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Performance of cumulative count of conforming chart with variable sampling intervals in the presence of inspection errors

Mohammad Saber Fallah Nezhad[1*], Yousof Shamstabar[1]
[1]*Industrial Engineering Department, Yazd University, Iran*
Fallahnezhad@yazd.ac.ir, y.shamstabar@gmail.com

Abstract

In high quality industrial processes, the control chart is design based on cumulative count of conforming (CCC) items is very useful. In this paper, the performance of CCC-r chart with variable sampling intervals (CCC-r_{VSI} chart) in the presence of inspection errors is investigated. The efficiency of CCC-r_{VSI} chart is compared with CCC-r chart with fixed sampling interval (CCC-r_{FSI} chart). The comparison results show thatthe VSI scheme can performs better than the FSI scheme. In addition, analysis and discussion of the results are presented to illustrate the effect of input parameters on the performance of CCC-r_{VSI} chart.

Keywords: high quality processes; CCC-r chart; variable sampling interval; inspections errors; average time to signal

1- Introduction

As one of the basic Statistical Process Control (SPC) tools, control chart is useful in maintaining stability and improving quality through variability reduction in the production process. The cumulative count of conforming (CCC) control chart is based on the cumulative number of conforming items between two consecutive nonconforming items that is the random variable with the geometric distribution (Calvin, 1983; Goh, 1987).This control chart is mostly applied for high- quality processes. In the automated and discrete manufacturing systems, very low level of non-conforming items is produced. As a result the CCC chart has received considerable attention from the industry (Joekes et al., 2016).The CCC-*r* chart is developed based on the CCC chart which considers the cumulative count of conforming items until observing a fixed number "*r*" of nonconforming ones that follows the negative binomial distribution(Ohta et al.,2001; Kudo et al.,2004).

The scheme of variable sampling interval (VSI) is designed in order to improve the efficiency of the CCC control chart (Liuet al., 2006).The current researches on the application of VSI scheme have denoted the better performance of this scheme in comparison with FSI scheme. $\overline{X}$ control chart with variable sampling intervals was proposed by some researchers (see for example, Reynolds et al.,1988; Reynolds and Arnold , 1989; Runger and Pignatiello Jr, 1991; Runger and Montgomery, 1993; Amin and Miller, 1993; Zhang et al.,2012). Aparisi and Haro, (2001) and Villalobos et al. (2005) studied a VSI multivariate shewhart chart. Reynolds *et al.*, (1990) and Luo and Wang (2009) investigated the VSI- CUSUM control chart. VSI- EWMA charts have been studied by some researchers (see for example, Shamma et al., 1991; Saccucci et al, 1992;Castagliolaet al, 2006; Castagliolaet et al, 2006).

*Corresponding author.
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Some studies have been performed for VSI- CCC (CCC_{VSI}) chart and they concluded that their proposed design is more efficient than FSI- CCC (CCC_{FSI} chart). First, Liu et al. (2006) proposed CCC chart with variable sampling intervals (CCC_{VSI} chart). Chen et al. (2011) investigated CCC chart with variable sampling intervals and variable control limits ($CCC_{VSI/VCL}$ chart) and Chen (2013) studied CCC chart with variable sampling intervals for correlated samples ($GCCC_{VSI}$ chart). Zhang et al. (2014) investigated the performance of CCC_{VSI} chart with estimated control limits.

The strategy of using 100% inspection is proposed for the implementation of the CCC chart and many studies on this charts have assumed that inspection is perfect accurate, but inspection errors exist in the process. Burke *et al.*(1995) discussed inspection errors and their impact on control charts and denoted their important effect on the results. Lu *et al.* (2000) calculated the adjusted control limits for the *CCC* chart in the presence of inspection errors based on the relationship between the true and observed values of the nonconforming proportion. Ranjan et al. (2003) designed a procedure to set control limits for *CCC* charts considering the inspection errors to obtain the maximum ARL. Some other studies about inspection errors have been done by Case (1980), Lindsay (1985), Suich (1988),Huang et al. (1989), Suich (1990), Johnson *et al.*(1991), Cheng and Chung(1994), Wang and Chen (1997, Ryan (2011), Nezhad and Nasab (2012) , Fallahnezhad and Babadi(2015) and Fallah Nezhad et al.(2015).

The goal of this paper is to develop a model to consider the inspection errors in implementation of variable sampling interval scheme for CCC-r control chart. So, this paper considers the adjusted control limits for the CCC-rVSI chart that can reduce for the effects of inspection errors. In section 2, the CCC-r $_{VSI}$ Chart in the presence of inspection errorshas been described .In section 3, the comparison study between the CCC-r_{VSI} chart and the CCC-r_{FSI} chart is performed and the results are elaborated using sensitivity analysis. In section 4, a practical case study for the implementation of the CCC-r_{VSI} chart in the presence of inspection errors is described. Finally, we have concluded the paper.

2- Description of the CCC-r $_{VSI}$ chart

2-1- Notations

p_{0t}	the observed value of the in-control nonconforming fraction,
p_{1t}	the observed value of the out-of-control nonconforming fraction,
p_0	the true value of the in-control nonconforming fraction,
p_1	the true value of the out-of-control nonconforming fraction,
$\alpha_{disired}$	the probability of false alarm,
X_i	the cumulative count of items inspected after observing the (i-1)$_{th}$ nonconforming item until the i$_{th}$ nonconforming item (including the last nonconforming item),
n	the number of different intervals for the CCC-r_{VSI} chart,
d_j	j =1,2,…,n. Sampling interval lengths for the CCC-r_{VSI} chart, , i.e., the time between inspections of two consecutive items ($d_n<d_{n-1}<\ldots<d_2<d_1$)
d	the sampling interval length for the CCC-r_{FSI} chart,
IL_i	i =1,2,…,n-1. the interval limits in the CCC-r_{VSI} chart which divide the region between UCL and

LCL into n sub-regions $I_1; I_2; \ldots; I_n$ $\left(IL_{n-1} < IL_{n-2} < \ldots < IL_1\right)$,

L_i the sampling interval length which is used to obtain X_i,

ARL_a the adjusted average run length,

ATS_a the adjusted average time to signal,

ATS_{aV} the adjusted in-control *ATS* of the CCC-r_{VSI} chart,

ATS_{aF} the adjusted in-control *ATS* of the CCC-r_{FSI} chart,

ATS'_{aV} the adjusted out-of-control *ATS* of the CCC-r_{VSI} chart,

ATS'_{aF} the adjusted out-of-control *ATS* of the CCC-r_{FSI} chart,

I improvement factor, defined as $I = ATS'_V \,/\, ATS'_F$,

q_j the probability that point X_i falls region I_j when the process nonconforming fraction is p_0 ,

q'_j the probability that point X_i falls within the region I_j when the process nonconforming fraction is p_1

2-2- CCC-r_{VSI} control chart in the presence of the inspection errors

The relationship between the true and observed value of nonconforming proportion in presence of inspection errors is as follows (Burke et al., 1995):

$$p = p_t\left(1-e_2\right) + \left(1-p_t\right)e_1 \tag{1}$$

The observed (estimated) non-conforming fraction is p_t and p is the true value of nonconforming proportion, while $e_1(Type\ I\ error)$ and $e_2(Type\ II\ error)$ denote, respectively, the probability of classifying a conforming item as nonconforming and the probability of classifying a nonconforming item as conforming. So, we have:

in control state: $p_0 = p_{0t}\left(1-e_2\right) + \left(1-p_{0t}\right)e_1$

out of control state: $p_1 = p_{1t}\left(1-e_2\right) + \left(1-p_{1t}\right)e_1$

If the acceptable risk of false alarm is $\alpha_{desired}$, then the control limits and the center line can be determined as follows (Xie et al., 2012):

$$\sum_{i=r}^{UCL-1}\binom{i-1}{r-1}(1-p_{0t})^{i-r}\,p_{0t}^{\ r} = 1-\alpha_{disired}\,/\,2 \tag{2}$$

$$\sum_{i=r}^{CL}\binom{i-1}{r-1}(1-p_{0t})^{i-r}\,{p_{0t}}^{r}=0.5 \tag{3}$$

$$\sum_{i=r}^{LCL}\binom{i-1}{r-1}(1-p_{0t})^{i-r}\,{p_{0t}}^{r}=\alpha_{desired}/2 \tag{4}$$

Lu et al. (2000) proposed the adjusted acceptable risk of false alarm in the presence of inspection errors. Thus a control chart is modified to provide a first type error   that is closer to the one under error-free inspection in order to reduce the impact of inspection errors. In the presence of inspection errors,   can be obtained as following,

$$\alpha^*_{desired}=\frac{\alpha_{desired}P_{0t}}{P_0} \tag{5}$$

As the result, in the presence of inspection errors, the adjusted control limits and the center line can be determined as follows:

$$\sum_{i=r}^{UCL_a-1}\binom{i-1}{r-1}(1-p_0)^{i-r}\,{p_0}^{r}=1-\alpha^*{}_{disired}/2 \tag{6}$$

$$\sum_{i=r}^{CL_a}\binom{i-1}{r-1}(1-p_0)^{i-r}\,{p_0}^{r}=0.5 \tag{7}$$

$$\sum_{i=r}^{LCL_a}\binom{i-1}{r-1}(1-p_0)^{i-r}\,{p_0}^{r}=\alpha^*{}_{desired}/2 \tag{8}$$

The ARL (average run length) is defined as the average number of points plotted until receiving an out-of-control signal. Thus, ARL_a can be obtained as following,

$$ARL_a=\frac{1}{1-\sum_{LCL_a}^{UCL_a}\binom{i-1}{r-1}p^r(1-p)^{i-r}} \tag{9}$$

ANI (average number of items) is defined as the expected value of the number of items inspected until the chart signals an alarm .ANI_a can be computed for CCCG-r$_{FSI}$ and CCCG-r$_{VSI}$ chart by applying the following equation:

$$ANI_a=\frac{r}{p}ARL_a \tag{10}$$

When the CCC-r$_{VSI}$ chart is applied, then the time between inspections of two consecutive items would be $d_1, d_2, \ldots, d_n$ $(d_1 > d_2 > \ldots > d_n)$. These interval lengths should be determined considering the practical conditions of production system. As an example, the minimum value of interval length is not less than the time lag between productions of two successive items. The maximum value of interval length can be obtained with regards to the maximum amount of time that is allowed for the process to run without inspection. The interval limits $IL_1, IL_2, \ldots, IL_n$ are determined in the CCC-r_{VSI} chart, so that the interval between UCL_a and LCL_a is divided into n different intervals $I_1, I_2, \ldots, I_n$. Thus following framework is used for sampling from the process,

$$L_i = \begin{cases} d_1, X_{i-1} \in I_1 = (IL_1, UCL_a) \\ d_2, X_{i-1} \in I_2 = (IL_2, IL_{1a}) \\ . \\ . \\ . \\ d_n, X_{i-1} \in I_n = (LCL_a, IL_n) \end{cases} \tag{11}$$

The interval limits $IL_1, IL_2, \ldots, IL_{n-1}$ can be determined as the following that F^{-1} is inverse function of the negative binomial distribution function with r and p_0 parameters :

$$q_1 = p(IL_1 + 1 \le x < UCL) = P(x \ge IL_1 + 1) - p(x \ge UCL)$$

$$= \sum_{x=IL_1+1}^{\infty} \begin{bmatrix} x-1 \\ r-1 \end{bmatrix} p_0^r (1-p_0)^{x-r} - \sum_{x=UCL}^{\infty} \begin{bmatrix} x-1 \\ r-1 \end{bmatrix} p_0^r (1-p_0)^{x-r}$$

$$= \sum_{x=IL_1+1}^{UCL-1} \begin{bmatrix} x-1 \\ r-1 \end{bmatrix} p_0^r (1-p_0)^{x-r} = 1 - F(IL_1) - \frac{\alpha^*_{desired}}{2}$$

$$IL_1 = F^{-1}(1 - q_1 - \frac{\alpha^*_{desired}}{2})$$

So we have:

$$IL_1 = F^{-1}(1 - q_1 - \frac{\alpha^*_{desired}}{2})$$

$$IL_2 = F^{-1}\left(1 - q_1 - q_2 - \frac{\alpha^*_{desired}}{2}\right)$$

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$$IL_{n-1} = F^{-1}(1 - q_1 - q_2 - \cdots - q_n - \frac{\alpha^*_{desired}}{2}) \tag{12}$$

This scheme continues until the *IL* values falls between UCL_a and LCL_aas following:

$$LCL_a < IL_{n-1} < IL_{n-2} < \ldots < IL_2 < IL_1 < UCL_a \tag{13}$$

ATS is the average length of time that is needed to observe a signal in a control chart. Also, ATS_{aF} and ATS_{aV} can be determined as following,

$$ATS_{aF} = ANI_a \times d = ARL_a \times \frac{r}{p} \times d \tag{14}$$

$$ATS_{aV} = ARL_a \times \frac{r}{p} \times \frac{d_1 q_1 + d_2 q_2 + \ldots. + d_n q_n}{q_1 + q_2 + \ldots. + q_n} \tag{15}$$

3- Performance comparisons between the CCC-r_{VSI} and the CCC-r_{FSI} chart

The performance of CCC-$r_{VSI}$is compared with the CCC-r_{FSI} chart in this section. Note that the same values of nonconforming fraction p_0 and false alarm probability α are assumed for both the CCC_{FSI} and the CCC_{VSI} chart. In order to compare these charts, the design parameters for the CCC-r_{VSI} and the CCC-r_{FSI} chart are determined so that the equation $ATS_{aF} = ATS_{aV}$ is satisfied at the in control state. On the other hand, when the process nonconforming fraction changes to $p_1 (> p_0)$, the values of ATS'_{aF} and ATS'_{aV} should be evaluated. The control chart with smaller value of out-of-control ATS'_a will have the better performance.

Let $ATS_{aF} = ATS_{aV}$, thus,

$$d = \frac{d_1 q_1 + d_2 q_2 + \ldots. + d_n q_n}{q_1 + q_2 + \ldots. + q_n} = \frac{d_1 q_1 + d_2 q_2 + \ldots. + d_n q_n}{1 - \alpha} \tag{16}$$

It is assumed that the sampling interval length of the CCC-r_{FSI} chart is adjusted to be equal 1 (d = 1), the values of $(d_1,\ d_2,\ \ldots,d_n)$ and $(q_1,\ q_2,\ \ldots,q_n)$ are determined so that Eq. (16) is satisfied then the matched CCC-r_{VSI} and CCC-r_{FSI} chart are obtained that have the same in-control value of *ATS*. Then, when the nonconforming fraction changes to p_1, the performance of the CCC-r_{VSI} chart can be analyzed by computing the value of *I*, that is equal to the ratio of out-of-control ATS_a of the CCC-r_{VSI} and the CCC-r_{FSI} chart:

$$I = \frac{ATS'_{aV}}{ATS'_{aF}} = \frac{d_1 q'_1 + d_2 q'_2 + \ldots. + d_n q'_n}{q'_1 + q'_2 + \ldots. + q'_n} \tag{17}$$

Based on Equation (17), if the value of Improvement factor is less than 1.00, variable sampling interval scheme can produce a signal more quickly than fixed sampling interval scheme when the process is out of control. So, when *I* is less than 1.00, it denotes that the CCC-r_{VSI} chart performs better than the CCC-r_{FSI} chart.

The values of q'_j can be calculated as following,

$$\begin{cases} q'_1 = \sum_{x=IL_1+1}^{UCL_a-1} \binom{x-1}{r-1} p_1^{\ r} (1-p_1)^{x-r} \\ q'_2 = \sum_{x=IL_2+1}^{IL_1} \binom{x-1}{r-1} p_1^{\ r} (1-p_1)^{x-r} \\ . \\ . \\ . \\ q'_{n-1} = \sum_{x=IL_{n-1}+1}^{IL_{n-2}} \binom{x-1}{r-1} p_1^{\ r} (1-p_1)^{x-r} \\ q'_n = \sum_{x=LCL_a+1}^{IL_{n-1}} \binom{x-1}{r-1} p_1^{\ r} (1-p_1)^{x-r} \end{cases} \tag{18}$$

The performance of CCC-r_{VSI} chart is analyzed based on the number of sampling interval (n) assuming equal probabilities for each interval:

$$q_1 = q_2 = \ldots = q_n = \frac{1-\alpha}{n} \tag{19}$$

By substituting d=1 in Equation (16), we have,

$$1-\alpha = d_1 q_1 + d_2 q_2 + \ldots. + d_n q_n = \frac{1-\alpha}{n}(d_1 + d_2 + \ldots + d_n) \longrightarrow$$

$$d_1 + d_2 + \ldots + d_n = n \tag{20}$$

4- Comparative study of CCC-r_{VSI} chart in the presence of inspection errors

In this paper, we apply the input data in the numerical study of Liu et.al (2006)for comparison study. This data is as following: $\alpha_{disired} = 0.0027, p_{0t} = 0.0005$ and sampling interval lengths $(d_1, d_2, ..., d_n)$ with the fixed value of $d = 1$ can be chosen as follows:

$n = 2, d_1 = 1.9, d_2 = 0.1; n = 3, d_1 = 1.9, d_2 = 1, d_3 = 0.1; n = 4, d_1 = 1.9, d_2 = 1.2, d_3 = 0.8, d_4 = 0.1;$
$n = 5, d_1 = 1.9, d_2 = 1.5, d_3 = 1, d_4 = 0.5, d_5 = 0.1; \ldots$

4-1- Improvement factors for different process shifts

Now, we study the performance of CCC-r_{VSI} chart in the presence of inspection errors for different value of process shifts and several values of e_1 and e_2.First, the value of corresponding improvement factors I are computed and the results are shown in Table 1. As can be seen when the nonconforming ratio (p_{1t}/p_{0t}) increases then, the improvement factor I decreases, and thus the CCC-r_{VSI} chart performs better than CCC-r_{FSI} chart in the presence of the inspection errors.

Table 1. Improvement factors for different process shifts with n=2 and r=3

p_{1t}/p_{0t}	e_1	e_2				
		0.0001	0.0005	0.001	0.005	0.01
	0.0001	0.815298	0.815451	0.815425	0.815520	0.815619
	0.0005	0.886032	0.885843	0.886173	0.886103	0.886719
1.2	0.001	0.922658	0.922839	0.923065	0.923523	0.923764
	0.005	0.977602	0.977653	0.977717	0.978230	0.978870
	0.01	0.988620	0.988647	0.988681	0.988951	0.989289
	0.0001	0.588719	0.588873	0.588864	0.589064	0.589299
	0.0005	0.732450	0.732290	0.732647	0.732838	0.733772
1.5	0.001	0.814567	0.814774	0.815033	0.815751	0.816318
	0.005	0.946067	0.946130	0.946209	0.946837	0.947622
	0.01	0.971979	0.972013	0.972055	0.972389	0.972808
	0.0001	0.422636	0.422773	0.422773	0.423006	0.423287
	0.0005	0.601449	0.601318	0.601677	0.602049	0.603173
1.8	0.001	0.716255	0.716479	0.716759	0.717677	0.718502
	0.005	0.915172	0.915245	0.915338	0.916076	0.916998
	0.01	0.955506	0.955545	0.955595	0.955992	0.956489
	0.0001	0.340892	0.341014	0.341017	0.341244	0.341521
	0.0005	0.526532	0.526418	0.526769	0.527220	0.528406
2.0	0.001	0.656343	0.656574	0.656863	0.657881	0.658840
	0.005	0.894944	0.895024	0.895125	0.895933	0.896943
	0.01	0.944618	0.944662	0.944717	0.945155	0.945703

When e_1 (the probability of classifying a nonconforming item as conforming) increaseas, the improvement factor I also increseas and with increasing e_2, the value of improvement factor increases. Thus it is concluded that the superiority of CCC-r_{VSI} chart over CCC-r_{FSI} chart improves by increasing the enspection errors.

4-2- Improvement factors for different CCC-r_{VSI} control chart

In this subsection, we fix the number of sampling intervals (n=2), and process shift (p_{1t}/p_{0t}=2) then for different possible values of parameter r, the results are shown in Table 2. The improvement factor decreases by increasing the parameter r in the all cases.

Table 2. Improvement factors for different CCC-r_{VSI} control chart with n=2 and (p_{1t}/p_{0t}=2)

r	e_1	e_2				
		0.0001	0.0005	0.001	0.005	0.01
1	0.0001	0.605403	0.605636	0.605928	0.606050	0.606224
	0.0005	0.735948	0.736124	0.736345	0.737161	0.737476
	0.001	0.814998	0.815131	0.813868	0.815191	0.815423
	0.005	0.946441	0.946484	0.946537	0.946963	0.947496
	0.01	0.972932	0.972955	0.972984	0.973214	0.973501
2	0.0001	0.439389	0.439393	0.439469	0.439835	0.440073
	0.0005	0.611414	0.611659	0.611417	0.612233	0.613130
	0.001	0.722723	0.722055	0.722296	0.723364	0.724063
	0.005	0.918428	0.918493	0.918574	0.916049	0.916857
	0.01	0.956441	0.956476	0.956519	0.956869	0.957306
3	0.0001	0.340892	0.341014	0.341017	0.341244	0.341521
	0.0005	0.526532	0.526418	0.526769	0.527220	0.528406
	0.001	0.656343	0.656574	0.656863	0.657881	0.658840
	0.005	0.894944	0.895024	0.895125	0.895933	0.896943
	0.01	0.944618	0.944662	0.944717	0.945155	0.945703
4	0.0001	0.275905	0.275931	0.275903	0.276193	0.276462
	0.0005	0.462490	0.462483	0.462551	0.463416	0.464143
	0.001	0.604573	0.604831	0.604626	0.605640	0.606773
	0.005	0.878854	0.878948	0.879065	0.880006	0.879073
	0.01	0.935730	0.935781	0.935845	0.936357	0.936998

4-3-Improvement factors for different numbers of sampling intervals

In order to investigate the overall performance of CCC-r_{VSI} chart based on the number of sampling intervals, we fix the parameter r=3, and process shifts (p_{1t}/p_{0t}) = 2. The results in Table 3 indicate that for different values of e_1 and e_2, the number of sampling intervals is efficient on the improvement factor, *I*. for example, if e_1=0.0001 and e_2=0.0001, then CCC-r $_{VSI}$ chart with n=2, is more efficient and if e_1=0.005 and e_2=0.0001, then CCC-r $_{VSI}$ chart with n=5, is more efficient.

Table 3. Improvement factors for different numbers of sampling intervals with r=3 and p_{1t}/p_{0t}=2

n	e_1	e_2				
		0.0001	0.0005	0.001	0.005	0.01
2	0.0001	0.340892	0.341014	0.341017	0.341244	0.341521
	0.0005	0.526532	0.526418	0.526769	0.527220	0.528406
	0.001	0.656343	0.656574	0.656863	0.657881	0.658840
	0.005	0.894944	0.895024	0.895125	0.895933	0.896943
	0.01	0.944618	0.944662	0.944717	0.945155	0.945703
3	0.0001	0.398213	0.398269	0.398213	0.398515	0.398715
	0.0005	0.571905	0.572057	0.571976	0.572629	0.573556
	0.001	0.689961	0.689730	0.689796	0.690421	0.691381
	0.005	0.905632	0.905705	0.905796	0.905631	0.906544
	0.01	0.948151	0.948191	0.948241	0.948639	0.949137
4	0.0001	0.513543	0.513568	0.513577	0.513647	0.513731
	0.0005	0.577744	0.577714	0.577820	0.578016	0.578466
	0.001	0.631454	0.631556	0.631623	0.632071	0.632461
	0.005	0.752696	0.752742	0.752801	0.753267	0.753851
	0.01	0.782404	0.782430	0.782464	0.782731	0.783066
5	0.0001	0.561937	0.561957	0.561943	0.561988	0.562034
	0.0005	0.596664	0.596717	0.596723	0.596886	0.597067
	0.001	0.627734	0.627795	0.627854	0.628042	0.628227
	0.005	0.703274	0.703304	0.703342	0.703646	0.704026
	0.01	0.722546	0.722564	0.722586	0.722764	0.722986

4-4- Improvement factors based on different lengths of sampling interval

Based on the above analysis, we investigate the effect of interval length on the performance of CCC-r_{VSI} chart when the number of sampling intervals is n=2.Four different cases of sampling interval lengths are analyzed. As can be seen in Table 4 the larger values for the differences between interval lengths, (d_1, d_2) leads to better performance of CCC-r_{VSI} chart. Also, in all cases, the value of I is less than 1, thus the performance of CCC-r_{VSI} chart is better than CCC-r_{FSI} chart in all cases.

Table 4. Improvement factors based on different lengths of sampling interval with n=2 ,r=3 and p_{1t}/p_{0t}=2

(d_1,d_2)	d_1-d_2	e_1	e_2				
			0.0001	0.0005	0.001	0.005	0.01
(1.9,.1)	1.8	0.0001	0.340892	0.341014	0.341017	0.341244	0.341521
		0.0005	0.526532	0.526418	0.526769	0.527220	0.528406
		0.001	0.656343	0.656574	0.656863	0.657881	0.658840
		0.005	0.894944	0.895024	0.895125	0.895933	0.896943
		0.01	0.944618	0.944662	0.944717	0.945155	0.945703
(1.7,0.3)	1.4	0.0001	0.487361	0.487455	0.487458	0.487635	0.487850
		0.0005	0.631747	0.631659	0.631932	0.632282	0.633205
		0.001	0.732711	0.732891	0.733116	0.733908	0.734653
		0.005	0.918289	0.918352	0.918431	0.919059	0.919845
		0.01	0.956925	0.956959	0.957002	0.957343	0.957769
(1.5,0.5)	1.0	0.0001	0.633829	0.633896	0.633898	0.634025	0.634179
		0.0005	0.736962	0.736899	0.737094	0.737344	0.738003
		0.001	0.809080	0.809208	0.809368	0.809934	0.810466
		0.005	0.941635	0.941680	0.941736	0.942185	0.942746
		0.01	0.969232	0.969257	0.969287	0.969531	0.969835
(1.2,0.8)	0.4	0.0001	0.853532	0.853559	0.853559	0.853610	0.853671
		0.0005	0.894785	0.894760	0.894838	0.894938	0.895201
		0.001	0.923632	0.923683	0.923747	0.923974	0.924187
		0.005	0.976654	0.976672	0.976695	0.976874	0.977099
		0.01	0.987693	0.987703	0.987715	0.987812	0.987934
(1,1)=FSI	0.0	0.0001	1	1	1	1	1
		0.0005	1	1	1	1	1
		0.001	1	1	1	1	1
		0.005	1	1	1	1	1
		0.01	1	1	1	1	1

4-5- Improvement factors for different probability allocations

The above overall performance of CCC-r_{VSI} chart is analyzed based on the equal in control probability allocations. In order to investigate the overall performance of CCC-r_{VSI} chart when the condition $q_1 = q_2 = \cdots = q_n$ is not satisfied, we fix n=2 and d_1=1.9, and only change the values of in control probability, q_1 as proposed by Liu et al. (2006). It should be noted that $q_1+q_2=1-\alpha$. The value of d_2 can be obtained using the following equation:

$$d_2 = \frac{1-\alpha-d_1q_1}{q_2} > 0 \tag{21}$$

As shown in Table 5, when (q_1-q_2) decreases, improvement factor *1* decreases and the performance of CCC-r_{VSI} chart in the presence of the inspection errors becomes better.

Table 5. Improvement factors for different probability allocations with n=2, r=3 and p_{1t}/p_{0t}=2

q	q_1-q_2	e_1	e_2				
			0.0001	0.0005	0.001	0.005	0.01
(0.1,0.8973)	0.7973	0.0001	0.903176	0.903177	0.903180	0.903187	0.903199
		0.0005	0.913818	0.913824	0.913837	0.913890	0.913971
		0.001	0.927470	0.927500	0.927496	0.927584	0.927707
		0.005	0.971056	0.971075	0.971100	0.971298	0.971239
		0.01	0.983816	0.983828	0.983843	0.983963	0.984114
(0.2,0.7973)	0.5973	0.0001	0.791976	0.791986	0.791989	0.792020	0.792063
		0.0005	0.826065	0.826073	0.826097	0.826236	0.826430
		0.001	0.860662	0.860732	0.860697	0.861030	0.861296
		0.005	0.949416	0.949452	0.949496	0.949857	0.950308
		0.01	0.972467	0.972488	0.972514	0.972724	0.972987
(0.3,0.6973)	0.3973	0.0001	0.663879	0.663890	0.663878	0.663961	0.664075
		0.0005	0.733464	0.733585	0.733611	0.733945	0.734344
		0.001	0.794450	0.794325	0.794470	0.794913	0.795411
		0.005	0.930786	0.930837	0.930901	0.930247	0.930887
		0.01	0.962606	0.962635	0.962671	0.962960	0.963323
(0.4,0.5973)	0.1973	0.0001	0.514631	0.514717	0.514657	0.514828	0.514995
		0.0005	0.633797	0.633757	0.633765	0.634299	0.634870
		0.001	0.726008	0.726178	0.726391	0.726862	0.727767
		0.005	0.911538	0.911604	0.911686	0.912346	0.913172
		0.01	0.951742	0.951779	0.951825	0.952190	0.952646
(0.49865,0.49865)	0	0.0001	0.340892	0.341014	0.341017	0.341244	0.341521
		0.0005	0.526532	0.526418	0.526769	0.527220	0.528406
		0.001	0.656343	0.656574	0.656863	0.657881	0.658840
		0.005	0.894944	0.895024	0.895125	0.895933	0.896943
		0.01	0.944618	0.944662	0.944717	0.945155	0.945703

5- Conclusion

In manufacturing technology, many production processes today are producing a very small proportion of nonconforming items. Thus, many process control methods have been proposed, such as CCC chart that has received considerable attention from the industry. In this paper, we have investigated the performance of CCC-r_{VSI} control chart in the presence of the inspection errors for high quality processes. Some sensitivity analysis was done and the results demonstrated that the CCC-r_{VSI} chart is more efficient than CCC-r_{FSI} chart and when the parameter r increases then, the efficiency of CCC-r_{VSI} chart will be enhanced. Also the superiority of CCC-r_{VSI} increases by increasing the difference between the interval lengths and uniform probability allocation is more efficient. For future woks, we suggested developing the CCC-r chart with the variable sampling intervals under group inspection in the presence of inspection errors.

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