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Adaptive optimization model based on supply function equilibrium in modern power markets

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Abstract

In this paper, an adaptive optimization model based on a closed-loop control system is developed to regulate the strategic bidding process of generation companies (GenCOs) in day-ahead electricity markets. Each day, the bidding problem of each GenCO is submitted in the form of a supply function consisting of 24 sub-problems, one for each hour of the next day. The hourly market clearing price and the total demand of the next day are the unknown values in the bidding problem that should be estimated by the concerned GenCO. The GenCOs, as the main players in the market, receive feedback signals for market clearing price and demand for each hour of the previous day, based on which they set their bidding for the next day. In the optimization model, the limitations on the production level and production change rate are considered in terms of the minimum and maximum quantities constraints. To better adapt to the market demand and price dynamics beforehand, we also used an adaptive forecasting algorithm for the next day's demand and clearing price. Using this adaptive dynamic model, the network operator can clear the market based on the bids received from the GenCOs and the consumers. As we concentrated on the GenCO side, as the most influential player of electricity markets, the bids from the demand side are considered here as a whole and modeled by a linear function. Finally, the real market data from the day-ahead Nordic electricity market (Nord Pool) are used as the case study to verify the effectiveness of the proposed model and its adaptive algorithm. The results show that the GenCO that uses the proposed model can gain more profit in comparison to those that take non-strategic behavior (naive strategy) in the market.

Keywords: Day-ahead electricity market, supply function equilibrium, strategic bidding, Generation Company (GenCO), adaptive control system

1- Introduction

In the last decades, power markets around the world have experienced reconstruction intending to improve their efficiency and competitiveness. In new reconstructed electricity markets, market players compete for more shares of the market to maximize their profits. Despite plenty of efforts to push power markets towards perfect competition, the constraints that are inherent to electricity make these markets more akin to an oligopoly with a few dominated generators rather than perfect competition.

The main intrinsic constraints are 1) non-storable in large amounts, 2) needing physical transmission lines for energy transmission, and 3) high capital cost and considerable time for establishing new generation units and transmission lines.

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In this structure of comparative electricity market, an individual supplier can manipulate electricity price by taking a strategic bidding scenario so it has a form of the power market. Regarding the oligopolistic nature of the electricity market, the generator companies (GenCOs) will have a power market with dominated effect on the market clearing conditions. Therefore, studying the behaviour of GenCOs is quite important in the market analysis.

Most of the current modern electricity markets are pool-based markets, in which both the consumers and generation companies submit their bids for the market independent system operator (ISO). Then the ISO clears the market by intersecting the total demand and total supply curve, through which the price and quantities of energy (produced or consumed) are specified.

Game theory is one of the popular methods for analysing the interaction among players in the reconstructed electricity market. The models based on this method are categorized into three groups: Bertrand, Cournot, and Supply Function Equilibrium (SFE). Because of complexity, SFE models have rarely been used in comparison to the others. However, since it reflects the supplier behaviour in a more realistic view, it provides more precise models. Accordingly, we developed an adaptive closed-loop strategic bidding process in an SFE based market. The developed model is adapted to the market trend by getting the feedbacks of the forecasted price and demand.

After Schweppe (1988) for the first time discovered the spot nature of the electricity price, the process of GenCos optimized bidding have frequently been noticed by researchers, regarding its significance in electricity market operation. One of the best models introduced for this purpose is the work by Liu, Y et al. (2006) that is based on control system approach. In this paper a similar model with new features as below has been developed for the analysis of day ahead electricity market with oligopolistic structure:

1. The bidding functions are in form of supply function. This helps the supplier to decide simultaneously about the production and price in such a way that is very close to what happen in real world.
2. A forecasting method based on adaptive recursive filters have been implemented in the model for predicting the next day cleared price and quantity. This makes the model to capture the real values in advance so the proposed bidding strategy by Gencos will be more realistic.
3. In the proposed model the minimum and maximum quantity of production by each Genco have been implemented. These constraints are one the most important technical constraints in the electricity market which are not covered in some similar works.

The rest of this paper is organized in four chapters. At chapter 2 a review of related literature has been given. In chapter 3 the proposed model and its structure has been given in detail. In chapter 4 a case study based on data from NordPool market has been simulated by the proposed model and the results have been discussed there. At the last, in chapter 5 the conclusions have been discussed.

2- Literature review

For the first time, Schweppe noticed the dynamicity of electricity prices in some of the power markets (Schweppe, 1988). Afterward, the problem of strategic bidding for suppliers has been increasingly addressed. Various models have been introduced for solving these problems, which generally can be categorized into three groups: 1- Optimization for one GenCO, 2- simulation-based models, and 3- market equilibrium (Ventosa, et al., 2005). In the first group, the strategic behaviour of just one GenCO (as the main market player) to gain the maximum profit is surveyed and the effect of the other parts is modelled based on some assumptions. In the two later ones, the interaction among all the market players and their impacts on market trends are surveyed to demonstrate market dynamics and equilibrium. The general form of the equilibrium models in the electricity market is linked with use of game theory in which the Nash equilibrium of the game is sought. Here we briefly review the literature of each of these groups.

2-1-Optimization-based models

Generally, in optimization models, to find the optimal solution of the so-called optimization problem, the mathematical programming techniques such as Non-Linear Programming (NLP), Dynamic Programming (DP), Multi-Objective Linear Programming (MOLP), Mixed Integer Programming (MIP), and Stochastic Mixed-Integer Linear Programming (SMILP) are applied. In these models, usually, a complex parameter such as opponent behaviour, demand function, interactions between market players, and so on are simplified with some assumptions such as defining exogenous variables or ignoring the impact of some components (Bunn et al., 2010). The works by De la Torre & Conejo (2002) Fleten (2007), Sen (2006), Gross and Finlay (1999), Wen (2001) and Rahimiyan (2007) are some good examples of such works. . In the study conducted by Sharifi et al. (2020), a bi-level Stackelberg-based model between an electricity retailer and consumers is presented. They proposed a stochastic optimization model to deal with a price-maker retailer's strategic bidding (in a flexible demands manner) in the day-ahead electricity market. Their result indicates that their proposed model enhances the retailer's profit.

2-2-Simulation-based models

By the deregulation of the electricity market along with the emergence of new components such as renewable sources of generation, the electricity markets have become more complex than before (Ventosa et al., 2005). In contrary to the static models, they take into account the fact that stakeholders decide on the historical data. In modelling such complexities, two main simulation techniques have been used till now: Agent-Based Models (ABM) and System Dynamics (SD). These models have generally been used to analyse market issues and strategic behaviour of the market players in deregulated electricity markets (Teufel et al., 2013). Agent-Based Models have been used in a variety of applications, the first of which goes back to the late 1940s. These models can be divided based on the learning algorithm they use. Some of these algorithms are Genetic Algorithm (GA), adaptive algorithm, and numerical analysis. Sheble (2001) has used GA for solving the proposed Agent-Based Model (Gao and Sheble, 2010). In Naghibi Sistani et al. (2006) market players use Q-learning algorithms for their optimization problems and learn from their past situation in the market to improve their future position.

System dynamics introduced by Forester are used mostly for analysing complex mechanisms. From the 1970s, various models have been introduced in the fields of energy. For example, Roger Neil has introduced COAL1 and COAL2 models for studying fossil resources, which have been used later in the US energy markets. Wang et al. (2019) investigated all the players' bidding behaviour in the electricity market and proposed a hybrid simulation model (HSM). The proposed HSM model incorporated the agent-based simulation (ABS) and system dynamics simulation (SDS), in which input variables of one kind of simulation gathered from another's output.

2-3-Equilibrium models

The equilibrium models in the electricity market are mostly presented by game theory models, whose objective is to find the Nash Equilibrium, the point at which no player can individually increase its payoff (Song, et al.,2003). The game theory technique provides a framework for the analysis and explanation of the power market operation especially when there is a strategic bidding behaviour among the rational market players. To cope with the underlying problems, usually, some assumptions are used in these models, among which is the rationality of all the players, which usually does not take place in practice (Gao and Sheble, 2010). Another assumption is the necessity of common knowledge on actual generation costs of all GenCOs (Song, et al.,2003).

In imperfect competitive markets, GenCOs use three main models to form their bidding strategy respective to the level of competition: Bertrand model, Cournot model, and SFE model. In Bertrand model, as the most competitive one, the GenCOs compete with each other in an oligopolistic framework considering the price as the strategic variable and neglecting the production capacity constraints. In Cournot models, as the least competitive one, the GenCOs compete with each other by considering the production quantity as the strategic variable with the assumptions of homogenous products and price-dependent demand, so that an MCP is generated by the intersection of aggregated supply and market demand curves. Lastly, in the SFE models, we have the GenCOs in competition with each other through the simultaneous decision of price and production quantity in the form of supply functions. Therefore, the resulting level of competition and the price equilibria comes generally between the Bertrand model and Cournot model (Younes and Ilic, 1999). The SFE approach in the electricity market was first developed by Klemperer and Meyer (1989). Then, Green and Newbery (1992) developed it by incorporating production capacity constraints, used for the privatization of the British electricity market. In Baldick et al. (2004) through some discussions about the advantages of SFE models, it is shown that they can better benefit the GenCOs in comparison to the equivalent Bertrand and Cournot models. Also, for analysing the market mechanism and strategic bidding of GenCOs with incomplete information, Li and Shahidehpour (2005) suggested an SFE model comprising of a two-level optimization problem, in which the individual GenCO maximization problem and market clearing process are solved at the first and the second level, respectively. Abapour et al. (2020) proposed a game theory method to optimize demand response aggregators' bidding strategies in the energy market based on customer benefit function and price elasticity. They modelled the interaction between the system operator and aggregators and solved the game by applying the Nash equilibrium idea. Guo et al. (2021) attempted to model the electricity market as a double-sided non-cooperative game and consider multiple electricity firms and customers by employing the supply function equilibrium model. On the supply hand, four parameterization approaches of the supply function equilibrium model are investigated. On the demanding hand, they employed a comprehensive model to advantage each customer and lead the electricity utilization in reality.

Based on what mentioned above, for the sake of simplifying the mathematical works and having a more efficient bidding process, we propose an adaptive mechanism based on a closed-loop control system for the analysis of the bidding strategy of individual GenCOs in a Day-ahead market. Since GenCOs are the main players in the recent power market, the rationale behind this model, which is mostly concentrated on the generation side, can be justifiable.

3- Modelling process

To adapt the model with the actual trend of the market instead of using the estimated data, using the historical data published by the market every day, we apply two adaptive filters based on Recursive Least Square (RLS) algorithm for forecasting market demand and clearing price. The predicted data will be applied for building the SFE-based bids in the process of the dynamic control system.

A. *GenCOs' bids*

In short-term period, the variable cost of power generation should be considered. Basically, the variable cost of power generation is the fuel cost. Usually such variable cost of generator *i* is assumed to be a quadratic and concave function as below (Liu, 2006):

$$C_i\big(q_i(t)\big) = b_i(t) + a_i(t)\, q_i(t) + \frac{1}{2} c_i(t) q_i(t)^2; \quad i = 1, \dots, n \tag{1}$$

Where $q_i(t)$ is the total amount of production (MWh) of GenCO *i* and $a_i(t)$, $b_i(t)$, and are $c_i(t)$ time-variant non-negative coefficients as they are dependent on time-variant parameters such as fuel prices. However, for the sake of simplicity, we assume them constant here. This assumption seems to be rational because it makes sense that the corresponding parameters such as fuel price be constant during successive days. Therefore, the marginal cost of GenCO *i* is:

$$C_i'\big(q_i(t)\big) = b_i(t) + C_i(t) q_i(t); \; i = 1, \dots, n \tag{2}$$

The supply function of generator *i* is represented by $q_i(t)$ (in MWh), which is a monotonically increasing linear function of $P(t)$ (system price) as follow:

$$q_i(t, p) = \alpha_i(t) + \beta_i(t)\, P(t); \; i = 1, \dots, n \tag{3}$$

The strategic bid that each GenCO submits in the electricity market is represented by its supply function. Therefore, the optimal values of α and β at time t are the variables of the objective function of the GenCOs' bidding strategy.

In practice, the generation companies have limitations on the upper and lower levels of the production due to technical constraints shown below:

$$q_{i\,min} \le q_i \le q_{i\,max}\;;\; i = 1, \dots, n \tag{4}$$

B. *Demand side model*

Since in this paper we are concentrated on the GenCO side, the total market demand is represented by the linear function, as in some existing literature such as Song et al. (2003). This function is considered as follows:

$$D(t,P) = N - \theta P(t) \tag{5}$$

By definition, both N and θ are positive since demand is a decreasing function of price and the price is always positive even at D(t)=0.

C. *Market clearing mechanism*

After receiving all the demand bids and supply offers, the ISO determines the optimal dispatching scheme in the day-ahead electricity market and clears the market. Regarding the non-storable nature of electricity and so the instantaneous and mandatory balance between supply and demand of electricity, the market clearing condition (market equilibrium condition) at time *t* in these markets can be represented as:

$$\sum_{i=1}^{n} q_i = D(P)$$
$$\text{s.t.}\qquad q_{i\,min} \le q_i \le q_{i\,max}\;;\; i = 1,2, \dots, n \tag{6}$$

To solve this set of equations, at first, the constraints related to the upper and lower bounds of supply are not considered, and the solution will be as follows:

$$MCP = P = \frac{N - \sum_{i=1}^{n}\alpha_i}{\sum_{i=1}^{n}\beta_i + \theta}$$
$$q_i = \alpha_i + \beta_i P \qquad i = 1,2, \dots, N \tag{7}$$

Then as a result, if each of the quantities gets values beyond the boundaries, it will be fixed on its upper or lower limits, respectively, when it is greater than $q_{i\,max}$ or lower than $q_{i\,min}$ and the corresponding supplier is excluded from the equation and the set of equations will be solved again for the other suppliers.

D. *GenCO's Problem Formulation*

The objective of each GenCO is the maximization of the net profit in the market, and as mentioned below this will be followed by choosing the optimal values for α and β (denoted by α* and β*) as the variables of GenCOs' bidding strategy. Since at the very low amount of production (the small values of qi) the supplier is quite willing to increase its market share to bid very close to its marginal cost, we can make an approximation of α_i/β_i, as the intercept of the reverse supply function, which is equal to b_i as the intercept of the marginal cost function, i.e., $-\alpha_i/\beta_i = b_i$.

Therefore, the problem of determining the GenCO's bidding strategy is just to choose the optimal value for β_i in each bidding stage:

$$\max_{\beta_i} \pi_i(t) \tag{8}$$

Where

$$\pi_i(t) = q_i(t)p_i(t) - C_i(q_i(t))$$

$$-\frac{\alpha_i}{\beta_i} = b_i(t) \tag{9}$$

From the GenCOs viewpoint in oligopolistic markets, the maximum profit is the point at which the marginal revenue is equal to the marginal cost, which means the marginal profit at this point is zero:

$$\frac{\partial \pi_i(t)}{\partial \beta_i(t)} = 0 \tag{10}$$

The above condition expresses the optimal behaviour of a competitive GenCO in a liberalized market, which is usually being used in the related literature (e.g. Liu, (2006), Ocaña and Romero, (1998)). Besides, collusive bidding and power network constraints can affect the decisions of the market players and thus result in a different optimal condition. However, they have been excluded from this work since the former one is usually avoided by the laws and the latter is required for the sake of simplicity. After some algebra, the marginal profit can be rewritten as below:

$$\varphi_i = \frac{\partial \pi_i}{\partial \beta_i} = \frac{\alpha_i \beta_{-i} + \alpha_i \theta + \beta_i\, N - \beta_i \alpha_{-i}}{(\beta_i + \beta_{-i} + \theta)^3} . (\beta_{-i}\, N + N\theta - \alpha_{-i}\beta_{-i} - \alpha_{-i}\theta - \alpha_i \beta_{-i} - \alpha_i \theta) . (N - \alpha_i - \alpha_{-i} - b_i \beta_i - b_i \beta_{-i} - b_i \theta - c_i \alpha_i \beta_{-i} - c_i \alpha_i \theta - c_i \beta_i\, N + c_i \alpha_{-i} \beta_i)$$

Where

$$\alpha_{-i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \alpha_j \quad , \quad \beta_{-i} = \sum_{\substack{j=1 \\ j \neq i}}^{n} \beta_j \tag{11}$$

As we assumed that the opponents' decision is independent of that of GenCO i, the derivative of the opponent's decision parameters is zero. Practically, it is so hard for a GenCO at each bidding stage to adjust its generation at the optimal condition because this would lead to a sudden change in energy production at an infinite speed. Instead, a more practical approach could be moving towards the optimal condition by the repeated adjustments of the bid parameter $\beta_i(t)$ in successive rounds. In this regard, an appropriate model could be proposed regarding the fact that the GenCO *i* alter its bid decision based on the derivative of the profit function $\pi_i(t)$ at the next bidding stage:

$$\beta_i(t+1) - \beta_i(t) = k_i(t+1) \left. \frac{\partial \pi_i(t+1)}{\partial \beta_i(t+1)} \right|_{\beta_i(t+1) = \beta_i(t)} \qquad k_i(t+1) > 0 \tag{12}$$

Where $\beta_i(t)$ is the bided supply function coefficient of GenCO *i* at a specific hour of the present day and $\beta_i(t+1)$ is the same-hour supply function coefficient needed to be bid for the next day, as the adjustment is an hourly-based process in compliance with the day-ahead market. The reason for proposing such an approach is based on the practice that the energy generator moves towards the increase in their profit. This model comprises 24 different subproblems corresponding to each hour of the day. This hourly division is also consistent with the fact that the energy demand has a periodic nature making it to be approximately unchanged at the same hours through consecutive days. However, the market demand figure during the weekends is usually different from the weekdays, so 24 new splits should be defined for each hour of the weekends. Finally, there will be 48 distinct splits in total (24 ones for each hour of the weekdays, and 24 ones for each hour of the weekends).

It needs to be noticed here that the marginal profit of the bid on the next day is calculated by keeping the bid value of GenCO *i* unchanged because it needs to know how its profit can increase if it doesn't change its bid value, therefore the bid will be adjusted towards increasing the profit.

The parameter k_i represents a kind of velocity of the adjusting process by which the GenCO *i* adjust its production level respective to the likely increase or decrease in the profit.

By substituting equation (8) into equation (9) and making some algebraic arrangement, we will have the following equations as the strategic bidding problem:

$$\beta_i(t+1) = \beta_i(t) + k_i\ \varphi[\alpha_i(t), \alpha_{-i}(t+1), \beta_i(t), \beta_{-i}(t+1), b_i, c_i, N(t+1), \theta(t+1)]\ . \tag{13}$$

Therefore, the marginal profit of GenCO i at the next round bidding, shown here is a function of the demand function coefficients, cost function coefficients, and GenCO's bidding parameters (which are the coefficients of supply function bid).
The bid for supply function coefficient $\beta_i(t+1)$ of GenCO *i* at the next day is therefore a function of its bid value at the current day $\beta_i(t)$, the sum of the next-day bid values of the other generators $\beta_{-i}(t+1)$, and the coefficients of demand function of the next day.
From the equations of the market equilibrium and with the assumption that the parameters of all GenCO's cost function are the same (i.e.,$b_i = b;\ i = 1, \dots, N$), we can calculate $\beta_{-i}(t+1)$ as below:

$$\beta_{-i}(t+1) = \frac{D(t+1)-(1-b)\beta_i(t)}{P(t+1)-b} \tag{14}$$

Where $D(t+1)$ and $P(t+1)$ indicate the total demand and market price at time t+1 (next day), the information that should be estimated or predicted by using the market-available historical data.

The other parameters that should be estimated in equation (13) are the coefficients of market demand function at the next round, i.e. $N(t+1)$ and $\theta(t+1)$. These parameters can also be calculated by the historical data of the total demand and market price. However, here for the sake of simplicity, we choose a fixed value of $\theta = 1500$ based on the existing literature, such as Ocaña and Romero, (1998). and then N can be easily calculated as follows:

$$N(t+1) = D(t+1) + \theta.P(t+1) \tag{15}$$

To incorporate the SFE into an adaptive bidding algorithm corresponding to a control system approach, a formulation is given here for learning and adjusting the GenCO bidding process. The ISO clears the day-ahead market each day at 12 PM for all the 24 hours of the day based on the received bids for supply and demand and then publishes the resulting market data. This is repeated over time for the subsequent days. The whole process has been illustrated in figure 1.

$$-b_i\beta_i(t) - b_{-i}\beta_{-i}(t+1) + [\beta_i(t) + \beta_{-i}(t+1)].P(t+1) = D(t+1) \tag{16}$$

The aggregate supply function of the market can be rewritten by separating the bid parameters of GenCO i and its competitors as below:

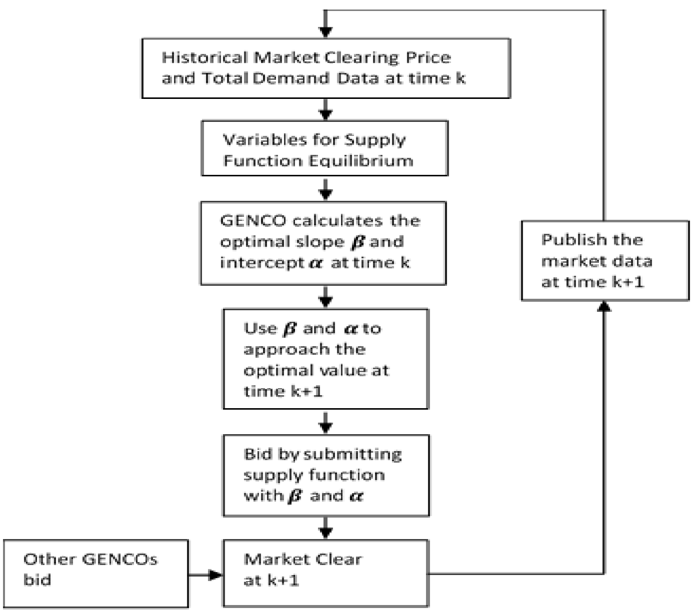

Fig. 1. Dynamic model based on closed-loop system

At day $k+1$, the GenCO makes the new optimal bid in supply function form by using the condition of supply function equilibrium of day k.

E. *Electricity market demand and price prediction*

As previously mentioned, the dynamic adjusting process based on SFE can model the dynamics of the bidding process in power markets. In this model each GenCO uses the optimal supply function bided in the previous round to move towards the optimal bidding strategy belonging to the next round, in a closed loop structure. For better capturing the real situation the model will adapt the real situation by using a forecasting method based on an exponentially weighted Recursive Least Square algorithm, for market clearing price and total demand. This forecasting method is a kind of adaptive filters which have been considered as effective tools for modelling non-stationary signals.

In these filters, the coefficients are updated in order to capture the signal instability. One of the most frequently-used form of adaptive filters is Recursive Least Square (RLS) that we focuses here on introducing and using it for predicting clearing price and demand of the electricity I order to be implemented in the process of GenCOs' strategic bidding.

Considering $\widehat{P}(t)$ and $\widehat{D}(t)$ represent the predicted price and demand value at time t, and the vectors $\overline{X}(t)$ and $\overline{Y}(t)$ are p historical and individual data respectively for demand and price:

$$\overline{X}(t) = [D(t), \dots \ , D(t-p+1)\ ]^T$$
$$\overline{Y}(t) = [P(t), \dots \ , P(t-p+1)\ ]^T \tag{17}$$

The equations between the predicted and the historical data are as follows:

$$\widehat{D}(t+1) = \sum_{k=0}^{p-1} u_t(k) D(t-k) = U_t^T \overline{X}(t)$$
$$\widehat{P}(t+1) = \sum_{k=0}^{p-1} v_t(k) P(t-k) = V_t^T \overline{Y}(t) \tag{18}$$

In which parameter p is denoted as the filter order, the vectors U_t and V_t are the filter coefficients that are written as follows:

$$U_t = [u_t(1), \dots, u_t(p)]^T \tag{19}$$

$$V_t = [v_t(1), \dots, v_t(p)]^T \tag{20}$$

Now error functions over time are defined by comparing the estimated value and the actual value, as follows:

$$e_D(\tau) = D(\tau) - \widehat{D}(\tau) = D(\tau) - U_\tau^T \overline{X}(\tau)$$
$$e_P(\tau) = P(\tau) - \widehat{P}(\tau) = P(\tau) - V_\tau^T \overline{Y}(\tau) \tag{21}$$

Then, these errors need to be implemented into the RLS filter for updating the coefficients so that the cost function, that is the weighted least square error, can be minimized by the updated coefficients, the formulation can be like below:

$$\varepsilon_D(t) = \sum_{\tau=0}^{t-1} \lambda^{t-\tau} e_D(\tau)^2$$
$$\varepsilon_P(t) = \sum_{\tau=0}^{t-1} \mu^{t-\tau} e_P(\tau)^2 \tag{22}$$

In which λ and μ are the weighting factors considered because of the forgetting principle that means the recent samples have more effect on the minimization of the error. Whenever the new optimal parameters are calculated, they will be used for calculating the new prediction by (15). The detailed mathematical formulation can be followed in Hayes (2009). Here the RLS algorithm for prediction of market demand, as an example is given as follows:

$$\textit{Input Parameters}\begin{cases} \gamma = \textit{order of the filter} \\ \lambda = \textit{exponencially weighting factor} \\ \delta = R(0)\ \textit{Start value} \end{cases} \tag{23}$$

$$\text{Start}\begin{cases} W_0 = 0 \\ R(0) = \delta^{-1}I \end{cases}$$
$$\text{For}\quad \tau = 1, 2, 3, \ldots, t$$
$$Z(\tau) = R(\tau - 1)X(\tau)$$
$$K(\tau) = \frac{1}{\lambda + X^T(\tau)z(\tau)}Z(\tau)$$
$$\alpha(\tau) = D(\tau) - U_{\tau-1}^T X(\tau)$$
$$U_\tau = U_{\tau-1} + \alpha(\tau)K(\tau)$$
$$R(\tau) = \frac{1}{\lambda}[R(\tau - 1) - k(\tau)Z^H(\tau)]$$
$$\text{End}$$

Where W_τ is the filter coefficients, K(t) denoted the gain vector, R(t) is a $(\gamma + 1) \times (\gamma + 1)$ matrix that is the inverse of the exponentially weighted deterministic autocorrelation matrix, and $\alpha(\tau)$ represents the error that is the difference between the desired value of the market demand and the estimated value formed by applying the previous set of to the new $X(\tau)$. By selecting the filter order, γ equals to four, as an example, the linear predictors will be:

$$\begin{aligned} \widehat{D}(t+1) &= u_t(1)D(t-1) + u_t(2)D(t-2) + u_t(3)D(t-3) + u_t(4)D(t-4) \\ \widehat{P}(t+1) &= v_t(1)P(t-1) + v_t(2)P(t-2) + v_t(3)P(t-3) + v_t(4)P(t-4) \end{aligned} \tag{24}$$

In which, $u_t(1{\sim}4)$ and $v_t(1{\sim}4)$ are updated over time based on the adaptive approach, then the market variables of the next round are calculated based on the updated values of these coefficients. As a result, the predicted market variables of price and demand at the certain time step can be estimated by applying the previous data.

F. *Adaptive control system*

In the previous section it is shown that the electricity demand and price can be predicted beforehand by using the RLS method, therefore GenCOs will be able to regulate their strategic bid and obtain the more optimal supply function parameters (β and α) based on the predicted market variables which are much closer to the actual data.

By implementing the predicting approach, the GenCO strategic bidding as a closed-loop control system (figure 1) will be an adaptive process so the SFE model proposed here for electricity market operation will be an adaptive controller as illustrated in figure 2.

Again by selecting the order of the filter, P, equal to four, the mentioned adaptive closed-loop system applies the four historical values of market price and demand data ($\overline{X}(t)$ and $\overline{Y}(t)$) as the inputs of the control system for estimating the future data of market price and demand data ($\widehat{D}(t)$ and $\widehat{P}(t)$). As the new values of data are estimated by this adaptive filtering process, they will be compared with the actual market variables (i.e. P(t) and D(t)) that are routinely made published by the ISO as market clearing outputs in order to calculate the errors (e_D and e_D) and then errors are applied into the filters in order to update their coefficients in such a way that the cost functions (weighted least square errors) become minimum by the new coefficients.

The newly predicted values are used in the SFE bidding process for calculating the optimal supply function parameters (β and α) that follows the market equilibrium equation. The results in terms of the supply function parameters (β and α) which are the outputs of the control system, will then be used by the GenCO as the bidding decision parameters for participating in the competition of the day-ahead electricity market. After collecting all the bid, the ISO will then makes the market settled by clearing procedure.

After clearing the market, the actual values of market price and demand (P(t) and D(t)) are determined by the ISO. In order to test the adaptive closed-loop algorithm of bidding strategy, the actual data can be used in SFE to calculate the optimum profit at the market equilibria point. This is the profit the GenCO ideally can earn based on the actual data, so it will be compared with the profit calculated based on the estimated data and then the error will be calculated in order to be used for adjusting the parameters of the controller in the direction of minimizing the so called error. Besides, the actual market variables are also injected back to RLS filter to renew the coefficients of the filter in the next time step.

In this adaptive closed-loop system, the theory of control system, signal processing and game theory have been used simultaneously to model the bidding process and its dynamics. It has an auto-adjusting nature for effective analysis of the power market dynamics.

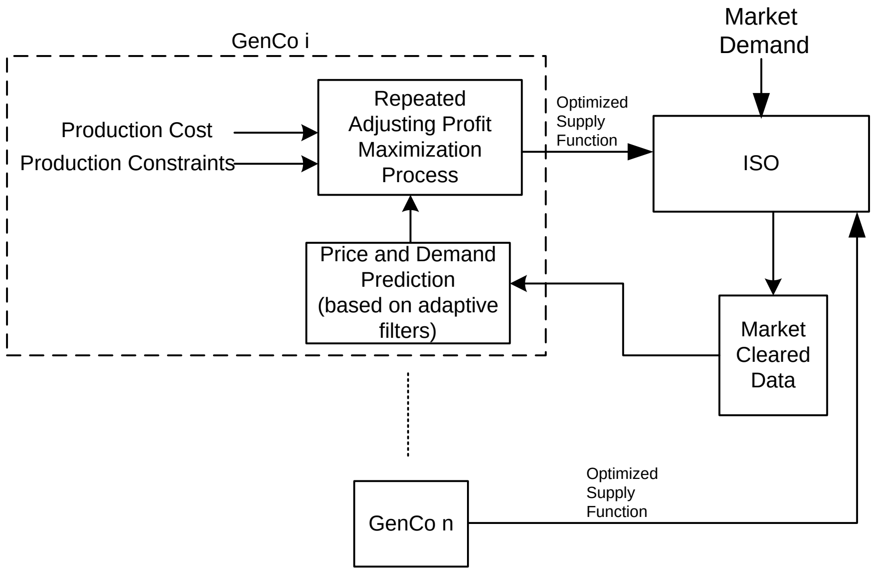

Fig. 1. The adaptive controller model of electricity market operations

Finally, ISO clears the market and assigns the amount of power each GenCO wins. The process of market clearing can be modelled as a nonlinear optimization problem, which the ISO tries to minimize total payment cost based on GenCO's bids. For this purpose, offers are ranked in an increasing order beginning with the least expensive and continuing until the demand is satisfied. On the other hand, since suppliers and consumers are connected through the transmission network, congestion should be considered in the market-clearing process. Consequently, the process of market clearing can be shown by equation (4)

4- Results and discussions

In this section, using a simulation study, we assess the effectiveness and performance of the proposed model. In this regard, the real market data from the day-ahead Nordic electricity market (Nord Pool) are used to test the results of the adaptive bidding model.

The entire day time is split into 24 slots, one for each hour of the day, so the bidding strategy comprises 24 sub-problems, corresponding to these 24 splits.

The implementation of the proposed strategies requires that the GenCO has some information of the market to estimate the parameters such as supply function and market demand coefficients, which reflect the dynamicity of the market.

For the sake of simplicity, only two producers with the same cost functions are considered here, which means n = 2 subscribers.

Similar to the values used in Liu (2006), the producers' cost function parameters are assumed to be 10, 1.5, and 0.001 for a_i, b_i, and c_i respectively. This assumption seems to be realistic to some degree because these are approximately the same cost parameters of a thermal generator which provides about one-half of the production of the Nordic electricity market.

The historical data published in the NORDIC day-ahead electricity market is used to predict the market demand and clearing price of the next day to be applied in the RLS process, previously proposed as part of the adaptive bidding model (figure 2).

Despite the model formulated for the GenCO's strategic bidding for all hours of the day, it is simplified to average daily values of the market demand and clearing price, as shown in Figures 3 and 4. These average values can be a good representative of the market dynamics to develop and simulate the closed-loop adaptive control system and verify the proposed bidding process. However, it will easily be possible to extend the simulation to every hour of the day.

The simulation is performed for the whole month of April 2007. This month was selected as a good representative of the average behaviour of the market because it doesn't have extreme conditions such as the coldest, warmest, darkest, or the brightest days of the year. Besides, the sudden spikes in the electricity price as a result of unpredictable events (e.g. failure of transmission lines) are very rare in this period.

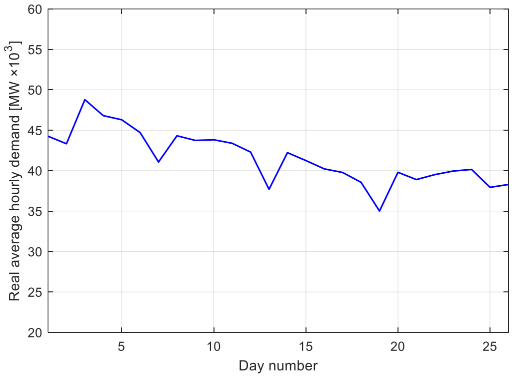

Fig. 2. Real daily average market demand

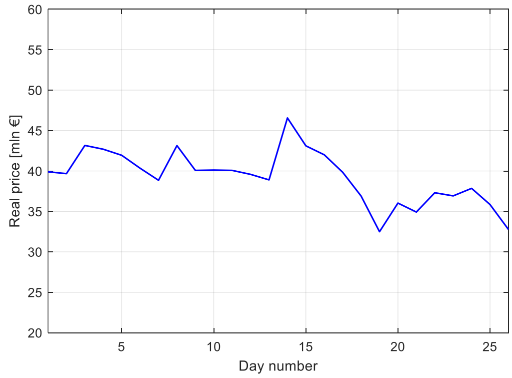

Fig. 3. The real daily average market clearing price

The simulation is evaluated here using some averaged parameters: the generated quantity (measured in MWh), the daily profit, the cleared price, and the daily social welfare (all measured in €). The meaning of these parameters is straightforward except for social welfare. As discussed in Amelin (2004) the social welfare in the market can be defined as the integral of the difference between the inverse market demand function and the aggregate marginal cost function of all the generators over the total quantity from the origin to the point specified by the ISO as the market cleared quantity, shown respectively as the dark and light grey shaded areas in figure 5. The market social welfare thus denotes the whole benefit of the society, which is the aggregate benefit of consumers for purchasing the energy at the cleared market price P_{ISO} (dark grey area in figure 5) and the aggregate profit of the producers for selling the energy at the system price (light grey area in figure 5)

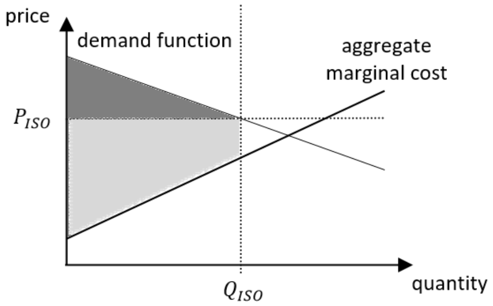

Fig. 4. Social welfare as a result of difference integral between the inverse demand function and the aggregate marginal cost of all the producers over the quantity

The results show that the GenCOs can adapt their strategic bidding to the real market by implementing the predicted SFE, therefore can bid better values of the bidding parameters. This algorithm is developed based on two 24-adaptive filter models given in (14) and (15), using the four historical data of the four previous days of the estimated day. In figures 6 and 7, the historical data is shown by the dotted line and the estimated data is shown by the solid line. The objective of the adaptive filter at a specific time is to estimate the real data (the solid line) by using the given historical data (the dotted line). As shown in figures 6 and 7, the adaptive filter is a useful tool for the estimation of future data because the estimated data closely follows the real data over time.

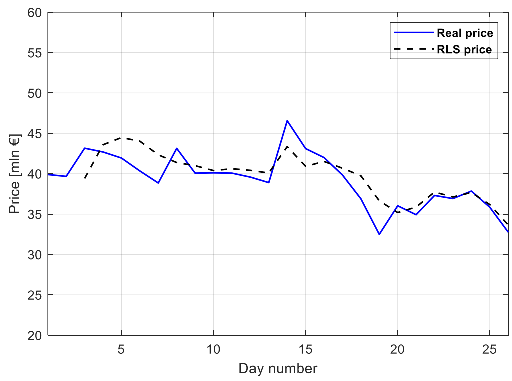

Fig. 5. Actual price and RLS price

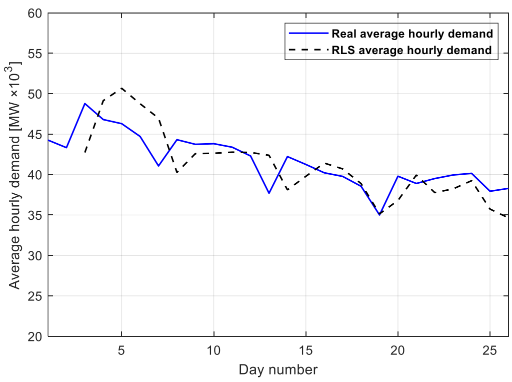

Fig. 6. Actual demand and RLS demand

We consider another scenario as a "naive" strategy to represent the behaviour of a producer that does not take the strategic bidding in the market. This simple strategy is supposed to be akin to a GenCO in a competitive market that bids at its marginal cost that is:

$$c_i'\left(q_i(t)\right) = P \tag{25}$$

By substituting (25) in the equations, we will have:

$$\beta_i(t+1) = \frac{P(t+1)-b}{c.(b-P(t+1))} \tag{26}$$

Following the discussion made in the previous section, the only parameter that needs to be chosen in the model is the GenCo bid adjusting speed of the generators, which has been defined by $k_i(t)$. It is assumed here that these parameters are constant over time since in practice each GenCO can find a constant optimal value for them based on the experience acquired during participation in the market.

To compare the result of the decisions made by each generator company, the simulation is done for three modes as table 1:

Table 1. Three cases defined to compare the result of the decisions made by each generator company

Scenario	GenCO A	GenCO B
I	Naive Bid	Naive Bid
II	Strategic Bid	Naive Bid
III	Strategic Bid	Strategic Bid

The fourth scenario as naive and the strategic bids respectively for GenCO A and GenCO B can be discarded due to the assumed symmetry among the GenCOs. In table 2, the results of the proposed simulations have been summarized.

Table 2. The simulation results for the three cases

Scenario	I	II	III
Profit Gen.1 (mln €)	0.371	0.374	0.385
Profit Gen.2 (mln €)	0.371	0.392	0.385
Quantity Gen.1 (MWh)	21473.65	18947.83	19296.06
Quantity Gen.2 (MWh)	21473.65	23708.91	19296.06
Price (€)	24.10	24.56	25.19
Social welfare (mln €)	39.21	42.98	39.21

The results show that when going from scenario 1 to 2, in which GenCO A switches his bid from the naive to the strategic, this GenCO gains about 4.22% increase in its profit. This happens because of the increase in the market price when its production is lower. On the other hand, the second GenCO will generate more power but less profit. In the third scenario, both GenCOs gain equally more profit than the other scenarios with less generation than the first scenario. It can be concluded that the third scenario corresponds to the Nash equilibrium and Prison dilemma because none of the producers will gain more profit if they change their scenario individually.

5- Conclusions

In this work, we proposed an adaptive algorithm to model the generators' strategic bidding and pricing system in day-ahead electricity markets, which are more akin to an oligopolistic structure with a limited number of producers and unlimited number of consumers than complete completive. The proposed algorithm is based on the profit maximization of the players and it has the capability of capturing the market dynamics using the embedded forecasting part, based on an adaptive filter and its closed-loop structure. A case study of the Nord Pool electricity market was also considered to verify the results of the model. Based on the simulation results, the following conclusions are made:

- The adaptive closed-loop control system with the forecasting tools based on the RLS algorithm can model the bidding strategy of supply function equilibrium, which helps GenCOs to increase their market profit.
- The prediction algorithm can estimate the electricity market demand and the price with an appropriate accuracy, through which the GenCOs can bid for the next day more realistically.
- The results verify that GenCOs don't need to know the competitor's production costs. The adaptive closed-loop model for the strategic bidding is capable to capture the market dynamics in advance, providing more profit than the conventional supply function equilibrium bidding models. In fact, this approach can provide the GenCOs with market data near to actual values to be used for the bidding strategy. As a result, GenCOs will be able to acquire more profit during the electricity market operation.
- This research showed that the control theory is an appropriate technique for analysing and modelling the electricity markets mostly because of the embedded closed-loop structure, by which the strong dynamics and stochastic properties can be modelled.

There are also some suggestions for future works in this area, including different types of markets, as real-time markets, as well as issues such as transmission constraints and the dynamics corresponding to the renewable sources of energy.

References

Abapour, S., Mohammadi-Ivatloo, B., & Hagh, M. T. (2020). A Bayesian game theoretic based bidding strategy for demand response aggregators in electricity markets. Sustainable Cities and Society, 54, 101787.

Amelin, M. (2004). On Monte Carlo simulation and analysis of electricity markets (Doctoral dissertation, KTH).

Baldick, R., Grant, R., & Kahn, E. (2004). Theory and application of linear supply function equilibrium in electricity markets. *Journal of regulatory economics*, *25*(2), 143-167.

Bunn, D. W., Martoccia, M., Ochoa, P., Kim, H., Ahn, N. S., & Yoon, Y. B. (2010). Vertical integration and market power: A model-based analysis of restructuring in the Korean electricity market. *Energy Policy*, *38*(7), 3710-3716.

de la Torre, S., Arroyo, J. M., Conejo, A. J., & Contreras, J. (2002). Price maker self-scheduling in a pool-based electricity market: a mixed-integer LP approach. *IEEE Transactions on Power Systems*, *17*(4), 1037-1042.

Fleten, S. E., & Kristoffersen, T. K. (2007). Stochastic programming for optimizing bidding strategies of a Nordic hydropower producer. *European Journal of Operational Research*, *181*(2), 916-928.

Gao, F., & Sheble, G. B. (2010). Electricity market equilibrium model with resource constraint and transmission congestion. *Electric Power Systems Research*, *80*(1), 9-18.

Green, R. J., & Newbery, D. M. (1992). Competition in the British electricity spot market. *Journal of political economy*, *100*(5), 929-953.

Gross, G., Finlay, D. J., & Deltas, G. (1999, January). Strategic bidding in electricity generation supply markets. In IEEE Power Engineering Society. 1999 Winter Meeting (Cat. No. 99CH36233) (Vol. 1, pp. 309-315). IEEE.

Guo, N., Wang, Y., & Yan, G. (2021). A double-sided non-cooperative game in electricity market with demand response and parameterization of supply functions. International Journal of Electrical Power & Energy Systems, 126, 106565.

Hayes, M. H. (2009). *Statistical digital signal processing and modeling*. John Wiley & Sons.

Liu, Y. (2006). Network and temporal effects on strategic bidding in electricity markets. HKU Theses Online (HKUTO).

Klemperer, P. D., & Meyer, M. A. (1989). Supply function equilibria in oligopoly under uncertainty. *Econometrica: Journal of the Econometric Society*, 1243-1277.

Li, T., & Shahidehpour, M. (2005). Strategic bidding of transmission-constrained GENCOs with incomplete information. *IEEE Transactions on power Systems*, *20*(1), 437-447.

Naghibi-Sistani, M. B., Akbarzadeh-Tootoonchi, M. R., Bayaz, M. J. D., & Rajabi-Mashhadi, H. (2006). Application of Q-learning with temperature variation for bidding strategies in market-based power systems. Energy Conversion and Management, 47(11-12), 1529-1538.

Ocaña, C., & Romero, A. (1998). A simulation of the Spanish electricity pool. Comision Nacional del Sistema Electrico, Madrid WP DT, 5, 98.

Rahimiyan, M., & Mashhadi, H. R. (2007). Risk analysis of bidding strategies in an electricity pay as bid auction: A new theorem. *Energy conversion and management*, *48*(1), 131-137.

Sheblé, G. B. (2001). Economically destabilizing electric power markets for profit. In *2001 IEEE Power Engineering Society Winter Meeting. Conference Proceedings (Cat. No. 01CH37194)* (Vol. 1, pp. 50-54). IEEE.

Schweppe, F. C. (1988). Management of a spot price based energy marketplace. *Energy policy*, *16*(4), 359-368.

Shahidehpour, M., Yamin, H., & Li, Z. (2003). Market operations in electric power systems: forecasting, scheduling, and risk management. John Wiley & Sons.

Sharifi, R., Anvari-Moghaddam, A., Fathi, S. H., & Vahidinasab, V. (2020). A bi-level model for strategic bidding of a price-maker retailer with flexible demands in day-ahead electricity market. International Journal of Electrical Power & Energy Systems, 121, 106065.

Sen, S., Yu, L., & Genc, T. (2006). A stochastic programming approach to power portfolio optimization. *Operations Research*, *54*(1), 55-72.

Song, Y., Ni, Y., Wen, F., Hou, Z., & Wu, F. F. (2003). Conjectural variation based bidding strategy in spot markets: fundamentals and comparison with classical game theoretical bidding strategies. *Electric Power Systems Research*, *67*(1), 45-51.

Teufel, F., Miller, M., Genoese, M., & Fichtner, W. (2013). Review of System Dynamics models for electricity market simulations (No. 2). Working paper series in production and energy.

Ventosa, M., Baıllo, A., Ramos, A., & Rivier, M. (2005). Electricity market modeling trends. *Energy policy*, *33*(7), 897-913.

Wang, J., Wu, J., & Che, Y. (2019). Agent and system dynamics-based hybrid modeling and simulation for multilateral bidding in electricity market. Energy, 180, 444-456.

Wen, F., & David, A. K. (2001). Optimal bidding strategies and modeling of imperfect information among competitive generators. *IEEE transactions on power systems*, *16*(1), 15-21.

Younes, Z., & Ilic, M. (1999). Generation strategies for gaming transmission constraints: will the deregulated electric power market be an oligopoly?. *Decision support systems*, *24*(3-4), 207-222.