

Benders Decomposition Algorithm for Competitive Supply Chain Network Design under Risk of Disruption and Uncertainty

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Abstract

In this paper, a bi-level programming is proposed for designing a competitive supply chain network. A two-stage stochastic programming approach has been developed for a multi-product supply chain comprising a capacitated supplier, several distribution centers, retailers and some resellers in the market. The proposed model considers demand's uncertainty and disruption in distribution centers and transportation links. A Stackelberg game is used to formulate the competition among the components of supply chain. A bi level mixed integer programming mathematical model is used to formulate the supply chain and the impacts of the strategic facility location on the operational decisions such as inventory and shipments have been investigated via the mathematical model. To solve the model, Bender's decomposition algorithm is used. Some discussions through several numerical examples and some managerial insight are suggested for the situations similar to the assumed problem.

Keywords: Competition, Supply chain network design, Disruption, Benders decomposition algorithm

1- Introduction

The field of disruption in supply chain management had attracted the interest of many researchers and very important articles on this subject have been published by different journals. Natural disruption (earthquakes, floods, etc.) and human activities, intentionally or unintentionally (operator errors, terrorist attacks, etc.) have a great effect on the performance of supply chains and in some cases enormous costs have been arisen. As Snyder et al (2012), disruption has attracted more attention for the following factors. First, the various events such as the 11th September attacks, Katrina Hurricane and the tsunami in Japan caused disruptions which their impacts on the society and the global economy are understood well. The second factor was an increase in the use of JIT (Just in time) production, lean production and existence of many uncertainties in dynamic systems which have greatly increased the vulnerability of systems.

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Furthermore, because reducing the vertical integration of supply chains besides globalization, the possible failures of each of the facilities could affect the performance of other sectors and resulting large losses in total supply chains.

Nowadays, intense competition in the markets makes the organizations and firms to make certain about their reliable and correct performance at different periods of time and implement their strategic and operational planning by considering the potential risk in the competitive environments. Small crash in systems could cause a different kind of disruption and also can be seen as a severe threat to the society and environment. Excessive complexity of supply chains makes them highly sensitive to disruptions and failures, and minor disturbance in one component of the chain could cause harmful effects on the whole supply chains. Disruption in the supply chain components' can cause a loss of market share in a competitive market and will create irreparable damages or harm. Therefore, taking possible disruptions into account in supply chains at the planning period could reduce costs, increase competitiveness, service and reliability. Supply chain network design (SCND) considers some strategic decisions such as the location of facilities and their capacities and also operational decisions such as inventory and distribution policies. In most of the literature in supply chain network design, it is considered that the supply chain performs exclusively and the existence of competitors is ignored in most cases. It should be considered that supply chains compete to gain market share and even if there was no rival at the time, supply chains should be prepared for competitive condition in the future (Farahani et al. 2014b). One of the factors that could affect the competition between supply chains and also among the components of each supply chain is disruption and the way the supply chains are facing them at different times. Therefore, in this paper, supply chain network is designed by considering two important factors, competition and disruption.

The remainder of this work is organized as follows. In Section (2) the literature will be reviewed. Then, Section (3) and Section (4) describe the problem, and presents the proposed model, Section (5) presents the Benders' decomposition algorithm for solving the proposed model. Next, Section (6) describes a computational study for implementing the feasibility of the applied model and discusses the significant findings for the practical application of the proposed model. Finally, conclusions are drawn in Section (7) and some topics for future research in this area is given.

2- Literature review

In this section, the recent researches in the literature of competitive and reliable supply chain network design are reviewed and the main contributions of this work are specified in comparison to the recent literature.

2-1- Literature on Competitive Supply Chain Network Design (CSCND)

Intense competition in the markets requires organizations to act as a member of the supply chain. Being a member of a chain helps firms in their field of expertise to respond quickly to customer needs changes and improve their agility and flexibility. The ultimate goal of supply chain management is to improve the competitiveness of the entire chain. Integrating different parts of the supply chain to form a network for managing the flow of materials through the structure, plays an important role on the chain performance and competitiveness (Bernstein and Federgruen, 2004). When the supply chain has no rival in the market, it has a monopoly on the market and gain all the market share. When there are other competitors in the market, considering the monopoly for the supply chain is an unrealistic assumption in many cases (Zhang and Rushton, 2008).

Modeling efforts, mainly optimization and simulation models, have tried to address some of the competitive supply chain network design. Competitions in the context of the SC can be classified into three categories: (1) competition among firms in one tier of a SC. (2) Competition among firms from different SC tiers and (3) competition among independent SCs.

Seifert and Langenberg (2011) investigated that if supply chains want to remain in competitive markets, they should be sensitive to environmental changes and quickly adapt themselves with these changes. Viswanadham and Gaonkar (2003) proposed a mixed integer programming model considering all of the chain members shared their information for obtaining the maximum profit and then the efficiency of the supply chain is evaluated. Boyaci and Gallego (2004) considered a market with two competing supply chains that supply only one kind of product and the competition is based on the service level of each one for having more consumers. Shen(2005) designed a supply chain for maximizing the profit of an organization and expressed that it is not necessary to satisfy all of customers demands. Therefore, it is better to satisfy the demands which maximize the profit and rest of the demands should be assigned to other competitors. The model is solved by branch and bound algorithm.

Zhang (2006) proposed a model for the formulation of competition in supply chains. An economic supply chain network of companies are involved in procurement, production and distribution activities is designed. The proposed model investigates some heterogeneous supply chains with different number of echelons and the winner of the competition is determined in the equilibrium condition. Gurnani et al.(2007) have considered a two echelon supply chain including producer and retailers. Also, it is assumed that advertisement, quality level of the products, price of retailers and price of producers have some effect on demand of consumers. Both of the players want to optimize their profits.

Xu et al.(2008) proposed a model for designing a supply network taking into account the impact of price and delivery time on utility function for customer satisfaction and competitiveness in organizations. They assumed three strategies for distribution of products. A game theory approach is used for modeling the supply network design. Most of the papers considers the competition just among independent firms and focuses on the integration of supply chain. The competition among different members of each layer and the other layers of supply chain do not have attracted many attentions. Some papers such as the following have been studied the competition among supply chains. Xiao and Yang (2008) considered two bilayer supply chain with uncertain demands, sensitive to retail price and service competition. Supply chains consist of a risk neutral supplier and a risk averse retailer. Products are not quite the same and suppliers sell their products through retailers to customers and then the impact of risk for retailers on the wholesale price and selected strategies of the players was examined.

Rezapour and Farahani (2010) proposed an equilibrium model for supply chain design. They have considered that demand is certain and the price of the products depends on demand .In this model, the supply chain competitors have been produced the same products that can be replaced by each other and the role of strategic decisions in the tactical decisions such as facility location and transportation are being seen. In recent years, more attention has been made to the competition in supply chain network design.

Farahani et al.(2014a) have designed a two-level supply chain network that determines the location of retailers and capacity of competing supply chains in the market. In this paper the competition is assumed to be static with having final certain demands. The model in this paper can be used in the automotive industry, fuel and tires as well as luxury goods for tensile and non-tensile demand. To solve the model in the small-scale, a branch-and-bound algorithm is used and for large-scale problems a heuristic approach has been developed. In another study, Rezapour and Farahani (2014) designed a new supply chain with regard to price and service level and also restrictions on production. The main goal of the multilevel proposed model was to achieve the maximum profit in the future. Fallah et al.(2015) considered the competition between two closed-loop supply chains, including manufacturers, retailers and recycling centers in an uncertain situation. In this study, retail price and the cost paid by the consumers to buy recycled products are assumed as competitive factors. The proposed model is used to examine the Stackelberg game and competition between the chains with respect to price dependent demand.

2-2- Literature on Supply Chain Network Design under Disruption and Uncertainty

Considering disruption and uncertainties helps organizations and companies to service and supply in a better manner. In the last decade an excessive research have been attended the possible disruptions and uncertainties in supply chains. In this section the recent research in the context of disruption and uncertainties in SCND is investigated.

There are several uncertainties in supply chain network including operational uncertainties and unexpected disruptions with low probabilities of occurrence. The first category are related to uncertainties in demand of customers, supply and procurement process and also uncertain quantities of raw material prices, etc. These uncertainties have significant impacts on the performance of supply chains and they occur frequently. There has been much work in literature of designing supply chain considering operational uncertainties only in demand-side (Shen and Daskin, 2005), (Romeijn et al. 2007), (Ko and Evans, 2007), (You and Grossmann,2008), (Pan and Nagi,2010), (Park et al. 2010), (Cardona-Valdés et al.2011),(Hsu and Li,2011), (Rezaee et al. 2015), (Rezaee et al. 2015), (Han et al. 2015), (Yin et al. 2015), (Cardoso et al. 2015)). Also supply-side uncertainty management which affect operational uncertainty beside demand uncertainty has a rich literature (Santoso et al. 2005), (Yu et al. 2009), (Bode and Wagner 2015), (Giri and Bardhan, 2015),(Jabbarzadeh et al. 2015)). Difficulties with supply in one entity can disrupt the performance of all entities of the supply chain leading to lost sales and poor service level and long-term demand attenuation (Rezapour et al. 2015).

In the recent years an excessive research have been attended the possible disruptions in SCND. The first research which considers disruption in facility location was proposed by Drezner (1987). He introduced two models in his study. The first is P-median problem where it is assumed that the nodes fail with a certain probability and each node and also connection links are likely to fail. The aim is to minimize the distance traveled for satisfying expected weighted demand. The second model is a (p-q) center problem in which the location of P facilities are determined and the assumption that in the future Q facilities may be failed. The objective is to minimize the distance between the closest facilities to their customers. Snyder and Daskin (2005) investigated a facility location problem by considering reliability and fixed costs for establishing then in P-median problem. Both proposed model used a bi objective programming in which the first objective minimized the total expected costs in normal condition and the second one minimized the total expected costs with considering expected failure. Peng et al.(2011)proposed a model for designing a reliable network performed after failure like normal condition (without disruption) as much as possible. Using P-robust for reducing the disruption risk is the main contribution of this study. In fact, by using mixed integer programming in association with P-robust concept, the total costs are minimized by considering the maximum regret for each scenario. Jabbarzadeh et al.(2012) have been studied a supply chain network design by assuming risk of disruption in facilities. They proposed a nonlinear mixed integer programming for determining the number and location of facilities, the allocation of customers to facilities and also the flow of product in supply chain. Finally, they used Lagrangian relaxation for solving the proposed model. Azad and Davoudpour (2013) have been considered a location routing problem with disruption risks. To deal with operational risk in the supply chain network, robust optimization is used. In this paper location routing problem at three levels: customer, distributor and the supplier, was investigated. For the first time, conditional value at risk is used in routing problem. In fact, the main purpose of this model is to show the advantage of using the conditional value to control risk in the supply chains. To solve the model, a hybrid approach, Tabu search and simulated annealing algorithm is used. In some recent studies, Torabi et al.(2015) proposed a two stage stochastic programming for order allocation and supplier selection in which the operational and disruption risks are considered for making the supply chains more resilient. They applied several proactive strategies for enhancing the resilience level of selected supply base. Sadghiani et al.(2015) proposed a robust retail network design under disruption risk, they proposed multiple deterministic set covering models and they developed a robust scenario based model for designing retail network. Eventually, it has shown that designing the retail supply chain without considering the disruption is misleading.

Investigating the literature of CSCND and SCND under risk of disruption demonstrates that competition and disruption are considered simultaneously in a few studies. Most of the studies ignore this fact that any disruptions could impress the competitive position of each supply chain in markets and could cause loss of market share. A modeling effort for SCND under risk of disruption and uncertainty has not been considered the competition among the components of supply chain to the best of our knowledge. One general reason could be the difficulty of solving the competitive model for example bi-level programming which is used in this paper (Saharidis and Ierapetritou, 2009). In this study it has been considered that disruptions affect the competition between components of supply chain which have ignored in the most studies. This paper contributes to the era of CSCND and SCND under risk of disruption and uncertainty in the following way. First we present a stochastic bi level supply chain network design model for optimization of the profit of each supply chain's member. Second, we present Benders' decomposition approach for solving the bi level proposed model. Third, a number of numerical tests are implemented to investigate (1) the performance of the proposed solution method, (2) impact of considering disruption and competition simultaneously, and (3) sensitivity of numerical result to change in the input parameters. In the next section, the assumed problem will be described and a bi-level programming will be proposed for competitive and reliable supply chain network design.

3- Problem description

Consider a supply chain including a producer, distribution centers (DC's) and resellers which are in contract with producer to order their needs of products only from the producer. But in some periods to obtain more profit, they will be eager to buy their requirements from other suppliers which are in the market or for the sake of disruption in connection link; resellers have to buy their requirement from other suppliers. If resellers buy their needs from other suppliers instead of the main producer, it makes the manufacturer lose and reduce its profits. To prevent this situation, the producer could expand its supply chain by increasing number of DCs or by balancing the price of products. In other word, the price of the products and the location of the DCs are affecting the profit of the producer and even resellers. So in this paper, the main purpose is to redesign the existing supply chain and to determine the location and the number of new distribution centers as strategic decisions, besides the flow of products in supply chain and the price of the products under risk of disruption in connection link and distribution centers.

For modeling the explained problem, a bi level programming is proposed for modeling the competition between the main producer and resellers when the producer is the leader of the competition and the resellers are their followers. The structure of the supply chain is illustrated in figure 1. As it is obvious, resellers could gain their product needs form DCs of the main producer and other suppliers in markets. In addition to existing DCs, some locations are candidate for establishing new DCs. After opening new facilities, the flow of products will be changed. The assumptions of the proposed model are as follows:

- Disruption is considered in existing DCs and there is no disruption in new DCs.
- Disruption is considered to be partial in DCs and complete in connection links.
- The capacity of the DCs is known.
- Demand of each reseller is uncertain.
- Resellers could gain their need form the suppliers in market by some limitations and also from DCs.

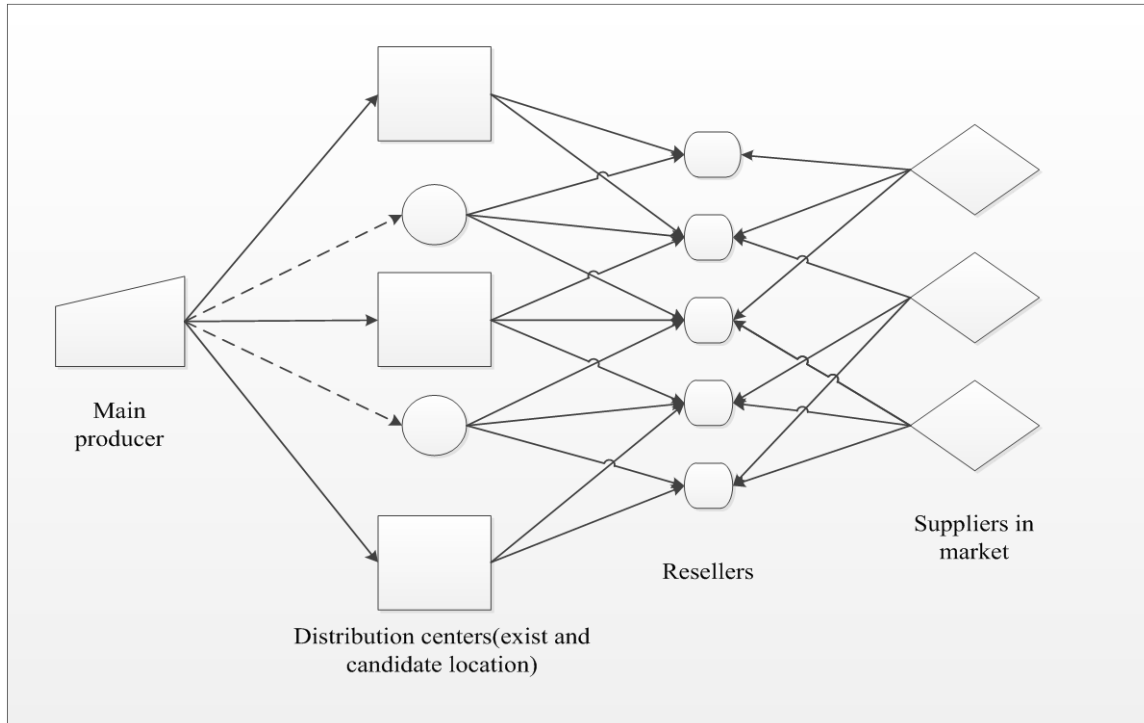


Figure 1. Structure of the supply chain

- The producer has a limited capacity for supplying products.
- Each of the resellers should order a minimum level of products from the producer.
- Each reseller sells the products at the price which the leader determined also the product obtained from suppliers in the market.

The model aims to determine the following decisions at each period of the planning horizon:

- The number of new DC should be established.
- The location of new DCs.
- The flow of products in supply chain under each scenario.
- The price of the product sold at each reseller point under each scenario.
- The quantity of product which have been bought form other suppliers in the market by resellers under each scenario.
- The product inventory level in DCs at the end of each period under each scenario.

4- Mathematical model

In this section, after introducing the nomenclature, the proposed model is presented and the model is linearized.

4-1- Nomenclature

Indices:

I	Set of existing DCs indexed by i
I'	Set of candidate location for new DCs indexed by i'
L	Set of all existing and candidate location for DCs indexed by $l, I \cup I'$
J	Set of sellers indexed by j
K	Set of suppliers in the market indexed by k
R	Set of routes between main producer and DCs indexed by r
M	Set of routes between DCs and sellers indexed by m
S	Set of disaster scenarios indexed by s
T	Set of time periods indexed by t
P	Set of products

Parameters:

$f_{i'}$	Fixed cost of establishing a DC at candidate location i'
opc_{plr}	Unit transportation cost of product p from producer to DC l under scenario s
tr_{ljmp}	Unit transportation cost of product p from DC l to reseller j under scenario s
$Scap_p$	Production capacity of producer for product p
$Dcap_l$	Capacity of DC l
$Rcap_j$	Capacity of reseller j
mp_{kp}	Maximum supply of product p by supplier k
ord_j	Minimum order of reseller j from the producer at each period
pr_{jkp}	Unit cost of supplying product p for reseller j from supplier k at each period
η_{isrt}	A binary parameter, equal to one if the routes r between producer and DC i is disrupted under scenario s at period t ; 0 otherwise
λ_{ist}	Percent of disruption at DC i under scenario s at period t
γ_{ljsmt}	A binary parameter, equal to one if the routs m between DC l and reseller j is disrupted under scenario s at period t ; 0 otherwise
d_{jspt}	Demand for product p at reseller j in period t under scenario s
tt_{jlm}	Travel time from DC l to reseller j in rout m
ttf_{jk}	Travel time from supplier k to reseller j
$Budg$	Maximum budget for establishing new DC
hd_l	Unit holding cost at DC l
θ	Percent of the profits from product sales at each reseller
LB_p	The minimum price of the product p

Decision variables:

PRS_{pjs}	Price of product p at reseller j under scenario s
XD_{lpsrt}	Quantity of product p delivered at DC l in rout r under scenario s at period t
Y_{ljpsmt}	Quantity of product p delivered at reseller j from DC l in rout m under scenario s at period t
Xo_{kjpsst}	Quantity of product p bought from supplier k by reseller j under scenario s at period t
In_{plst}	Product p inventory level at DC l under scenario s at the end of period t
$X_{i'}$	A binary variable, equal to 1 if a DC is located at site i' ; 0 otherwise

4-2- Objective functions

The leader objective function maximizes the expected profit of the producer and the follower objective function maximizes profit of resellers.

$$Leader : Max \sum_s \pi_s \left(\sum_l \sum_j \sum_p \sum_t \sum_m (1-\theta) PRS_{pjs} Y_{ljpsmt} - \sum_p \sum_t hd_l In_{plst} - \left(\sum_t \sum_r \sum_l \sum_p opc_{plr} XD_{lpsrt} + \sum_t \sum_m \sum_l \sum_j \sum_p tr_{ljmp} Y_{ljpsmt} \right) \right) - \sum_{i'} f_{i'} X_{i'} \quad (1)$$

$$Follower : Max \sum_t \sum_s \pi_s \left(\sum_m \sum_l \sum_j \sum_p \theta PRS_{pjs} Y_{ljpsmt} + \sum_k \sum_j \sum_p PRS_{pjs} Xo_{kjpsst} - \sum_k \sum_j \sum_p pr_{jkp} Xo_{kjpsst} \right) \quad (2)$$

The leader objective function components under each scenario include the profit of selling the products by resellers, inventory costs, transportation costs and costs of establishing new DCs. Also, the follower objective function components include the expected profit of selling products which have been supplied from the producer and other suppliers and the other term is the costs of supplying products from other suppliers.

Decision variable $X_{i'}$ is the only scenario-independent variable. More specifically, decision on the location of DCs are made before a scenario occurs and hence the value of $X_{i'}$ is not reliant on the scenario realization. In our two-stage programming approach the value of decision variable $X_{i'}$ is determined in *stage 1* and the other decisions are made in *Stage 2* when a decision scenario is realized.

4-3- Model constraints

The objective functions formulated in Section 4.3 are subject to the following constraints.

$$\sum_r \sum_l XD_{lpsrt} \leq Scap_p \quad \forall p \in P, \forall s \in S, \forall t \in T \quad (3)$$

$$\sum_p XD_{lpsrt} \leq Dcap_l (1 - \eta_{lsrt}) \quad \forall l \in L, \forall s \in S, \forall r \in R, \forall t \in T \quad (4)$$

$$\sum_r \sum_p XD_{i'psrt} \leq Dcap_{i'} X_{i'} \quad \forall i' \in I', \forall s \in S, \forall t \in T \quad (5)$$

$$\sum_r \sum_p XD_{ipsrt} \leq Dcap_i \lambda_{ist} \quad \forall i \in I, \forall s \in S, \forall t \in T \quad (6)$$

$$In_{lps,t-1} + \sum_r XD_{lpsrt} = \sum_r \sum_j Y_{ljpsrt} + In_{lpst} \quad \forall l \in L, \forall p \in P, \forall s \in S, \forall t \in T \quad (7)$$

$$Y_{ljpsmt} \leq Rcap_j \gamma_{ljpsmt} \quad \forall j \in J, \forall s \in S, \forall t \in T \quad (8)$$

$$\sum_m \sum_l \sum_p Y_{ljpsmt} \leq Rcap_j \quad \forall j \in J, \forall s \in S, \forall t \in T \quad (9)$$

$$\sum_r \sum_l Y_{ljpsrt} + \sum_k Xo_{kjpst} = d_{jpst} \quad \forall j \in J, \forall p \in P, \forall s \in S, \forall t \in T \quad (10)$$

$$\sum_j Xo_{kjpst} \leq mp_{kp} \quad \forall k \in K, \forall p \in P, \forall s \in S, \forall t \in T \quad (11)$$

$$\sum_m \sum_p \sum_l Y_{ljpsmt} \geq ord_j \quad \forall j \in J, \forall s \in S, \forall t \in T \quad (12)$$

$$LB_p \leq PRS_{pjs} \leq UB_p \quad \forall j \in J, \forall p \in P, \forall s \in S \quad (13)$$

$$\sum_{i'} f_{i'} X_{i'} \leq budg \quad (14)$$

$$PRS_{pjs}, XD_{lpsrt}, Y_{ljpsmt}, Xo_{kjpst}, In_{plst}, \alpha_{jkpst} \geq 0, X_{i'} \in \{0,1\} \quad (15)$$

Constraint (3) ensures that the quantity of products in each period does not exceed the production capacity of producer for each product. Constraint (4) enforces having no flow of product in the routes if they have been disrupted. Constraint (5) makes sure to have no flow of product to new DCs if they have not been established. Constraint (6) considers the capacity of existing DCs by considering partial disruption. Constraint (7) represents product inventory balance constraints at DCs. Constraint (8) and (9) determine the capacity of resellers by respect to disruption in connection routes. Constraint (10) ensures that demands in resellers are fulfilled under each scenario. Constraints (11) express the maximum capacity each supplier could supply for each product. Constraint (12) presents that each reseller should order a minimum quantity of products from the producer. Constraints (13) and (14) express the minimum and maximum price for each product and the available budget for establishing new DCs respectively. At last, constraints (15) define the eligible domains of the decision variables.

4-4- Linearization of the model

As it is obvious in objective functions, two continuous variables are multiplied each other, $PRS_{pjs} Y_{ljpsmt}$ and $PRS_{pjs} X_{okjpst}$, so the model is nonlinear. For linearization of the model we use the approach introduced by Vidal and Goetschalckx (2001). In this approach, first a lower bound and an upper bound should be determined as follows for each of the continuous variables have multiplied. Constraints (16) and (17) determine lower bound and upper bound for variables X_{okjpst} and Y_{ljpsmt} which is equal to demand of each product at resellers.

$$0 \leq X_{okjpst} \leq d_{jps} \quad \forall p \in P, k \in K, j \in J, s \in S, t \in T \quad (16)$$

$$0 \leq Y_{ljpsmt} \leq d_{jpst} \quad \forall p \in P, l \in L, j \in J, s \in S, m \in M, t \in T \quad (17)$$

Then the nonlinear phrase is replaced with new continuous variables as illustrated below:

$$W_{jkpst} = PRS_{pjs} X_{okjpst} \quad \forall p \in P, k \in K, j \in J, s \in S, t \in T \quad (18)$$

$$V_{ljpsmt} = PRS_{pjs} Y_{ljpsmt} \quad \forall p \in P, l \in L, j \in J, s \in S, m \in M, t \in T \quad (19)$$

And these constraints are added to the model:

$$0 \leq W_{jkpst} \leq d_{jpst} PRS_{pjs} \quad \forall p \in P, k \in K, j \in J, s \in S, t \in T \quad (20)$$

$$LB_p X_{okjpst} \leq W_{jkpst} \leq UB_p X_{okjpst} \quad \forall p \in P, k \in K, j \in J, s \in S, t \in T \quad (21)$$

$$0 \leq V_{ljpsmt} \leq d_{jpst} PRS_{pjs} \quad \forall p \in P, l \in L, j \in J, s \in S, m \in M, t \in T \quad (22)$$

$$LB_p Y_{ljpsmt} \leq V_{ljpsmt} \leq UB_p Y_{ljpsmt} \quad \forall p \in P, l \in L, j \in J, s \in S, m \in M, t \in T \quad (23)$$

5- Solution procedure

In bi level optimization, two optimization level exists which are leader and follower optimization levels. The feasible region of the upper level optimization is determined by its own constraints and the follower constraints. In addition, in bi level programming some of the variables are controlled by leader and some of them are under the control of the followers. The model can be considered as a two-person game where the leader knows the cost function of the followers who may or may not know the cost function of the leader (Saharidis and Ierapetritou, 2009). As Sun et al.(2008) claimed, in general, it is difficult to solve the bi-level programming problem for the following reasons. First, the bi level programming problem is an NP-hard as Ben-Ayed et al.(1988) investigated, also bi level models are possible to be a non convex problem while the upper level and lower level are convex. In the literature, the exact method for solving mixed integer bi-level linear problem addressed a very restricted class of problem. In our model both of objective functions are linear and the objective function includes some binary variables. We used reformulation method for solving the model. The reformulation techniques transform the bi-level model into a single level one, for example by using Karush–Kuhn–Tucker (KKT) optimality conditions of the lower level and adding them as constraints in the upper level problem. In this paper, we used benders decomposition algorithm (BDA) for resolution of proposed bi-level mixed integer model. The proposed algorithm is developed by Saharidis and Ierapetritou (2009). BDA is used in a variety of contexts of supply chain management, such as Pishvae et al. (2014), Üster and Agrahari (2011), Keyvanshokoh et al. (2015). In BDA that was first introduced by Benders (1962) the model is decomposed into series of sub-problems for facilitating the solution of complex mixed integer models. One of the sub-problem is slave problem (SP) which is obtained by fixing the binary variables in the bi-level model and the second one is restricted master problem (RMP) which gives the optimal solution after the addition of cuts. The

assumption which is added in this algorithm for solving mixed integer bi-level problem (MIBLP) is that although integer variables could appear in both levels they should be controlled by leader optimization problem. Let fix the binary variable in the proposed bi-level model to given values ($X_{i'} = \overline{X}_{i'}$) then the slave problem is formulated as follows which has been named SP1:

$$\begin{aligned}
 \mathbf{SP1: Leader} : \text{Max} \sum_s \pi_s & \left(\sum_{l,j,p,t,m} (1-\theta) PRS_{pjs} Y_{ljpsmt} - \sum_{p,t} hd_l In_{plst} \right) \\
 & - \left(\sum_{t,r,l,p} opc_{plr} XD_{lpsrt} + \sum_{t,m,l,j,p} tr_{ljmp} Y_{ljpsmt} \right) - \sum_{i'} f_{i'} \overline{X}_{i'} \\
 \mathbf{Follower} : \text{Max} \sum_t \sum_s \pi_s & \left(\sum_{m,l,j,p} \theta PRS_{pjs} Y_{ljpsmt} + \sum_{k,j,p} PRS_{pjs} Xo_{kjpst} \right. \\
 & \left. - \sum_{k,j,p} pr_{jkp} Xo_{kjpst} \right)
 \end{aligned} \tag{25}$$

Subject to:

$$\sum_r \sum_p XD_{rpsrt} \leq Dcap_{i'} \overline{X}_{i'} \quad \forall i' \in I', \forall s \in S, \forall t \in T \tag{26}$$

And constraints(3,4,6,7,8,9,10,11,12,13,15,20,21,22,23).

Then the SP1 can be transformed into a single level problem by using KKT optimality condition. The follower objective function is introduced by $FF(V, W, Xo)$ and the set of continuous variables is demonstrated by $CV = \{W, V, Xo, PRS, Y, XD\}$, also u_i represents the dual variables of corresponding constraints of SP1. The constraints of the model are in the shape of $G(cv) \leq 0$. Then the SP2 is created as follow. The constraints of SP2 for optimality condition are given in appendix.

SP2:

$$\text{Max} \sum_s \pi_s \left(\sum_{l,j,p,t,m} (1-\theta) PRS_{pjs} Y_{ljpsmt} - \sum_{p,t} hd_l In_{plst} \right) - \sum_{i'} f_{i'} \overline{X}_{i'} \tag{27}$$

Subject to:

Stationary conditions:

$$\nabla_{cv} FF(V, W, Xo) - \sum_i v_i \nabla_{cv} G_i(cv) = 0 \tag{28}$$

Complementary conditions:

$$v_i G_i(cv) = 0 \tag{29}$$

And constraints{3,4,6,7,8,9,10,11,12,13,15,,20,21,22,23,26,27,28}.

After solving SP2, the active constraint has been determined, for example, if constraint (3) specified as an active constraint, it is added to later sub-problem (SP3) as an equal constraint as follows:

SP3: The objective function of equation (24)

Subject to:

Active constraints for example:

$$\sum_r \sum_l XD_{lpsrt} = Scap_p \quad \forall p \in P, \forall s \in S, \forall t \in T$$

And constraints {4,6,7,8,9,10,11,12,13,15,,21,22,23,24}.

Now for achieving the lower bound at each iteration, dual of SP3 (DSP3) is solved and then the lower bound of the algorithm is updated. The dual sub-problem formulation is as follow:

$$\begin{aligned} \text{Min Dual Sub - problem: } & \sum_p \sum_s \sum_t v_{pst}^3 Scap_p + \sum_l \sum_s \sum_r \sum_t v_{lsrt}^4 Dcap_l (1 - \eta_{lsrt}) + \sum_{i'} \sum_s \sum_t v_{i'st}^5 Dcap_{i'} X_{i'} \\ & + \sum_i \sum_s \sum_t v_{ist}^6 Dcap_{i'} \lambda_{ist} + \sum_l \sum_j \sum_s \sum_m \sum_t v_{jst}^8 Rcap_j \gamma_{ljst} + \sum_j \sum_s \sum_t v_{jst}^9 Rcap_j + \sum_j \sum_p \sum_s \sum_t v_{jpst}^{10} d_{jpst} \\ & + \sum_k \sum_p \sum_s \sum_t v_{kpst}^{11} mp_{kp} + \sum_j \sum_s \sum_t v_{jst}^{12} (-ord_j) + \sum_p \sum_j \sum_s v_{jst}^{13} (-LP_p) + \sum_p \sum_j \sum_s v_{jst}^{14} UP_p \end{aligned} \quad (30)$$

Subject to:

$$v_{pst}^3 + v_{lsrt}^4 + v_{ist}^5 + v_{lst}^6 + v_{lpst}^7 \geq -\pi_s opc_{plr} \quad \forall l, p, s, r, t \quad (31)$$

$$-v_{lpst}^7 + v_{ljst}^8 + v_{jst}^9 + v_{jpst}^{10} - v_{jst}^{12} + LB_p v_{ljpsmt}^{23} - UB_p v_{ljpsmt}^{\square 23} \geq \pi_s tr_{ljmp} \quad \forall l, j, p, s, m, t \quad (32)$$

$$v_{jpst}^{10} + v_{kpst}^{11} + LB_p v_{pkjst}^{21} - UB_p v_{pkjst}^{\square 23} \geq 0 \quad \forall j, k, p, s, t \quad (33)$$

$$-v_{jps}^{13} + v_{jps}^{\square 13} - \sum_k \sum_t v_{pkjst}^{20} - \sum_k \sum_t d_{jpst} v_{pkjst}^{20} - \sum_l \sum_m \sum_t d_{jpst} v_{ljpsmt}^{22} \geq 0 \quad \forall l, j, m, p, s, t \quad (34)$$

$$v_{jkpst}^{20} - v_{pkjst}^{21} + v_{pkjst}^{\square 21} \geq \pi_s \quad \forall l, j, p, s, m, t \quad (35)$$

$$v_{jkpst}^{22} - v_{ljpsmt}^{23} + v_{ljpsmt}^{\square 23} \geq (1 - \theta) \pi_s \quad \forall l, j, p, s, m, t \quad (36)$$

$$-v_{lpst}^7 + v_{lps(t+1)}^7 \geq \pi_s hd_l \quad \forall l, p, s, t \quad (37)$$

$$v_{pst}^3, v_{lsrt}^4, v_{i'st}^5, v_{ist}^6, v_{jst}^8, v_{jst}^9, v_{jpst}^{11}, v_{jst}^{12}, v_{jst}^{13}, v_{jst}^{14}, v_{pkjst}^{20}, v_{ljpsmt}^{22}, v_{ljpsmt}^{23} \geq 0 \quad \forall p, l, j, s, r, t \quad (38)$$

$$v_{jpst}^{10}, v_{lpst}^7 \text{ free } \forall p, l, j, s, t \quad (39)$$

Now, according to DSP3 solution, the RMP is created for obtaining the upper bound at each iteration.

$$Max RMP : \xi - \sum_{i'} f_{i'} X_{i'} + constant \quad (40)$$

Subject to:

$$\begin{aligned} \xi \leq & \sum_p \sum_s \sum_t v_{pst}^3 Scap_p + \sum_l \sum_s \sum_r \sum_t v_{lsrt}^4 Dcap_l (1 - \eta_{lsrt}) + \sum_{i'} \sum_s \sum_t v_{i'st}^5 Dcap_{i'} X_{i'} + \sum_i \sum_s \sum_t v_{ist}^6 Dcap_{i'} \lambda_{ist} \\ & + \sum_l \sum_j \sum_s \sum_m \sum_t v_{jst}^8 Rcap_j \gamma_{ljst} + \sum_j \sum_s \sum_t v_{jst}^9 Rcap_j + \sum_j \sum_p \sum_s \sum_t v_{jpst}^{10} d_{jpst} + \sum_k \sum_p \sum_s \sum_t v_{jpst}^{11} mp_{kp} \\ & + \sum_j \sum_s \sum_t v_{jst}^{12} (-ord_j) + \sum_p \sum_j \sum_s v_{jst}^{13} (-LP_p) + \sum_p \sum_j \sum_s v_{jst}^{14} UP_p \end{aligned} \quad (41)$$

$$\begin{aligned} & \sum_p \sum_s \sum_t \bar{v}_{pst}^3 Scap_p + \sum_l \sum_s \sum_r \sum_t \bar{v}_{lsrt}^4 Dcap_l (1 - \eta_{lsrt}) + \sum_{i'} \sum_s \sum_t \bar{v}_{i'st}^5 Dcap_{i'} X_{i'} + \sum_i \sum_s \sum_t \bar{v}_{ist}^6 Dcap_{i'} \lambda_{ist} \\ & + \sum_l \sum_j \sum_s \sum_m \sum_t \bar{v}_{jst}^8 Rcap_j \gamma_{ljst} + \sum_j \sum_s \sum_t \bar{v}_{jst}^9 Rcap_j + \sum_j \sum_p \sum_s \sum_t \bar{v}_{jpst}^{10} d_{jpst} + \sum_k \sum_p \sum_s \sum_t \bar{v}_{jpst}^{11} mp_{kp} \\ & + \sum_j \sum_s \sum_t \bar{v}_{jst}^{12} (-ord_j) + \sum_p \sum_j \sum_s \bar{v}_{jst}^{13} (-LP_p) + \sum_p \sum_j \sum_s \bar{v}_{jst}^{14} UP_p \geq 0 \end{aligned} \quad (42)$$

$$\sum_{i'} f_{i'} X_{i'} \leq budg \quad (43)$$

Constraint (40) and (41) represent the optimality and feasibility cut and v_{\circ} and \bar{v}_{\circ} indicates the extreme points and rays which have obtained by solving the DSP3. In the first iteration the variables $X_{i'}$ are assumed equal to one caused SP2 to be feasible at first iteration. While the gap of lower and upper bound becomes smaller than a specific value the algorithm will be continued.

6- Computational results

In this section some experiments have been designed to (1) evaluate the performance of the proposed solution method, (2) examine the impact of disruption and (3) investigate the competition among the producer and resellers and (4) also sensitive analysis on the capacity of facilities is being done. All computational experiments are completed using mathematical modeling software. Data for parameters of sets are generated randomly for a large scale problem as shown in Table 1.

Table 1. Characteristics of the dataset used in all experiments

	$ L $	$ I $	$ I' $	$ J $	$ R $	$ M $	$ P $	$ T $	$ S $
Dataset	11	6	5	18	4	4	5	4	4

6-1- The initial numerical results

For three set of random parameters the model is solved by the proposed algorithm (Benders' decomposition) and the number of iterations for convergence of the model, running time and lower bound and upper bound of the solution is reported in table2.

Table 2. Numerical results for the three datasets

Experiments	Number of iterations	Run time(Minutes)	Lower bound	Upper bound
1	17	37	23412	23412
2	19	41	33569	33570
3	15	36	43120	43120

The related experimental results are reported in figure 2, as the experimental results illustrated the running time of the bi-level model is appropriate for this large scale problem. Figure 2 shows the convergence of the model more clearly for three mentioned data sets.

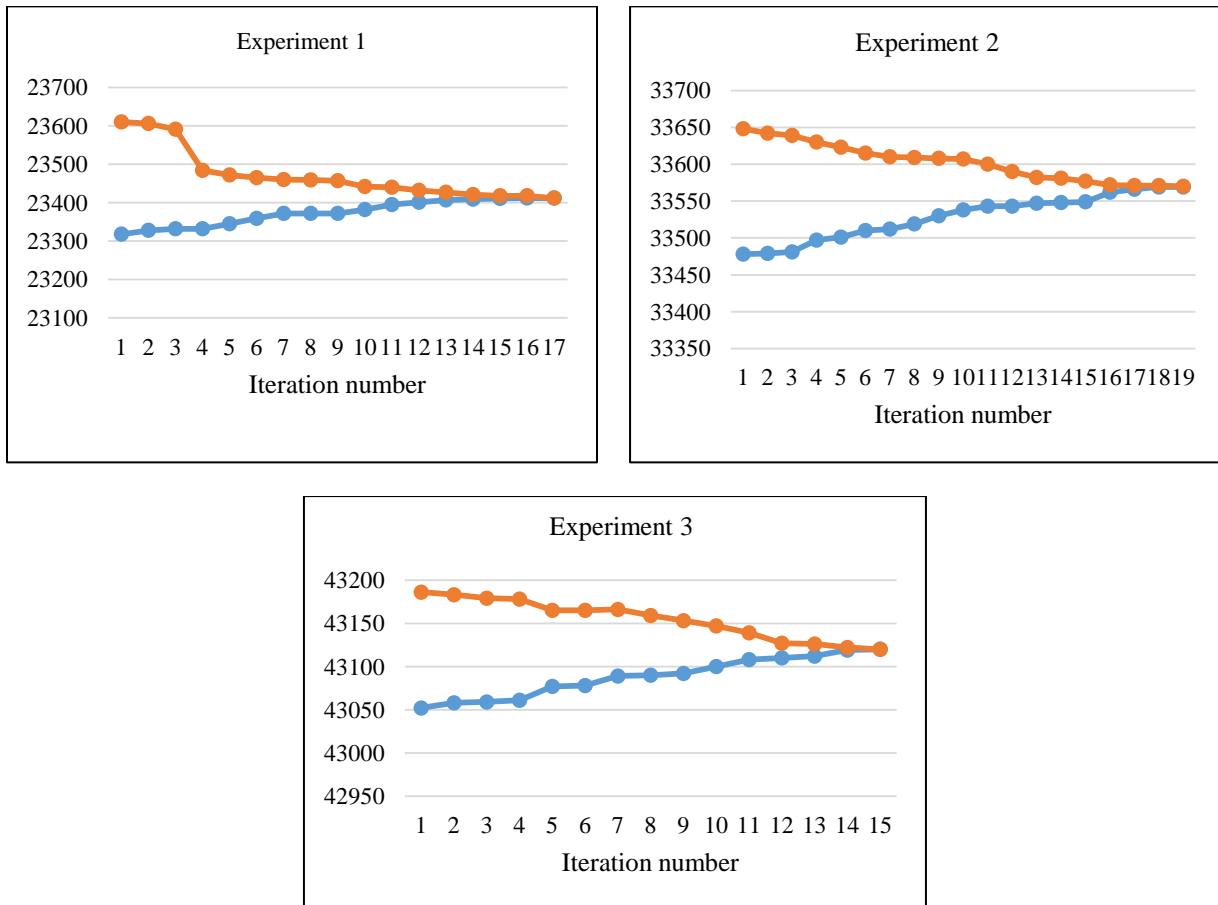


Figure 2. Convergence of benders decomposition algorithm for experiments 1-3

6-2- Impact of disruption

This section aims to examine the impact of considering disruption in competitive supply chain network design. First the model is solved without any disruption in distribution centers and transportation links and then disruptions are considered individually. The importance level of each disruption and their impact on the profit of producer and resellers is determined. As it has being illustrated in figure 3 the producer

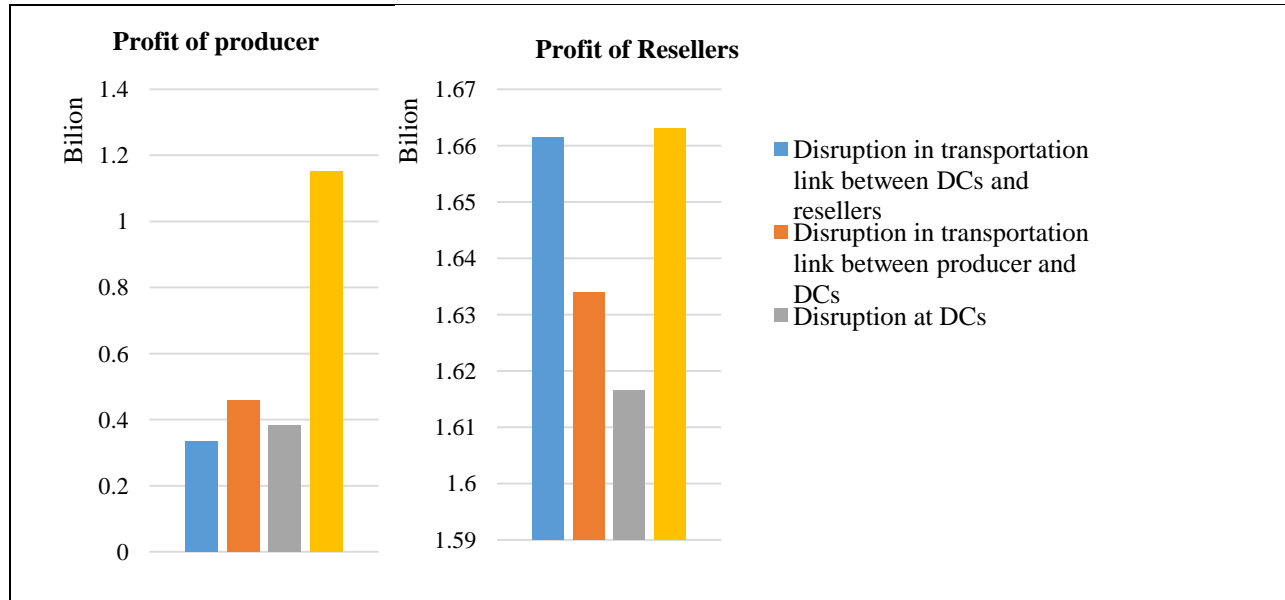


Figure 3. Impact of Disruption

and also reseller's profits are reduced when disruption is considered in SCND, but the impact of the disruption on transportation links between DCs and resellers are more obvious on the profit of producer while this disruption makes the resellers obtain more profit in comparison with other case of disruptions. As it is obvious, disruption at DCs will be more effective on the performance of supply chain and decreases the profit of producer and resellers more visibly.

It can be concluded that each disruption has a different impact on the performance of supply chain and considering these disruptions in SCND could cause recognizing them easier.

6-3- Considering competition

As mentioned before, the competition has been considered among producer and resellers. For analyzing the competition among them, the proposed model is solved first without considering the competition. For this purpose, each of objective functions is optimized and then the results are compared with competitive condition.

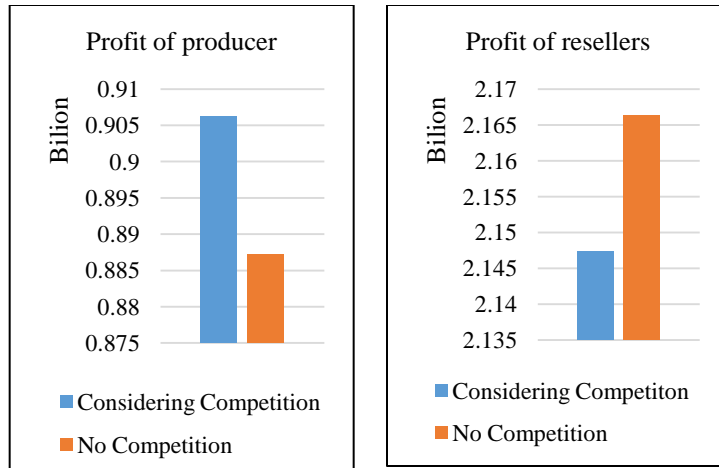


Figure 4. Competition between Producer and resellers

As figure 4 illustrates when competition is considered, the leader decides for the location of new DCs and the price of products which makes the profit of resellers reduce. When more DCs are established the resellers are eager to provide their needs more form the producer which makes the profit of producer increase. So as a leader the producer should be considered the market's competitive environment to achieve more profits.

6-4- Capacity of facilities

Now sensitivity analysis will be done to examine whether facility capacity adjustment can be used as a strategy to improve Profit for the leader or not. A general observation is that increased capacity of DCs results in reduced supply chain cost and increase in profit for leader. Similar patterns can be observed for the three datasets. The cost savings (curve steepness) is indeed a function of “inventory cost over transportation cost” ratio. As figure 5 show, dataset 2 holds the lowest ratio implying a greater transportation cost and smaller inventory cost which allows the network to take advantage of increased capacity of facilities to reduce the frequency and quantity of shipments between supply chain nodes.

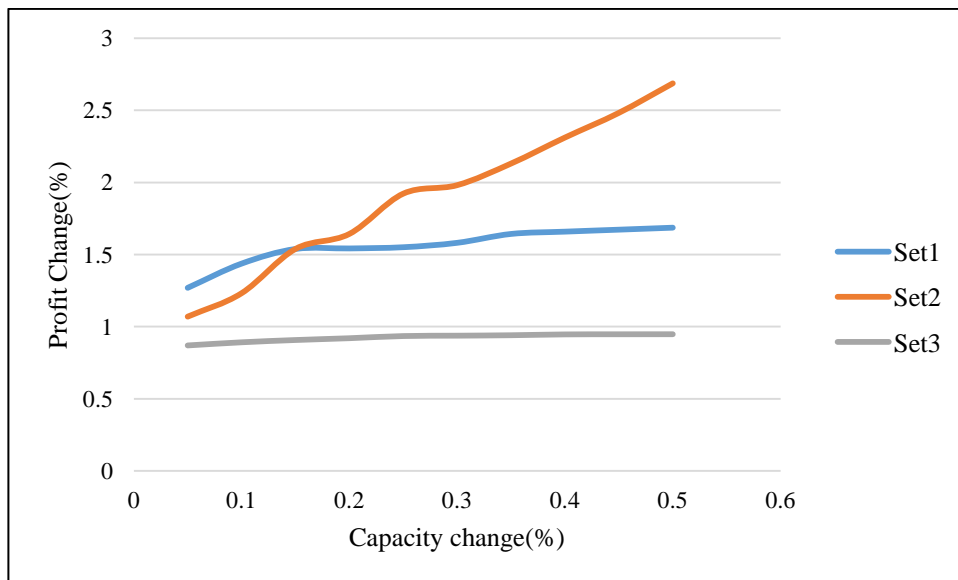


Figure 5. Analysis on capacity of facilities

7- Conclusion and future research

Competition and disruption both have a great impact on the correct performance of organizations and firms. Small crash in systems could cause different kind of disruptions and also can be seen as a severe threat to the society and environment. Disruption in the supply chain components can cause a loss of market share in a competitive market and will create irreparable damages or harm. Therefore, taking possible disruptions into account in supply chains at the planning period could reduce costs, increase competitiveness, service and reliability. In this paper, a bi level programming was used to design a reliable competitive supply chain network by considering risk of disruption in distribution centers and communication links. The competition has been considered among two main components of the chain (the main provider agencies or retail-sales). Then the impact of each disruption and competition among supply chain members' has been investigated. Also Benders' decomposition algorithm was used for solving the proposed model and the efficiency of the proposed solution is investigated for large scale problem.

The modeling effort in this paper can set the stage for additional research in the area of competitive supply chain network design. Future researches could investigate the application of the model and solution method presented in this paper to manage actual supply chain challenges. In addition, more sophisticated models and solution techniques such as multi-objective programming and also robust optimization can be used for developing the presented model in this paper.

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Appendix A

In this section the constraints of SP2 (Slave problem 2) for KKT optimality condition are given:

Complementary condition

$$v_{pst}^3 \left(\sum_r \sum_l XD_{lpsrt} - Scap_p \right) = 0 \quad \forall p \in P, \forall s \in S, \forall t \in T \quad A1$$

$$v_{lsrt}^4 \left(\sum_p XD_{lpsrt} - Dcap_l (1 - \eta_{lsrt}) \right) = 0 \quad \forall l \in L, \forall s \in S, \forall r \in R, \forall t \in T \quad A2$$

$$v_{ist}^5 \left(\sum_r \sum_p XD_{ipsrt} - Dcap_i \bar{X}_i \right) = 0 \quad \forall i' \in I', \forall s \in S, \forall t \in T \quad A3$$

$$v_{ist}^6 \left(\sum_r \sum_p XD_{ipsrt} - Dcap_i \lambda_{ist} \right) = 0 \quad \forall i \in I, \forall s \in S, \forall t \in T \quad A4$$

$$v_{lpst}^7 \left(In_{lps,t-1} + \sum_r XD_{lpsrt} - \sum_r \sum_j Y_{ljpsrt} - In_{lpst} \right) = 0 \quad \forall l \in L, \forall p \in P, \forall s \in S, \forall t \in T \quad A5$$

$$v_{jst}^8 \left(Y_{ljpsmt} - Rcap_j \gamma_{ljpsmt} \right) = 0 \quad \forall j \in J, \forall s \in S, \forall t \in T \quad A6$$

$$v_{jst}^9 \left(\sum_m \sum_l \sum_p Y_{ljpsmt} - Rcap_j \right) = 0 \quad \forall j \in J, \forall s \in S, \forall t \in T \quad A7$$

$$v_{jpst}^{10} \left(\sum_l Y_{ljpsrt} + \sum_k Xo_{kjpst} - d_{jpst} \right) = 0 \quad \forall j \in J, \forall p \in P, \forall s \in S, \forall t \in T \quad \text{A8}$$

$$v_{kpst}^{11} \left(\sum_j Xo_{kjpst} - mp_{kp} \right) = 0 \quad \forall k \in K, \forall p \in P, \forall s \in S, \forall t \in T \quad \text{A9}$$

$$-v_{lpst}^{12} \left(\sum_m \sum_p \sum_l Y_{ljpsmt} - ord_j \right) = 0 \quad \forall t \in T \forall j \in J, \forall s \in S \quad \text{A10}$$

$$v_{jps}^{13} \left(LB_p - PRS_{pjs} \right) = 0 \quad \forall j \in J, \forall p \in P, \forall s \in S \quad \text{A11}$$

$$v_{jps}^{\square 13} \left(PRS_{pjs} - UB_p \right) = 0 \quad \forall j \in J, \forall p \in P, \forall s \in S \quad \text{A12}$$

$$v_{pkjst}^{20} \left(W_{jkpst} - d_{jpst} PRS_{pjs} \right) = 0 \quad \forall p \in P, k \in K, j \in J, s \in S, t \in T \quad \text{A13}$$

$$v_{pkjst}^{21} \left(LB_p Xo_{kjpst} - W_{jkpst} \right) = 0 \quad \forall p \in P, k \in K, j \in J, s \in S, t \in T \quad \text{A14}$$

$$v_{pkjst}^{\square 21} \left(W_{jkpst} - UB_p Xo_{kjpst} \right) = 0 \quad \forall p \in P, k \in K, j \in J, s \in S, t \in T \quad \text{A15}$$

$$v_{ljpsmt}^{22} \left(V_{ljpsmt} - d_{jpst} PRS_{pjs} \right) = 0 \quad \forall p \in P, l \in L, j \in J, s \in S, m \in M, t \in T \quad \text{A16}$$

$$v_{ljpsmt}^{23} \left(LB_p Y_{ljpsmt} - V_{ljpsmt} \right) = 0 \quad \forall p \in P, l \in L, j \in J, s \in S, m \in M, t \in T \quad \text{A17}$$

$$v_{ljpsmt}^{\square 23} \left(V_{ljpsmt} - UB_p Y_{ljpsmt} \right) = 0 \quad \forall p \in P, l \in L, j \in J, s \in S, m \in M, t \in T \quad \text{A18}$$

Stationary Condition

$$-v_{pst}^3 - v_{lsrt}^4 - v_{lst}^5 - v_{lst}^6 - v_{lpst}^7 = 0 \quad \forall l, p, s, r, t \quad \text{A19}$$

$$+v_{lpst}^7 - v_{lst}^8 - v_{jst}^9 - v_{jpst}^{10} + v_{jst}^{12} - LB_p v_{ljpsmt}^{23} + UB_p v_{ljpsmt}^{\square 23} = 0 \quad \forall l, j, p, s, m, t \quad \text{A20}$$

$$v_{jpst}^{10} + v_{kpst}^{11} + LB_p v_{pkjst}^{21} - UB_p v_{pkjst}^{\square 23} = -\pi_s pr_{jkp} \quad \forall j, k, p, s, t \quad \text{A21}$$

$$-v_{jps}^{13} + v_{jps}^{\square 13} - \sum_k \sum_t v_{pkjst}^{20} - \sum_k \sum_t d_{jpst} v_{pkjst}^{20} - \sum_l \sum_m \sum_t d_{jpst} v_{ljpsmt}^{22} = 0 \quad \forall l, j, m, p, s, t \quad \text{A22}$$

$$v_{jkpst}^{20} - v_{pkjst}^{21} + v_{pkjst}^{\square 21} = \pi_s \quad \forall l, j, p, s, m, t \quad \text{A23}$$

$$v_{jkpst}^{22} - v_{ljpsmt}^{23} + v_{ljpsmt}^{\square 23} = \theta \pi_s \quad \forall l, j, p, s, m, t \quad \text{A25}$$

$$-v_{lpst}^7 + v_{lpst(t+1)}^7 = 0 \quad \forall l, p, s, t \quad \text{A26}$$