this paper, our focus is on developing a loyalty model based on perceived quality and the quality of electronic banking (e-banking) services. The proposed model should be able to evaluate the relationship between e-banking services and customer loyalty. The factors affecting customer loyalty in banks are as follows:

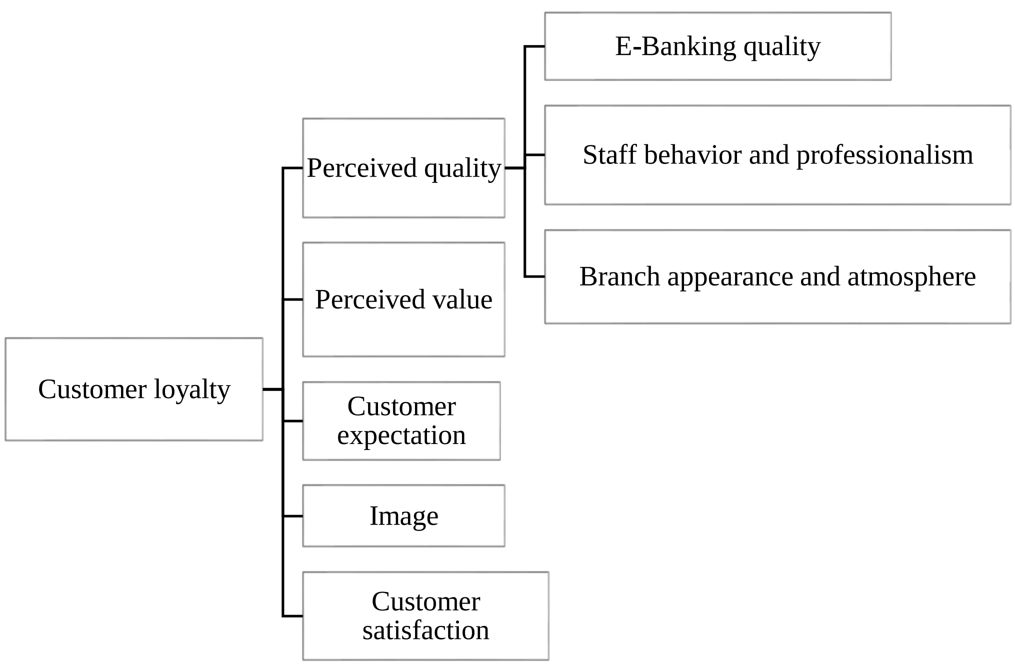

Fig 3. Customer LF in banks

E-banking services use modern digital technologies to provide direct services to customers at all hours. The idea was first developed and implemented in the United States in 1995 and was adopted rapidly by other countries (Amade & Jafarpour, 2009). Numerous studies have thus far been published on e-banking, most of which concluding that e banking leads to enhanced service quality and, ultimately, to customer loyalty. Our research also indicates that banks, which implement effective e-banking practices, are able to keep their customer base satisfied and thereby loyal. Ribbink et al. (2004) show that e banking results in customer satisfaction and leads to long-term profitability. Turban et al. (2004) concluded that customers who find it easy to work with e-banking services are more loyal compared to those do not use e banking. Yang and Fang (2004) identified the qualitative attributes of online service and assessed the relationship between these attributes and customer satisfaction. The research conducted on the impact of e banking on customer loyalty have often used questionnaires. However, in this paper, we adopt a computational method and, providing a mathematical loyalty-measurement model, we examine the impact of e banking on customer loyalty as a key long-term determinant of success.

3-Proposed model for measuring loyalty factor

Camanho and Dyson (2005) proposed a model for calculating CE in which the relative value of input weights was equal to the relative value of input prices. Adopting this view, we change the weighting structure of the basic CCR model in a way that each output's weight is equal to its relative loyalty value. Loyalty value is a numerical composite measure of banking codes in e-banking services. These codes have been extracted and evaluated after numerous studies on e-banking accounting heads in bank branches and indicate the number and values of e-banking transactions.

As mentioned earlier, our goal is to examine the impact of e-banking services quality on customer loyalty. To this end, considering production approach in bank branches efficiency measurement (Paradi & Zhu, 2013), location index and personnel expenses are used as the model's inputs, while deposits, services, and loans are the outputs. Since improving the quality of services leads to higher sustainability of deposits in bank branches, we consider the *r* value as the relative weight of deposits, one of the outputs, in the proposed loyalty model. Therefore, the larger the value of *r*, the higher the quality of e-banking services and, consequently, the larger the amount of resources attracted by each branch. This means that the LF in branches with a large *r* index value is optimal. This factor is related to the importance of the loyalty of the branch customers and includes all the factors involved in the level of loyalty. But because we do not have access to the real amount, we estimate it using service quality parameters, especially e-banking. In fact, this factor tries to estimate the qualitative concept of customer loyalty of each branch as a quantitative parameter to prioritize the weight of deposits in the proposed model. Finally, the loyalty value is obtained after performing mathematical computations on the respective codes. The *r* index could be normalized to take non-zero values between 0 and 1. Further details on how to calculate the *r* index are provided in subsection 5.2.

As seen in figure 4, Since it is not possible to take into account all influencing factors, we consider r as an estimate of R that is the LF encompassing all influencing criteria.

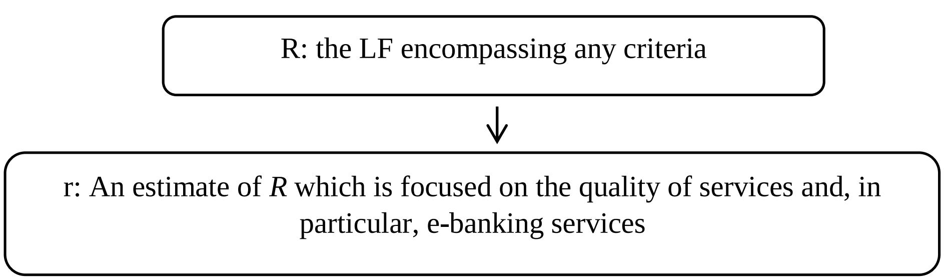

Fig 4. r as an estimate of R

Suppose we have *n* DMUs with *m* inputs and *s* outputs such that for DMU_j we have $X_j = (x_{1j}, \ldots, x_{ij}, \ldots, x_{mj})$ and $Y_j = (y_{1j}, \ldots, y_{rj}, \ldots, y_{sj})$, where $j = 1, \ldots, n$. It is assumed that

$X_j \geq 0$, $Y_j \geq 0$, $X_j \neq 0$, and $Y_j \neq 0$ for all DMUs. The DEA models for the CRS and VRS technologies are denoted by CCR and BCC in honor of their developers, respectively. The CCR and BCC models to measure technical efficiency (TE) and pure efficiency (PE) for a given DMU$_p$, where $p = 1, \ldots, n$ are given below:

$$
\begin{aligned}
&\theta_{CCR} = Min \sum_{i=1}^{m} v_i x_{ip} \\
&s.t. \sum_{r=1}^{s} u_r y_{rp} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} \leq 0 \quad j = 1, 2, \ldots, n \\
&u_r \geq \varepsilon, \quad r = 1, 2, \ldots, s \\
&v_i \geq \varepsilon, \quad i = 1, 2, \ldots, m.
\end{aligned} \tag{1}
$$

and

$$
\begin{aligned}
&\theta_{BCC} = Min \sum_{i=1}^{m} v_i x_{ip} - u_0 \\
&s.t. \sum_{r=1}^{s} u_r y_{rp} = 1 \\
&\sum_{r=1}^{s} u_r y_{rj} - \sum_{i=1}^{m} v_i x_{ij} - u_0 \leq 0 \quad j = 1, 2, \ldots, n \\
&u_r \geq \varepsilon, \quad r = 1, 2, \ldots, s, \\
&v_i \geq \varepsilon, \quad i = 1, 2, \ldots, m \\
&u_0 free
\end{aligned} \tag{2}
$$

Where u_r and v_i are the weights assigned to output r and input i, respectively, and $\varepsilon > 0$ is a non-Archimedean infinitesimal that prevents zero weights. The CCR and BCC models are based on the CRS and VRS technologies, respectively.

Adopting production approach in bank branch measurement and considering two inputs of personnel expenses and location index, and three outputs of deposits, loans, and services, the proposed CRS Loyalty Model (LM) is defined as follows:

$$
\begin{aligned}
&\theta_{LM} = Min \sum_{i=1}^{2} v_i x_{ip} \\
&s.t. \sum_{r=1}^{3} u_r y_{rp} = 1 \\
&\sum_{r=1}^{3} u_r y_{rj} - \sum_{i=1}^{2} v_i x_{ij} \leq 0 \quad j = 1, 2, \ldots, n \\
&\frac{u_1}{u_i} = r_p, i = 2, 3 \\
&v_i \geq \varepsilon, \quad i = 1, 2 \\
&u_r \geq \varepsilon, \quad r = 1, 2, 3
\end{aligned} \tag{3}
$$

Where r_p is the relative weight of output 1 as deposits for DMU_p and $\varepsilon > 0$ is a non-Archimedean infinitesimal that prevents zero weights. We know that r is the loyalty value. The constraints

$\frac{u_1}{u_i} = r_p, i = 2,3$ are called loyalty constraints which contain the loyalty value of r_p . These constraints are assurance region type I weight restrictions and so the model (3) is always feasible. When imposing ARI there will always exist at least one efficient DMU. Moreover, whether the output or input orientation is used, a DEA model incorporating ARI produces the same relative efficiency scores. We note that in model (3), we assign value of one to the weights u_2 and u_3 .

Definition 1. The LF of a DMU is defined as the ratio of LM to CCR efficiency scores that measures the changes between two frontiers before and after adding the loyalty constraints.

The LF of each DMU is different from other DMUs and indicates the loyalty factor of that specific unit, such as bank branches. In fact, when the LF of a DMU is equal to 1, it means that the new efficiency frontier has not changed with respect to the LF. If, however, the LF of a DMU is less than one, it means that the DMU's customers are not completely loyal and, consequently, it needs to improve in terms of loyalty.

In figure 5, the distance between the two boundaries CCR and LM represents the LF, which for DMU A is equal to 1. This means that DMU A is efficient both in terms of technical performance and loyalty, and its customers are completely loyal. Contrastingly, DMU B is only technically efficient and performs poorly in terms of the loyalty index. DMU E is not efficient either technically or in terms of loyalty.

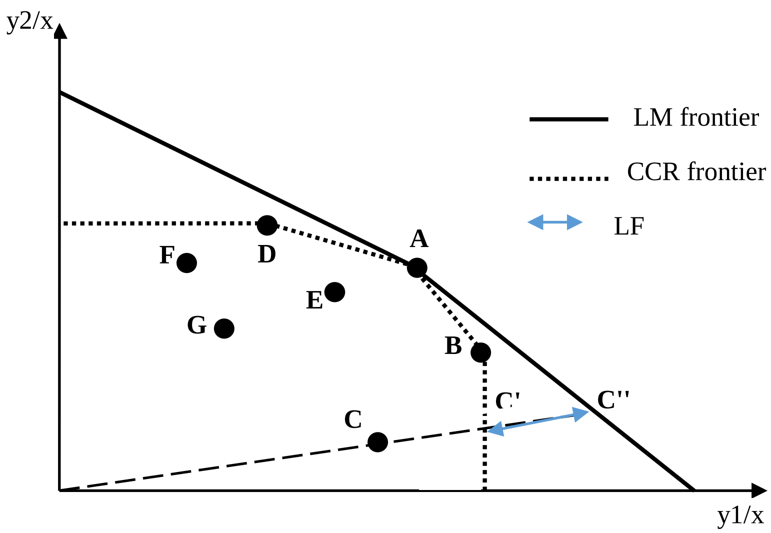

Fig 5. LF of a DMU

4-Extended Malmquist index (EMI)

Assume x and y as the input and output of the DMUs, while *t* and *t+1* are the two time-periods during which the MI is assessed. The basic formulation of the MI is as follows:

$$M_I^G = \sqrt{M_I^t M_I^{t+1}} = \left(\frac{D_I^t\left(x^{t+1}, y^{t+1}\right)}{D_I^t\left(x^t, y^t\right)} \times \frac{D_I^{t+1}\left(x^{t+1}, y^{t+1}\right)}{D_I^{t+1}\left(x^t, y^t\right)} \right)^{\frac{1}{2}} \tag{4}$$

Where $D^{t+1}\left(x^t, y^t\right)$ is an output distance function representing the distance between observations over time-period *t* and the technology related to time-period *t + 1*, and is obtained as follows:

$$D^{t+1}\left(x^t, y^t\right) = \min\left\{\phi > 0 : \left(x^t, \frac{y^t}{\phi}\right) \in T^{t+1}\right\} \tag{5}$$

Where technology *T* can be both CRS and VRS, which are defined as $D_{CRS}^{t+1}\left(x^t, y^t\right)$ and $D_{VRS}^{t+1}\left(x^t, y^t\right)$, respectively. From (5), the following relationship is satisfied between the output distance function and output-oriented DEA models:

$$\begin{aligned}
& D_{CRS}^{t+1}\left(x_p^t, y_p^t\right)^{-1} = \min \sum_{i=1}^{m} v_i^{t+1} x_{ip}^t \\
& s.t. \sum_{r=1}^{s} u_r^{t+1} y_{rp}^t = 1 \\
& \sum_{r=1}^{s} u_r^{t+1} y_{rj}^{t+1} - \sum_{i=1}^{m} v_i^{t+1} x_{ij}^{t+1} \leq 0 \\
& u_r^{t+1} \geq \varepsilon \quad r = 1, 2, \ldots, s \\
& v_i^{t+1} \geq \varepsilon \quad i = 1, 2, \ldots, m
\end{aligned} \tag{6}$$

The two-component decomposition of MI is obtained by decomposing (4) into the two components efficiency change (EC) and technological change (TC), as follows:

$$MI = \frac{D_{CRS}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)}{D_{CRS}^{t}\left(x_p^{t}, y_p^{t}\right)} \left[\frac{D_{CRS}^{t}\left(x_p^{t+1}, y_p^{t+1}\right)}{D_{CRS}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)} \frac{D_{CRS}^{t}(x_p^{t}, y_p^{t})}{D_{CRS}^{t+1}\left(x_p^{t}, y_p^{t}\right)}\right]^{\frac{1}{2}} = EC \times TC \tag{7}$$

Three-component decomposition of MI requires VRS technology measurement of efficiency obtained by BCC model along with CRS technology measure from CCR model. Pure Efficiency (PE) is defined as the measure obtained from BCC model. The decomposition is composed of Pure Efficiency Change (PEC), Scale Efficiency Change (SEC) and TC. Scale Efficiency (SE) is calculated as the ratio of CCR to BCC efficiency scores. This decomposition is presented as follows:

$$\begin{aligned}
MI = & \left[\frac{D_{VRS}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)}{D_{VRS}^{t}\left(x_p^{t}, y_p^{t}\right)}\right] \times \left[\frac{D_{VRS}^{t}\left(x_p^{t}, y_p^{t}\right)}{D_{CRS}^{t}\left(x_p^{t}, y_p^{t}\right)} \times \frac{D_{CRS}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)}{D_{VRS}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)}\right] \times \\
& \left[\frac{D_{CRS}^{t}\left(x_p^{t+1}, y_p^{t+1}\right)}{D_{CRS}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)} \frac{D_{CRS}^{t}\left(x_p^{t}, y_p^{t}\right)}{D_{CRS}^{t+1}\left(x_p^{t}, y_p^{t}\right)}\right]^{\frac{1}{2}} = PEC \times SEC \times TC
\end{aligned} \tag{8}$$

Now consider the LM as a new technology replacing CCR in two-component decomposition, and the new Malmquist index decomposition is computed as follows:

$$
\begin{aligned}
&EMI = EEC \times ETC \\
&EEC = \frac{D_{LM}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)}{D_{LM}^{t}\left(x_p^{t}, y_p^{t}\right)} \\
&ETC = \left[\frac{D_{LM}^{t}\left(x_p^{t+1}, y_p^{t+1}\right)}{D_{LM}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)} \frac{D_{LM}^{t}\left(x_p^{t}, y_p^{t}\right)}{D_{LM}^{t+1}\left(x_p^{t}, y_p^{t}\right)}\right]^{\frac{1}{2}}
\end{aligned}
\tag{9}
$$

Where x_p^t, y_p^t are the input and output vectors for DMU_p in period t, respectively. The distance function $D_{LM}^{t+1}\left(x_p^t, y_p^t\right)$ is computed as follows:

$$
\begin{aligned}
&D_{LM}^{t+1}\left(x_p^t, y_p^t\right)^{-1} = \min \sum_{i=1}^{m} v_i^{t+1} x_{ip}^{t} \\
&s.t. \sum_{r=1}^{s} u_r^{t+1} y_{rp}^{t} = 1 \\
&\sum_{r=1}^{s} u_r^{t+1} y_{rj}^{t+1} - \sum_{i=1}^{m} v_i^{t+1} x_{ij}^{t+1} \leq 0 \\
&u_{r_a}^{t+1} - \frac{r_{r_a}^{t+1}}{r_{r_b}^{t+1}} u_{r_b}^{t+1} = 0 \\
&r_a < r_b \ , r_a, r_b = 1, 2, \ldots, s
\end{aligned}
\tag{10}
$$

Based on the concept described in definition 1, a novel three-component decomposition that specifies the impact of loyalty factor change (LFC) on productivity is developed as follows:

$$
LFC = \left[\frac{D_{CRS}^{t}\left(x_p^{t}, y_p^{t}\right)}{D_{LM}^{t}\left(x_p^{t}, y_p^{t}\right)} \times \frac{D_{LM}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)}{D_{CRS}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)}\right]
\tag{11}
$$

LFC represents changes in the performance of DMU_p between two points in time, including any changes occurring after applying the loyalty index. EC, based on CRS, LFC, and ETC, defines a new decomposition of the MI:

$$
\begin{aligned}
&EMI = EC \times LFC \times ETC \\
&LFC = \frac{LF^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)}{LF^{t}\left(x_p^{t}, y_p^{t}\right)} = \left[\frac{D_{LM}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)}{D_{CRS}^{t+1}\left(x_p^{t+1}, y_p^{t+1}\right)} \times \frac{D_{CRS}^{t}\left(x_p^{t}, y_p^{t}\right)}{D_{LM}^{t}\left(x_p^{t}, y_p^{t}\right)}\right]
\end{aligned}
\tag{12}
$$

Where $D_{LM}^{t}\left(x_{p}^{t},y_{p}^{t}\right)=D_{CRS}^{t}\left(x_{p}^{t},y_{p}^{t}\right)\times LF^{t}\left(x_{p}^{t},y_{p}^{t}\right)$. We can easily make sure that $EEC = EC \times LFC$. Determining the role of LFC in MI may be regarded as the main contribution of this paper. It is worth noting that this aspect of LFC has not been examined in the literature on the MI thus far and, using the findings of the present study, it can henceforth be accurately analyzed and evaluated by fellow researchers.

The results for both traditional and extended Malmquist productivity index shows that discrimination power of extended index in determining the contribution of each component, including loyalty factor changes, in productivity rate is increased.

In addition, if the VRS technology is considered along with LM and CCR, other novel four-component decompositions of EMI can be obtained that breaks down EMI into PEC, SEC, LFC, ETC, as follows:

$$EMI = PEC \times SEC \times LFC \times ETC \tag{13}$$

5-Case study

In this section, the proposed method is used for 177 branches of the Maskan Bank of Iran, one of the largest state banks in the country, which mainly operates in the housing sector. The data used in the case study are taken from two research projects carried out independently for this bank in 2018 and 2019.

5-1-Input and output data

The Maskan Bank has over 1,100 branches in Iran. This study was conducted on the 177 branches in the capital city of Tehran. The inputs include location index and personnel expenses, while the outputs are deposits, services, and loans. The location of the branch refers to its status in terms of several qualitative and quantitative factors. Computing the location index for all the branches was done as part of the research project 'Model design and implementation for Maskan Bank branches location, contract No. 48-90-2612, dated 13/07/2011, which was also conducted by the authors of the present study. In addition, calculating the outputs i.e. deposits, loans, and services, involved monitoring of the number and values of the transactions on a daily basis. Moreover, the outputs were calculated hierarchically for each of the levels (see subsection 5.2). This hierarchical process was applied separately for the number and values of transactions. Finally, the data are normalized and the indicators are obtained.

The descriptive statistics of the data are shown in table 1. Except for personnel expenses, the indices do not require units as they have been normalized.

Table 1. Statistical data

		2018				2019			
		Min	Max	Average	STD	Min	Max	Average	STD
Inputs	Personnel expenses (1000 IR Rials)	1921242	8573092	3661868	1544549	1896171	11495457	4080219	1861055
	Location index	632.6	1168	1018.368	100.268	632.6	1168	1018.368	100.268
Outputs	Deposits	516.9	8522	1564.222	1402.326	341.3	4840	1359.339	901.4543
	Loans	64.87	2218	748.5359	464.0763	75.62	2075	754.7907	427.9015
	Services	520.1	10250	1310.776	1561.774	482.3	5574	1060.724	880.3856

5-2-Loyalty codes related to outputs

In this subsection, we will calculate the *r* index. As previously mentioned, we use the e-banking codes of branches to calculate the *r* index. After examining 10 codes, the *r* index is obtained as follows: each of the codes consists of two parts: one in Iranian Rial and the other as a number. We first sum up the Rial codes and then the numerical ones. Next, using AHP coefficients approved by banking experts, we normalize the resulting values and obtain the normalized codes. At this point, there is only one normalized code assigned to each branch. In order to normalize the codes, each was divided by their maximum value. Consequently, an *r* index whose value is between 0 and 1 is obtained for each branch. Since our focus is on the deposits, we should find out how loyalty leads to deposits' growth in number and value.

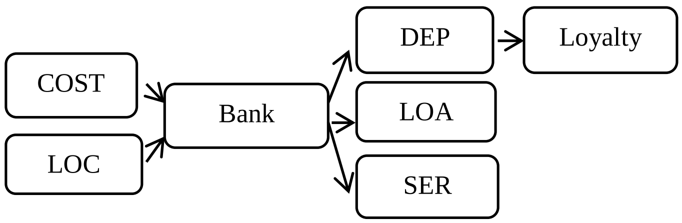

Fig 4. Inputs and outputs

The deposits of the branch whose *r* index is larger are found to have been more durable. In other words, branches with higher service quality appear to have more loyal customers and, as a result, lose fewer deposits. Conversely, branches with relatively lower service quality have customers who tend to withdraw their deposits after a while, lowering the total value of the branches' resources.

Table 2. Descriptive statistics of e-banking codes

	2018				2019			
	Min	Max	Average	STD	Min	Max	Average	STD
E-banking related codes	0.006806	1	0.331308	0.194406	0.000748	1	0.320969	0.193214

5-3-LM construction

In this section, we begin constructing the model. The relative weights assigned to loans and services are set to 1 for all the branches and the weights of the deposits is equal to the *r* index as calculated in the previous subsection. The proposed model for evaluating DMU_p is hence defined as follows:

$$
\begin{aligned}
&\theta_{LM} = \min \sum_{i=1}^{2} v_i x_{ip} \\
&s.t. \sum_{r=1}^{3} u_r y_{rp} = 1 \\
&\sum_{r=1}^{3} u_r y_{rj} - \sum_{i=1}^{2} v_i x_{ij} \le 0 \quad j = 1,2,\ldots,177 \\
&u_1 - r_p u_2 = 0 \\
&u_1 - r_p u_3 = 0 \\
&v_i \ge \varepsilon \quad i = 1,2 \\
&u_r \ge \varepsilon \quad i = 1,2,3
\end{aligned} \tag{14}
$$

Where r is the weight of the deposits of DMU_p and u_1, u_2, and u_3 are the weights of deposits, loans, and services, respectively. In addition, v_1 and v_2 are the weights of personnel expenses and location index, respectively. The LF for DMU_p is calculated according to Definition 1.

5-4-LM results

Efficiency scores calculated through the CCR model and LM along with the LF for 7 selected branches in 2015 are shown if table 3. According to the loyalty codes defined in this study, a LF of 1 means that the branch's performance in terms of technical efficiency and ensuring customer loyalty is perfect. The difference between the CCR and LM scores shows the effect of adding new constraints in the model (5.1). Units with smaller changes in CCR and LM scores after adding the new loyalty constraints displayed better loyalty behavior in related loyalty codes.

Table 3. Results of CCR, LM and LF for selected branches

Branches	CCR Model	LM	LF
1	65.05301	30.04525	0.46
5	47.39608	15.23021	0.32
9	72.68236	30.98521	0.42
11	39.5729	10.35269	0.26
16	60.07314	28.36581	0.47
19	90.36221	40.01236	0.44
40	100	100	1

As can be seen in table 3, branches 40 and 11 have the highest and lowest LF, respectively. A LF of 1 (branch 40) means that performance of the related branch in both perspectives of technical efficiency and loyal behavior with respect to considered loyalty codes is perfect. The difference between the CCR and LM scores shows the effect of adding new constraints to the model (5-1). Branches that have lower scores after adding the new loyalty constraint display better customer loyalty behavior with respect to the loyalty codes. All models are solved by GAMS software.

Table 4. Comparison between deposits and r-index

Branches	Deposits	*r* index	CCR Model	LM
20	5500	45	65.35852	35.32514
45	2550	70	40.32587	15.21547

Branch 45 has fewer deposits than Branch 20, but its better loyalty performance compensated for the shortage of deposits.

5-5-EMI results

Four components of EMI given in (4-10) are computed for all 177 branches and the results for 7 branches selected in previous section are presented in table 5.

Table 5. MI, EMI and their components for selected branches

Branches	LFC	PEC	SEC	ETC	TC	MI	EMI
1	1.25	0.90	1.08	0.45	0.99	0.96	0.54
5	1.35	1.84	0.99	0.52	0.47	0.86	1.28
9	2.85	1.27	1.00	0.25	0.79	1.00	0.90
11	3.45	1.55	0.99	0.33	0.64	0.98	1.75
16	2.90	1.23	1.02	0.21	0.73	0.92	0.76
19	3.58	0.84	0.98	0.36	0.86	0.71	1.06
40	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Table 5 details the data on growth and decline of productivity scores in these branches throughout two consecutive periods between 2018 and 2019. By comparing the first four columns i.e. LFC, PEC, SEC, and ETC, we can determine each component's contribution to increasing or decreasing the EMI.

Taking the customer loyalty factor into account of productivity changes analysis, explores the role of this factor in four component decomposition of MI. The portion of changes in this factor in total factor productivity change is calculated quantitatively and as a percentage by the model objectively. By considering this factor, some branches changed their status from efficient to inefficient, and vice versa The results of EMI were presented to head office managers of branches to verify the validity of them. Fortunately, those closely correspond to the actual system definition. So, the reliability of model is confirmed.

6-Conclusion and future research directions

Commensurate with the growing interest in applications of efficiency analysis and the impact of customer loyalty on efficiency and productivity in banks, this study attempted to develop a DEA-based model by introducing a new index, named the loyalty factor (LF). We demonstrated that higher customer loyalty leads to increased efficiency and thereby productivity. In order to develop the proposed LF, we relied on e-banking, an increasingly significant in today's banking experience. The authors believe that examining other influential factors, such as off-site services, could be further explored in future research. Then we expanded the Malmquist index in the presence of the LF, show the need for such a new factor, and compare the traditional and extended Malmquist index.

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