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A New Optimization Model for Designing Acceptance Sampling Plan Based on Run Length of Conforming Items

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Abstract

The purpose of this article is to present an optimization model for designing an acceptance sampling plan based on cumulative sum of run length of conforming items. The objective is to minimize the total loss including both the producer and consumer losses. The concept of minimum angle method is applied to consider producer and consumer risks in the optimization model. Also the average number of inspection is considered in the constraint of the model. A practical case study has been done and a sensitivity analysis is performed for elaborating the effect of some important parameters on the objective function. The results of sensitivity analysis showed that the performance of the proposed model is logical, reliable in all the cases and also has better performance in comparison with classical method in most of the cases. A computational experiment is done to compare the different sampling schemes. The results of computational experiment showed that the proposed model has better performance due to smaller ANI value in all cases.

Keywords: Quality control, Conforming run length, Acceptance Sampling Plan, Minimum angle method, Taguchi loss function

1- Introduction

The purpose of this paper is to design an optimization model to determine the optimal sampling plan which minimizes the producer's loss plus the consumer's quality loss while considering the average sample number along with the producer's and consumer's risks. Acceptance sampling is a branch of quality control which provides decision rules for producers and consumers to make a decision about a lot of items. Different methods are available for designing economic acceptance sampling methods. The proposed model is developed to consider some key concepts in production environments. The first important concept is related to quality cost. Quality cost has two parts. One part is the cost of unsatisfied consumer which is the result of the deviation of quality characteristic from its target value which is known as the consumer loss. The second part is the quality cost for producer. It is obvious that rejecting an item has cost for producer which includes the cost of processing and reprocessing the item.

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Also inspecting the items has cost because of applying inspection equipment or employing inspectors. It is obvious that total inspection cost is a function of average number of conforming items that is abbreviated as *ANI*. Along with optimizing quantitative measures of sampling method, some qualitative properties of sampling plan should be optimized simultaneously.

Most important criterion for a sampling system is the risk to producer and the risk to consumer. These two risks can be considered in one objective function as performed in minimum angle method (Fallahnezhad, 2012). Thus we tried to develop a general optimization model which considers all these important criteria in one model. Thus all objectives are optimized simultaneously in contrast with previous models where usually only one or two of these objectives have been optimized together. It is clear that we may improve the performance of sampling system in any production environments by taking all important aspects of sampling plan into account.

Ferrell and Chhoker (2002) presented an economical acceptance sampling plan. Their plan has 3 options: (1) they used continuous loss function. (2) Inspection error is considered in their sampling plan. (3) Their model can be used for designing near optimal sampling plan. They constructed graphs in order to make their model more understandable for practitioners. Moskowiz and Tang (1992) presented a new acceptance sampling plan based on Taguchi loss function and Bayesian approach. Aslam and Fallahnezhad (2013) proposed an economical acceptance sampling plan based on Bayesian analysis. Wu *et al.* (2004) proposed an optimization design of control charts based on Taguchi loss functions and random process shifts and they minimized overall mean value of Taguchi loss function by adjusting the sample size and control chart limits. Elsayed and Chen (1994) proposed an economic design of control chart. They used Taguchi continuous quadratic loss function. Their objectives were to minimize the total quality cost and to determine the optimal parameters of control chart. Kobayashi*et al.* (2003) used Taguchi quadratic loss function for economical operation of $(\overline{x}, s)$ control chart. They considered sampling cost and the loss function in order to obtain total operation cost. Arizono *et al.* (1997) presented variable sampling plan for normal distribution based on Taguchi loss function.

Since there is always a need to produce an item on target thus Taguchi proposed that manufacturers should consider loss to consumer. The consumer bears quality loss either in repairs or the purchase of a new item but the manufacturer bears the costs of quality loss due to negative feedback from consumers. Thus any item manufactured away from target value would result in some loss to the manufacture (Taguchi *et al.* 2005).

A new type of control chart that has been successfully applied in many quality control problems is cumulative count of conforming control chart. The cumulative counts of conforming (CCC) control charts are very useful for controlling high-quality processes (Calvin,1983). In this research, a new model for designing economical sampling plan is proposed using Taguchi quadratic loss function based on the run length of conforming items.

In quality inspection methods, items are compared against some standards and then classified as conforming or nonconforming. The control of such inspection processes is usually performed by attribute control chart. The cumulative counts of conforming (CCC) control chart is a type of attribute control chart for determining whether the nonconforming proportion of high yield processes fall within standards (Calvin, 1983; Goh ,1987; Xie & Goh, 1992). The CCC chart is also named the conforming run length (CRL) chart.Classical CCC charts detect the shifts in nonconforming proportion based on cumulative number of conforming items between two successive nonconforming items. The CCC chart is simple, but it is relatively insensitive to process changes. In order to enhance the sensitivity of the CCC chart, Kuralmani *et al.* (2001) and Noorossana *et al.*(2007) proposed the conditional chart which detects the shifts of nonconforming proportion based on the previous data. Amiri and Khosravi (2012) presented maximum likelihood estimator for the change point of the nonconforming proportion with a linear trend. Then, they applied Monte Carlo simulation to evaluate the performance of the proposed estimator. Also they compared proposed estimator with the MLE of the nonconforming proportion based on a single step change. Zhang *et al.* (2014) analysed the performance of CCC chart with variable sampling intervals. They obtained optimal parameters of CCC chart with variable sampling intervals where nonconforming proportion is estimated with Bayesian estimator. Zhang *et al.* (2013) discussed standard deviation of the average run length (SDARL) as a new metric to analyse the performance of CCC chart based on Bayesian estimation. Zhang *et al.*(2010) proposed a Generalized CRL chart (namely GCRL chart) to control the mean value of a quality characteristic under 100% inspection. Chen (2013) applied the variable sampling interval (VSI) in order to increase the sensitivity of the generalized CCC (GCCC) chart. He assumed that data within each sample have correlation. Also he compared GCCC charts with VSI and fixed sampling interval (FSI).Chan *et al.* (2009) applied the concept of cumulative count of conforming chart (CCC chart) in inspection and maintenance planning for systems where minor inspection, major inspection, minor maintenance and major maintenance are available. Bersimis *et al.*(2013) applied a compound rule based on the number of conforming units observed between two successive nonconforming items for monitoring high quality processes.

Run length of conforming items has been applied for decision making about quality of received lots. In this type of sampling plans, sample size is not fixed and average sample number is determined based on the quality of process. Thus, in contrast with classical sampling method where the sample size is fixed, the sample size in the proposed approach is optimized by minimizing the total cost of the system. Bourke (2002),(2003) proposed a continuous sampling plan using sums of run-lengths of conforming items. The cost analysis of sampling method based run-lengths of conforming items has been discussed by Fallahnezhad and Niaki (2013).The concept of control threshold policy has been applied for decision making in this study. Fallahnezhad *et al.* (2014) presented an optimal iterative decision rule for minimizing total cost in designing a sampling plan for machine replacement problem using the approach of dynamic programming. They applied the control threshold policy. Aslam et al.(2013) proposed a decision making procedure for the Weibull distribution based on run lengths of conforming items using a control threshold policy. Fallahnezhad and Nasab (2013) proposed a new acceptance sampling method for lot sentencing problem when inspection is imperfect. They assumed that every defective item cannot be detected with complete certainty. First they determined the probability distribution function of the number of defective items in the batch through Bayesian inference then the probabilities of correct decision are evaluated. Fallahnezhad and Ahmadi Yazdi (2015) proposed a new sampling plan based on the run length of inspected items and Taguchi loss function.

Markov chain can be efficiently implemented in practical quality control problems (Bowling *et al*.(2004), Fallahnezhad and Ahmadi(2014)).Mirabi and Fallahnezhad (2012) presented the Markov analysis of an acceptance sampling plan in a single and two stages model. Fallahnezhad *et al*.(2012) proposed a decision tree approach to accept or reject a batch based on Bayesian modeling. Fallahnezhad *et al*.(2011) proposed a novel acceptance-sampling plan to decide whether to accept or reject a batch of items. In their plan, the items in the batch are inspected until two nonconforming items are found. Fallahnezhad and Nasab (2011) proposed a new control policy for the acceptance sampling problem. Decision in their study is made based on the number of defective items in an inspected batch. The objective of their model is to find a constant control threshold that minimizes the total costs, including the cost of rejecting the batch, the cost of inspection and the cost of defective items.

Bush *et al.* (1953) analyzed the sampling systems by comparing operation characteristic (OC) curve against the ideal OC curve. Their study was a motivation for constructing the concept of minimum angle method. Soundararajan and Christina (1997) proposed a method for the selection of optimal single stage sampling plans based on the minimum angle method. They were the first to use minimum angle method for designing an acceptance sampling plan. But few studies have been done on designing a sampling plan based on minimum angle method. Ahmadi Yazdi and Fallahnezhad (2014) proposed a new sampling optimization model based on run length of conforming items. They used minimum angle model as objective function. Also, they considered average number of inspection (ANI), first and second type of error and first derivation of ANI as constraint. But they did not consider consumer or producer’s loss in their model while these losses have an important impact on the sampling plans. In this research, we considered and calculated total loss in proposed sampling plan. To design a statistically optimal sampling plan, minimum angle method is considered in constraints. To optimize the number of inspections in sampling plan, the first derivation of ANI is considered as model constraint. Briefly, the main idea of this research is to optimize the sampling cost and risk including external costs, internal costs, consumer risk and producer risk in an optimization model simultaneously. This subject was not considered in the previous relevant studies like (2013), (2015), (2011), and (2014).

In designing economic model of acceptance sampling plan, three important types of cost should be considered. First cost is the consumer loss which is incurred due to deviation between quality characteristic and its target level. The second cost is the producer loss which is incurred due to rejecting the item and not selling the item in market and third cost is the inspection cost. The consumer loss is usually represented by quadratic function between the specifications limits and the producer loss is represented by a constant value. It is usually desired to determine standard limits for quality characteristic in the product design in order to check the conformance of the product with quality tolerance limits. These quality tolerance limits are applied in the inspection process. The objective of any product design model is to determine a tolerance limit for the quality characterises so that if its value was within these tolerances then the item would be conforming otherwise it would be nonconforming (Fallahnezhad and Fakhrzad (2012).In general two important factors exist in each production system. First one is the cost. Cost of any production system can be categorized in two types. 1) External cost 2) Internal cost. External cost is resulted from unsatisfied consumers. Measuring this cost is a challenging problem but Taguchi has suggested the concept of loss function for modeling this type of cost that is known as consumer loss function. Internal cost has two categories. One category is incurred due to scrapped items. Each rejected item in the inspection process leads to a cost. This type of cost is known as the producer loss. Second type of internal cost can be evaluated based on number of conforming items which is the total inspection cost. The second important factor in production environment is risk. Risk has two types. First one is the consumer risk that means selling bad lots to the consumer. The second one is producer risk that means rejecting good lots. We can use the minimum angle method for optimizing these types of risks simultaneously. We apply an objective function which considers both of these risks together. The summary of key factors in the proposed approach is denoted in table 1.

Table 1. Key factors in the proposed approach

Key factor	***Cost***			***Risk***	
Types	External cost	Internal cost		Consumer risk	Producer risk
Definition	Consumer loss	Producer loss	Inspection cost	Selling bad lot to consumer	Rejecting good lot by producer
Measurement criteria	Taguchi loss function	Cost of rejecting each item	Average Number Inspected	Probability of accepting bad lot	Probability of rejecting good lot
				Minimum Angle Method	

The well-known Dodge-Romig sampling plans only consider LTPD or risk to consumer thus this sampling method may be impractical in some complicated industrial environment where many other considerations exists. It may lead to additional cost and risk for the producer and it results in inefficient inspection process. Proposed model considers the requirements of both sides of contract. It means loss of producer and consumer, risk to producer and consumer. Also inspection cost has been explicitly considered in the model.

The idea behind this paper is to consider a simple sampling problem as an optimization problem with different objectives and constraints. This fact leads to a generalized sampling method which considers all

criteria of both sides of contract along with technical constraints in practical applications. Every constraint in optimization model reflects one important characteristic of sampling problem.
Thus the contributions of proposed approach are as follows:

- Proposing a general optimization model based on run length of conforming items;
- Considering some important factors in an acceptance sampling plan like costs and producer and consumer loss and risk together in proposed optimization model;
- Using minimum angle method based on run length of conforming items into an optimization model;
- Markov modelling of acceptance sampling plan is developed based on run length of conforming items;
- Comparison of proposed sampling system with classicalsingle stage sampling plan and showing its advantages over classical methods.

We obtained the mathematical formulations of new sampling model in section 2. A real case study is presented for illustrating the application of the new model in section 3. A sensitivity analysis of some important parameters is performed in section 4. A computational experiment is presented in section 5 to compare the performance of proposed model with classical single sampling method. The discussion came in section 6 and we concluded the paper in the last section.

2- Proposed sampling model

Following notations are used in the rest of the paper,
N : The number of items in the lot
δ : The optimal value of tolerance limit

U : The upper control threshold for run length of conforming items

L : The lower control threshold for run length of conforming items

c' : Cost of an inspection

$c_p(x)$: Producer loss that is defined as:

$$c_p(x) = B. \tag{1}$$

$c_c(x)$: The consumer loss that follows a quadratic loss function,

$$c_c(x) = A(x-\mu)^2. \tag{2}$$

AL : The cost of accepting the lot

RL : The cost of rejecting the lot

Where μ is the target value for the quality characteristic and x is the quality characteristic variable. Assume that the lot is inspected until r_{th} non-conforming item is detected. Let Y denotes the number of conforming items. Thus Y is defined as the cumulative run length of conforming items. In this sampling system, if $Y \geq U$ then the lot is accepted and if $Y \leq L$ then the lot is rejected. If $U > Y > L$ then the inspection continues. We define State 1 as the state of inspecting more items and State 2 as the state of accepting the lot and State 3 as the state of rejecting the lot. Let p_{kl} denotes the probability of going from state *k* to state *l* in a single step, we have,

$$p_{11} = P\{U > Y > L\},\ p_{12} = P\{Y \geq U\},\ p_{13} = P\{Y \leq L\}. \tag{3}$$

Where Y follows negative binomial distribution with parameters r, p.

$P(Y = i \mid r, p) = \binom{i-1}{r-1}(1-p)^{i-r} p^r \quad for\ i = r, r+1, \ldots$. The parameter *p* denotes the proportion of non-conforming items in the lot and parameter r denotes the number of detected non-conforming items in the inspected sample. It is obvious that state 2 and 3 are absorbing states.

Let m_{11} denote the expected number of visiting the transient state 1 before absorption occurs, given that the initial state is 1.It can be obtained as follows (Bowling *et al.*[29]):

$$m_{11} = \frac{1}{1 - p_{11}} = \frac{1}{1 - P\{U > Y > L\}}. \tag{4}$$

The long-run absorption probabilities are as follows (Bowling et al.[29]):

$$f_{12} = \frac{p_{12}}{1 - p_{11}}, \quad f_{13} = \frac{p_{13}}{1 - p_{11}}. \tag{5}$$

where f_{12}, f_{13} denote the probability of accepting and rejecting the lot respectively. Since m_{11} denotes the expected number visiting the transient state 1, also in each visit to this state, the average number of inspections is $\frac{r}{p}$ (The mean value of negative binomial distribution with parameters r, p), consequently the expected inspection loss is given by (Ferrell and Choker [2])

$$E(I) = \left(c' + \int_{-\infty}^{\mu-\delta} Bf(x)\,dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x)\,dx + \int_{\mu+\delta}^{\infty} Bf(x)\,dx\right) m_{11}\left(\frac{r}{p}\right). \tag{6}$$

where c' is the inspection cost and the total loss for each conforming items is the summation of inspection cost plus the consumer loss $\left(\int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x)\,dx\right)$ for each conforming item plus producer loss $\left(\int_{-\infty}^{\mu-\delta} Bf(x)\,dx + \int_{\mu+\delta}^{\infty} Bf(x)\,dx\right)$.

The total loss consists of the producer loss, consumer loss and inspection cost. Since $m_{11}\left(\frac{r}{p}\right)$ denotes the average number inspected therefore the expression $N - m_{11}\frac{r}{p}$ denotes the number of items in the lot that have not been inspected and the expression $\int_{-\infty}^{\infty} A(x-\mu)^2 f(x)dx$ is the loss of each item that has been accepted without inspection (consumer loss). Therefore the consumer loss is obtained as follows:

$$AL = \left(N - m_{11} \frac{r}{p} \right) \int_{-\infty}^{\infty} A(x-\mu)^2 f(x)dx. \tag{7}$$

The expected consumer loss $(\mathrm{E}(AL))$ is the consumer loss multiplied by the probability of the lot being accepted (i.e. f_{12}). If the lot is rejected then all items should be inspected therefore the producer loss is obtained as follows:

$$RL = \left(N - m_{11}(\frac{r}{p}) \right) \left(c' + \int_{-\infty}^{\mu-\delta} Bf(x)dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x)dx + \int_{\mu+\delta}^{\infty} Bf(x)dx \right). \tag{8}$$

Thus the expected producer loss $(\mathrm{E}(RL))$ is the producer loss multiplied by the probability of the lot being rejected (i.e. f_{13}).

Consequently, the total expected loss is determined as follows:

$$\begin{aligned} E(TC) &= E(I) + E(AL) + E(RL) = \left(\left(N - m_{11}(\frac{r}{p}) \right) \int_{-\infty}^{\infty} A(x-\mu)^2 f(x)dx \right) f_{12} + \\ & \left(N - m_{11}(\frac{r}{p}) \right) \left(c' + \int_{-\infty}^{\mu-\delta} Bf(x)dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x)dx + \int_{\mu+\delta}^{\infty} Bf(x)dx \right) f_{13} + \\ & m_{11}(\frac{r}{p}) \left(c' + \int_{-\infty}^{\mu-\delta} Bf(x)dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x)dx + \int_{\mu+\delta}^{\infty} Bf(x)dx \right). \end{aligned} \tag{9}$$

Substituting for f_{12} and m_{11}, the expected loss equation can be rewritten as follows:

$$\begin{aligned} E(TC) &= \left(\left(N - (\frac{r}{p}) \left(\frac{1}{1-p_{11}} \right) \right) \int_{-\infty}^{\infty} A(x-\mu)^2 f(x)dx \right) \frac{p_{12}}{1-p_{11}} + \left(N - (\frac{r}{p}) \left(\frac{1}{1-p_{11}} \right) \right) \\ & \left(c' + \int_{-\infty}^{\mu-\delta} Bf(x)dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x)dx + \int_{\mu+\delta}^{\infty} Bf(x)dx \right) \left(1 - \frac{p_{12}}{1-p_{11}} \right) + \\ & \left(c' + \int_{-\infty}^{\mu-\delta} Bf(x)dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x)dx + \int_{\mu+\delta}^{\infty} Bf(x)dx \right) (\frac{r}{p}) \left(\frac{1}{1-p_{11}} \right). \end{aligned} \tag{10}$$

where $p = \Pr\{\text{Reject the item}\}$ is obtained as follows:

$$\begin{aligned} & 1 - p = \Pr\{\text{Accept the item}\} = \int_{\mu-\delta}^{\mu+\delta} f(x)dx, \\ & p = \Pr\{\text{Reject the item}\} = 1 - \Pr\{\text{Accept the item}\}. \end{aligned} \tag{11}$$

where δ is the coefficient of specification limit (tolerance limit). According to the ANI graph, when the nonconforming proportion of lot is equal to its desired value then the first order derivation of the ANI

function at this point should be equal to zero, or in the other words, it is minimized. We try to consider this concept as a constraint in the optimization model and examine its impact on the optimal solution of the model. The first order derivative of ANI function is written as follows (Chen, 2013).

$$ANI = m_{11}\frac{r}{p} = \frac{1}{1-p_{11}}\frac{r}{p} = \frac{r}{p(1-p_{11})} = \frac{r}{k(p)} \tag{12}$$

$$ANI_p(p) = \frac{\partial}{\partial p}\frac{r}{k(p)} = \frac{-rk_p(p)}{k^2(p)}. \tag{13}$$

where,

$$\begin{aligned} &k(p) = p\{1-[F(U-1|r,p)-F(L|r,p)]\}, \\ &k_p(p) = 1-F(U-1|r,p)+F(L|r,p)+(L-1)f(L-1|r,p)-Uf(U|r,p). \end{aligned} \tag{14}$$

where F is cumulative distribution function of negative binomial distribution. Upper and lower limits for the first derivative of ANI function have been considered in the optimization model in order to guaranteethat its value is sufficiently close to zero. Since desired value of nonconforming proportion, p is an important parameter in decision making about the process thus this value is selected as reference value in constraint of ANI derivative. It is obvious that the lower limit is negative and the upper limit is positive. When the interval of these limits would be tighter then it will be closer to zero which is more favorable. This constraint is obtained as follows,

$$\lambda_1 \le ANI_p(p) \le \lambda_2. \tag{15}$$

Where λ_1, λ_2 are lower and upper limits respectively.

We apply concept of producer risk and consumer risk in order to construct the second constraint of optimization model. The purpose is to maximize the value of $\{P_a(AQL) - P_a(LQL)\}$ where $P_a(LQL)$, $P_a(AQL)$ are the probabilities of accepting the lot when the nonconforming proportion is respectively limiting quality level (LQL) and acceptable quality limit (AQL). It is obvious that $1-P_a(AQL)$ is the producer risk and $P_a(LQL)$ is the consumer risk. The method of maximizing the term $P_a(AQL) - P_a(LQL)$ is known as minimum angle method (Soundararajan and Christina[36]). The values of $P_a(LQL), P_a(AQL)$ are determined as follows,

$$p = AQL \to P_a(AQL) = f_{12}(AQL) = \frac{P\{U \le Y_i\}}{1-P\{U > Y_i > L\}}, \tag{16}$$

$$p = LQL \to P_a(LQL) = f_{12}(LQL) = \frac{P\{U \le Y_i\}}{1-P\{U > Y_i > L\}}. \tag{17}$$

It has been tried to reach the ideal OC curve in minimum angle method (MAM). Soundararajan and Christina[36]proposed equation (18) for guarantying that OC curve is closer to the ideal OC curve.

$$P_a(AQL) - P_a(LQL) \geq \omega. \tag{18}$$

Where ω is the threshold that denotes the degree of similarity with ideal OC curve. Now the optimization problem can be defined as follows,

$$\begin{aligned} & \underset{\delta, r, L, U}{Min}\ Z \\ & S.T. \\ & \lambda_1 \leq ANI_p(p) \leq \lambda_2 \\ & P_a(AQL) - P_a(LQL) \geq \omega. \end{aligned} \tag{19}$$

where objective function is as follows,

$$\begin{aligned} Z = E(TC) = & \left(\left(N - (\frac{r}{p}) \left(\frac{1}{1-p_{11}} \right) \right) \int_{-\infty}^{\infty} A(x-\mu)^2 f(x) dx \right) \frac{p_{12}}{1-p_{11}} + \\ & \left(N - (\frac{r}{p}) \left(\frac{1}{1-p_{11}} \right) \right) \left(c' + \int_{-\infty}^{\mu-\delta} Bf(x) dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x) dx + \int_{\mu+\delta}^{\infty} Bf(x) dx \right) \left(1 - \frac{p_{12}}{1-p_{11}} \right) + \\ & \left(c' + \int_{-\infty}^{\mu-\delta} Bf(x) dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x) dx + \int_{\mu+\delta}^{\infty} Bf(x) dx \right) (\frac{r}{p}) \left(\frac{1}{1-p_{11}} \right). \end{aligned} \tag{20}$$

The objective is to find the optimal value of L, U, r, δ in order to design an optimal acceptance sampling policy along with tolerance limits for inspecting product.
One of the meritsof proposed economic model for designing sampling system is to obtain the optimal value of tolerance limit for inspecting one specific product and optimal parameter of acceptance sampling plan, simultaneously.

After constructing proposed method optimization model, it is very beneficial to compare this new model with classical single stage sampling method. The optimization model forclassical single stage sampling method is as follows,

$$\begin{aligned} & \underset{\delta, n, c}{Min}\ Z' \\ & St. \\ & P_a(AQL) - P_a(LQL) \geq \omega. \end{aligned}$$

where the objective function of this model is as follows.

$$Z' = E(TC) = \left((N-n) \int_{-\infty}^{\infty} A(x-\mu)^2 f(x) dx \right) P_a(p) +$$
$$(N-n)\left(c' + \int_{-\infty}^{\mu-\delta} Bf(x)dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x)dx + \int_{\mu+\delta}^{\infty} Bf(x)dx \right)(1-P_a(p)) +$$
$$\left(c' + \int_{-\infty}^{\mu-\delta} Bf(x)dx + \int_{\mu-\delta}^{\mu+\delta} A(x-\mu)^2 f(x)dx + \int_{\mu+\delta}^{\infty} Bf(x)dx \right) n.$$

$P_a(p)$ denotes the probability of accepting the lot which is obtained by cumulative function of binomial distribution.

$$P_a(p) = \sum_{x=0}^{c} \binom{n}{x} p^x (1-p)^{n-x}$$

If the lot is accepted then $(N-n)$ items would be sent to the consumers thus the loss of consumer is multiplied by the probability of accepting the lot and if the lot is rejected then all items must be inspected thus its corresponding loss is multiplied by probability of rejecting the lot. At the end, the average inspected number should be multiplied with its corresponding loss function in order to consider the inspection cost.

It is obvious that the constraint regarding first derivation of ANI function has not been considered in the optimization model because ANI in classical single stage sampling methodis constant, ANI=n
The objective in this method is to find the optimal values of n, c, δ. The comparison between proposed methodology and classical single stage sampling method comes in the next sections.

3- Case study

For illustrating the application of proposed model, a case study in a juice production industry is presented and solved by visual basic programming in Microsoft Excel 2013 using grid search procedure. A juice production factory has produced a lot of $N = 100$ items. Amount of vitamin *C* in juice is inspected by experimenters. The cost of inspecting a juice is $C = 1\$$. The quality characteristic (amount of vitamin *C* in juice), x is assumed to follow a uniform distribution between -6 and 6 and has a target of zero ($\mu = 0$). It is obvious that amount of vitamin *C* in juice cannot be negative and its value is scaled to this interval (Ferrell *et al.* 2002).

$$x = U[-6,6] \to f(x) = \frac{1}{12}$$

The producer loss function is defined as a constant and the consumer loss function is defined as quadratic (Ferrell *et al.* 2002).

$$c_c(x) = 25x^2, c_p(x) = 0.04.$$

limit (AQL) is equal to 0.04 and limiting The values of $c_c(x)$ and $c_p(x)$ were defined based on Ferrell *et al.* (2002). Because the parameter λ was not used and specified in the literature , the value of this parameter was defined randomly in a logical interval. Then in the computational experiment section different values of λ is considered in the model and the effect of this parameter on the objective function is analyzed. Now the subject is to decide about accepting or rejecting this incoming lot when acceptable quality quality limit (LQL) is equal to 0.2. Designers would like toadjust $\omega = 0.9$ in order to guarantee sufficient similarity with ideal OC curve according to minimum angle method. The parameters $\lambda_1 = -400, \lambda_2 = 400$ have been determined as lower and upper limits for first derivation of ANI , (ANI_p) respectively in order to control its value at desired level. We solved this problem with proposed methodology and classical single sampling plan in order to compare the results. We used a grid search method for solving this problem and searched the optimal value of L, U, r, δ in intervals L= [1,2,3,& 10], U= [2,3,4,&,70], r= [1, 2, 3] and δ= [1 to 6 step 0.2]. We solved classical model using a grid search method in intervals n= [5,6,7,&, 50] and c= [1, 2,3,&,10].First we restricted our grid search space using the desirable intervals in order to reach optimal value of L and U and r. So First the feasible values of r and L and U will be determined and the optimal solution which minimizes the objective function will be selected among them. The results have been shown in table 2.

Table 2. Optimal solution of two methodologies

	Proposed Method							Classical single stage sampling				
	L	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)'*
Optimal values	5	26	2	4.8	0.9	35	**826 $**	45	5	4.8	0.9	828 $

According to table 2, proposed model has better performance in comparison with classical single stage sampling method because its cost objective function is less. In this case, the values of MAM constraint are equal for both methods but it is obvious that the value of ANI in proposed method is much less than sample size of single stage sampling. It is interesting to know the tolerance limits for both methods are identical.

4- Sensitivity analysis

We performed sensitivity analysis for illustrating the effect of each parameter on optimal solution. The model has been solved for three different values of each parameter in order to elaborate the effect of each parameter on optimal values of decision variables. Also proposed method and classical method are compared in each case. The results have been shown in tables (3-10).

Table 3. Sensitivity analysis for *AQL*

AQL	Proposed Method							Classical single sampling				
	L	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)'*
0.02	8	22	2	4.8	0.9	18	819	19	1	4.8	0.9	821
0.07	10	36	3	4.6	0.9	29.8	896	**Infeasible**				
0.1	5	46	3	3.8	0.9	31	1138	**Infeasible**				

Table 4. Sensitivity analysis for LQL

LQL	Proposed Method							Classical single sampling				
	L	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)*'
0.15	6	39	2	4.8	0.9	28	844	**Infeasible**				
0.2	5	26	2	4.8	0.9	35	826	45	5	4.8	0.9	828
0.3	8	19	3	4.8	0.9	34	715	37	7	4.8	0.9	722

Table 5. Sensitivity analysis for inspection cost

c'	Proposed Method							Classical single sampling				
	L	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)*'
0.1	5	26	2	4.8	0.9	35	741	45	5	4.8	0.9	742
10	5	26	2	4.8	0.9	35	1680	45	5	4.8	0.9	1682
100	5	26	2	4.8	0.9	35	10216	45	5	4.8	0.9	10236

Table 6. Sensitivity analysis for lot size

N	Proposed Method							Classical single sampling				
	L	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)*'
50	7	25	2	4.8	0.9	22	416	40	4	4.8	0.9	420
150	5	26	2	4.8	0.9	35	1233	45	5	4.8	0.9	1235
200	5	26	2	4.8	0.9	35	1640	45	5	4.8	0.9	1643

Table 7. Sensitivity analysis for *A*

A	Proposed Method							Classical single sampling				
	L	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)*'
0.7	5	26	2	4.8	0.9	35	1020.27	45	5	4.8	0.9	1020.64
10	10	70	3	1.6	0.98	5	2160	45	5	1.6	0.91	2160
40	10	70	3	1	0.98	4	2405	49	5	1	0.93	2405

Table 8. Sensitivity analysis for *B*

B	Proposed Method							Classical single sampling				
	L	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)*'
10	5	26	2	4.8	0.9	35	542.36	45	5	4.8	0.9	542.49
30	5	26	2	4.8	0.9	35	921.72	45	5	4.8	0.9	922.65
50	5	26	2	4.8	0.9	35	1301	45	5	4.8	0.9	1303

Table 9. Sensitivity analysis for λ_2 & λ_1

λ_1	λ_2	Proposed Method							Classical single sampling				
		L	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)'*
-3	3	9	47	2	1	0.9	3	2185	45	5	4.8	0.9	828
-100	100	10	29	2	4.6	0.9	12	897	45	5	4.8	0.9	828
-600	600	5	26	2	4.8	0.9	35	826	45	5	4.8	0.9	828
-1000	1000	5	26	2	4.8	0.9	35	826	45	5	4.8	0.9	828

Table 10. Sensitivity analysis for ω

ω	Proposed Method							Classical single sampling				
	L	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)'*
0.4	6	9	2	4.8	0.4	13	668	12	2	4.8	0.4	666
0.6	8	12	2	4.8	0.6	13	735	16	2	4.8	0.6	738
0.8	7	19	2	4.8	0.8	20	798	33	4	4.8	0.8	801

Table 3 shows that objective function increases by increasing the value of AQL. Also results in table 3 shows that proposed model performs better than classical single sampling method because it has smaller value for cost objective function in different values of AQL. Also proposed model has feasible solution in all cases but classical method doesn't have feasible solution for AQL =0.07 and AQL =0.1. The values of MAM constraints for both methods are equal in all cases. Table 4 shows that objective function increases by increasing the value of LQL. Also it is concluded that the proposed method performs better than classical method in all scenarios Table 5 shows that objective function increases by increasing the value of inspection cost. The results confirm the better performance of proposed method in comparison with classical method based on the measures like cost and average number inspected. Table 6 shows that the objective function increases by increasing the lot size. The classical method has larger value of cost objective function in the cases N=150 and 200. Note that proposed method has smaller ANI in comparison with classical method in all the cases. Table 7 shows that the objective function increases by increasing the value of *A*. also it is seen that even though proposed method and classical method both have equal value of objective function but proposed methodology performs better than classical method based on the MAM and ANI criteria. Table 8 shows that objective function increases by increasing the value of *B*. proposed method has smaller value of objective function and *ANI* in comparison with classical method in all scenarios.

Table 9 denotes that the objective function of proposed method decreases by widening the constraint limits of first derivation for ANI function and it gets closer to objective function of classical method. This result is logical because the cost objective function increases by adding new constraints and assuming tighter limits for the constraints. Since proposed model has two constraints and classical method has one constraint thus the second constraint of proposed model can play an important role in determining the optimal solution of proposed method. But we have seen that proposed method performsbetterthanclassical method based on the measures like MAM and ANI and cost in spite of having more constraints.

Table 10 shows that cost objective function increases by increasing the value of ω as was expected because of having tighter interval for constraint. Also proposed method has smaller value of ANI for $\omega = 0.6$ and 0.8 .Also it is seen that in most of analyzed cases above, the tolerance limit of proposed method and classical method are the same that this result can confirm the validity of results. Another merit of proposed method is to generate feasible solutions for all analyzed cases.

5- Computational experiment

In this section we are going to compare the results of proposed method with classical single sampling in 100 different scenarios of input parameters. 100 different scenarios of parameters are randomly generated by uniform distribution. The results are summarized in table “A” in appendix.

According to Table “A”, ANI values of proposed method is less than their corresponding values in classical single sampling plan (*n*) for all of the cases. Also it is seen that classical method does not have any infeasible solution in 12% of cases while proposed method has feasible solution in all of the cases. Also proposed method has better objective function in 6% of cases but proposed model is worse than classical method in 4% of cases and for the rest of the cases, the objective function of these two methods are equal. Also the optimal values of δ in proposed method and classical method are equal in 96% of cases.

As mentioned, minimum angle method (*MAM*) is an important characteristic of any sampling system thus it’s necessary to compare the results of two methods regarding this important criterion ($P_a(AQL) - P_a(LQL)$). According to Table “A”, the proposed model has larger value of *MAM* measure in 77% of cases and two methods have the same performance in the 4 % of cases thus the proposed method performs better than traditional method in approximately 80% of cases. This is another advantage and merit of proposed methodology.

6- Discussion

Since the proposed model has three integer decision parameters thus optimization method for solving proposed sampling model is simple and just by a simple search method we can easily determine the optimal solution but the main focus of proposed model is to optimize different aspects of quality control simultaneously. First idea is to use conforming run length as a measure for decision making about quality of items. This idea has been successfully applied in many quality problems but applying this measure in an optimization model is not addressed before. Second contribution is to consider qualitative criteria and quantitative criteria of sampling model in an optimization model. The model considers all types of cost and risk functions which may occur in a sampling system. Even though there are some optimization models for sampling system in literature but most of them applied classical approach in a very limited model. Their approach includes specifying two points on OC curve or designing economically optimal sampling system. These models cannot determine the optimal solution for decision maker who consider risk and cost together along with inspection process and loss for both sides of contract.

The main contribution of this research is to compare two different methods of acceptance sampling in quality control. We have shown that sampling based on CRL performed better than (*n,c*) classical sampling system. The comparison of these two sampling systems has been performed in several researches but we have shown that if all important aspects of an acceptance sampling problem were considered in an optimization model then sampling based on CRL would perform better than classical (n,c) design in most of the conditions. The resulted optimization model is not just obtained by considering some important key factors of sampling problems in a complex optimization model. The hidden and important idea is to propose a conceptual model for designing economically and statistically optimal acceptance sampling plan. As explained, CCC charts based on CRL has been widely used in process control but using this concept in acceptance sampling methods is not widely addressed and it is necessary to illustrate its advantages and disadvantages. Also, the criterion MAM is not widely applied in quality control problem while it is an important factor for designing statistically optimal sampling system. Other factors like risk and cost and loss have been considered in the model that helps the decision maker to select a proper decision by considering all important factors. All acceptance sampling models are simple optimization problems. The objective of these optimization problems is to minimize number of inspected items regarding LTPD or AOQ constraints. Acceptance sampling based on CRL can be a suitable alternative for classic l Dodge-Romig (*n,c*) sampling system (Fallahnezhad and Ahmadi Yazdi, 2015) .

The optimization model of classical (*n,c*) sampling systems is very limited with one objective function and one constraint. In this research a generalized economical optimization model based on CRL was developed for sampling system. The objective function and constraints are designed in order to consider most important characteristic of acceptance sampling problems.

Fallahnezhad and Niaki (2013) have compared classical (*n,c*) design with CRL method based on cost objective function. They have shown that these two methods do not have identical performance and the CRL method performs better than classical method in some cases. Aslam *et al.*[26] have compared CRL method and classical sampling system based on ARL objective function. They have shown that CRL method performs better than classical sampling method in most of the cases.

As for the best of author's knowledge, there has not been a general comparison study between these two sampling systems so that we cannot compare the risk, the cost and the sample size of these two methods. Thus, we have developed a generalized optimization problem considering these key factors to analyse and compare the performance of these two methods in order to identify the merits of them based on some simulated case studies. The results of comparison study are summarized in table 11. Second column denotes the percentage of simulated cases studies where proposed method performs better than classical sampling systems and the results in third column denotes the percentage of cases which the classical sampling systems has the better performance. It is observed that the results are in favour of applying CRL sampling systems based on all important criteria of a sampling system.

Table 11. The results of comparing classical model and proposed method

Comparison Indexes	***Proposed method***	***classical method***	***Equal performance***
Existing feasible solution	100%	87%	-
E(TC)	6%	4%	90%
MAM	77%	19%	4%
ANI	100%	0%	0%

The proposed model can be applied as a new tool in quality control environment where classical models cannot provide guarantee for both producer and consumer, also adding other constraints or objective to the optimization model and solving this model is very simple.

7- Conclusion

In this paper, a nonlinear optimization model is developed for obtaining the optimal threshold for tolerance design of products along with decision parameters of sampling method. The concept of cumulative run length of conforming items is employed for decision making in many cases. It is tried to optimize several objectives like the total loss, average number inspection, producer risk and consumer risk in one model where some objective are included in the constraints of the model by defining desired thresholds. It is observed that *ANI* values of proposed method are less than their corresponding values in classical single sampling plan for all of the simulated cases. Also it is seen that proposed method has better objective function in 6% of cases but proposed model is worse than classical method in 4% of cases and for the rest of the cases, the objective function of these two methods are equal. Also the optimal values of tolerance threshold in proposed method and classical method are equal in 96% of cases. Also it is seen that proposed model has larger value of *MAM* measure in 77% of cases and two methods have the same performance in the 4 % of cases thus we can sure that proposed method can perform better that purposed method in approximately 80% of cases. Also it is seen that classical method does not have any infeasible solution in 12% of cases while proposed method has feasible solution in all of the cases.

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Appendix:

Table “A”. Proposed method VS. Classical single sampling

Scenarios	***Input Parameters***								***Proposed Method***							***Classical single sampling***				
	AQL	*LQL*	C_1	*N*	*A*	*B*	λ	*w*	*L*	*U*	*r*	δ	*MAM*	*ANI*	*E(TC)*	*n*	*c*	δ	*MAM*	*E(TC)*
1	0.02	0.21	7.7	63	44.2	28	6	0.87	10	70	3	1	0.99	4	2093	50	6	1	0.92	2093
2	0.04	0.15	88.1	81	22.5	19	539	0.9	8	48	2	1	0.9	3	8553	**Infeasible**				
3	0.03	0.16	11.8	64	17.1	49	498	0.79	10	70	3	1.6	0.99	4	3274	48	5	1.6	0.79	3274
4	0.1	0.23	91.1	196	18.9	36	588	0.52	10	58	3	1.4	0.51	4	23876	50	10	1.4	0.58	23876
5	0.09	0.25	15.7	200	16.8	54	530	0.78	3	34	3	4.6	0.81	5	10489	50	6	1.8	0.83	11812
6	0.02	0.27	62.8	91	9.8	49	29	0.65	10	44	3	2.2	0.99	5	9032	45	2	2.2	0.93	9032
7	0.05	0.24	82.3	85	6.2	29	381	0.64	7	11	2	2.2	0.64	3	8836	11	1	2.2	0.66	8836
8	0.08	0.22	23.5	60	11.2	39	59	0.75	6	36	2	1.8	0.74	3	3251	50	7	1.8	0.84	3251
9	0.06	0.2	4	70	29	52	107	0.72	10	39	2	1.4	0.72	3	3343	45	6	1.4	0.81	3343
10	0.07	0.13	90.5	146	5.2	26	359	0.56	6	33	2	2.2	0.6	3	16068	**Infeasible**				
11	0.02	0.24	77.9	91	25	20	406	0.87	10	70	3	1	0.99	4	8695	43	6	1	0.92	8695
12	0.05	0.14	31	129	33.1	56	546	0.78	8	70	3	1.2	0.95	4	10196	**Infeasible**				
13	0.02	0.24	6.6	190	43.7	12	465	0.7	10	70	3	1	0.99	4	3656	50	9	1	0.77	3656
14	0.02	0.19	82.6	186	11.5	13	18	0.72	10	70	3	1	0.99	4	17584	37	4	1	0.84	17584
15	0.08	0.2	73.4	166	19.8	39	360	0.9	8	52	3	1.4	0.9	4	17640	**Infeasible**				
16	0.05	0.26	12.2	150	9.5	36	505	0.64	3	34	3	4.6	0.88	5	3241	50	4	2	0.9	6083
17	0.06	0.2	8.2	102	29.7	13	553	0.57	10	70	3	1	0.9	4	2134	50	8	1	0.67	2134
18	0.04	0.21	3	104	3	29	7	0.9	9	47	2	2.6	0.9	4	2683	50	4	3.2	0.93	2639
19	0.01	0.21	11.6	121	14.4	60	402	0.76	5	70	1	2	0.86	2	6993	46	3	2	0.98	6993
20	0.08	0.23	41.9	196	44.4	60	391	0.89	8	49	3	1.2	0.9	4	18464	**Infeasible**				
21	0.1	0.24	26.7	121	14	33	73	0.67	3	70	3	1.6	0.96	10	6544	49	7	1.6	0.84	6531
22	0.03	0.25	0.7	111	14.4	56	87	0.59	10	70	3	2	0.99	5	4932	50	4	22	0.98	4932
23	0.04	0.19	75.9	127	45.7	49	193	0.89	9	70	3	1	0.98	4	15148	50	5	1	0.9	15148
24	0.03	0.24	62.3	119	2.2	38	577	0.73	10	18	3	4.8	0.74	25	7528	29	5	4.8	0.73	7593
25	0.07	0.23	4.4	99	7.7	31	146	0.72	7	70	3	2	0.89	5	2800	50	5	2	0.82	2800
26	0.02	0.16	11.4	125	15.7	57	39	0.57	10	70	3	2	0.99	5	7091	49	4	2	0.91	7091
27	0.02	0.26	31	132	4.5	23	193	0.91	10	70	3	2.2	0.99	5	6328	47	3	2.2	0.99	6328
28	0.03	0.28	78.3	136	15.9	44	518	0.59	10	70	3	1.6	0.99	4	15521	49	8	1.6	0.95	15521
29	0.08	0.2	83.2	193	23.9	41	209	0.51	10	70	3	1.4	0.67	4	22801	50	9	1.4	0.54	22801
30	0.09	0.19	82.6	67	19	52	488	0.87	3	70	3	1.6	0.97	10	8321	**Infeasible**				

31	0.04	0.14	28.5	81	9.1	34	221	0.77	10	70	3	2	0.97	5	4480	50	4	2	0.79	4480
32	0.05	0.23	48.8	93	40.4	17	155	0.77	6	70	3	1	0.99	4	6018	50	8	1	0.85	6018
33	0.04	0.17	94.3	76	35.1	36	145	0.73	10	70	3	1	0.98	4	9590	50	6	1	0.77	9590
34	0.05	0.26	48.1	197	25.1	32	576	0.52	10	70	3	1.2	0.97	4	14929	49	10	1.2	0.75	14929
35	0.08	0.17	36	62	31.5	55	225	0.65	10	70	3	1.4	0.66	4	5112	49	5	1.4	0.65	5112
36	0.04	0.23	13.8	192	15	13	74	0.87	9	70	3	1	0.98	4	4916	49	7	1	0.91	4916
37	0.08	0.2	38.7	85	37.7	49	183	0.51	8	70	3	1.2	0.7	4	6956	47	8	1.2	0.6	6956
38	0.09	0.15	91.9	65	31.2	46	516	0.52	10	60	3	1.2	0.53	4	8506	**Infeasible**				
39	0.03	0.14	90	135	46.8	13	143	0.68	10	70	3	1	0.97	4	13943	50	5	1	0.69	13943
40	0.07	0.27	48.3	155	41.1	55	210	0.84	7	70	3	1.2	0.94	4	14871	47	9	1.2	0.85	14871
41	0.1	0.17	87.9	62	21.7	28	384	0.52	9	63	3	1.2	0.57	4	6907	50	7	1.2	0.54	6907
42	0.06	0.24	58.9	192	4.3	52	227	0.8	8	23	3	4.8	0.79	45	15878	42	7	4.8	0.8	15953
43	0.04	0.25	26.2	169	19.6	10	214	0.79	8	70	3	1	0.99	4	6056	47	8	1	0.85	6056
44	0.06	0.28	83.7	196	5.5	30	174	0.53	6	15	3	4.8	0.55	30	16531	32	8	4.8	0.54	16384
45	0.04	0.24	82.6	138	14	47	34	0.81	7	70	3	1.8	0.99	5	16507	50	7	1.8	0.94	16507
46	0.1	0.23	72.3	187	30.5	30	33	0.53	10	58	3	1	0.53	4	18579	49	10	1	0.59	18579
47	0.07	0.3	90.3	200	9.7	21	424	0.63	10	70	3	1.4	0.74	4	21565	43	7	1.4	0.95	21565
48	0.04	0.29	87.4	102	41.8	56	400	0.89	10	70	3	1.2	0.99	4	13947	47	9	1.2	0.92	13947
49	0.07	0.22	57	133	42.1	53	540	0.84	10	70	3	1.2	0.85	4	13778	41	5	1.2	0.86	13778
50	0.03	0.14	72.4	124	8.5	56	422	0.63	3	15	1	2.6	0.64	2	13897	19	1	2.6	0.63	13897
51	0.06	0.15	83.3	130	4.8	47	435	0.72	6	29	2	3.2	0.72	5	14756	49	4	3.2	0.72	14756
52	0.02	0.16	40.3	166	18.2	24	35	0.86	5	70	2	1.2	0.98	3	10221	50	4	1.2	0.9	10221
53	0.05	0.16	66.9	150	38.3	27	202	0.55	10	70	3	1	0.96	4	13763	45	6	1	0.56	13763
54	0.05	0.18	43.9	123	18.2	22	469	0.52	10	70	3	1	0.94	4	7744	50	8	1	0.52	7744
55	0.09	0.16	42.8	184	12.2	30	482	0.58	10	65	3	1.6	0.57	4	12369	**Infeasible**				
56	0.08	0.24	4.5	130	39.7	56	308	0.76	8	70	3	1.2	0.82	4	6844	50	9	1.2	0.77	6844
57	0.04	0.28	51.3	92	26.3	24	592	0.73	10	70	3	1	0.97	4	6681	50	10	1	0.85	6681
58	0.04	0.15	46	174	38.8	18	282	0.5	7	70	3	1	0.98	4	10961	41	5	1	0.61	10961
59	0.01	0.12	15.2	186	18.1	55	398	0.77	9	70	3	1.8	0.94	4	11044	50	3	1.8	0.87	11044
60	0.03	0.21	20	172	5.7	11	347	0.58	3	70	2	1.4	0.99	3	5075	32	2	1.4	0.88	5075
61	0.06	0.26	2.7	85	16.2	39	369	0.68	9	70	3	1.6	0.92	4	3017	50	7	1.6	0.95	3017
62	0.04	0.22	11.5	72	46.8	45	520	0.82	6	70	3	1	0.99	4	3676	46	7	1	0.82	3676
63	0.08	0.21	20.1	135	45.9	36	241	0.9	7	60	3	1	0.9	4	7121	**Infeasible**				
64	0.06	0.21	22.9	107	10.3	35	163	0.69	10	70	3	1.8	0.93	4	5491	50	6	1.8	0.9	5491
65	0.07	0.21	76.9	137	49.4	59	440	0.63	9	70	3	1	0.79	4	17682	50	8	1	0.73	17682
66	0.02	0.17	30.5	178	12.7	21	125	0.72	10	70	3	1.2	0.99	4	8693	50	6	1.2	0.75	8693
67	0.07	0.13	62.1	130	24.8	18	288	0.61	6	70	3	1	0.85	4	10184	**Infeasible**				
68	0.01	0.16	23.8	72	25.3	59	144	0.59	3	70	3	1.6	0.91	10	5253	50	6	1.6	0.73	5253

69	0.09	0.29	19.9	109	16	28	471	0.69	10	49	3	1.4	0.7	4	4749	50	9	1.4	0.92	4749
70	0.03	0.27	84.6	163	32.7	26	290	0.89	9	70	3	1	0.99	4	17680	50	9	1	0.9	17680
71	0.03	0.26	91.3	178	45.2	41	265	0.87	9	70	3	1	0.99	4	22742	50	8	1	0.93	22742
72	0.02	0.28	38.2	73	6.8	53	442	0.57	6	52	2	2.8	0.99	4	5484	50	1	2.8	0.79	5484
73	0.06	0.17	37.6	128	34.2	14	512	0.76	4	70	3	1	0.98	4	6519	50	5	1	0.8	6519
74	0.04	0.18	46.7	187	14.3	21	256	0.83	9	70	3	1.2	0.98	4	12193	48	5	1.2	0.87	12193
75	0.03	0.29	66.4	189	18	51	388	0.52	3	70	3	1.6	0.99	10	20466	49	7	1.6	0.98	20466
76	0.05	0.24	9.9	167	6.9	16	332	0.75	3	70	3	1.6	0.99	10	3911	40	4	1.6	0.9	3911
77	0.01	0.25	90.3	128	23.8	45	287	0.74	10	70	3	1.4	0.99	4	16461	50	10	1.4	0.75	16461
78	0.05	0.26	11.8	199	46.4	43	120	0.68	10	70	3	1	0.97	4	9961	45	9	1	0.78	9961
79	0.06	0.29	23.5	89	9.6	53	471	0.92	10	70	3	2.4	0.93	5	5552	50	5	2.4	0.93	5552
80	0.06	0.16	16.2	149	42.6	20	352	0.59	10	70	3	1	0.91	4	5203	50	6	1	0.65	5203
81	0.07	0.22	37.6	141	4.6	38	343	0.57	9	11	2	3	0.56	4	8931	15	2	3	0.59	8934
82	0.03	0.27	75.4	136	28.9	24	445	0.87	10	70	3	1	0.99	4	13174	44	8	1	0.88	13174
83	0.09	0.22	3.8	198	20.2	28	283	0.68	8	70	3	1.2	0.69	4	5520	50	9	1.2	0.68	5520
84	0.06	0.16	98.1	73	47.3	50	112	0.54	7	63	2	1	0.55	3	10451	50	6	1	0.63	10451
85	0.02	0.14	11.7	170	1.7	56	7	0.53	6	8	1	3.6	0.54	3	6499	14	1	4.8	0.56	5307
86	0.03	0.3	25	162	38.4	23	103	0.77	9	70	3	1	0.99	4	7504	50	10	1	0.91	7504
87	0.04	0.13	43.4	170	32.5	29	411	0.8	10	70	3	1	0.95	4	11796			**Infeasible**		
88	0.07	0.25	4.4	173	4.4	37	565	0.58	10	70	3	3	0.77	6	5096	50	4	3	0.69	5096
89	0.04	0.21	48.5	76	47.4	13	524	0.57	6	70	3	1	0.99	4	4730	43	8	1	0.56	4730
90	0.03	0.15	58.7	139	7.1	19	68	0.86	9	70	2	1.6	0.95	3	10300	50	4	1.6	0.87	10300
91	0.08	0.25	77.5	72	40	26	217	0.88	10	45	3	1	0.88	4	7293	50	8	1	0.89	7293
92	0.09	0.28	88.7	165	41.9	32	52	0.54	10	60	3	1	0.54	4	19441	50	10	1	0.85	19441
93	0.04	0.28	54.9	108	2.1	12	564	0.72	4	12	2	4.8	0.72	22	5191	25	5	4.8	0.72	5224
94	0.04	0.13	99	145	42	59	256	0.52	4	70	2	1.2	0.95	3	21811	36	3	1.2	0.64	21811
95	0.02	0.16	63.6	116	38.4	41	526	0.67	10	70	3	1	0.99	4	11585	29	3	1	0.72	11585
96	0.01	0.17	56.1	100	43.4	42	58	0.76	9	70	3	1	0.99	4	9307	50	6	1	0.78	9307
97	0.07	0.16	37.3	100	33.6	25	233	0.64	10	70	3	1	0.8	4	5970	50	6	1	0.65	5970
98	0.08	0.24	15.2	55	9.2	15	110	0.9	9	49	3	1.2	0.9	4	1536			**Infeasible**		
99	0.04	0.26	75.1	66	36.8	20	451	0.8	7	70	3	1	0.99	4	6213	50	10	1	0.8	6213
100	0.01	0.14	91.2	187	41.1	34	461	0.63	9	70	3	1	0.98	4	22777	50	5	1	0.74	22777