

Cross dock scheduling under multi-period condition

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Abstract

This paper proposes a truck scheduling model in a cross dock system under multi-period, multi-commodity condition with fixed outbound departures. In an operational truck scheduling problem, outbound trucks leave the cross dock terminals at predetermined times and delayed loads are kept as inventory that are sent at the next period (a time slot in a day). The proposed model optimizes the inbound truck scheduling problem through the minimizing cross dock operational costs. Accelerated Benders decomposition technique based on Covering Cut Bundle (CCB) strategy and a heuristic approach are developed to solve the model. Finally, numerical analysis introduces the sensitivity of the input parameters to the objective value.

Keywords: Cross dock, scheduling, heuristic algorithm, sensitivity analysis.

1-Introduction

By expanding the business criteria for reducing distribution costs, in the last few years, cross docking has played an important role in the transportation areas. Cross dock center refers to the facility in which products move from the manufacturing plant to the customers with no or little storage levels. Indeed, cross docking can decrease distribution network costs by reducing products' delivery time, material handling, and inventory holding costs (Kaboudani et al., 2018; Theophilus et al., 2019; Zarandi et al., 2014). Bartholdi & Gue (2004) expressed that firms use cross dock centers to reduce inventories and transportation costs in the midst of fierce price competition. Successful implementing of cross dock concept in different companies such as Wal-Mart (Stalk et al., 1992), Toyota (Witt, 1998), and Eastman Kodak Co. (Cook et al., 2005) have shown cross docking importance in competitive advantages. However, in spite of these advantages, there might be some difficulties in scheduling of inbound and outbound trucks, labors, and material resources. Chen & Song (2009) studied two-stage hybrid cross docking scheduling problem and assumed that multiple trucks can be loaded or unloaded in parallel machines at inbound and outbound stages. Moreover, in the cross dock scheduling systems, Soltani & Sadjadi (2010) proposed two hybrid meta-heuristic approaches to minimize the total system flow time.

In cross dock systems, products are stored in temporary storages and inventory holding costs significantly are reduced (Bodnar et al., 2017; Luo et al., 2019; Rahmazadeh Tootkaleh et al., 2016; Shahmardan and Sajadieh, 2020; Wang and Alidaee, 2019; Yu et al., 2015). Alpan et al. (2011) proposed a model based on the cross dock total inventory costs, in which a bounded dynamic programming approach was used to solve the model. Moreover, Forouharfard & Zandieh (2010) scheduled receiving and shipping trucks in cross docking systems to decrease temporary storages.

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Products are stored in cross dock centers for less than 24 hours (see, e.g., Buijs et al., 2014; Golshahi-Roudbaneh et al., 2019; Guemri et al., 2019; Ladier and Alpan, 2016; Nassief et al., 2016; Rahbari et al., 2019; Shiguemoto et al., 2014). Consequently, scheduling inbound trucks affects the cross dock system costs, especially the temporary storage costs.

On the other hand, some industries, especially postal services and retailing industries use fixed outbound departure times for the steady and constant flow of products. Boysen & Fliedner (2010) expressed that in the large hub-and-spoke networks with the multi cross docking stages, a steady flow of trucks is essential. They correspondingly represented that in the postal services and less-than-truck load (LTL) services, the cross docks are scheduled based on the fixed outbound departure times. Moreover, according to Belle et al. (2012) research, outbound trucks have to depart in a certain due date in some cases. Furthermore, it has been explained that outbound trucks leave cross dock centers at a fixed departure time in the parcel delivery sectors. The delayed parcels must wait to the next truck departure for the same destination. Recently, Boysen et al. (2013) developed a model in the truck scheduling with fixed outbound departures and moreover, they presented a prescheduled point of time and a lost profit penalties for departure due dates and delayed loads respectively. They consider that the delayed loads must be stocked in the temporary storages until the next day. They assume that the outbound trucks with a determined destination leave the terminal gates only in one departure time and several departure times are not allowed to a specific destination in a planning horizon (24 hours). Indeed, lost profit of delayed loads is not clearly confirmed in cross dock systems (Ladier and Alpan, 2013) and delayed loads are stored in the temporary storage until the next outbound truck departure time for the same destination (next period) (Van Belle et al., 2012).

In many postal services, there are several predetermined departure times less than 24 hours to a specific destination. Subsequently, it is acceptable for managers to send delayed loads with the next outbound trucks in a planning horizon. Therefore, this paper proposes an inbound trucks scheduling model considering a fixed departure time for outbound trucks and temporary storage of loads in a cross dock facility. Additionally, it is assumed that delayed loads are stored until the next period (a time slot in a day). Moreover, it is considered in which each inbound truck shipment may include multiple products. Furthermore, it is not necessary to all trucks be available at the beginning of the day and it would be acceptable that trucks be available before the unloading period. Despite Boysen et al. (2013) research, it is assumed that the delayed loads are not stocked until the next day, but they are sent to the customer with the next outbound trucks with the same destination. Therefore, multi-periods (outbound departure times) are considered in a day and inbound truck scheduling is prepared for multi-period, multi-commodity problem. Additionally, in this paper, a mixed integer linear programming (*MILP*) model that minimizes total holding costs of delayed loads in a planning horizon (often in a day) is proposed. To the best knowledge of present authors, this work is the first paper that deals with multi-period, multi-commodity in a cross dock scheduling problem for the fixed outbound trucks departure times without substitution condition. This paper is a complementary research on Rahmanzadeh et al. (2016) study that investigates substitution condition in a cross dock scheduling problem. This paper assumes that the information about all inbound and outbound trucks is available before the cross dock daily scheduling. Furthermore, capacity of outbound trucks is considered as real operational constraint that can affect the scheduling problem.

The paper is organized as follows. Section 2 provides modeling assumptions and notations and development of the model. Section 3 examines the problem complexity and introduces our Benders decomposition approach. A heuristic algorithm for large-size problems is also presented in this section. Section 4 shows sensitivity analysis of the input parameters and finally, Section 5 is devoted to the conclusions and further research directions.

2-Problem statement

In a cross dock system, inbound trucks unload their shipments and products are then moved to the outbound terminals for consolidation and loading on outbound trucks. As shown in Figure 1, different product are sorted and moved to the outbound gates and consolidated on the shipping trucks.

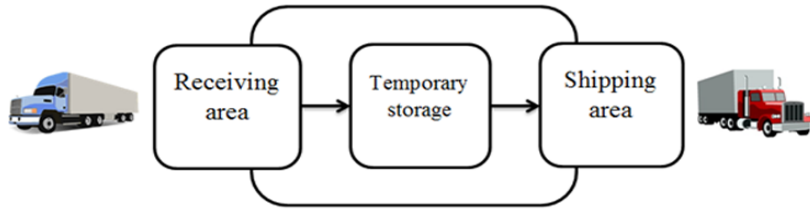


Fig 1. Cross docking distribution center

Each of the inbound trucks carries different product types and the shipment is unloaded based on the scheduling problem. The products are then sorted and sent to the outbound gates for consolidation and loading on the outbound trucks, according to the fixed predefined departure times. If the shipments arrive late, they would miss the outbound trucks and must be stocked until the next outbound trucks with the same destination. This paper proposes a scheduling model for the inbound trucks, in which the objective function minimizes total inventory holding costs. Several papers assume that all inbound trucks are available at the beginning of time horizon (e.g. 24 hours). This assumption is not realistic and different arrival times of inbound trucks is more sensible. Thus, in this model, it is assumed that inbound trucks are available only at the beginning of the unloading period. Moreover, it is assumed that the number of inbound trucks is higher than the number of inbound terminal gates. Additionally, information of inbound trucks' loads corresponding to the type and quantity of products are available before the scheduling time horizon. For simplifying the model, it is assumed that transshipment time of all commodities depends only to the related inbound and outbound gates. Splitting of the truck loads is not allowed, i.e. the inbound trucks must unload the whole shipments, which are then stored near the corresponding outbound. In each period, the shipments arriving after the departure time of outbound trucks are stored and moved to load and consolidate at the next periods. Consequently, the model is formulated as a MILP problem. The notations used in this model are presented in the next section.

2-1- Notations

Indices

| | |
|-------|--|
| T | Set of scheduling periods |
| I_t | Set of inbound trucks in period t ; $i=1, 2, \dots, I_t$ |
| O | Set of outbound trucks assigned to a specific outbound gate. |
| K | Set of inbound gates |
| N | Set of commodity types |

Parameters

| | |
|------------|---|
| P_{tin} | Processing time to unload total commodity n from inbound truck i in period t |
| d_{to} | Departure time of outbound truck o in period t (<i>it includes inbound trucks loading process time</i>) |
| t_{ko} | Time for moving from inbound gate k to outbound truck o |
| L_{tion} | Number of commodity n delivered by inbound truck i dedicated to outbound truck o in period t |
| M | Large number |
| h_{tn} | Holding cost rate of commodity n in period t (per unit per period) |
| Cap_{to} | Capacity of outbound truck o in period t |

Decision variables

| | |
|-----------|--|
| Q_{ton} | Continuous variable; quantity of commodity n that is loaded in the outbound truck o in period t |
| S_{ton} | Continuous variable; quantity of commodity n for the outbound truck o in period t stored until the next period |

| | |
|-------------------|--|
| C_{ti} | Continuous variable; end time for unloading shipment of inbound truck i in period t |
| x_{tij}^k | Binary variable; 1 if inbound truck i is processed before inbound truck j in inbound gate k in period t ; 0 otherwise |
| x_{t0i}^k | Binary variable; 1 if inbound truck i is the first truck processed in inbound gate k in period t ; 0 otherwise |
| $x_{ti(l_t+1)}^k$ | Binary variable; 1 if inbound truck i is the last truck processed in inbound gate k in period t ; 0 otherwise |
| y_{tio} | binary variable; 1 if the shipment of inbound truck i arrives after the departure time of outbound truck o in period t ; 0 otherwise |

Below, the MILP model is explained:

2-2-Model

Problem (1)

$$\text{Min } Z_1 = \sum_{t \in T} \sum_{n \in N} h_{tn} \cdot \sum_{o \in O} S_{ton}$$

S.t.

$$\sum_{k \in K} \sum_{\substack{i \in I_t \cup \{0\} \\ i \neq j}} x_{tij}^k = 1 \quad \forall j \in I_t, t \in T \quad (1)$$

$$\sum_{i \in I_t} x_{t0i}^k \leq 1 \quad \forall k \in K, t \in T \quad (2)$$

$$\sum_{\substack{i \in I_t \cup \{0\} \\ i \neq j}} x_{tij}^k = \sum_{\substack{j \in I_t \cup \{l_t+1\} \\ i \neq j}} x_{tji}^k \quad \forall j \in I_t, k \in K, t \in T \quad (3)$$

$$C_{ti} \geq C_{tj} + \sum_{n \in N} P_{tin} - M \cdot (1 - x_{tji}^k) \quad \forall i \in I_t, j \in I_t \cup \{0\}, t \in T, k \in K \quad (4)$$

$$y_{tio} \cdot M > C_{ti} - d_{to} + \sum_{k \in K} t_{ko} \times \left(\sum_{\substack{j \in I_t \cup \{0\} \\ j \neq i}} x_{tji}^k \right) \quad \forall i \in I_t, o \in O, t \in T \quad (5)$$

$$S_{ton} + Q_{ton} = S_{(t-1)on} + \sum_{i \in I_t} L_{tion} \quad \forall t \in T, o \in O, n \in N \quad (6)$$

$$Q_{ton} \leq S_{(t-1)on} + \sum_{i \in I_t} (1 - y_{tio}) \cdot L_{tion} \quad \forall t \in T, o \in O, n \in N \quad (7)$$

$$\sum_{n \in N} Q_{ton} \leq \text{Cap}_{to} \quad \forall t \in T, o \in O \quad (8)$$

$$\sum_{t \in T} C_{t0} = 0 \quad (9)$$

$$x_{tij}^k \in \{0, 1\} \quad \forall i, j \in I_t, t \in T \quad (10)$$

$$x_{t0j}^k \in \{0, 1\}, x_{ti(l_t+1)}^k \in \{0, 1\} \quad \forall j \in I_t, t \in T \quad (11)$$

$$y_{tio} \in \{0, 1\} \quad \forall i \in I_t, o \in O, t \in T \quad (12)$$

$$Q_{ton} \geq 0 \quad \forall t \in T, o \in O, n \in N \quad (13)$$

$$S_{ton} \geq 0 \quad \forall t \in T, o \in O, n \in N \quad (14)$$

$$C_{ti} \geq 0 \quad \forall t \in T, \quad \forall i \in I_t \quad (15)$$

In this model, it is assumed that outbound gates are assigned to the fixed destination for a long period of time (e.g. 6 months). Furthermore, outbound trucks are assigned to the outbound gates on a mid-term horizon. Accordingly, outbound trucks destinations are predetermined on long plan horizon and the proposed model schedules inbound trucks on a short time horizon (24 hours).

The objective function minimizes total inventory holding costs on the planning horizon. Set of constraints (1) show that each inbound truck must be assigned to the gates once per each period. Set of constraints (2) express that not more than one queue of trucks is scheduled to unload shipments in each inbound gate. Set of constraints (3) define the sequence of inbound trucks at each gate and set of constraints (4) show the completion time of unloading process of inbound trucks. Set of constraints (5) determine if the inbound truck shipment arrives before the departure time of outbound trucks or not. Set of constraints (6) check the inventory balance between the periods. In this model, each outbound gate is allocated to a certain destination and dedicating outbound trucks to the outbound gates is predefined. However, L_{tion} (quantity of commodity n that must be transmitted to the appropriate destination regarding outbound truck o in the period t) is determined before the problem is scheduled. Set of constraints (7) express that the quantity of inbound truck loads must be less than sum of stored loads from the previous periods and without delayed loads in the current period. Set of constraints (8) examine that the quantity of outbound truck loads must be less than the truck capacity. Constraint (9) expresses that the starting time of truck 0 must be zero. To minimize the total commodity stored in the cross dock center, a very large penalty applied on the inventory of the last period.

To create a connection between the periods, an inventory balancing concept was applied. As shown in figure 2, total incoming loads in a period must be equal to the total output loads in that period. This concept is considered in constraints (6). Furthermore, it is not acceptable to apply $|T|$ separate minimized model to find the optimum solution in the planning horizon. Inbound trucks scheduling in each period is depends on the outbound trucks departure times in the next periods. Hence, the proposed model finds the optimum solution considering inbound and outbound trucks dependency in a planning time horizon (e.g. 24 hours).

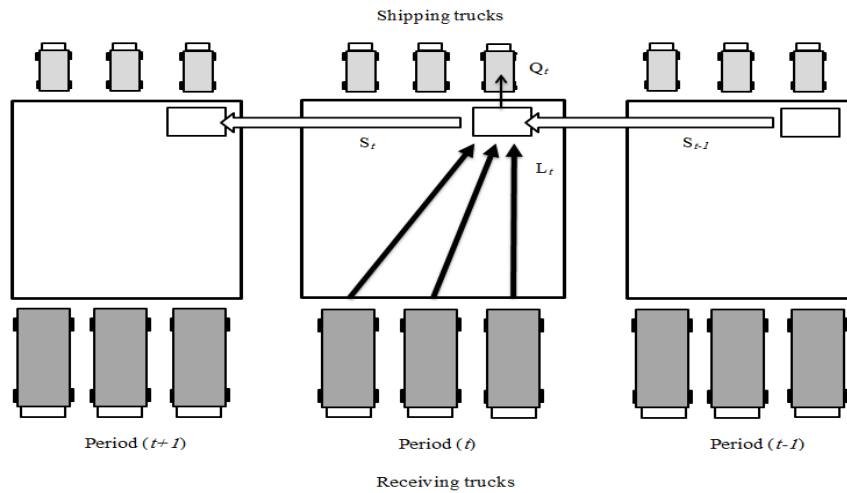


Fig 2. Inventory balancing in a cross dock center

3-Solution algorithm

The proposed model schedules inbound trucks unloading process based on minimizing cross dock inventory holding costs. To solve the model, first it is shown that the model is strongly NP-hard and then to find the model's optimum solution, an accelerated Benders' decomposition technique is applied. As computation time of Benders algorithm is too long, a heuristic algorithm is proposed to find near optimum solution. Furthermore, computational experiments are applied to compare Benders decomposition technique with heuristic algorithm.

3-1- Computational complexity

As shown before, the mathematical model developed in this paper was formulated as *MILP* model. There are several papers that study cross docking problems in the format of *MILP* as a NP-hard problem (see, e.g., Feo and Resende 1989; Mosheiov 1989). This paper extended Boysen et al. (2013) model to multi-period, multi-commodity types. Therefore, the model presented in this paper is more complex. Moreover, Boysen et al. (2013) proved that their model is NP-hard in the strong sense. Hence, the presented model is strongly NP-hard as well.

In this paper, a Benders algorithm was proposed for small size problems and a heuristic approach was given for medium and large size problems.

3-2- Benders algorithm

Benders algorithm is a decomposition technique, which is significantly applied in *MILP* to solve mathematical models and find optimal solutions (Benders, 1962). In more detail, Benders algorithm determines a lower bound and an upper bound on the optimum value. After solving the Master Problem (*MP*), based on the primal minimization (maximization) problem, a lower bound (upper bound) is presented on the objective function. On the other hand, by solving the Sub-Problem (*SP*), an upper bound (lower bound) is created on the original problem. Geoffrion (1972) proposed the theory of finite convergence of upper and lower bounds on the Benders algorithm.

As a brief explanation for Benders algorithm, first *MP* is solved using a feasible solution of uncomplicated variable. Then a lower bound (upper bound in the primal maximization problem) and integer variables values (complicated variables) are obtained. In the second step, based on the integer variables calculated in *MP*, *SP* is solved and an upper bound (lower bound in the primal maximization problem) on the original problem was built. The gap between upper and lower bounds comprised with a predetermined value the algorithm is finished when the calculated gap to be less. Otherwise, at the next iteration in the Benders algorithm, a Bender's cut based on the *SP* dual variables is added to the *MP*. Hence, the *MP* is updated and the new bounds and new values of variables are determined. Finally, this procedure is continued until the appropriate gap achieved. By increasing the problem size, Benders algorithm iterations increase significantly and accordingly, in large size problems, classical Benders algorithm converges to the optimum solution slowly. In the last decade, various techniques were developed to accelerate classical Benders algorithm. In this paper, we use CCB strategy to restrict Benders number of iterations and solution space of decomposed problems. Our accelerated Benders algorithm is based on (Saharidis et al., 2010) research on CCB generation method. As a brief description of CCB generation method, the problem is decomposed into Restricted Master Problem (*RMP*) and *SP*. *RMP* is the deduced form of *MP* in which some constraints of *MP* are relaxed. Therefore, the optimum solution of *RMP* is not greater than *MP* solution. A necessary and sufficient condition that *RMP* solution be an optimal solution of *MP* is *RMP* solution which satisfies all constrains of *MP*. In CCB strategy, a bundle of cuts is appended to *RMP* based on α -covered conditions. The detailed procedure of CCB method is presented by (Saharidis et al., 2010). Furthermore, an auxiliary primal problem (*APP*) is used in CCB generation concept. Consequently, first the *RMP* is solved and based of *RMP* variable solution, multiple of strength cuts based on the α -covered conditions are added to *RMP* and it is resolved again to optimality. The *RMP* form of our proposed model is presented as follows:

RMP

Min Z_1

S.t.

$$\sum_{k \in K} \sum_{\substack{i \in I_t \cup \{0\} \\ i \neq j}} x_{tij}^k = 1 \quad \forall j \in I_t, t \in T \quad (16)$$

$$\sum_{i \in I_t} x_{t0i}^k \leq 1 \quad \forall k \in K, t \in T \quad (17)$$

$$\sum_{\substack{i \in I_t \cup \{0\} \\ i \neq j}} x_{tij}^k = \sum_{\substack{j \in I_t \cup \{l_t+1\} \\ i \neq j}} x_{tji}^k \quad \forall j \in I_t, k \in K, t \in T \quad (18)$$

$$\begin{aligned} Z_2 \geq & \sum_t \sum_i \sum_{j \neq i} \sum_k \left(\sum_n P_{tin} - M \cdot (1 - x_{tji}^k) \right) \cdot \vartheta_1(i, j, t, k) + \sum_t \sum_i \sum_o (y_{tio} \cdot M + d_{to} - \\ & \sum_k (t_{ko} \cdot \sum_{j \in I \cup \{0\}} x_{tji}^k)) \cdot \vartheta_2(t, i, o) + \sum_t \sum_o \sum_n (\sum_i L_{tion}) \cdot \vartheta_3(t, o, n) + \sum_t \sum_o \sum_n (\sum_i (1 - \\ & y_{tio}) \cdot L_{tion}) \cdot \vartheta_4(t, o, n) + \sum_t \sum_o (Cap(t, o)) \cdot \vartheta_5(t, o) \end{aligned} \quad (19)$$

$$\begin{aligned} & \sum_t \sum_i \sum_{j \neq i} \sum_k \left(\sum_n P_{tin} - M \cdot (1 - x_{tji}^k) \right) \cdot U_1(i, j, t, k) + \sum_t \sum_i \sum_o (y_{tio} \cdot M + d_{to} - \\ & \sum_k (t_{ko} \cdot \sum_{j \in I \cup \{0\}} x_{tji}^k)) \cdot U_2(t, i, o) + \sum_t \sum_o \sum_n (\sum_i L_{tion}) \cdot U_3(t, o, n) + \sum_t \sum_o \sum_n (\sum_i (1 - \\ & y_{tio}) \cdot L_{tion}) \cdot U_4(t, o, n) + \sum_t \sum_o (Cap(t, o)) \cdot U_5(t, o) < 0 \end{aligned} \quad (20)$$

$$x_{tij}^k \in \{0, 1\} \quad \forall i, j \in I_t, t \in T \quad (21)$$

$$x_{t0j}^k \in \{0, 1\}, x_{ti(l_t+1)}^k \in \{0, 1\} \quad \forall j \in I_t, t \in T \quad (22)$$

$$y_{tio} \in \{0, 1\} \quad \forall i \in I_t, o \in O, t \in T \quad (23)$$

Z_1 is a continuous variable

In this model, constraint sets (16), (17), and (18) are the same as the primal model (Problem (1)) constraint sets (1), (2), and (3), respectively. In constraints (19) and (20), Bender's optimality and feasibility cut are added to *RMP*. ϑ_i and U_i $\{i \in 1, 2, \dots, 5\}$ are sub-set of extreme points and extreme rays of dual *SP* related to constraint sets (23), (24), (25), (26), and (27), respectively. Furthermore, *APP* form of proposed model is presented as follows: (**APP**)

$$\begin{aligned} Max Z_2 = & -\xi - \sum_{t \in T} \sum_{i \in I_t} \sum_{j \in I_t} \sum_{k \in K} UB^1_{tijk} \times \theta^1_{tijk} - \sum_{t \in T} \sum_{i \in I_t} \sum_{o \in O} UB^2_{tio} \times \theta^2_{tio} \\ & + \sum_{t \in T} \sum_{i \in I_t} \sum_{j \in I_t} \sum_{k \in K} LB^1_{tijk} \times \mu^1_{tijk} + \sum_{t \in T} \sum_{i \in I_t} \sum_{o \in O} LB^2_{tio} \times \mu^2_{tio} \end{aligned}$$

S. t.:

$$C_{ti} - M \cdot \theta^1_{tijk} + M \cdot \mu^1_{tijk} - \xi \geq C_{tj} + \sum_n P_{tin} - M \cdot (1 - x'^k_{tji}) \quad \forall i \in I_t, t \in T, j \in I_t \cup \{0\}, k \in K \quad (24)$$

$$y'_{tio} \cdot M - \sum_k t_{ko} \cdot \left(\sum_{\substack{j \in I_t \cup \{0\} \\ j \neq i}} \theta^1_{tijk} \right) + \sum_k t_{ko} \cdot \left(\sum_{\substack{j \in I_t \cup \{0\} \\ j \neq i}} \mu^1_{tijk} \right) + M \cdot \theta^2_{tio} - M \cdot \mu^2_{tio} - \xi \\ > C_{ti} - d_{to} + \sum_k t_{ko} \cdot \left(\sum_{\substack{j \in I_t \cup \{0\} \\ j \neq i}} x'^k_{tji} \right) \quad \forall i \in I_t, o \in O, t \in T \quad (25)$$

$$S_{ton} + Q_{ton} = S_{(t-1)on} + \sum_{I_t} L_{tion} \quad \forall t \in T, o \in O, n \in N \quad (26)$$

$$Q_{ton} - \sum_{I_t} \theta^1_{tio} \times L_{tion} + \sum_{I_t} \mu^1_{tio} \times L_{tion} - \xi \\ \leq S_{(t-1)on} + \sum_{I_t} (1 - y'_{tio}) \cdot L_{tion} \quad \forall t \in T, o \in O, n \in N \quad (27)$$

$$\sum_{n \in N} Q_{ton} \leq Cap_{to} \quad \forall t \in T, o \in O \quad (28)$$

$$\sum_{t \in T} C_{t0} = 0 \quad (29)$$

$$S_{ton}, Q_{ton}, \theta^1_{tijk}, \mu^1_{tijk}, \theta^2_{tio}, \mu^2_{tio} \geq 0 \quad \forall t \in T, o \in O, n \in N, \quad i, j \in I_t \quad (30)$$

$$C_{ti}, \quad \xi \geq 0 \quad \forall t \in T, i \in I_t \cup \{0\} \quad (31)$$

In the *APP* model, constraints (24) - (29) correspond to constraints (4) - (9) in the primal model (Problem (1)), respectively. Parameters x'^k_{tji} and y'_{tio} are *RMP* solutions for x^k_{tji} and y_{tio} variables and additionally, UB^1 (UB^2) and LB^1 (LB^2) are the upper bound and lower bound on the coefficient of the variable decision x^k_{tij} (y_{tio}). In the *CCB* strategy not only the *SP* is solved but also a successive resolution of *APP* using the same optimal solution of the current *RMP* is created. In each resolution of *APP*, the parameters LB^1 , LB^2 , UB^1 and UB^2 are changed and fixed to a certain value for the generation of a new cut. The *CCB* procedure stops when predetermined maximum number of cuts has been added or when all possible decision variables of the *RMP* have been α -covered. After this, *RMP* is solved and optimality condition is checked by *SP* optimal solution. Consequently, we present *SP* as follows: **(SP)**

$$\text{Min } Z_3 = \sum_{t \in T} \sum_{n \in N} h_{tn} \cdot \sum_{o \in O} S_{ton}$$

S.t.:

$$C_{ti} \geq C_{tj} + \sum_{n \in N} P_{tin} - M \cdot (1 - x'^k_{tji}) \quad \forall i \in I_t, t \in T, j \in I \cup \{0\}, k \in K \quad (32)$$

$$y'_{tio} \cdot M > C_{ti} - d_{to} + \sum_{k \in K} t_{ko} \cdot \left(\sum_{\substack{j \in I_t \cup \{0\} \\ j \neq i}} x'_{tji}^k \right) \quad \forall i \in I_t, o \in O, t \in T \quad (33)$$

$$S_{ton} + Q_{ton} = S_{(t-1)on} + \sum_{i \in I_t} L_{tion} \quad \forall t \in T, o \in O, n \in N \quad (34)$$

$$Q_{ton} \leq S_{(t-1)on} + \sum_{i \in I_t} (1 - y'_{tio}) \cdot L_{tion} \quad \forall t \in T, o \in O, n \in N \quad (35)$$

$$\sum_{n \in N} Q_{ton} \leq Cap_{to} \quad \forall t \in T, o \in O \quad (36)$$

$$\sum_{t \in T} C_{t0} = 0 \quad (37)$$

$$S_{ton}, Q_{ton} \geq 0 \quad \forall t \in T, o \in O, n \in N \quad (38)$$

$$C_{ti} \geq 0 \quad \forall t \in T, i \in I_t \cup \{0\} \quad (39)$$

By solving *SP* model, an upper bound and dual variables are calculated. If the gap between upper and lower bounds is greater than a predetermined value (ε), the algorithm would go to the next iteration and the dual variables obtained in *SP* model would be properly applied in *RMP*. To solve *RMP*, a feasible solution of *SP* at the first iteration is needed. By solving the *RMP*, x_{tij}^k , y_{tio} and Z_2 values are obtained, in which Z_2 represents the primal model's lower bound and x'_{tji} and y'_{tio} are used in *APP* as the input parameters. In this paper, a heuristic algorithm was applied to find the initial feasible solution. As a conclusion, the algorithm steps for the model are explained below:

Algorithm

- Step 0** Initialize the input parameters and set the error $gap(\varepsilon)$
 - Step 1** Employ a heuristic algorithm to find a feasible solution for *RMP*
 - Step 2** Solve *RMP* and determine the solution of the lower bound (Z_2) and binary variables (x_{tji}^k, y_{tio})
 - Step 3** Setup *APP*'s objective function to produce multiple α -covered cuts for *RMP*
 - Step 4** Solve *SP* and obtain the upper bound (model's objective function) and dual *SP* extreme points and extreme rays. (ϑ_i and U_i).
 - Step 5** Calculate the gap between upper and lower bounds and, if $Z_3 - Z_2 < \varepsilon$ (initializing in the step 0) stop. Otherwise, go to Step 2.
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As mentioned before, Benders algorithm converges to the optimal value at finite iterations. Benders algorithm is a good approach for finding an optimal solution in the small and medium problem sizes. The solution time of Benders algorithm increases exponentially with the size of problem. Therefore, in this paper, a heuristic algorithm is proposed to solve the model in large problem sizes.

3-3- Heuristic algorithm

Computation time of Benders algorithm in the real operational cases is too long and in this paper, a heuristic algorithm was proposed for solving the large scale problems. To find the appropriate Heuristic algorithm, first, the effect of the input parameters on the objective function value was

evaluated and more, an expression for sorting of inbound trucks in a cross dock system was then proposed. In the algorithm, start time and place (inbound gate) of the inbound trucks for unloading process were scheduled based on the inventory holding costs. Indeed, a priority was set to the trucks based on their load quantities as well as their product types. Hence, in this heuristic algorithm, a decision expression was proposed to evaluate and sorting inbound trucks. The decision expression (equation 40) that is used to score the inbound trucks is expressed as below:

$Sc(ti)$ is defined as the score of inbound truck i in period t .

$$Sc(ti) = \frac{\sum_o \left(\frac{\sum_n L_{tion}}{d_{to}} \right)}{\sum_n P_{tin}} \quad (40)$$

where input parameters are as follows:

L_{tion} is quantity of commodity n transmitted from inbound truck i to outbound truck o in period t . Certainly, the importance of a truck is related to the quantity of its loads, therefore, L_{tion} was considered as a positive factor in the present decision expression.

d_{to} is departure time of outbound truck o in period t . As explained before, the loads arrived after the departure time of outbound trucks are stored to be sent at the next periods. By decreasing d_{to} , the objective function increases and consequently, there is a reverse relationship between d_{to} and the truck priority.

P_{tin} is processing time of unloading total commodity n from inbound truck i in period t . When a truck utilizes a long unloading process time, other trucks unload commodities with a delayed timeout. Therefore, there is a reverse relationship between P_{tin} and truck priority.

In more detail, the quantity of truck loads plays a direct role in the decreasing of the inventory holding costs and it is reasonable to unload large load trucks first. Similarly, total inventory holding costs are decreased by considering the trucks' processing time as a negative feature in the scheduling process. Therefore, L_{tion} and P_{tin} were applied to $Sc(ti)$ expression in direct and reverse form, respectively. Finally, the heuristic algorithm is proposed as follows:

Algorithm

- Step 0** Determine the values of input parameters and assign $t=1$.
 - Step 1** Find $Sc(ti)$ for inbound truck i in period t .
 - Step 2** Sort the inbound trucks as follows:
In period t :
For $i, j \in I$, if $Sc(ti) > Sc(tj)$, then inbound truck i is unloaded first.
 - Step 3** Allocate the first sorted truck to the first inbound gate and the second one to the second inbound gate and so on. When the entire inbound gates are allotted once, the next inbound truck is scheduled to the first gate and it is continued until the allocation of the last inbound truck.
 - Step 4** If $t=T$, the algorithm is finished; otherwise, $t=t+1$ and go to Step 2.
-

3-4- Computational experiments

To analyze the efficiency of the heuristic algorithm, it was compared with Benders decomposition in small and medium problem sizes. In practice, to determine the input parameters, Boysen et al. (2013) presented an instance generation section and proposed some input parameter formulation according to real-world operational consideration. These formulations were used to produce input parameters' values to generate the computational experiments. In this section, two experiment sets considering different parameter series were used. Input parameters are considered as follows:

For each inbound truck, its processing time is randomly drawn following normal distribution $N(\mu, \sigma^2)$ with $\mu=30$ minutes and $\sigma=5$. If $rnd[x, y]$ represent a uniformly distributed random number between x and y , h_m is chosen according to

$$h_{tn} = \begin{cases} rnd[0.2, 0.8] & \text{if } t \text{ is not last period number} \\ rnd[2, 8] & \text{if } t \text{ is last period number} \end{cases}$$

Transshipment times t_{ko} are randomly drawn from interval $[15; 30]$ minutes with equal distribution. Furthermore, departure time d_{to} is determined by

$$d_{to} = \frac{\sum_{i=1} \sum_{n=1} p_{tin}}{|K|} \times rnd[0.6, 1.2]$$

where $|K|$ is the cardinality of inbound gates. Then, L_{tion} is generated based on $rnd[30, 50]$ and Cap_{to} is defined as follows:

$$Cap_{to} = \frac{\sum_{i=1} \sum_{n=1} L_{tion}}{|O|} \times rnd[0.8, 1.1]$$

Where $|O|$ is the cardinality of outbound destination.

The problem sizes were considered as follows:

$|O| = 3$, $|T| = 6$, and $|K| = 10$. In experiment 1, $|I| \in \{10, 20, 30, 40\}$, $|N|=2$ and in experiment 2, $|N| \in \{1, 2, 3, 4\}$, $|I|=20$ are considered. The Benders algorithm is programmed with C# language in Visual studio 2010 software with CPLEX 10.1 library. The experiments implemented on a PC computer with 2.6GHz CPU and 4GB RAM.

As shown in table 1 and table 2, heuristic algorithm indicates a good fitness to the optimal values calculated from Benders decomposition technique. However, there were some notations that are defined below:

T_B is the time spent by Benders decomposition technique.

T_h is the time spent by heuristic algorithm.

Heuristic algorithm's optimality gap is defined as follows:

$$Err(\%) = \frac{(\text{Heuristic algorithm solution} - \text{Benders technique solution})}{\text{Benders technique solution}} \times 100 \quad (41)$$

Table 1. Comparing Benders technique and heuristic algorithm via changing the number of inbound trucks

| Example | Number of inbound truck | Classical Benders | CCB algorithm | Heuristics algorithm | |
|---------|-------------------------|-------------------|------------------|----------------------|---------|
| | | technique | ($\alpha=0.1$) | Time (sec) | Err (%) |
| | | Time (sec) | Time (sec) | Time (sec) | Err (%) |
| Test 1 | 10 | 20 | 9.02 | 7.10 | 0 |
| Test 2 | 10 | 24 | 12.96 | 6.59 | 0 |
| Test 3 | 10 | 18 | 9.18 | 7.10 | 0 |
| Test 4 | 10 | 13 | 5.59 | 6.86 | 0 |
| Test 5 | 10 | 25 | 11.50 | 7.09 | 0 |
| Test 6 | 10 | 25 | 13.25 | 7.23 | 0 |
| Test 7 | 10 | 10 | 4.71 | 6.63 | 0 |
| Test 8 | 10 | 24 | 12.96 | 6.80 | 0 |
| Test 9 | 10 | 12 | 5.16 | 6.59 | 0 |
| Test 10 | 10 | 15 | 6.33 | 7.26 | 0 |
| Test 11 | 10 | 12 | 6.12 | 6.98 | 0 |
| Test 12 | 10 | 20 | 10.40 | 7.10 | 0 |
| Test 13 | 10 | 24 | 12.96 | 6.59 | 0 |
| Test 14 | 10 | 18 | 8.82 | 7.10 | 0 |
| Test 15 | 10 | 13 | 5.72 | 6.86 | 0 |
| Test 16 | 20 | 137 | 56.17 | 7.71 | 0.98 |
| Test 17 | 20 | 111 | 55.52 | 7.76 | 0.83 |
| Test 18 | 20 | 129 | 63.21 | 7.79 | 0.85 |
| Test 19 | 20 | 123 | 54.12 | 8.52 | 0.69 |
| Test 20 | 20 | 145 | 68.15 | 8.60 | 0.79 |
| Test 21 | 20 | 101 | 51.51 | 8.23 | 0.69 |
| Test 22 | 20 | 95 | 50.35 | 8.42 | 0.73 |
| Test 23 | 20 | 125 | 58.75 | 8.53 | 1.19 |
| Test 24 | 20 | 109 | 49.05 | 8.21 | 0.51 |
| Test 25 | 20 | 115 | 54.05 | 7.62 | 1.07 |
| Test 26 | 20 | 143 | 70.07 | 8.42 | 0.89 |
| Test 27 | 20 | 137 | 61.65 | 7.71 | 0.98 |
| Test 28 | 20 | 111 | 53.28 | 7.76 | 0.83 |
| Test 29 | 20 | 129 | 63.21 | 7.79 | 0.85 |
| Test 30 | 20 | 123 | 51.66 | 8.52 | 0.69 |
| Test 31 | 30 | 858 | 454.74 | 8.9 | 1.37 |
| Test 32 | 30 | 660 | 349.8 | 8.74 | 1.87 |
| Test 33 | 30 | 797 | 422.41 | 8.00 | 1.68 |
| Test 34 | 30 | 924 | 425.04 | 8.47 | 1.87 |
| Test 35 | 30 | 970 | 465.6 | 8.21 | 1.34 |
| Test 36 | 30 | 638 | 299.86 | 8.77 | 1.51 |
| Test 37 | 30 | 992 | 446.4 | 8.10 | 1.49 |
| Test 38 | 30 | 795 | 373.65 | 8.36 | 1.67 |
| Test 39 | 30 | 992 | 486.08 | 8.45 | 1.33 |
| Test 40 | 30 | 657 | 295.65 | 8.34 | 1.46 |
| Test 41 | 30 | 829 | 356.47 | 8.49 | 1.33 |
| Test 42 | 30 | 858 | 471.9 | 8.90 | 1.37 |
| Test 43 | 30 | 660 | 310.2 | 8.74 | 1.87 |
| Test 44 | 30 | 797 | 350.68 | 8.02 | 1.68 |
| Test 45 | 30 | 924 | 462.04 | 8.47 | 1.87 |
| Test 46 | 40 | 5951 | 3035.01 | 9.71 | 1.95 |
| Test 47 | 40 | 4543 | 2044.35 | 10.12 | 1.66 |
| Test 48 | 40 | 5541 | 2770.5 | 9.55 | 2.07 |
| Test 49 | 40 | 4899 | 2302.53 | 9.55 | 1.88 |
| Test 50 | 40 | 4579 | 2060.55 | 9.91 | 2.07 |
| Test 51 | 40 | 6405 | 3266.55 | 9.41 | 1.57 |
| Test 52 | 40 | 4365 | 2269.8 | 9.73 | 2.16 |
| Test 53 | 40 | 5292 | 2116.8 | 9.67 | 1.52 |
| Test 54 | 40 | 6615 | 3307.5 | 9.30 | 1.78 |
| Test 55 | 40 | 5718 | 2801.82 | 9.44 | 1.94 |
| Test 56 | 40 | 5158 | 2836.9 | 9.79 | 1.96 |
| Test 57 | 40 | 4951 | 2425.99 | 9.73 | 1.95 |
| Test 58 | 40 | 7543 | 3997.79 | 10.12 | 1.66 |
| Test 59 | 40 | 6541 | 3008.86 | 9.55 | 2.07 |
| Test 60 | 40 | 3899 | 1598.59 | 9.82 | 1.88 |

Table 2. Comparing Benders technique and heuristic algorithm via changing the number of commodity types

| Example | Number of commodity type | Classical Benders technique | CCB algorithm ($\alpha=0.1$) | Heuristics algorithm | |
|---------|--------------------------|-----------------------------|--------------------------------|----------------------|---------|
| | | Time (sec) | Time (sec) | Time (sec) | Err (%) |
| Test 1 | 1 | 20 | 4.75 | 7.15 | 0 |
| Test 2 | 1 | 24 | 8.41 | 6.65 | 0 |
| Test 3 | 1 | 22 | 4.47 | 6.66 | 0 |
| Test 4 | 1 | 31 | 8.09 | 7.54 | 0 |
| Test 5 | 1 | 34 | 8.87 | 7.65 | 0 |
| Test 6 | 1 | 30 | 9.30 | 7.44 | 0 |
| Test 7 | 1 | 31 | 10.85 | 7.81 | 0 |
| Test 8 | 1 | 24 | 4.80 | 7.32 | 0 |
| Test 9 | 1 | 28 | 7.00 | 7.20 | 0 |
| Test 10 | 1 | 30 | 6.30 | 7.53 | 0 |
| Test 11 | 1 | 25 | 8.75 | 7.59 | 0 |
| Test 12 | 1 | 27 | 5.67 | 7.27 | 0 |
| Test 13 | 1 | 21 | 4.20 | 7.24 | 0 |
| Test 14 | 1 | 36 | 12.24 | 7.13 | 0 |
| Test 15 | 1 | 31 | 9.92 | 7.13 | 0 |
| Test 16 | 2 | 81 | 18.66 | 6.71 | 1.4 |
| Test 17 | 2 | 104 | 30.34 | 8.75 | 4.1 |
| Test 18 | 2 | 112 | 28.15 | 7.99 | 2.93 |
| Test 19 | 2 | 142 | 41.35 | 8.37 | 1.84 |
| Test 20 | 2 | 123 | 32.06 | 7.75 | 0.54 |
| Test 21 | 2 | 96 | 29.76 | 7.84 | 0.74 |
| Test 22 | 2 | 130 | 39.00 | 8.11 | 0 |
| Test 23 | 2 | 149 | 43.21 | 7.68 | 0.36 |
| Test 24 | 2 | 112 | 22.40 | 7.40 | 0.57 |
| Test 25 | 2 | 106 | 33.92 | 7.97 | 1.37 |
| Test 26 | 2 | 105 | 36.75 | 7.32 | 0 |
| Test 27 | 2 | 104 | 24.96 | 7.88 | 1.54 |
| Test 28 | 2 | 141 | 35.25 | 7.79 | 0.69 |
| Test 29 | 2 | 103 | 36.05 | 7.42 | 0.72 |
| Test 30 | 2 | 123 | 39.36 | 7.42 | 0.84 |
| Test 31 | 3 | 389 | 105.28 | 9.80 | 1.23 |
| Test 32 | 3 | 419 | 113.18 | 8.01 | 3.21 |
| Test 33 | 3 | 401 | 140.36 | 8.60 | 0.13 |
| Test 34 | 3 | 736 | 221.04 | 6.99 | 0.98 |
| Test 35 | 3 | 542 | 119.27 | 7.12 | 0.96 |
| Test 36 | 3 | 381 | 87.63 | 7.72 | 1.34 |
| Test 37 | 3 | 765 | 191.25 | 8.34 | 0 |
| Test 38 | 3 | 687 | 240.45 | 8.55 | 0.63 |
| Test 39 | 3 | 642 | 160.50 | 8.50 | 0.63 |
| Test 40 | 3 | 649 | 181.72 | 8.55 | 1.02 |
| Test 41 | 3 | 347 | 114.51 | 8.47 | 0.21 |
| Test 42 | 3 | 735 | 154.35 | 8.50 | 1.03 |
| Test 43 | 3 | 666 | 173.16 | 7.83 | 0.52 |
| Test 44 | 3 | 407 | 81.40 | 8.00 | 0.29 |
| Test 45 | 3 | 418 | 137.94 | 8.05 | 1.21 |
| Test 46 | 4 | 1436 | 359.18 | 8.36 | 2.17 |
| Test 47 | 4 | 1585 | 412.14 | 7.62 | 2.64 |
| Test 48 | 4 | 1694 | 457.44 | 8.04 | 0.168 |
| Test 49 | 4 | 2150 | 559.25 | 8.10 | 1.19 |
| Test 50 | 4 | 1721 | 482.01 | 8.02 | 1.23 |
| Test 51 | 4 | 2097 | 482.31 | 7.92 | 1.43 |
| Test 52 | 4 | 1795 | 628.25 | 8.54 | 0.54 |
| Test 53 | 4 | 1514 | 454.20 | 8.23 | 1.74 |
| Test 54 | 4 | 2287 | 708.97 | 8.35 | 0.87 |
| Test 55 | 4 | 2298 | 528.54 | 7.98 | 1.51 |
| Test 56 | 4 | 1749 | 454.74 | 8.01 | 0.65 |
| Test 57 | 4 | 2358 | 636.66 | 8.29 | 2.34 |
| Test 58 | 4 | 1770 | 407.10 | 8.27 | 0.89 |
| Test 59 | 4 | 1482 | 459.42 | 8.12 | 1.31 |
| Test 60 | 4 | 1482 | 474.24 | 8.12 | 1.36 |

In experiment 1, 60 test problems were implemented with different input parameters and based on increasing inbound trucks number. As shown in table 1, Benders classical algorithm, CCB algorithm with $\alpha=0.1$, and heuristic algorithm were compared with each other in terms of the model's running time and the heuristic algorithm solution error. Furthermore, in experiment 2, the results of the Benders algorithm (classical and CCB method) and heuristic algorithm were calculated based on the change in the number of commodity types. Additionally, running time in Benders technique increased exponential by growth in the model's size. However, because of the nature of time consumption of Benders algorithm in the large size problems, it would be acceptable to apply the heuristic algorithm to find a near optimum solution.

4-Sensitivity analysis

This section examines the effect of input parameters' values on the model objective function. In the cross dock sensitivity analysis, (Rahmanzadeh Tootkaleh et al., 2014) proposed sensitivity analysis on the cross dock network's truck capacity based on the branch and bound algorithm. Furthermore, they presented a bound on the objective function value based on different truck capacities. In this section, effects of holding cost rate (h_{to}), outbound truck capacity (Cap_{to}), transshipment time (t_{ko}), and outbound truck departure time (d_{to}) were analyzed on the optimal solution. For this purpose, four experiments (table 3) were employed, and the results of which are presented in figures 3-a, 3-b, 3-c, and 3-d. In these experiments, I_t , O , N , T , and K were considered to be equal to 30, 3, 3, 3, and 3, respectively. Moreover, in table 3, Z_{tc} is defined as the problem objective value and $P_{n/b}$ is calculated

as $\frac{\text{parameter new value}}{\text{parameter base value}}$ expression.

Table 3. Sensitivity analysis range of the parameters

| Parameter | Sensitivity analysis range based on $P_{n/b}$ |
|------------|--|
| h_{to} | { 0.5, 0.8, 0.9, 0.95, 1, 1.05, 1.1, 1.2, 1.3, 1.5, 2, 5, 10 } |
| Cap_{to} | { 0.9, 0.95, 1, 1.05, 1.1, 1.2, 1.3, 1.5, 2, 3, 4, 5, 10 } |
| t_{ko} | { 0.5, 0.8, 0.9, 0.95, 1, 1.05, 1.1, 1.2, 1.3, 1.5, 2, 3, 5 } |
| d_{to} | { 0.5, 0.8, 0.9, 0.95, 1, 1.05, 1.1, 1.2, 1.3, 1.5, 2, 3, 5 } |

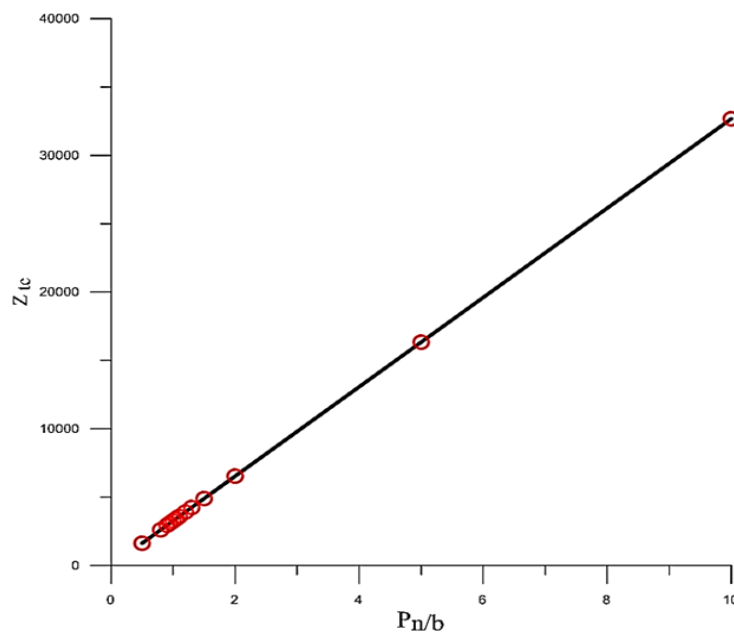


Fig 3-a. Sensitivity analysis of the holding cost rate

As demonstrated in figure 3-a, the problem's objective function value linearly increased by growing of cross dock holding cost rate. In more detail, the model only minimizes the holding cost rate and change of holding cost effects on the objective value directly.

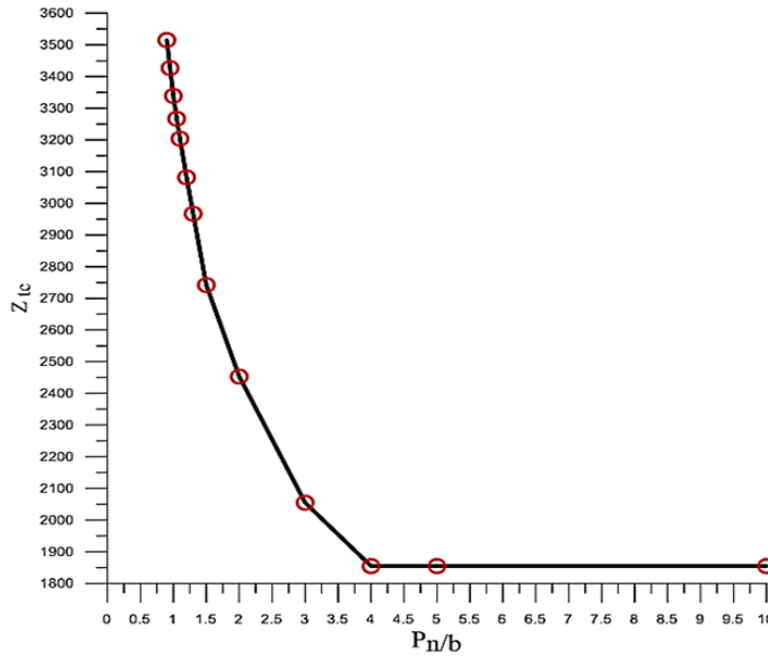


Fig 3-b. Sensitivity analysis of the truck capacity

Figure 3-b shows that by increasing the capacities of outbound trucks, first, the objective value are decreased significantly and then decreasing slop is reduced and the objective function fixed at a certain point. Indeed, by increasing the capacities of outbound trucks, undelayed loads that must be stored according to the trucks' capacity limitations are loaded and sent and the inventory level is decreased. This process is continued until sending the whole undelayed loads and, afterwards, increasing the trucks' capacities would not decrease the cross dock inventories.

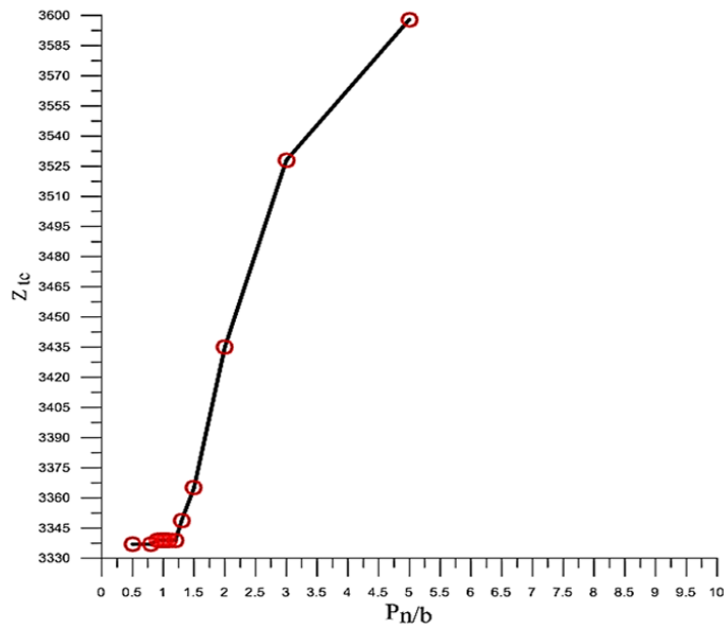


Fig 3-c. Sensitivity analysis of the transshipment time

The experiment on the sensitivity analysis of the transshipment time expressed that the cross dock transshipment time increment will affect the problem's objective value directly. Indeed, by increasing the transshipment time, the number of delayed loads and the cross dock inventory costs are increased as well.

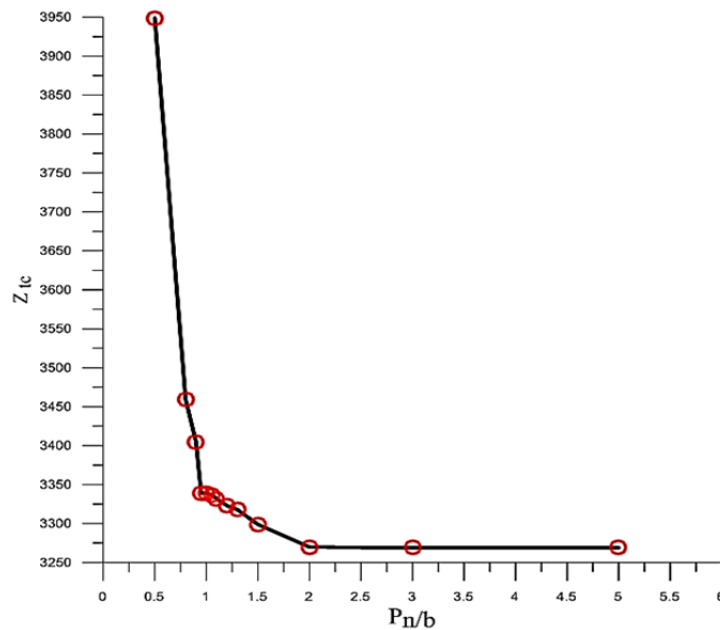


Fig 3-d. Sensitivity analysis of the outbound truck's departure time

Figure 3-d shows, by postponing the outbound truck's departure time, first, the objective value was reduced with a considerable slope and the decreasing rate gradually reduced. As is clear, by delaying the outbound truck's departure time, delayed loads were prepared for the loading and consolidating process and consequently, inventory holding costs were decreased.

In these experiments, values of input parameter were changed according to their operational values. As it is shown, Figures 3-a, 3-b, 3-c, and 3-d indicate the relationship between optimal value and the change in the parameter's value. As a managerial result, these experiments explained several ways for changing the cross dock holding cost. Thus, managers could decide about the parameters changes based on the problem conditions.

In another experiment, a cross dock with 15 inbound trucks, 3 outbound trucks, 1 inbound gate, and 3 types of commodity were considered. Results showed that by increasing the holding cost rate of each commodity, the trucks with the higher density of costly commodities are scheduled first. Ratio of commodity type i amount to the total truck loads is defined as $\alpha(i)$ for each inbound truck as follows:

$$\alpha(i) = \frac{\text{amount of commodity type } i}{\text{sum of all trucks' loads}} \quad (42)$$

Results demonstrated that by increasing the holding cost rate of commodity type i , the inbound trucks with the lower amount of $\alpha(i)$ unloaded their loads at the end of scheduling period, and moreover, the corresponding loads usually stocked until the next periods. In Figure 4, holding cost rate of commodity type 1 increased from 0.2 to 0.4. As is clear, the inbound trucks with higher $\alpha(I)$ unloaded their shipments earlier than others by increasing holding cost rate of commodity type 1. Furthermore, in Figure 4, the truck index is written on each rectangular.

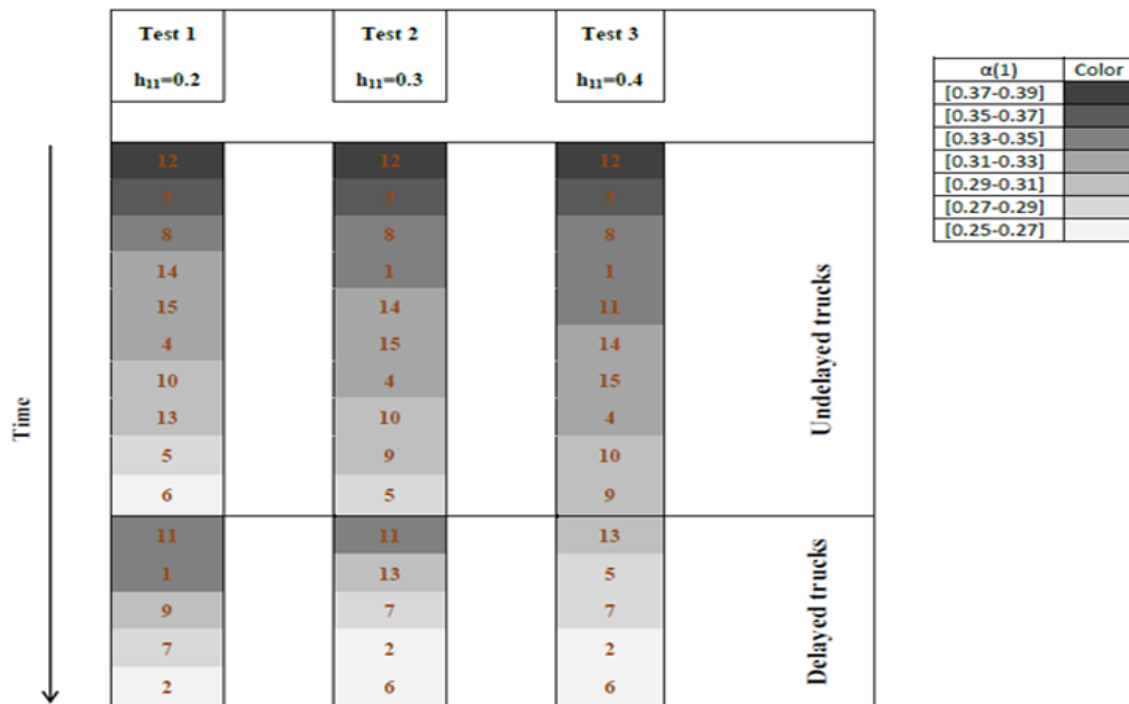


Fig 4. Comparing the inbound trucks' unloading time by increasing the holding cost rate of commodity type 1

4-Conclusion and future works

This paper reviewed some truck scheduling papers in cross dock centers and investigated truck scheduling with fixed outbound trucks departure time in more detail. Previous studies have proposed single period cross dock scheduling problems based on the cross dock strategic, tactical, and operational costs. In this paper, according to low inventory policy in cross dock systems, a multi-period, multi-commodity scheduling model was prepared. Moreover, a mixed integer linear mathematical model was proposed and a heuristic algorithm and Benders decomposition technique were extended for this model in the large and small problem sizes, respectively. Accuracy of heuristic algorithm was tested and compared to the optimal value of Benders technique in two distinct series of experiments. Sensitivity of input parameters was evaluated and their effect on the objective function solution was examined. For more management decisions, effects of holding cost rate, outbound truck capacities, transshipment time, and outbound trucks' departure time on the cross dock holding cost rate were indicated. Furthermore, it was shown that by changing the holding cost rate of commodities, the trucks' scheduling changed according to the commodity ratios of inbound trucks.

In future works, the model can be extended to the cases in which operational resource constraints considered as vital requirements in the cross dock centers. Stochastic concepts in the unloading process and arriving times will be good ideas for future works. Moreover, adding material handling and delay costs of trucks are recommended topics for future studies.

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