Journal of Industrial and Systems Engineering
Vol. 13, No. 4, pp. 124-141
Autumn (November) 2021

Integrating time and cost in dynamic optimization of supply chain recovery

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Abstract

The occurrence of disruptions has undeniable impacts on supply chain (SC) performance and severely affects its costs and revenues. SC resilience (SCR) reduces the impacts of these disruptions. Among the issues in the SCR, although the recovery of the SC after the disruption is of vital importance, it has not been considered as it should be. To fill this gap, this paper enumerates some important issues in SC recovery planning and proposes a dynamic model for it. One of the features of the proposed model is to consider the recovery time and cost in order to achieve the pre-disruption SC performance. Then, we demonstrate the application of this model in the recovery of a two-echelon poultry SC. Since the developed model is a nonlinear dynamic model, we use the direct collocation method to solve it. The outputs of the sensitivity analysis show that changes in many parameters result in significant changes in model variables. Based on the results, it can be said that the development of appropriate models plays an important role in the analysis of possible alternatives for SC recovery and can help SC managers to deal with disruptions by comparing alternative recovery options.

Keywords: Supply chain recovery, supply chain dynamics, supply chain resilience, optimal control, reactive measures, disruption risk

1-Introduction

In order to achieve a high level of customer service as well as to keep the related costs down, different decisions have to be made regarding the flow of material in the supply chain (SC). In every SC, this flow is influenced by various factors that must be identified and taken into account when making decisions and optimization. This flow of material is affected by SC disruptions and may impact the SC performance (Sawik, 2020).

There is no doubt that SC disruption management is an important part of SC decision making at all levels. Although SC disruption has always been a concern, some issues have made it increasingly important in recent years. First, various factors such as outsourcing, globalization, and greater attention to SC efficiency have made SCs more complex. Second, the occurrence of disruptions in the SCs of different industries has had an upward trend. Third, disruptions may have undesirable effects on SC performance and affect their revenues and costs (Gurnani et al., 2012; Sawik, 2020; Dolgui et al., 2018).

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Given the growing trend of disruptions and their significant impacts on SC performance, SCs need to be resistant to disruptions as well as being able to return to pre-disruption state if affected. SC resilience (SCR) is the concept that deals with this issue (Kamalahmadi and Mellat Parast, 2016).

Although SCR may be a part of the SC risk management, considering the limitations of traditional risk management, such as the emphasis on identifying risks and using statistical data, makes SCR different from it. SCR can be seen as a complement to traditional SC risk management (Fiksel et al., 2015). In the literature, there are various definitions of SCR that one of the prominent issues in almost all of them is the SC recovery. For example Melnyk et al. (2014) define SCR as “the ability of a supply chain to both resist disruptions and recover operational capability after disruptions occur”. According to this definition, dealing with SC disruptions can be accomplished in two important and complementary phases. The first phase is related to pre-disruption measures that create resistance capacity and minimizes the potential effects of the disruption. The second phase, which constitutes the recovery capacity, is a function of the post-disruption measures and returns the SC to its pre- disruption state.

In other words, in order to achieve SCR, two types of measures can be planned: pre-disruption or proactive measures, and post-disruption or reactive measures (Olivares-Aguila and ElMaraghy, 2020). Since disruption risks are difficult to predict (Simchi-Levi et al., 2014), and it is not always possible to achieve SCR through proactive measures, sufficient attention should be paid to the recovery phase and reactive measures. The importance of recovery phase is such that some authors believe that the focus should be on reactive measures, regardless of the type of the disruption (Ivanov et al., 2017).

Another issue that is of particular importance in dealing with SC disruptions is the dynamic nature of the SC. The SC is a dynamic system that changes over time (Ivanov and Sokolov, 2012). Since the SC experiences many changes during and after the disruption, it is necessary to pay adequate attention to these changes when planning for SCR, especially for its recovery (Khamseh et al., 2020).

In this research, we propose a general dynamic model for SC recovery using optimal control theory and demonstrate its application in a case study. In this general model, the dynamics of the SC is modeled by the state space representation and the differential equations that represent the changes of the SC state over the time.

In order to mathematically model SCR problems, the most commonly used techniques include mathematical programming, simulation, and multi-criteria decision making.

1-1- Mathematical programming

The problem of SC design in order to determine the number and location of backup facilities in the occurrence of natural and man-made disruptions investigated by Ratick et al. (2008). By exploiting location set covering model, they developed a model that existing facilities may act as backup facilities to increase flexibility. Hasani and Khosrojerdi (2016) investigated the resilience of a global SC design with respect to the uncertainty of customer's demand and procurement cost and considering correlated disruptions. Wang et al. (2016) investigated the contingent rerouting strategy on a multiple supplier SC from the perspectives of supplier selection and product allocation. They considered different criteria including production capacity, product quality, production cost, as well as decision maker’s preferences to model supplier selection and product allocation problem.

Rezapour et al. (2017) proposed a nonlinear mixed integer programming model for resilient SC design under conditions of competition and supplier disruption risk. In order to improve the resilience of the SC at the tactical level of the production-distribution planning problem, a two-stage mixed stochastic-possibilistic programming developed by Khalili et al. (2017), considering the operational and disruption risks. Sawik (2017) modeled the integrated problem of supplier selection and order quantity allocation, and production scheduling under disruption conditions using stochastic mixed integer programming. Namdar et al. (2018) proposed a scenario-based stochastic optimization model for supplier selection and quantity allocation in order to build a resilient supply base when SC faces disruption risks.

1-2- Simulation

Allen et al. (2006) considered a multi-product SC in the presence of production and distribution capacity constraints, and presented an agent-based simulation (ABS) model to improve the resilience of the SC. Another research used ABS model for addressing SCR in a multi-product, multi-country SC is the work done by Datta et al. (2007). According to their research, one of the key factors for improving operational resilience in dealing with changes in demand is the flexibility of production and distribution operations. In order to assess SCR to disasters, a quantitative approach based on simulation developed by Falasca et al. (2008) incorporating the concept of resilience into the process of SC design. In their paper, SCR quantified by minimizing resilience triangle (Tierney and Bruneau 2007), but they did not conduct experiments.

Colicchia et al. (2010) addressed SCR from the perspective of international transportation and considered the variability of supply lead time as a measure of the international SC resiliency. Using flexibility and redundancy strategies, a simulation model for designing a resilient SC proposed by Carvalho et al. (2012). They considered different scenarios for a three-echelon SC in the automotive industry and examined their effects on improving resilience capability. Schmitt and Singh (2012) considered customer fill rate measure to assess different strategies of backup and inventory location in the case of supply disruptions in a multi-level SC using a simulation model. In order to analyze the ripple effect a simulation model for designing a multi-stage SC facing capacity disruption developed by Ivanov (2017). Lohmer et al. (2020) used an agent-based simulation model to examine the impact of potential risk-related applications of blockchain technology on SCR.

1-3- Multi-criteria decision making

Ivanov et al. (2013) proposed a multi-objective and multi-period model for a distributed production-distribution network problem with different types of capacity constraint where the capacity of facilities and demand for products may change in periods. Adtiya et al. (2014) used fuzzy TOPSIS method for the selection of suppliers in a resilient SC in conditions of uncertainty and incomplete information. Torabi et al. (2015) proposed a multi-objective model incorporating the concept of business continuity for supplier selection and order allocation problem and its impact on achieving resilience in global SCs in the presence of operational and disruption risks. Jabarzadeh et al. (2018) developed a hybrid approach containing a stochastic bi-objective optimization model for designing a sustainable and resilient SC.

Taking into account temporary unavailability of some SC elements and their recovery, a multi-objective formulation including linear programming and system dynamic for re-planning material flow developed by Ivanov et al. (2016) to decide on the balance between service level and cost of SC facing a disruption. Margolis et al. (2018) developed a deterministic multi-objective optimization model for designing resilient SC networks to investigate the trade-off between total SC network cost and its connectivity. To achieve SCR, Hosseini et al. (2019) first calculated the likelihood of different disruption scenarios and then developed a stochastic bi-objective mixed integer programming model for decision making about supplier selection and optimal order allocation. The problem of designing a green and resilient SC to determine the optimal number of facilities using a fuzzy multi-objective programming model investigated by Mohammed et al. (2019).

1-4- Other techniques

The effect of system dynamics and different control policies on SCR examined by Spiegler et al. (2012) and they proposed integral of the time absolute error (ITAE) measure in control engineering to quantify it. Xu et al. (2014) proposed a structural evolution mechanism to assess SCR against supply disruption using cell resilience theory and resilience triangle. Bayesian network theory applied by Hosseini and Baker (2016) to develop a model for supplier selection considering traditional, green, and resilience criteria. Pavlov et al. (2018) used a fuzzy-probabilistic approach to analyze the resiliency of the SC by incorporating the ripple effect and structure reconfiguration. Chakraborty et al. (2020) developed a game-theoretic model to examine mitigating and pricing strategies for a retailer and suppliers in a two-echelon SC under uncertain stochastic demand in the presence of disruption risks.

Some research gaps can be identified by reviewing the existing literature on SCR. First, most models developed to manage SC disruptions focus on the design and planning phases, and the operational level and execution phase have received less attention. This is while addressing executive and operational issues takes up a significant portion of the time of SC managers (Ivanov and Sokolov 2012). Second, in most cases, the SC was modeled as a static system, while it is a dynamic system with various dynamic characteristics such as inventory dynamics and structural dynamics (Ivanov and Sokolov 2012). Third, recovery time and cost are neglected in dealing with disruptions, and they require more analysis (Ivanov et al. 2017). When deciding on the implementation of SC recovery measures, various criteria should be considered, including the effectiveness and efficiency of reactive measures. Effectiveness refers to the effect of a reactive measure in restoring the SC performance to its pre-disruption state, and efficiency indicates the cost of implementing the reactive measures. In addition, more details such as the timing and degree of application of reactive measures should be specified in decision making at the operational level.

In this study, we try to simultaneously address the three aforementioned research gaps and develop a dynamic model for SC recovery. This model is general and with regard to the context of every SC, can be used to recover it. This research extends the work of Khamseh et al. (2020) by considering more alternatives to recover the SC. This results in a nonlinear optimal control problem. Since the application of the proposed model is illustrated using a case study in a two-echelon poultry SC, the results of this study can be used by SC managers to analyze different SC recovery options and make better use of resources in order to recover the SC after a given disruption.

The remainder of this paper is organized as follows. Section 2 describes the problem statement and the conceptual model for dynamic SC recovery. Section 3 presents computational experiments including a numerical example based on a poultry SC along with its solution, sensitivity analysis, and managerial insights. Section 4 concludes this study and outlines future directions.

2- Problem statement and conceptual model

2-1- Problem statement

All SCs are exposed to a variety of risks that can disrupt their flow of material. SCR is a function of proactive and reactive measures, and each category has its own importance. No matter how much concentration and investment on proactive measure, disruption events are inevitable. Therefore, SCs must pay close attention to reactive measures to recover them. So the crucial problem that needs to be investigated is, if the SC is affected by a disruption, how should the reactive measures be used to recover the state and performance of the SC and return it to the time before the disruption occurrence?

To address the above problem, according to figure 1, three basic issues must be considered. First, the SC is a dynamic system that changes over time. This issue becomes even more important when disruptions occur. As it can be seen in figure 2, due to disruption the performance of the SC deteriorates, which should be compensated for by the reactive measures during the recovery period and be approached to pre-disruption condition. Therefore, in the recovery period, the SC performance will experience more variation.

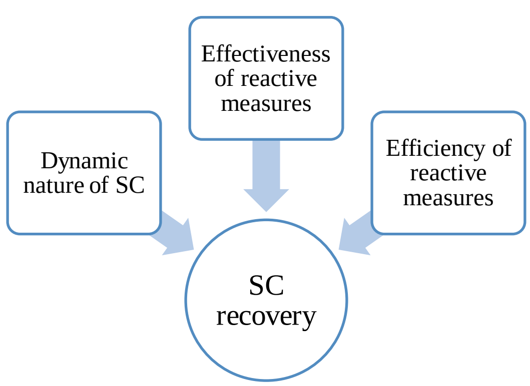

Fig 1. Basic issues in SC recovery

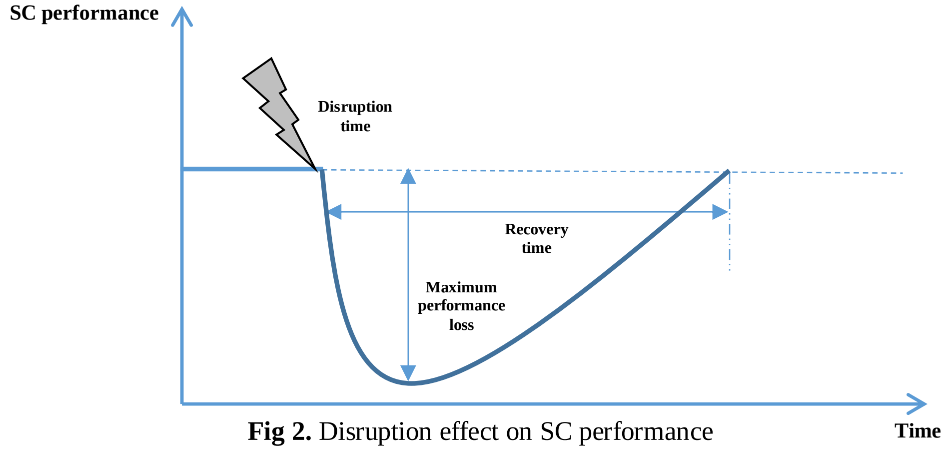

Fig 2. Disruption effect on SC performance

The second is the effectiveness of the reactive measures. Every SC, depending on its context, may implement different reactive measures in recovery phase. These measures do not act in the same way for restoring the SC state to its pre-disruption state. Reactive measures should be selected as far as possible to maximize recovery speed. The third is the cost or efficiency of reactive measures. The implementation of any reactive measure involves costs, and SC should seek to minimize these costs.

Therefore, the main subject of this research is the problem of dynamic SC recovery, with regard to the effectiveness and efficiency of various reactive measures. The effect of reactive measures on restoring the SC performance to the pre-disruption state and the cost of their implementation are considered as criteria for the effectiveness and efficiency of measures, respectively.

2-2- Conceptual model

The purpose of this section is to propose a conceptual decision making model to assist SC managers in implementing reactive measures to recover SC in order to achieve SCR using optimal control theory. Optimal control theory is a branch of control theory related to the optimization of dynamic systems (Sethi and Thompson 2000). Every dynamic system may have three types of variables: input (control), output, and state. The purpose of optimal control theory is to determine the inputs of a dynamic system to optimize a specific performance index. The development steps of the general model are illustrated in figure 3. The explanations of these steps are as follows:

2-2-1- Defining the assumptions

- There is only the possibility of occurrence of one disruption in the SC, and during recovery from this disruption no other disruption occurs in the SC.
- The severity of the disruption is known immediately after its occurrence.
- The occurred disruption does not result in SC collapse.
- There are a limited number of reactive measures to recover the SC.
- The SC is centralized.
- The ideal SC performance is constant over time.
- All parameters are deterministic and constant.
- Recovery time is fixed and known.

2-2-2- Defining the notations

Consider the following notations that are used to describe the conceptual model.

$t: independent\ variable\ (time)$

$x(t): vector\ of\ state\ variables\ at\ time\ t$

$u(t): vector\ of\ control\ variables\ at\ time\ t$

$y(t)$: $vector\ of\ output\ (response)\ variables\ at\ time\ t$
$r(t)$: $vector\ of\ reference\ values\ at\ time\ t$
f, h, w: $appropriate\ functions$

2-2-3- Considering the crucial issues in SC recovery planning

According to figure 1, three issues of the dynamic nature of the SC, the effectiveness of reactive measures, and the efficiency of reactive measures should be taken into account when planning SC recovery.

Dynamic nature of the SC: In continuous mode, the dynamic nature of the SC can be represented by the differential equations. The dynamic nature can be modeled by the state space representation. The general form of the state space representation is provided in equation (1) ($\dot{x}(t) = \frac{dx}{dt}$).

$$\dot{x}(t) = f(x(t), u(t), t) \tag{1}$$

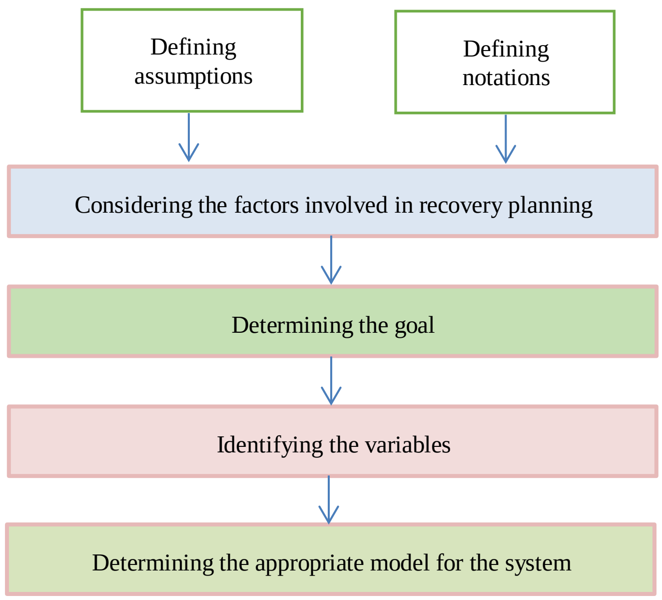

Fig 3. Steps of dynamic SC recovery planning

Effectiveness of reactive measures: This issue can be assessed by how effective the measures are in restoring SC performance. Consequently, any reactive measures that bring the current performance closer to the performance before the disruption is more effective. Therefore, the smaller the value of the expression (2), the greater the effectiveness of the reactive measures (recovery of SC or achievement of SCR).

$$\int_{t_0}^{t_f} h\{y(t) - r(t)\}\, dt \tag{2}$$

Efficiency of reactive measures: The cost of implementing reactive measures is a criterion that can be used to assess their efficiency. Reactive measures that require less cost during the recovery period are more efficient. As a result, the smaller the expression (3), the more efficient the reactive measures. The reason that $x(t)$ included in the expression (3) is that the reactive measures ($u(t)$) affect the state of the system, which can lead to some costs in SC.

$$\int_{t_0}^{t_f} w\{u(t), x(t)\}\, dt \tag{3}$$

2-2-4- Determining the goal

For the control system, different goals can be defined. As mentioned earlier, here, the goal is maximizing the SCR efficiently with regard to the reactive measures that can be supposed.

2-2-5- Identifying the variables

In order to model the recovery planning problem using the optimal control theory, we need to identify three types of variables, namely input (control), state, and output variables in the system (SC). Input or control variables are values that are controlled and considered as decision variables in the system. These variables are highly problem-specific and can be completely different depending on the context of the problem.

2-2-6- Determining the appropriate model

The following general model for dynamic SC recovery can be developed considering the explanations in Sections 2-2-1 to 2-2-5.

$$Min\ \ Total\ Cost\ \equiv Max\ Resilience + Min\ (Control\ Costs + \ Operational\ Costs) \\ = min\int_{t_0}^{t_f}(h\{y(t)-r(t)\}+w\{u(t),x(t)\})dt \tag{4}$$

$Subject\ to$:

$$\dot{x}(t) = \ f(x(t),u(t),t) \tag{5}$$

$$y(t) = \ g(x(t),u(t),t) \tag{6}$$

$$u_i(t) \geq \ u_{i(\min)} \tag{7}$$

$$u_i(t) \leq \ u_{i(max)} \tag{8}$$

$$x(t_0) = x_0 \tag{9}$$

$$x(t_f) = x_f \tag{10}$$

$$Other\ Constraints \tag{11}$$

In the above model, equation (4) represents the total cost to recover the SC. This performance criterion consists of two terms: the former represents the resilience, which is also called the recovery in this study, and the latter states the control and the operational costs.

According to figure 2, maximizing resilience can be defined as the minimization of a function of the difference between the actual output of the system ($y(t)$), and the expected output or reference ($r(t)$). This is shown in equation (12). The smaller the integral, the higher the recovery.

$$Max\ \ Resilience\ \equiv Min\int_{t_0}^{t_f}h\{y(t)-r(t)\}\,dt \tag{12}$$

The second term of the performance criterion shows the costs of applying controls (inputs) needed to recover the SC and other SC's costs (such as holding costs), which are shown as operational cost. Since the various controls applied to the system impose different costs, this term is included as a part of the costs of dealing with disruption in performance criterion.

Equation (5) is the state equation, representing the dynamics of the system under study. Equation (6) shows the system output as a function of system state, input, and time. The output of the system can be expressed in terms of product fill rate, SC profit and so on. Expressions (7) and (8) show operational constraints on control variables. Equations (9) and (10) represent the initial state and the final state of the system, respectively. $x(t_0)$ shows the state of the system after the disruption, and $x(t_f)$ indicates the final state of the system. $x(t_0)$ is actually a function of proactive measures and represents the remaining capacity after the disruption. Expression (11) shows other possible constraints. Although expressions (7) and (8) can be regarded as specific cases of expression (11), since they appear separately in the problem of case study, they are presented separately.

3- Computational experiments

3-1- Numerical example

This section presents an application of the general recovery model developed in Section 2-2 for dynamic recovery of a real case. For this purpose, a case study in the poultry SC is used, and after describing the problem and modeling it, the optimal solution is calculated.

3-1-1- Case study (poultry SC)

Poultry SC consists of several stages. These stages are shown in figure 4 (Khamseh et al. 2020). Owing to the complexities of this SC, only two stages, including broiler farms and slaughterhouses, have been selected to apply the dynamic recovery model. These stages are marked with a red dashed line in figure 4.

In broiler farms, one-day-old chicks produced in the previous stage (parent farms) are reared for a defined period of time. The average rearing period for one-day-old chicks at this stage is between 40 and 45 days. Mature chickens weigh about 2.8 kilograms. After this period, the chickens are sent to the next stage for slaughter, the slaughterhouse, from where they are transported to the consumer market (Khamseh et al. 2020).

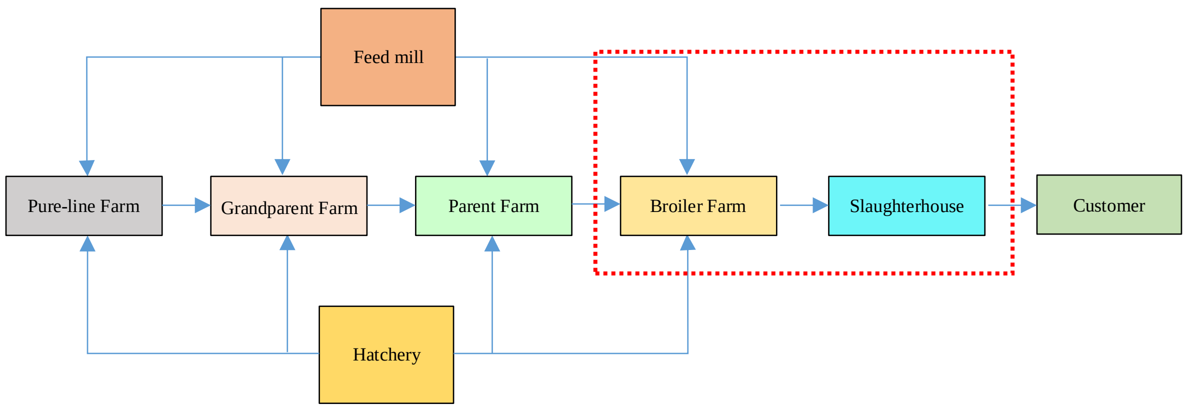

Fig 4. Poultry SC

Various disruptions are conceivable in this two-echelon SC. One of the most important of them is the incidence of diseases in broiler farms. When a disease occurs, depending on its type and severity, a certain percentage of the chicks in the farms cannot be slaughtered and must be eliminated. As a result, the SC performance (sales here) is affected, and the balance between supply and demand is lost. Appropriate reactive measures must be planned to recover the SC in a timely manner and with the minimum cost.

In the following, the application of the proposed general dynamic recovery model is illustrated through the recovery of one of the largest poultry SCs in Iran. Due to confidentiality reasons, the company that owns this SC is called Alpha. The company Alpha meets the daily demand of 200 tons of poultry meat. The percentage of meat extraction in the Alpha Company is 76%. According to the plans made to meet the market demand, the stock of ready for slaughter chicks in broiler farms should be about 2,878,650 kilograms. Assume that 30% of this inventory is eliminated cannot be slaughtered due to disease.

The SC manager wants the recovery to take place within 14 days with the minimum cost. The four possible solutions (reactive measures) for the company Alpha to fulfill this desire are to purchase chicks for the broiler farms, to purchase chickens ready for slaughter, to slaughter chicks from the broiler farms, and to enrich the nutrition. In addition, the company Alpha is subject to the following constraints to recover the SC after the disruption:

- weight of input purchased chicks to the farms
- weight of slaughtered chicks from the farms
- amount of nutrition enrichment

3-1-2- Modeling

In order to develop the dynamic recovery model for the problem under study, following notations are defined:

Parameters:

r_1: $unit\ purchase\ cost\ of\ input\ chicks\ to\ the\ broiler\ farms$
r_2: $unit\ purchase\ cost\ of\ chickens\ ready\ for\ slauther$
r_3: $fixed\ coefficient\ of opportunity\ cost\ of\ premature\ slaughtered\ chickens$
r_4: $unit\ cost\ of\ nutrition\ enrichment\ per\ unit\ weight$
r_5: $operational\ costs\ of\ farms\ per\ unit\ weight$
r_6: $fixed\ coefficient\ \ of\ losses\ caused\ by\ nutrition\ enrichment$
r_7: $shortage\ costs\ per\ unit\ wight$
$d(t)$: $market\ demand\ at\ time\ t$
d_2: $weight\ conversion\ constant\ of\ chicks\ to\ be\ produced$
d_3: $weight\ conversion\ coefficient\ of\ chicks\ to\ be\ purchased$
a: $growth\ rate$
b_1: $farms'\ inventory\ increase\ rate\ caused\ by\ chicks\ to\ be\ purchased$
b_3: $farms'\ inventory\ decrease\ rate\ caused\ by\ chicks\ to\ be\ slaughtered$
b_4: $farms'\ inventory\ increase\ rate\ caused\ by\ nutrition\ enrichment$
b_5: $fixed\ coefficient\ \ of\ mortality\ caused\ by\ nutrition\ enrichment$
u_{i1}: $lower\ bound\ of\ u_i$
u_{i2}: $upper\ bound\ of\ u_i$
t_0: $recovery\ start\ time$
t_f: $recovery\ finish\ time$
TC: $total\ cost$

Variables:

$x(t)$: $total\ weight\ of\ chicks\ in\ the\ farms\ at\ time\ t\ (state\ variable)$
$u_1(t)$: $weight\ of\ chicks\ to\ be\ purchased, input\ to\ the\ farms\ at\ time\ t\ (control\ variable)$
$u_2(t)$: $weight\ of\ chickens\ to\ be\ purchased, ready\ for\ slauther\ at\ time\ t\ (control\ variable)$
$u_3(t)$: $weight\ of\ chicks\ to\ be\ slaughtered, from\ the\ farms\ at\ time\ t\ (control\ variable)$
$u_4(t)$: $percentage\ of\ nutrition\ enrichment\ at\ time\ t\ (control\ variable)$
$y(t)$: $weight\ of\ final\ product\ being\ shipped\ to\ the\ market\ at\ time\ t\ (output\ variable)$

Considering figure 5, the dynamic SC recovery model is as follows:

$$
\begin{aligned}
Min\ TC(x(t), u_i(t), t) \\
&= \int_{t_0}^{t_f} [r_1u_1(t) + r_2u_2(t) + r_3u_3(t)^2 + r_4x(t)u_4(t) + r_5x(t) + r_6u_4(t)^2x(t) \\
&+ r_7(y(t) - d(t))^2]dt
\end{aligned}
$$

$$
\begin{aligned}
&Subject\ to: \\
&\dot{x}(t) = ax(t) + b_1u_1(t) - b_3u_3(t) + (b_4u_4(t) - b_5u_4(t)^2)x(t) \\
&y(t) = d_2u_2(t) + d_3u_3(t) \\
&u_{11} \leq u_1(t) \leq u_{12} \\
&u_{31} \leq u_3(t) \leq u_{32} \\
&u_{41} \leq u_4(t) \leq u_{42} \\
&x(t_0) = x_0 \\
&x(t_f) = x_f
\end{aligned}
$$

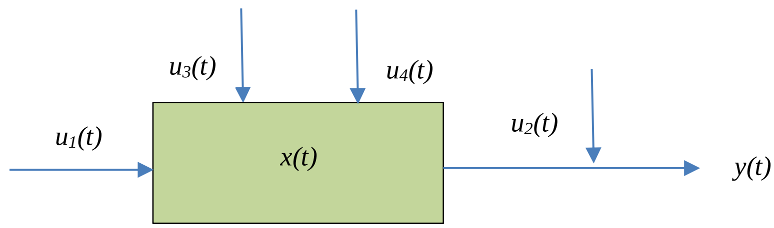

Fig 5. Dynamic representation of the problem

In this model, it is assumed that the mortality caused by nutrition enrichment can be estimated by a quadratic function of nutrition enrichment percentage. According to the parameters obtained from the SC of the company Alpha, the final dynamic recovery model will be as follows:

$$
\begin{aligned}
&Min\ TC(x(t), u_i(t), t) \\
&\qquad = \int_0^{14} \{17000u_1(t) + 18300u_2(t) + 0.0036u_3(t)^2 + 678.75x(t) \\
&\qquad + 565.6x(t)u_4(t) + 9675u_4(t)^2 x(t) \\
&\qquad + 2475(0.76(u_2(t) + u_3(t)) - 2 \times 10^5)^2\} dt \\
&Subject\ to: \\
&\dot{x}(t) = 0.046x(t) + u_1(t) - u_3(t) + (0.0714u_4(t) - 0.43u_4(t)^2)x(t) \\
&125000 \le u_1(t) \le 170000 \\
&184000 \le u_3(t) \le 263000 \\
&0 \le u_4(t) \le 0.1 \\
&x(0) = 2015050 \\
&x(14) = 2878650
\end{aligned}
$$

3-1-3- Solution approach: direct collocation

The dynamic model developed to recover this SC is nonlinear, and it is unlikely to find an analytical solution due to the structure and the number of control and state variables. For this reason, numerical methods is used to solve the model. Numerical methods to solve optimal control problems can be divided into direct methods and indirect methods (Betts 2010). Using direct methods to solve optimal control problems is very common. These methods avoid the complexity of calculating derivatives related to the necessary optimality conditions and have higher flexibility in applying various constraints to the problem. In the direct methods, the optimal control problem is transformed into a nonlinear programming problem (NLP) using the discretization process. Hence, these methods are known as first discretize, then optimize. One of the most common direct methods is the direct collocation method, which is used to solve the dynamic recovery model in this research. Direct collocation methods are robust in relation to inaccurate guess of initial conditions and have good convergence properties. In the direct collocation method, by converting the dynamic expressions of the optimal control problem into simple algebraic expressions, the process of converting the optimal control problem into NLP is performed directly. To this end, the planning horizon is divided into $N - 1$ subintervals via N nodes, known as collocation points. Then both control and state variables are discretized using time node values. Therefore, if we use N, n_u and n_y to display the number of time nodes, the number of control variables, and the number of state variables, respectively, the number of optimization variables in NLP will be $N\ (n_u + n_y)$. Conventional methods of integrating differential equations such as Euler and the trapezoidal methods, can be used to transform state equations, objective functions, and constraints into discrete form (Rao 2009; Diehl et al. 2006).

In this study, we use the trapezoidal collocation method to solve the dynamic recovery model developed in section 3.1.2. This method is notable for its ease of use, powerful convergence properties, and straightforward analysis. In trapezoidal method, the following formulas are used to parameterize the integral and differential expressions in the objective function and system dynamics (Betts 2010):

$$\int_{t_0}^{t_f} w\, dt = \sum_{k=0}^{N-1} \frac{d_k}{2}(w_k + w_{k+1}) \tag{13}$$

$$\dot{x} = f(x,u,t) \;\Rightarrow\; x_{k+1} - x_k = \frac{d_k}{2}(f_k + f_{k+1}) \tag{14}$$

where
$x_k = x(t_k)$: state variable at node point k
$d_k = t_{k+1} - t_k$: duration of subinterval k
f_k: system dynamics at node point k

The solution procedure for solving developed dynamic recovery model was coded in MATLAB, using FMINCON's sequential quadratic programming (SQP) algorithm as the NLP solver. The results of state and control variables changing over time are shown in figure 6. The value of the objective function is equal to 7.6586×10^{10}.

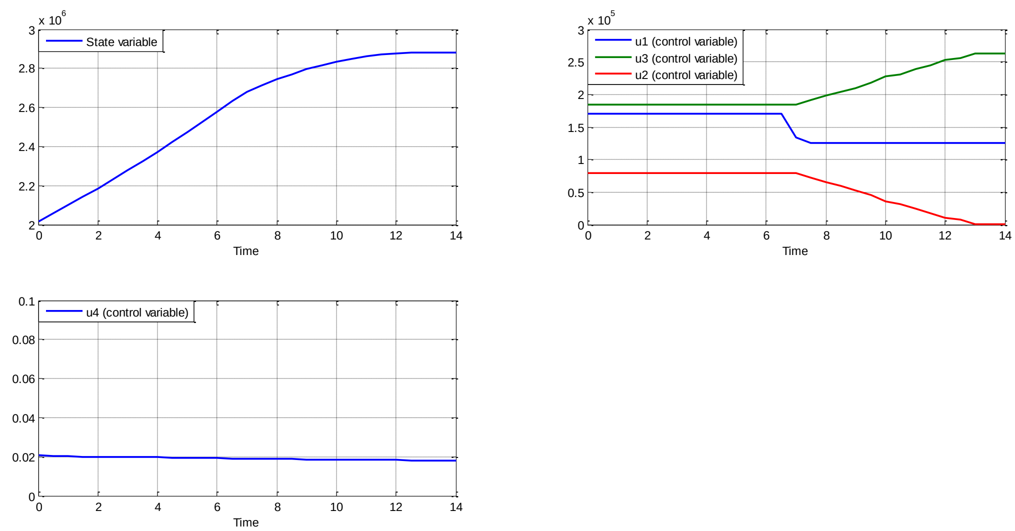

Fig 6. Optimal trajectory for the dynamic recovery problem

3-2- Sensitivity analysis

In the development of mathematical models, there is always some uncertainty in the parameters. In order to investigate the effects of these uncertainties on the output of the model, a set of sensitivity analysis was performed on four model parameters including a, b_5, r_2, and r_3. In the sensitivity analysis, compared with the original values, the values of each parameter changed by 10 and 20 percent. The results of this analysis are depicted in figures 7 to 10. In these figures, the purple and orange lines show the results of a 10 and 20 percent decrease in the parameter value, respectively, and the red and blue lines show the results of an increase of a 10 and 20 percent in the parameter value. The green line also shows the value of the control variables without changing the parameters.

As can be seen from the outputs of the sensitivity analysis, changes in the studied parameters have different effects on the control variables. Although the control variables are very sensitive to the changes of parameters a, r_2, and r_3, changes in parameter b_5 have little effect on other control variables except u_2.

The control variables behave quite differently with respect to the changes of r_2 and r_3. In general, with increasing r_2, the variables u_1, u_3, and u_4 increase, and the variable u_2 decreases. However, increasing parameter r_3 reduces the values of variables u_1, u_3, and u_4, and increases u_2. Compared with the first two parameters, the control variables show different behaviors in the changes of a.

The changes of u_3 are in the same direction as changes of a, while changes of u_2 and u_4 are in the opposite direction. Although u_1 is very sensitive to the changes of a, its changes do not have a general direction. The only control variable affected by changes in parameter b_5, is u_4, which changes in the opposite direction to the changes in parameter b_5.

A summary of the changes in the control variables relative to the changes in the parameters studied in the sensitivity analysis is given in table 1.

Table 1. Behavior control variables in relation to parameters

Parameters	Control variables			
	u_1	u_2	u_3	u_4
a	●	−	+	−
b_5	+	●	●	−
r_2	+	−	+	+
r_3	−	+	−	−

+: same direction   −: opposite direction   ●: no specific direction

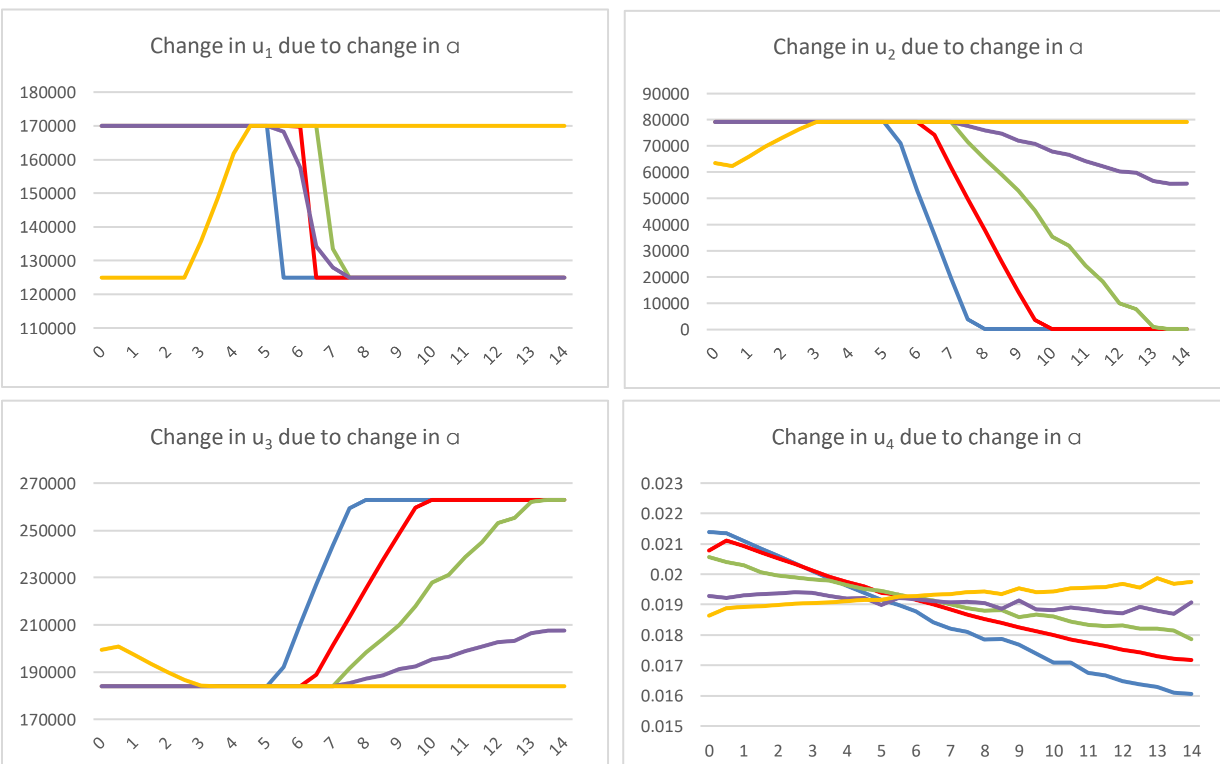

Fig 7. The effect of changes in parameter a on control variables

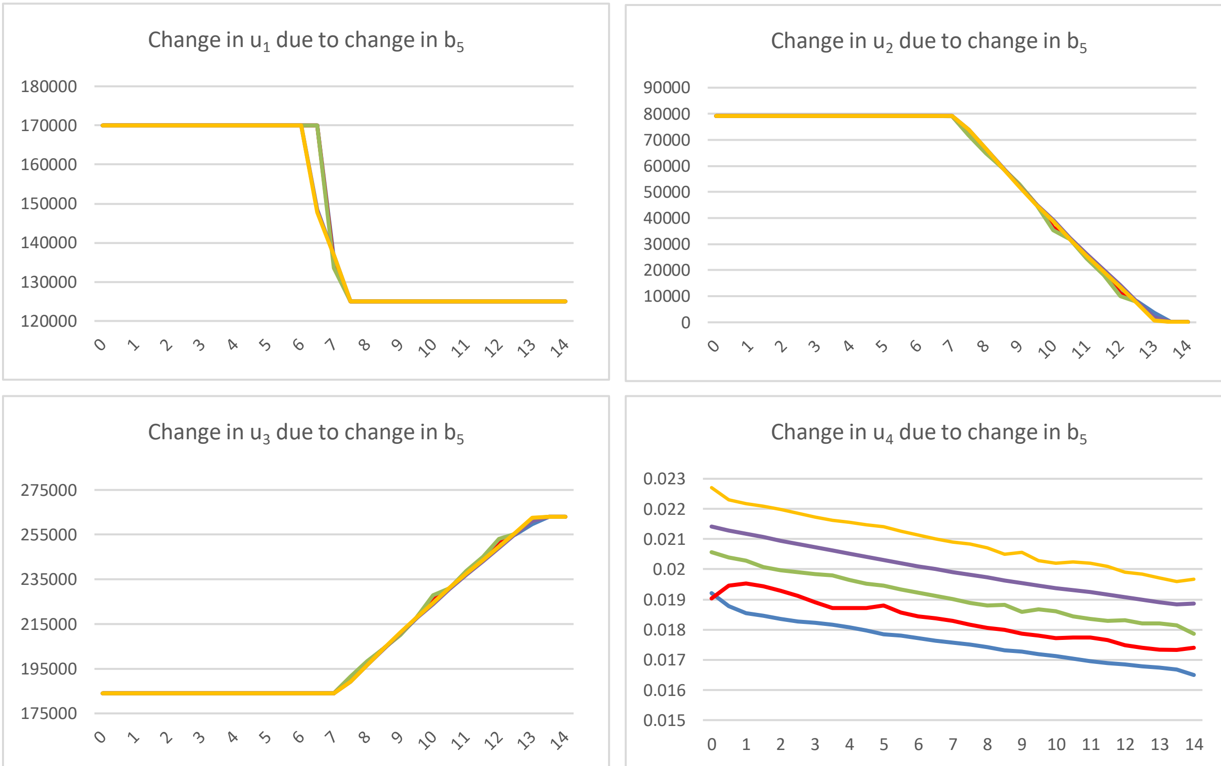

Fig 8. The effect of changes in parameter b_5 on control variables

Fig 9. The effect of changes in parameter r_2 on control variables

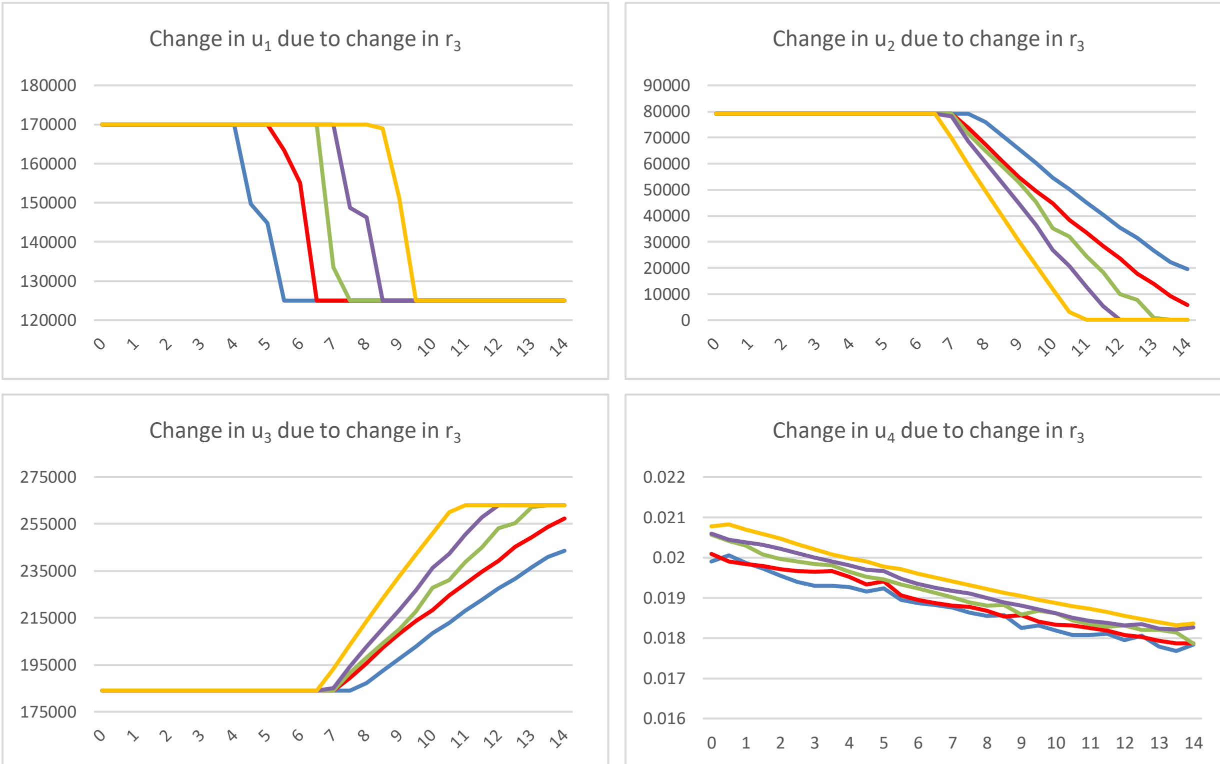

Fig 10. The effect of changes in parameter r_3 on control variables

3-3- Managerial insights

Some important managerial insights for enhancing the resilience of SCs in real-world recovery problems can be derived from this research.

- *The need for appropriate decision making tools and models to recover SCs*: SC recovery has its own complexities, and SC managers need tools to support them in decision making. It may seem obvious that in order to recover the SC of the case study, the control variables u_1 and u_3 should be set at their upper and lower limits respectively, but the output of the recovery model does not confirm this. In other words, decisions in this area should be made by considering various criteria, including the effectiveness and efficiency of reactive measures, as well as paying attention to existing constraints. In addition to having experience, this requires the use of appropriate decision making models in that regard.
- *The Consideration of all possible alternatives for dealing with disruptions*: In the example of the case study, four alternatives were identified to recover the SC, and the output of the model indicates the simultaneous use of them. Therefore, in order to manage SC disruptions, it is better to consider all possible options first, and make sure that no alternative is missed. Using techniques such as brainstorming, before and after a disruption, can increase the potential and capacity to manage the disruption.
- *The specificity of every SC (impossibility of some alternatives)*: It has been mentioned in many papers that considering more capacity in the design phase can help to enhance the resilience of SCs (Fattahi et al. 2020). However, not all capacity increasing alternatives are possible in all SCs. One example of greater capacity is holding of more inventory. However, in the SC of the case study, it is not possible to store the final product to be shipped to the market, and daily demand must be met in the same period.

4- Conclusion and future works

SC disruption is inevitable, so a proper recovery plan must be in place. Reactive measures are those actions taken after the occurrence of disruptions. Their purpose is to restore the SC performance to its pre-disruption state. Various criteria may be considered for planning reactive measures. The effect of the reactive measure on improving SC performance and the cost of implementing it can be considered as the effectiveness and the efficiency of measures, respectively. In addition, it should be noted that the SC is a dynamic system and decisions must be made with this in mind. Therefore, it is best to use a dynamic model for SC recovery.

In this research, the problem of dynamic SC recovery with respect to the recovery time and cost was investigated, and a general model based on optimal control theory was proposed for it. The purpose of the model was to restore SC performance to the pre-disruption state at a specified time with the minimum cost. Since every reactive measure has its own effect on the recovery of the SC performance, the time and extent of their implementation should be determined in such a way that in addition to bringing the SC performance to the pre-disruption state at the specified time, their implementation costs are also minimized.

The application of the proposed model was illustrated by recovering a two-echelon poultry SC after a disease disruption. Four applicable reactive measures recognized to recover the SC. These included the weight of purchased chicks (input to the farms), the weight of purchase chickens ready for slaughter, the weight of chicks to be slaughtered, and the percentage of nutrition enrichment. The outputs of the model recommended the simultaneous use of all four reactive measures. Based on the results of applying the dynamic recovery model on the case study and the conducted sensitivity analysis, it can be said that since every SC is widely specific, managers should use appropriate decision making tools to analyze all possible options for SC recovery.

Various developments can be enumerated for the present study by relaxation some of the assumptions considered. SCs can be managed at different levels of centralization. The SC can be thought of as decentralized where every member acts independently using their control variables to recover the SC. We considered the possibility of only one disruption, while SCs are prone to different disruptions. Considering multiple disruptions, which may occur simultaneously or sequentially, can lead to the definition of new problems. Uncertainty is one of the most significant challenges in SCs. One example is the uncertainty in the model parameters. Therefore, by assuming the nondeterministic parameters, more realistic models and solutions can be obtained. Moreover, in this study, only one supplier has been assumed to buy ready-to-slaughter chickens, while ready-to-slaughter chickens can be purchased from several sources with different constraints. In many SCs, multiple products are shipped to the market simultaneously, and the current model can be extended to multi-product conditions.

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