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A game theoretic approach to Pricing and Cooperative advertising in a multi-retailer supply chain

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Abstract

During the past few decades, there have been tremendous efforts in cooperative advertising. In spite of many practical applications in real life, cooperation in advertising and pricing strategies in a one-manufacturer and multi-retailer supply chain is almost overlooked in the literature. Hence, this paper seeks to investigate optimum co-op advertising and pricing decisions in a B2B relationship for a supply chain consist of a manufacturer and numerous multiple retailers in Iran as a case study. This paper introduces a game theoretic model containing pricing and cooperative advertising in a one-manufacturer and multi-retailer structure. Non-cooperative and cooperative game structures are used for analyzing the proposed model. The non-cooperative game structure uses Stackelberg game among the echelons and Nash game in the retailer echelon. Motivated by a real case study including an Iranian supply chain data of one manufacturer and 150 retailers, a novel model proposed to tackle the similar condition occurred in real life. The results indicate that the manufacturer prefers to suggest higher participation rate to smaller retailers. Sensitivity analysis is presented, and some managerial insights are finally derived from the results.
Keywords: Cooperative advertising, pricing, supply chain coordination, participation rate, game theory, retailer segmentation.

1-Introduction

Cooperation between independent members of a supply chain relationship has attracted more eyes of scholars and market governors. In this regard, the main aim of the supply chain's members is to maximize the profit of the whole supply chain channel in the course of cooperation. Due to the fact that cooperation in between supply chain members covers a wide spectrum of subjects including pricing, advertising, discount, order quantity, and so on, much research have been conducted in this field. In this connection, Cooperative advertising is typically known as a coordination mechanism of the supply chain marketing effort that the manufacturer can act in local advertising. As stated by Huang & Li (2001), advertising can be considered in two separate sections: local and national (global) advertising. National or global advertising mainly focuses on influencing potential consumers to consider a specific brand and developing a brand preference, whereas local advertising intends to encourage consumers' instant buying behavior. Nagler (2006) reports 0.9 billion-dollar investment in cooperative advertising in the United States in 1970 and 15 billion dollars in 2000. Lieb (2012) reports 50-500 billion dollars were invested in cooperative advertising in 2012.

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Aust & Buscher (2014) have provided a review of mathematical models on cooperative advertising and conduct literature study, which reveals that there were different meanings for the cooperative advertising term.

In this broad review, they have concentrated their analysis on vertical co-op advertising, which is one of the most popular advertising strategies. They have discussed the demand and cost functions in which the models are based on and analyzed the interaction between channel members with diverse primary power structures. Jorgensen & Zaccour (2014) also surveyed the literature on co-op advertising. They presented their paper in two parts: first one manufacturer one retailer supply chain and second more complex channel.

Numerous studies on advertising efforts and pricing strategies have focused on distribution channels formed by one manufacturer and one retailer. Yue et al. (2006), used a static model to study the cooperative advertising problem by considering a price discount in the demand function. Karray & Zaccour (2006), proposed a model to study the decision of a retailer and its effects on the manufacturer. They showed that the private label overview improves the profit of the channel and members. (Yue et al., 2006) studied the cooperative advertising when the manufacturer offers price deductions to consumers. And showed up co-op advertising performance for supply chain coordination.

In 2009, many articles were published in the cooperative advertising area. Xie & Neyret (2009) identified the pricing and co-op advertising strategies in four games (Nash, Manufacturer Stackelberg, Retailer Stackelberg, and Cooperative game). But they failed to analytically solve the Manufacturer-Stackelberg game. Therefore, they fixed some parameters and solved the manufacturer's price, by a numerical example and simply compared the differences in profits, advertising expenditures, and the prices for four scenarios. Buratto & Zaccour (2009) considered cooperative advertising between the licensor and licensee of the fashion business and complicated in a licensing contract.

Also, Xie & Wei (2009) considered four behaviors of channels by using different demand function. They considered no restriction on the effect of advertising on demand. After that, Aust & Buscher, (2012), Seyedesfahani et al., (2011) and Alirezaee & Khoshalhan (2014) followed the demand function and developed their models. He et al., (2009) modeled a one-manufacturer one-retailer channel as a stochastic Stackelberg differential game; they consider the demand function which depends on the retailer's price and advertising. They incorporated uncertainty in awareness share and derive optimal feedback procedures for both types of cooperative pricing advertising. Szmerekovsky & Zhang, (2009) considered a pricing and advertising dependent demand function in a two-member supply chain and obtained manufacturer- Stackelberg equilibrium. Aust & Buscher (2012) revealed that vertical cooperative advertising can help the manufacturer-retailer supply chain coordination for higher total profits and lower retail price for consumers.

Chen (2015) modeled a dual-channel supply chain and appraised pricing and cooperative advertising strategies and determined the optimal decision variables. Co-op advertising usually includes participation rate or accrual rate in the mathematical models. Zhang et al., (2017) established a differential game model with cooperative advertising that incorporates both the participation rate and the accrual rate. Sadjadi & Alirezaee, (2020) developed a game-theoretic model in a two-echelon supply chain to study the effect of pricing structure and CA decisions on the supply chain coordination performance with four possible scenarios.

Zhang & Zhang, (2018) thought about a situation that the manufacturer deal with two retailers, however it cooperates with just one retailer in local advertising, and another retailer decline to take part in cooperative advertising. (Xiao et al., 2019) proposed a hybrid horizontal and vertical strategy of CA and analyzed retailer's coalition and found out that all firms prefer to build up the largest coalition to achieve more benefits. Mokhlesian & Zegordi (2018) studied a supply chain consisting of one dominant retailer and multiple competitive manufacturers. For the first time they modelled the problem as a multi-follower bi-level programming model, then used metaheuristic method for solving. Wang et al., (2020) considered a supply chain consist of one manufacturer and two different retailers, one traditional offline manufacturer, the other one being online retailer. They used a variable (B) to divide demand between online and offline retailers and investigated the effects of global and cooperative advertising.

Most studies of cooperative advertising consider a single-manufacturer-single-retailer channel structure. This is a limiting issue, because a manufacturer, in real practices, would commonly deal with more than one retailers. Another form is to determine the optimal marketing strategies in terms of the relationship between the manufacturer and personal specifications. Cooperative advertising within a one-manufacturer two-retailer supply chain in a dynamic environment was considered by He et al., (2011) while Karray & Zaccour (2007) have assumed a static behavior in their research. Chutani & Sethi, (2012) and He et al., (2012) studied cooperative advertising, without pricing scheme in dynamic models. Chutai & Sethi, (2012) consider retail oligopoly where a manufacturer sells his/her product through N competing retailers and formulates it as a Stackelberg differential game. They obtain the Stackelberg feedback equilibrium and derive the settings they may or may not be any cooperative advertising. He et al., (2012) considered a manufacturer who sells products through a retailer in competition with outside retailers. They modeled the interaction between the manufacturer and his retailer as a Stackelberg differential game and the interaction between the competing retailers as a Nash differential game.

Ghadimi et al., (2013); Wang et al., (2011) and Zhang & Xie, (2013) considered a manufacturer and two retailers in their model and determined optimal advertising decisions of members under several conditions. Aust & Buscher, (2016) considered one manufacturer and two retailers in their model. Each player can choose his/her margin and advertising expenditures, but they fail to consider the diversity of retailers and assume similar initial demand as and similar participation strategy for the manufacturer against two retailers. Alirezaee & khoshAlhan (2014) considered two competing retailers in a supply chain and investigated the impact of the firm's marketing efforts (pricing and cooperative advertising).

Karray (2015) considered a channel where two competing manufacturers are selling compatible products through a common retailer. The channel members make their decisions over a two-period horizon. Two-stage game theoretic models are developed to analyze the long-term effects of retailer's promotions. In line with previous studies Zhang & Zhang (2016) considered one manufacturer and two retailers in their model, but they placed a fixed value on the manufacturer's CA participation rate while Xu et al., (2018) studied a supply chain comprise of one manufacturer and two competing retailers while firms makes their decision in two periods. Setak et al., (2018) investigated a supply chain that each retailer can add his own values (such as related services or product, software, maintenance, and etc.) to the product and propose them to consumers. They presented an incentive method for information sharing of retailers and compared different scenarios of information sharing decisions.

Zhang & Xie, (2012) studied multiple retailers in their model in two situations: symmetric and asymmetric retailers (in market sizes). They considered uniform participation rate of manufacturer and demonstrated that it tends to impose unfit encouragements for different retailers. Zhou et al., (2018) considered risk-averse agents in their model and realized the decentralized supply chain may perform better than the centralized one if the firms are more risk-seeking than the centralized supply chain. They found out the cooperative game can perform better than the Non-cooperative, only if decision making in a cooperative game is less risk averse than the other two independent firms.

According to this paper literature, our paper is the first approach of simultaneously analyzing pricing and cooperative advertising with a one-manufacturer multi-retailer supply chain in a static and deterministic environment. To encounter this situation, we present a comprehensive model that divides retailers into several clusters and then determines the best marketing strategy (include pricing and cooperative advertising policy) for the manufacturer of every cluster and retailer. The major contributions of the paper include:

- The proposed model has resulted in significant outcomes in the multiple-retailer supply chain with diverse activity level and initial demands.
- This paper has studied the marketing decisions in a new demand function.
- A real case study have finally been conducted to examine the proposed model and equilibrium.

The rest of this paper is organized as follows: Section 2 describes the demand function and game-theoretic model, Stackelberg-Equilibrium, and cooperative game are described in section 3. Numerical analysis for the general case is in section 4 and conclusions, managerial issues and future research are presented in section 5.

2-Models

In this section, the problem and related assumptions, as well as notations used in this research, are defined. Then the demand functions are introduced and their development in line with channel structure in leader-follower game models are presented in the proposed supply chain, there is a single-manufacturer and multiple-retailer channel in which the manufacturer sells specified branded products to the retailer's channel, and the retailers sell only the manufacturer's product to customers (see figure 1). The decision variables for the channel members are profit margins of the manufacturer as well as retailers; advertising efforts, and the co-op advertising repayment plans for the manufacturer.

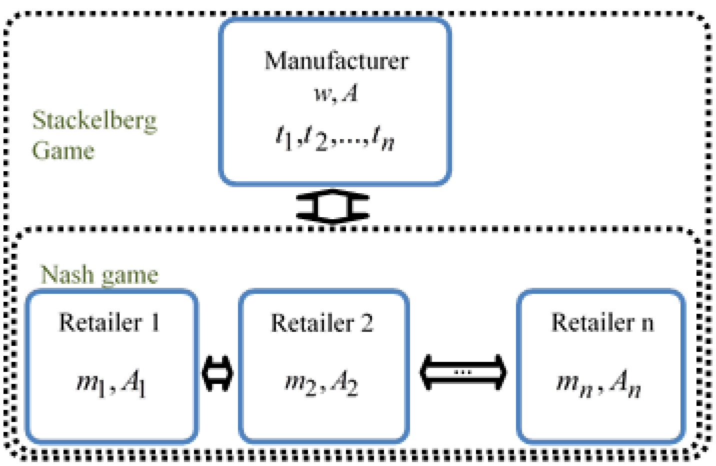

Fig. 1 Channel Structure

To investigate the performance of the supply chain, we consider a two-echelon supply chain with one manufacturer and multiple retailers, who operate in different areas. That is each retailer can sell the quantity demanded of a product, whereas the manufacturer serves as a single supplier for all retailers with a total demand of retailers. Each retailer's demand depends upon his local advertising and retail price as well as the manufacturer's global advertising. It is worthy to note for each retailer, the amount of product sold depends on mentioned three variables and independent of other retailer's pricing and advertising efforts.

Table 1. Notations

Π_M	The manufacturer's profit
Π_{R_i}	The retailer i's profit
Π_S	Total channel's profit
$D_i(p_i, A_i, A)$	The retailer i's demand function (depend on National Branding, Cooperative promotion and retail price)
p_i	Retail price of retailer i
w	Wholesale price
m_i	Profit margin of retailer i
A	Global advertising expenditures
A_i	Local advertising expenditures
t_i	Cooperative Advertising participation rate
γ_i	Distinct part of Initial base demand for retailer i
α	Constant part of Initial base demand
β	Intensity of saturation effect
k	effect of local advertising in consist of global advertising

We assume that the manufacturer does not apply price discrimination, i.e., he charges all retailers the same wholesale price w, while retailer i sells the product to customers in retail prices p_i. In addition to retail prices, the retailers' demand is influenced by the advertising expenditures of both manufacturer advertising and retailer's local one. Here, we represent manufacturer's global advertising expenditures by . A_i by i and the local advertising expenditures of retailer A

In this problem, the manufacturer can furthermore decide to offer a vertical cooperative advertising strategy to the retailers, whereby he shares a part of the local promotion costs of each retailer with a proportion t_i where $0 \le t_i \le 1$ and represent the level of manufacturers' participation in local advertising strategies. Apart from advertising costs, no further costs are considered. In other words, we ignore the production cost and yields a margin of w for the manufacturer and $m_i = p_i - w$ of for retailer i as well. The resultant profit functions of the manufacturer and retailer i are as follows:

$$\Pi_M(w, A', t_i) = \sum_{i=1}^{n} w.D_i - \sum_{i=1}^{n} t_i.A'_i - A' \tag{1}$$

$$\Pi_{r_i}(m_i, A'_i) = m_i D_i - (1 - t_i)A'_i \qquad i = 1..n \tag{2}$$

The demand of the customers is determined by a function that depends on both pricing and advertising efforts. Demand function is also influenced by retail prices as well as local and global advertising expenditures. We follow Karray (2015) and choose a demand pattern that is linearly influenced by price, whereas increases with a nonlinear trend by advertising. On the other side, the retailers' demand function decreases in price and is independent of other retailers pricing as well. Moreover, the demand of retailer i increases with local advertising effort and also manufacturer global brand advertising. The base demand function is shown in equation (3).

$$D_i = \gamma_i\left(\alpha - \beta p_i + k_m.\sqrt{A'} + k_r.\sqrt{A'_i}\right) \tag{3}$$

According to (Choi, 1991), we introduce the retailer margin m_i as a new decision variable by $m_i = p_i - w$. Splitting the retail price p_i into the wholesale price w and retailer margin m_i, the wholesale price also exerts an impact on the consumer demand, i.e. $D_i = \gamma_i\left(\alpha - \beta(w + m_i) + k_m.\sqrt{A'} + k_r.\sqrt{A'_i}\right)$. To simplify the demonstration, the sales response parameters k_m and k_r are considered, which describe the effectiveness of global and local advertising. Moreover, we introduce the ratio $k = k_m / k_r$. At any rate, demand conversion functions can be shown as:

$$D_i = \gamma_i\left(\alpha - \beta(w + m_i) + \sqrt{A'} + k\sqrt{A'_i}\right) \tag{4}$$

For the sake of simplicity, we replace A' with A^2 which means $A = \sqrt{A'}$. Hence, the total demand function for the retailer i is

$$D_i = \gamma_i\left(\alpha - \beta(w + m_i) + A + k.A_i\right) \tag{5}$$

$$D = \sum_{i=1}^{n} D_i = \sum_{i=1}^{n} \gamma_i\left(\alpha - \beta(w + m_i) + A + k.A_i\right) \tag{6}$$

The positive parameter $\alpha.\gamma_i$ denotes the initial base demand, where α is a constant factor and clearly, $\alpha.\gamma_i$ is different for each retailer whereas β, which describes the intensity of customers' saturation effect, is constant as well as k, which denotes the effect of local advertising in consisted of global advertising. Considering the demand function in the form of (6) and other stated assumptions and then substituting it into the profit functions, the functions will be converted to equations (7) and (8).

$$\Pi_M(w, A, t_i) = w.\sum_{i=1}^{n} \gamma_i\left(\alpha - \beta(w + m_i) + A + k.A_i\right) - \sum_{i=1}^{n} t_i.A_i^2 - A^2 \tag{7}$$

$$\Pi_{r_i}(m_i, A_i) = m_i\gamma_i\left(\alpha - \beta(w + m_i) + A + k.A_i\right) - (1 - t_i)A_i^2, i = 1 \ldots n \tag{8}$$

In this section as illustrated, we introduced parameters and decision variables, then demand function was consequently described and the model formulation was presented for channel member. In the next section, Stackelberg game model thereby, we first assume that all retailers act independently and make their decision about margin profit and local promotion expenditure.

3-Stackelberg equilibria

In this section, we model the decision process of supply chain members as a consecutive, non-cooperative game, considering the manufacturer as the leader and the retailers as the followers. Furthermore, we assume that the manufacturer holds the channel leadership. That is, he considers retailer's reactions and sets his wholesale price, global advertising and suggests participation rate to the retailers and also the reaction of the following retailers is taken into consideration. After that, the retailers have to determine their decision variables including retail price and local advertising expenditure. To determine the Stackelberg equilibrium by backward induction, we first solve the retailer i's optimal problem when the manufacturer's decision variables are given. Therefore, we obtain the following decision problem for each retailer (9).

Lemma3.1 *objective function (8) is a concave function with respect to* m_i *and* A_i .

Proof : To proof the optimality of the solutions of retailer i, we calculate the Hessian matrix, the second order partial derivatives are as follows:

$$H_{r_i} = \begin{pmatrix} \dfrac{\partial^2 \prod_{r_i}}{\partial m_i^{\,2}} & \dfrac{\partial^2 \prod_{r_i}}{\partial m_i \partial A_i} \\ \dfrac{\partial^2 \prod_{r_i}}{\partial A_i \partial m_i} & \dfrac{\partial^2 \prod_{r_i}}{\partial A_i^{\,2}} \end{pmatrix} = \begin{pmatrix} -2\beta\gamma_i & k\gamma_i \\ k\gamma_i & -2(1-t_i) \end{pmatrix} \tag{9}$$

The first principal minor of H_{r_i} is negative. $H_{r_i}^1 = -2\beta\gamma_i$ The second principal minor of H_{r_i} is $H_{r_i}^2 = (-2\beta\gamma_i)(-2(1-t_i)) - k\gamma_i.k\gamma_i = 4\beta\gamma_i(1-t_i) - k^2\gamma_i^{\,2}$ and is positive if $4\beta > k^2\gamma_i$ so, the principal minors of having alternating algebraic signs at the solution. It means that is negative definite and the profit of retailers is concave at this solution, which is a local maximum.

Proposition3.1 *Due to the lemma 3.1, we can set the partial first order derivatives* $\partial \prod_{r_i} / \partial m_i$ *and* $\partial \prod_{r_i} / \partial A_i$ *to zero, and by solving the resulted system of equations, the optimal variables for each retailer are obtained as equations (10, 11) as follows,*

$$m_i = \frac{2(1-t_i)(\alpha - \beta w + A)}{4\beta(1-t_i) - \gamma_i k^2}, \qquad i = 1,\ldots, n \tag{10}$$

$$A_i = \frac{\gamma_i k(\alpha - \beta w + A)}{4\beta(1-t_i) - \gamma_i k^2} \qquad i = 1,\ldots, n \tag{11}$$

In a Stackelberg game scheme, retailers’ reaction is known well by the manufacturer. Given this assumption, the manufacturer will maximize his profit by deciding the optimal wholesale price, global advertising and participation rate for each retailer. Regarding to the (10) and (11), we obtain the constraints of manufacturer’s decision problem as (12).

$$\begin{aligned} &Max \;\; \Pi_M(w, A, t_i) = \sum_{i=1}^{n} w.D_i - \sum_{i=1}^{n} t_i.A_i^2 - A^2 \\ &s.t. \quad m_i = \frac{2(1-t_i)(\alpha - \beta w + A)}{4\beta(1-t_i) - \gamma_i k^2}, \qquad i = 1 \ldots n \\ &A_i = \frac{\gamma_i k(\alpha - \beta w + A)}{4\beta(1-t_i) - \gamma_i k^2} \qquad i = 1 \ldots n \\ &0 < w \qquad 0 < A \qquad 0 \le t_i < 1 \end{aligned} \tag{12}$$

According to the demand function sign, we can calculate the feasible region for the problem (12) as $0 \le m_i \le (\alpha - \beta w + A + k.A_i)/\beta$ and $A_i \ge -(\alpha - \beta(w + m_i) + A)/k$, and also because of manufacturer effect on retailers, the feasible region for w and A is shown on (13).

$$0 \le w \le \frac{\sum_{i=1}^{n}\gamma_i(\alpha + A) - \beta\sum_{i=1}^{n}\gamma_i m_i + k\sum_{i=1}^{n}\gamma_i A_i}{\beta\sum_{i=1}^{n}\gamma_i}, A \ge -\frac{\sum_{i=1}^{n}\gamma_i(\alpha - \beta(w + m_i) + k.A_i)}{\sum_{i=1}^{n}\gamma_i} \tag{13}$$

To solve this problem, the retailer decision variables within the profit function have to be substituted by the constraints for m_i and A_i.

Lemma3.2 *The objective function of (12) is a concave function with respect to* w, A, t_i

Proof: To proof the optimality of the solutions of the manufacturer, we calculate the Hessian matrix,

$$H_M = \begin{pmatrix} \frac{\partial^2 \Pi_M}{\partial A^2} & \frac{\partial^2 \Pi_M}{\partial A \partial w} & \frac{\partial^2 \Pi_M}{\partial A \partial t_1} & \cdots & \frac{\partial^2 \Pi_M}{\partial A \partial t_n} \\ \frac{\partial^2 \Pi_M}{\partial w \partial A} & \frac{\partial^2 \Pi_M}{\partial w^2} & \frac{\partial^2 \Pi_M}{\partial w \partial t_1} & \cdots & \frac{\partial^2 \Pi_M}{\partial w \partial t_n} \\ \frac{\partial^2 \Pi_M}{\partial t_1 \partial A} & \frac{\partial^2 \Pi_M}{\partial t_1 \partial w} & \frac{\partial^2 \Pi_M}{\partial t_1^2} & \cdots & \frac{\partial^2 \Pi_M}{\partial t_1 \partial t_n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 \Pi_M}{\partial t_n \partial A} & \frac{\partial^2 \Pi_M}{\partial t_n \partial A} & \frac{\partial^2 \Pi_M}{\partial t_n \partial t_1} & \cdots & \frac{\partial^2 \Pi_M}{\partial t_n^2} \end{pmatrix} \tag{14}$$

The second order partial derivatives are as follows:

$$\begin{gathered} \frac{\partial^2 \Pi_M}{\partial A^2} = -2, \frac{\partial^2 \Pi_M}{\partial A \partial w} = \sum_{i=1}^{n}\gamma_i, \frac{\partial^2 \Pi_M}{\partial w^2} = -2\beta\sum_{i=1}^{n}\gamma_i \\ \frac{\partial^2 \Pi_M}{\partial t_i^2} = 0, \frac{\partial^2 \Pi_M}{\partial A \partial t_i} = 0, \frac{\partial^2 \Pi_M}{\partial w \partial t_i} = 0 \end{gathered} \tag{15}$$

So the Hessian matrix is (16)

$$H_M = \begin{pmatrix} -2 & \sum_{i=1}^{n}\gamma_i & 0 & \cdots & 0 \\ \sum_{i=1}^{n}\gamma_i & -2\beta\sum_{i=1}^{n}\gamma_i & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 \end{pmatrix} \tag{16}$$

The first principal minor of H_M is negative. $H_M^1 = -2$. The second principal minor of H_M is $H_M^2 = (-2).\left(-2\beta\sum_{i=1}^{n}\gamma_i\right) - \left(\sum_{i=1}^{n}\gamma_i\right)^2 = \left(\sum_{i=1}^{n}\gamma_i\right)\left(4\beta - \sum_{i=1}^{n}\gamma_i\right)$ and is positive if $4\beta > \sum_{i=1}^{n}\gamma_i$ and other principal minors are equal to zero, so the matrix is semi-definite and Manufacturer's decision problem is concave and this means that there exists a unique solution and is a local maximum.

Proposition3.2 *we can set the partial first-order derivatives* $\partial\prod_M/\partial w$, $\partial\prod_M/\partial A$ *and* $\partial\prod_M/\partial t_i$ $\quad i=1,\dots,n$ *to zero, and by solving the resulted system of equations,* $\partial\prod_M/\partial t_i=0$ $\; i=1,\dots,n$, *we derive the unique optimal value of* t_i *shown on (17), the participation rate for each retailer is independent of retailers' number and their behaviors.*

$$t_i=\frac{\left(\gamma_i k^2-4\beta\right)}{4\beta}-\frac{w\left(\gamma_i k^2-4\beta\right)}{\left(\alpha+A+\beta w\right)}\qquad i=1\dots n \tag{17}$$

The optimal value for A and w are dependent on retailers' number. Therefore, we solve a problem for each n, and the results are as follows: when there is only one monopolistic retailer in the supply chain.

Proposition3.3 *when there is only one retailer in the channel, the optimal solution is as follows,*

$$\begin{aligned}A^{(1)}&=\frac{-4\alpha\gamma_1}{\left(9k^2+4\right)\gamma_1-32\beta},\\ w^{(1)}&=\frac{\alpha}{\beta}\frac{3k^2\gamma_1-16\beta}{\left(9k^2+4\right)\gamma_1-32\beta}\end{aligned} \tag{18}$$

Proposition3.4 *More retailers in channel increase complexity, the optimal solution of the equilibria with 2-4 retailers can achieve by solving the resulted system of equations* $\partial\prod_M/\partial w$, $\partial\prod_M/\partial A$ *by consideration of (10), (11) and (18).*

Proof : To calculate the optimal solution of the manufacturer decision in any problem, we can set the partial first-order derivatives $\partial\prod_M/\partial w$, $\partial\prod_M/\partial A$ and $\partial\prod_M/\partial t_i$ $\quad i=1,\dots,n$ to zero and by solving the resulted system of equations, and considering similar sections that shown on (19) we can achieve optimal solution (20-23).

$$\begin{aligned}&F_1=\sum_{i=1}^{n}\gamma_i\;\; F_2=\sum_{i=1}^{n}\gamma_i^2\;\; F_3=\prod_{i=1}^{n}\gamma_i\;\; F_4=\sum_{i=1}^{n}\gamma_i^2\prod_{j\neq i}\gamma_j\;\; F_5=\sum_{i=1}^{n}\sum_{j\neq i}\gamma_i^2\gamma_j\;\; F_6=\sum_{i=1}^{n-1}\sum_{j=i+1}^{n}\gamma_i\gamma_j\;\; F_7=\sum_{i=1}^{n}\left(\prod_{j\neq i}\gamma_j\right)\\ &F_8=\sum_{i=1}^{n}\sum_{j\neq i}\gamma_i^2\left(\prod_{k\neq i,j}\gamma_k\right)\;\; F_9=\sum_{i=1}^{n}\sum_{j\neq i}\sum_{k\neq i,j}\gamma_i^2\gamma_j\gamma_k\;\; F_{10}=\sum_{i=1}^{n}\sum_{j\neq i}\sum_{k\neq i,j}\gamma_i\gamma_j\gamma_k\end{aligned} \tag{19}$$

The optimal solution for two retailers in the channel is shown on (21).

$$\begin{aligned}&w^{(2)}=\frac{\alpha}{\beta}.\frac{U_{21}}{U_{23}},\; A^{(2)}=\frac{-\alpha}{\beta}.\frac{U_{22}}{U_{23}}\quad U_{21}=64\beta^2F_1-12\beta k^2F_2-32\beta k^2F_3+3k^4F_4\\ &U_{22}=-4\beta F_1+2k^2F_3\\ &U_{23}=128\beta^2F_1-4\beta\left(9k^2+4\right)F_2-32\beta\left(2k^2+1\right)F_3\\ &\qquad+\left(9k^4+8k^2\right)F_4\end{aligned} \tag{20}$$

Also, the optimal solutions for more retailers on oligopoly situation are shown at (22) show the optimal solution in the supply chain with three retailers and, (23) shows it in the channel with four retailers.

$$w^{(3)} = \frac{-\alpha}{\beta}\frac{U_{31}}{U_{33}+U_{34}} \quad A^{(3)} = \frac{-\alpha}{\beta}\frac{U_{32}}{U_{33}+U_{34}}$$

$$\begin{aligned}
U_{31} &= 256\beta^3 F_1 - 48\beta^2 k^2 F_2 + 48\beta k^4 F_3 \\
&\quad - 3k^6 F_4 + 12\beta k^4 F_5 - 128\beta^2 k^2 F_6 \\
U_{32} &= 64\beta^2 F_2 - 96\beta k^2 F_3 + 12k^4 F_4 \\
&\quad - 32\beta k^2 F_5 + 128\beta^2 F_6 \\
U_{33} &= 512\beta^3 F_1 - 16\beta^2\left(4+9k^2\right)F_2 + 96\beta k^2\left(1+k^2\right)F_3 \quad U_{34} = -3k^4\left(4+3k^2\right)F_4 + 4\beta k^2\left(8+9k^2\right)F_5 \\
&\quad - 128\beta^2\left(1+2k^2\right)F
\end{aligned} \tag{21}$$

$$w^{(4)} = \frac{-\alpha}{\beta}\frac{U_{41}+U_{42}}{U_{44}+U_{45}} \quad A^{(4)} = \frac{-\alpha}{\beta}\frac{U_{43}}{U_{44}+U_{45}}$$

$$\begin{aligned}
U_{41} &= 1024\beta^4 F_1 - 192\beta^3 k^2 F_2 - 64\beta k^6 F_3 + 3k^8 F_4 \\
U_{42} &= 48\beta^2 k^4 F_5 - 512\beta^3 k^2 F_6 + 192\beta^2 k^4 F_7 - 12\beta F_8 \\
U_{43} &= 256\beta^3 F_2 - 16k^6 F_4 + 512\beta^3 F_6 + 48\beta k^4 F_8 \\
&\quad + 192\beta k^4 F_3 - 128\beta^2 k^2 F_5 - 384\beta^2 k^2 F_7 \\
U_{44} &= -2048\beta^4 F_1 + 64\beta^3\left(4+9k^2\right)F_2 \\
&\quad + 64\beta k^4\left(3+2k^2\right)F_3 - k^6\left(16+9k^2\right)F_4 \\
U_{45} &= -16\beta^2 k^2\left(8+9k^2\right)F_5 + 512\beta^3\left(1+2k^2\right)F_6 \\
&- 384\beta^2 k^2\left(1+k^2\right)F_7 + 12\beta k^4\left(4+3k^2\right)F_8
\end{aligned} \tag{22}$$

Proposition 3.5 *Optimal solution of the equilibria with 5 retailers can be achieved by solving the resulted system. The closed form is shown on (24)*

$$w^{(5)} = \frac{-\alpha}{\beta}\frac{U_{51}+U_{52}}{U_{55}+U_{56}+U_{57}} \quad A^{(5)} = \frac{-\alpha}{\beta}\frac{U_{53}+U_{54}}{U_{55}+U_{56}+U_{57}}$$

$$\begin{aligned}
U_{51} &= 4096\beta^5 F_1 - 768\beta^4 k^2 F_2 + 80\beta k^8 F_3 \\
&\quad - 3k^{10} F_4 + 192\beta^3 k^4 F_5 \\
U_{52} &= -2048\beta^4 k^2 F_6 - 256\beta^2 k^6 F_7 + 12\beta k^8 F_8 \\
&\quad - 48\beta^2 k^6 F_9 + 768\beta^3 k^4 F_{10} \\
U_{53} &= 1024\beta^4 F_2 - 320\beta k^6 F_3 + 20k^8 F_4 - 512\beta^3 k^2 F_5 \\
U_{54} &= 2048\beta^4 F_6 + 768\beta^2 k^4 F_7 - 64\beta k6 F_8 \\
&\quad + 192\beta^2 k^4 F_9 - 1536\beta^3 k^2 F_{10} \\
U_{55} &= 8192\beta^5 F_1 + \beta^4\left(1024 + 2304k^2\right)F_2 \\
&\quad - 160\beta k^8\left(k^2+2\right)F_3 + \left(9k^{10} + 20k^8\right)F_4
\end{aligned} \tag{23}$$

$$U_{56} = -\beta^3 k^2 \left(576 k^2 + 512\right) F_5 + 2048 \beta^4 \left(2k^2 + 1\right) F_6 + \left(768 \beta^2 k^4 + 512 \beta^2 k^6\right) F_7$$

$$U_{57} = -\left(36 \beta k^8 + 64 \beta k^6\right) F_8 + \left(114 \beta^2 k^6 + 192 \beta^2 k^6\right) F_9 - 1536 \beta^4 k^2 F_{10}$$

4-Cooperative games

In this section, we discuss different types of cooperation between the supply chain members focusing on a cooperative game structure. Here, the manufacturer and retailers reach a consensus to make their decisions in a way that the whole channel profit Cooperation between independent members of a supply chain relationship has attracted more eyes of scholars and market governors. In this regard, the main aim of supply chain's members is to maximize the profit of whole supply chain channel in the course of cooperation. Due to the fact that cooperation in between supply chain members covers a wide spectrum of subjects including pricing, advertising, discount, order quantity, and so on, much research have been conducted in this field maximized. The analytical solutions depend on $2n+1$ parameters, which describe customer behaviors. In addition to global advertising and brand power, in each state, retail price and local advertising influence customer demand functions. The total profit is given as (24), and therefore we have the following maximization problem which is a function of A and $p_i, A_i \quad i = 1...n$.

$$\begin{aligned} &Max \quad \Pi_S = \Pi_M + \sum_{i=1}^{n} \Pi_{r_i} = \sum_{i=1}^{n} p_i . D_i - \sum_{i=1}^{n} A_i^2 - A^2 \\ &s.t. 0 < p_i \qquad 0 < A_i \qquad 0 < A \end{aligned} \tag{24}$$

Lemma4.1 *Supply chain objective (24) is a concave function with respect to variables.*

Proof: To proof the optimality of the solutions of the supply chain, we calculate the Hessian matrix,

$$H_{SC} = \begin{pmatrix} \frac{\partial^2 \Pi_{SC}}{\partial A^2} & \frac{\partial^2 \Pi_{SC}}{\partial A \partial A_1} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial A \partial A_n} & \frac{\partial^2 \Pi_{SC}}{\partial A \partial p_1} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial A \partial p_n} \\ \frac{\partial^2 \Pi_{SC}}{\partial A_1 \partial A} & \frac{\partial^2 \Pi_{SC}}{\partial A_1{}^2} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial A_1 \partial A_n} & \frac{\partial^2 \Pi_{SC}}{\partial A_1 \partial p_1} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial A_1 \partial p_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 \Pi_{SC}}{\partial A_n \partial A} & \frac{\partial^2 \Pi_{SC}}{\partial A_n \partial A_1} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial A_n{}^2} & \frac{\partial^2 \Pi_{SC}}{\partial A_n \partial p_1} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial A_n \partial p_n} \\ \frac{\partial^2 \Pi_{SC}}{\partial p_1 \partial A} & \frac{\partial^2 \Pi_{SC}}{\partial p_1 \partial A_1} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial p_1 \partial A_n} & \frac{\partial^2 \Pi_{SC}}{\partial p_1{}^2} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial p_1 \partial p_n} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \frac{\partial^2 \Pi_{SC}}{\partial p_n \partial A} & \frac{\partial^2 \Pi_{SC}}{\partial p_n \partial A_1} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial p_n \partial A_n} & \frac{\partial^2 \Pi_{SC}}{\partial p_n \partial p_1} & \cdots & \frac{\partial^2 \Pi_{SC}}{\partial p_n{}^2} \end{pmatrix} \tag{25}$$

The second order partial derivatives are as follows:

$$H_{SC}=\begin{pmatrix} -2 & 0 & \dots & 0 & \gamma_1 & \dots & \gamma_n \\ 0 & -2 & 0 & 0 & k\gamma_1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & -2 & 0 & 0 & k\gamma_n \\ \gamma_1 & k\gamma_1 & 0 & 0 & -2\beta\gamma_1 & 0 & 0 \\ \vdots & 0 & \ddots & 0 & 0 & \ddots & 0 \\ \gamma_n & 0 & 0 & k\gamma_n & 0 & 0 & -2\beta\gamma_n \end{pmatrix} \tag{26}$$

The first principal minor of H_{SC} is negative $H_{SC}^1=-2$, and the second principal minor of H_{SC} is $H_{SC}^2=4$ and is positive. Additionally, the (n+1) principal minor is $H_{SC}^{n+1}=(-2)^{n+1}$ that is decussate negative and positive. In the following, $H_{SC}^{n+2}=(-2)^{n+2}\beta\gamma_1$ and the $(n+3)$ principal minor is $H_{SC}^{n+3}=(-2)^{n+3}\beta^2\gamma_1\gamma_2$. In the end, the $(2n+1)$ principal minor is $H_{SC}^{2n+1}=(-2)^{2n+1}\beta^n\prod_{i=1,\dots,n}\gamma_i$. Therefore, the principal minors of H_{SC} have alternating algebraic signs and the matrix are negative definite at the cooperative solution. It means that there exists a unique solution and is a local maximum.

Proposition 4.1 *Due to the lemma 4.1 we can set the partial first-order derivatives* $\partial\prod_S/\partial A$, $\partial\prod_S/\partial p_i$ *and* $\partial\prod_S/\partial A_i$ *to zero and by solving the resulted system of equations, the optimal value for decision variables for each retailer is obtained as equation (27) as follows,*

$$p_i{*}=\frac{\alpha+A+kA_i}{2\beta}\quad,\quad A_i{*}=\frac{k\gamma_i p_i}{2}\quad,\quad A{*}=\sum_{i=1}^{n}\frac{\gamma_i p_i}{2} \tag{27}$$

If $p_i, A_i \quad i=1...n$ and A are, respectively, equal to $p_i{*}, A_i{*} \quad i=1...n$ and $A{*}$, then the channel's profit is maximized with respect to $t_i \quad i=1...n$ being free to take any values between 0 and 1 as well as w to take any positive value. Apparently, the profit of manufacturer and the retailer is not independent of $t_i \quad i=1...n$ and w. None of the channel members agrees to maximize the system profit and to accept fewer profits with cooperation than those without cooperation. The relevancy of the cooperation game is discussed later in the next section with case study and results of two game structures.

5-Numerical example

In the previous sections, we considered two different forms of supply chain behavior. Though we were able to determine the two equilibriums analytically, the resulted expressions are too complicated for a meaningful interpretation. Hence, we apply a case study to get insights into the effects of multiple retailers in supply chains and also retail prices, advertising expenditures and profits in the framework of the two examined game approaches. Our research work is based on a supply chain with a manufacturer and multiple retailers in Iran. The monopolistic manufacturer sells his branded products to customers through the only retailer channels. We assumed that there is no other competitive brand product and according to the models, we consider only one product transactions. This manufacturer is located in Tehran with 150 retailers around the country; the retailers work in the separated area in different cities.

Our investigation focuses on information regarding a typical working day, with several orders from the retailers. Based on the available data, we generated a further set of 150 retailer's transaction data to evaluate the effectiveness of the proposed model and equilibrium approach in a wide range of different retailers. The analysis reported in the paper is based on transactional data from retailers with an annually purchasing order. The database contains the records from March 2017 to August 2018. We find from a C-suite interview that Maximum 5 segments of retailers would be in the supply chain. Therefore, we used the K-Mean Algorithm to assign each retailer to one segment according to their initial demands. K-means is the common clustering algorithm used as inputs a predefined number of clusters that is the K from its name. By preparation data, we consider $\alpha = 10000$ and translate each retailer's initial demand to $\alpha.\gamma_i$, and by utilization of this algorithm, we have determined 5 segments of retailers where each segment γ_i has been shown in table 2. Initial demand for each segment is average of its members' demand. By consideration of γ_i and α, parameters and also historical data we have $\beta = 88$. Due to an increase of local advertising impact, we assume $k = 1.1$.

Table 2. Case study parameters

Parameter	γ_1	γ_2	γ_3	γ_4	γ_5
Value	83.16	59.99	38.55	16.36	4.86

Figures 2-4 demonstrate dependency and sensitivity of the participation rate on essential parameters. According to figure 2, an increase in β (increase in dependency of demand on the price) lead to increases in participation rate between the manufacturer and the retailers, while a decrease in β, which is equal to fewer effect of variable price on demand, leads to more support for smaller retailer and less support for the bigger one. Also figure 3 shows that an increase in the effectiveness of environmental advertising (k), leads to rise in manufacturers support for small retail sellers and drop in his support for big retail sellers. A small value of k results in similar support for small and big retailers.

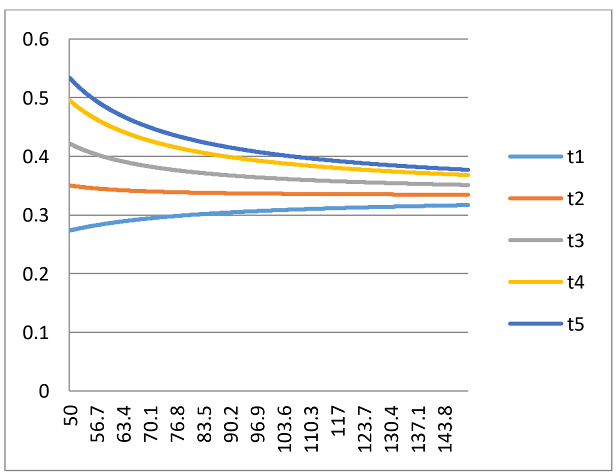

Fig 2. Sensitivity analysis of participation rates in consist of β

Figure 4 reflects the sensitivity of ratios of participation rate to the variable part of primary demand. According to this figure, if the primary demand of a retailer is high, it will gain low support percentage from the manufacturer and increase in its primary demand causes an increase in producer's support for other retailers.

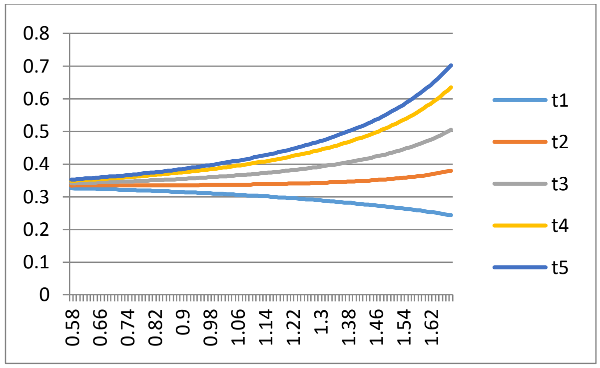

Fig 3. Sensitivity analysis of participation rates in consist of k

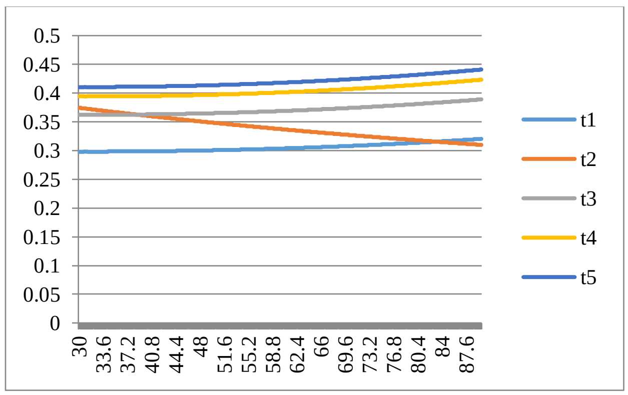

Fig 4. Sensitivity analysis of participation rates in consist of γ_2

Figures 5 to 7 demonstrate the variation of supply chain profit for two cases of centralized and decentralized. In the decentralized condition with manufacturer-Stackelberg equilibrium, profit of supply chain will always be less than its counterpart in centralized (cooperative game) condition. Regarding to figure 5, an increase in α causes a raise in supply chain profit for both cases. Figure 6 shows that lowering dependency of demand on the product price results in a nonlinear increase in supply chain benefit. In the condition of high dependency on the price, low supply chain coordination may enhance the performance. Also figure 7 shows that effective local advertising may enhance the chain enhancement in cooperative condition.

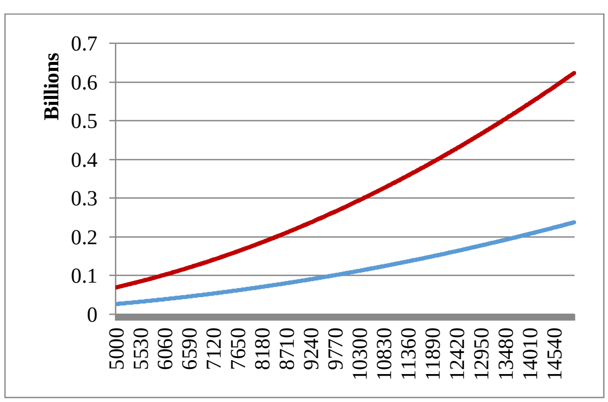

Fig 5. Sensitivity analysis of supply chains profit in consist of α

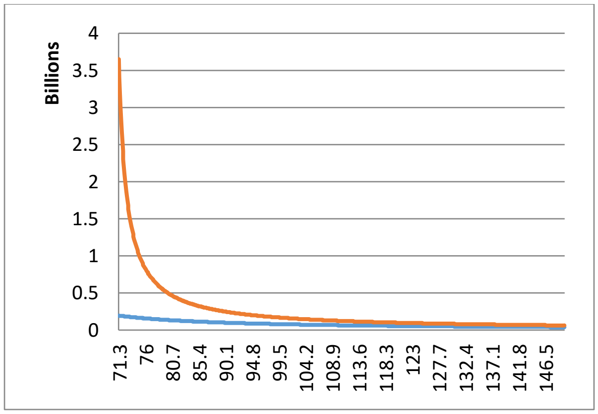

Fig 6. Sensitivity analysis of supply chains profit in consist of β

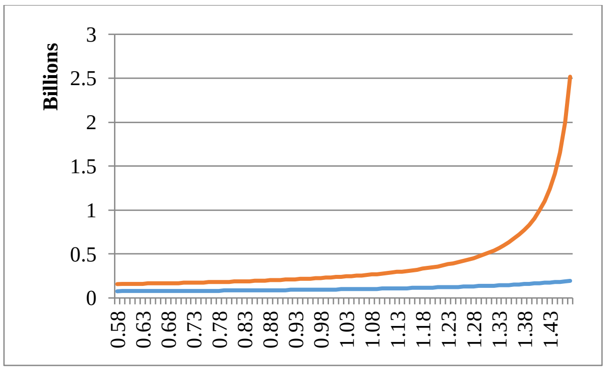

Fig 7. Sensitivity analysis of supply chains profit in consist of k

In the next section conclusions, managerial insights and future research are presented.

6-Conclusion and managerial insights

The manufacturer can determine different policies in the face of this paper attempted to investigate behaviors of a manufacturer and multiple retailers in the context of pricing and cooperative advertising. In this study, we applied a static model in a two-echelon supply chain. Most studies of cooperative advertising in the literature have focused on a relationship of a monopolistic manufacturer and a monopolistic retailer. Nevertheless, many manufacturers work with wide range of retailers as a sales channel around the world or country in practice. According to our recent research, this paper has created a first attempt to investigate this issue.

The proposed model was analyzed under two scenarios, and analytic solutions were obtained as well. In the first scenario, which was a leader-follower game, the manufacturer is the leader who determines the wholesale price, national advertising and participation rate of each retailer and then, the retailers, as the followers, determined the retail prices and local advertising expenditures. In the second scenario, a cooperative game was utilized for the optimal solutions. The proposed model can assist managers to develop better marketing strategies and advertising plans that completely apply the knowledge resulting from segmentation analysis and coordination supply chain's pricing and advertising functions.

The main obtained findings have provided the following new insights:

Different retailers, set a fixed wholesale price for all retailers, and relaxed retail price causes that bigger and stronger retailers sell the product at a high price. Due to the assumption (independent retail market and separate geographical place), this result is acceptable.

- When the manufacturer works with different retailers, he adopts different strategies, shares various participation rates, and accepts higher rates of local advertising with small retailers and lower rates in comparison to bigger ones.
- Depending on local advertising effectiveness versus global advertising (shown by k parameter) participation rate is different, whatever increase *k*, differences between maximum and minimum rate are increased.

There are three possible approaches for future research. First, the supply chain structure could be relaxed in a duopoly situation of manufacturers who sell their substitutable products through oligopolistic retailers. Second, in our analysis, we employed nonlinear sales response function to satisfy the saturation requirement. As indicated in the literature of channel studies, many important results in equilibriums depend on the shape of the demand function. Therefore, the use of other sales response function may yield different and interesting results in the analysis of cooperative advertising agreements, For example, demand functions that competitor retailers affect others demand function by pricing or advertising. Third, the pricing strategies can be different from our assumptions; in our analysis, the wholesale price of the manufacturer is identical for all retailers, and each retailer determined his retail price based on the market; on the other hand, some branded products have a unique retail price that is determined by the manufacturer. Future research may take this matter into consideration.

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